I
First Historical Introduction
A Preliminary History of Paraconsistent and Dialethic Approaches

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Although the notion of *system* was brought into prominence by Leibniz,[1] it is only in contemporary times that a clear conception of a formal or semantical system has developed. Thus recent definitions of paraconsistency[2] through such systems—in terms of systems which can tolerate some inconsistency without trivializing—are not strictly or directly applicable in a historical quest. Evidence of paraconsistent approaches in earlier times has accordingly to be more circumstantial.

There are however several good indicators of paraconsistent approaches of one sort or another which can be reliably used. For example, admission, or insistence, that some statement is both true and false, in a context where not everything is accepted or some things are rejected, is a sure sign of a paraconsistent approach—in fact of a dialethic approach. It involves not merely recognition of a non-trivial inconsistent theory, as with a (weaker) paraconsistent position, but the assumption that that is how things are, that, in effect, the world is inconsistent. A concession that both a statement, A say, and its negation, ~A, hold, works in a similar way, clearly revealing a strong paraconsistent approach. So does the concession that some statements A and ~A hold in a nontrivial theory or position, thereby revealing a weaker paraconsistent approach.

But often evidence is less direct. For instance, an author may not explicitly say that both A and ~A hold, or hold in a given theory, but what is said obviously implies that they do, and the author can be assumed to be aware that they do, or a case can be made that the author is aware of this. In such cases the approach is still *explicitly* paraconsistent. But an author may not be (clearly) apprised of what his or her position (obviously) implies, in which event the position will be either *implicitly* paraconsistent or else trivial, depending on the underlying logic adopted.

However determining the underlying logic assumed by an author, especially before contemporary times, but sometimes even now, is a particularly difficult and testing business. Fortunately it is often unnecessary to work out the underlying logic in much detail at all. From medieval times it is frequently enough to know where the author stood as regards paradoxes of implication (i.e. in effect, paradoxes of the consequence relation); and otherwise it may suffice to find out what the author thought about watershed principles such as consistency assumptions, for instance, that the world is perforce consistent, or, more strongly, that all that can be spoken of or described (non-trivially) is consistent.

This *universal* consistency hypothesis—to the effect that all non-trivial theories are consistent—emerges early in Greek thought and is embedded in the Eleatic tradition (sometimes explicitly described as "Eleatic logic", though no logic is ever outlined) emanating from Parmenides and running through the giant survivors (at least so far as representation of their work is concerned) of classical antiquity, Plato and Aristotle and into mainstream Western thought. If inconsistency is found in the work of such a mainstream author (as with little doubt inconsistency mostly can be[3]) then the philosophical theory is damaged perhaps severely, and requires not merely patching but some excision to restore consistency.

In classical antiquity, as subsequently, consistency theses take various, stronger or weaker forms—all of which we shall try to demolish. In strongest form, the one-world Eleatic position, *reference* (through which *truth* is determined) is confined to the one actual world (Being, or, in modern semantical symbolism, G), assumed consistent, and to certain (sub-)theories, ipso facto consistent, composed from its components. By the time of Plato, the classical exclusion picture of negation has emerged, ruling out the possibility of non-trivial inconsistency, but not restricting (consistent) theories to subtheories of, or to the confines of, the actual world G. In Aristotle however, a different connexive position, commonly confused with the classical position, also appears. According to this position, based on a cancellation view of negation, all propositions, and so derivatively all theories, are consistent.[4]

The attitude taken to inconsistency or contradiction is perhaps the clearest indicator of paraconsistent stance. Strong paraconsistent (i.e. dialethic) positions admit contradiction in the actual world, whereas weaker paraconsistent positions, though not conceding that much, allow inconsistency in some non-trivial theories, or their analogues, such as language or thought. In these terms, the official Soviet position seems to have moved from, first of all, a strong paraconsistent position, before the twenties, admitting contradiction in both the world and thought, to, secondly an anti-paraconsistent position, excluding contradiction both in the world and in theories, to in the third place, post-war, a weak paraconsistent position allowing inconsistency in thought, and "thought-constructions" such as theories, but not in the world. There are some signs now that the full cycle may be completed and the first strong position resumed. These shifts in official position can be determined, furthermore, without knowing what Soviet logic looks like in any detail.[5]

1. Western thought until the fall of the Roman Empire: paraconsistent elements

Eastern philosophy has generally been more tolerant of inconsistency, more amenable to paraconsistent approaches than Western. However, despite apparent dominance of classical Greek thought (from our present biased viewpoint) by consistency hypotheses, paraconsistent approaches were by no means exceptional in classical antiquity, but were assumed or espoused by several lesser philosophical schools.

In the first place, overtly paraconsistent approaches were adopted in classical antiquity, for example by the Megarians who held that some statements are both true and false. So it was that Chrysippus, the Stoic, devoted a book 'to replying to those who hold that Propositions may be at once False and True'.[6] Regrettably there are no further details of this book. But we can reasonably conjecture that Chrysippus's opponents were Megarians, who defended their position on the basis of such semantical paradoxes as the Liar. However, we really do not know exactly who the opponents were. And unfortunately this sort of lack of knowledge or uncertainty is virtually the norm as regards paraconsistent approaches in classical antiquity. Much, almost all, of the evidence has been lost or destroyed, not uncommonly at the hand of politically more powerful establishment positions, it would appear.[7] The entire history of paraconsistent approaches from this important period has to be pieced together from a few fragments, and—especially bad[8] for such alternative logical approaches as paraconsistent ones—from what the opposition had to say.

Paraconsistent thinking begins in the West, so far as we can determine, with Heraclitus of Ephesus. This was certainly the view of idealist historians (such as Hegel and more recently Stace), but the matter has been in doubt since antiquity. Thus Aristotle: 'It is, therefore, impossible to ever conceive that something is and is not, as some think Heraclitus said' (*Metaphysics* IV, 1005f.). Since then there have been repeated attempts to fit Heraclitus into the orthodox Western framework, which has long been underpinned by consistency hypotheses;[9] that is, attempts to render Heraclitus's position coherent, in the over-restrictive sense of consistent.

But Heraclitus' work—or rather the small but tantalising set of fragments that remain—resists such relocation, and is much more easily and naturally interpreted paraconsistently or dialethically (as Hegel was to interpret Heraclitus). Central in Heraclitus' thought is the theme of unity of opposites,[10] that opposites are united, at least in always being connected, but sometimes more strangely in being identical. Heraclitus is often saddled by commentators with the stronger, extravagant, thesis that all opposites are identical and indeed that all things are identical. The stronger thesis is justified by the principle, sometimes explicitly ascribed to Heraclitus,[11] that connection between the opposites implies their identity, i.e. in symbols

$xRy \rightarrow . \; x = y$. While such a fallacious reduction of connection of all sorts to identity was tempting to the Greeks (and apparently encouraged by their language)—as indeed a reduction of relations to properties (or to properties together with identity or instantiation) has remained tempting virtually to present times—there is insufficient evidence that Heraclitus adopted such a reduction of connection to identity (unless *being united* is simply construed narrowly as *being identical*, an inference English semantics does not justify). All that many examples of the unity of opposites reveal is unity in the sense of appropriate connections (as in United Nations or United Church),[12] and this is what analogies (such as the bow and the lyre) also suggest.[13] Moreover were Heraclitus committed by the unity of opposites to the identity of all opposites, he would be committed by the main truth expressed in the Logos, that all things are united,[14] to the thesis that all things are identical, and so (since there is something) to the monist thesis that there is exactly one thing—a thesis he did not hold but rejected, since evidently on his view there are *many* things in tension.[15]

The principal truth expressed in the Logos is none other than one of the main "laws" of ecology, that everything is connected to everything else,[16] not that there is only one thing. The Logos has in fact multiple roles, in somewhat the way that worlds do (especially the factual world T) in recent semantical theory: the Logos supplies both the general truths about things and the intelligent principles upon which they function (the basic information or axioms); and it takes the form of Fire.[17] The principle truth of the Logos, that everything is united, is supported by, but of course not entailed by, the unity of opposites. It is a part of the weaker theme, that some opposites are not merely connected but identical, a theme that suffices for dialethism given only familiar assumptions.

The central argument that Heraclitus' position is dialethic goes, when duly filled out, as follows:

(i) Some (suitable) opposites are identical. Let f and $\bar{f}$ be among such opposites (a predicate representation of opposites is convenient but not essential). Then $f = \bar{f}$. Now let x be some item that has f; then fx. For all ordinary predicates this follows from a suitable theory of objects (for let x be $\xi z f z$, i.e. an arbitrary object which is f). However, a less exotic route will serve. By excluded middle, fx, or else $\bar{f}x$, for any object x. Whichever alternative is assumed the argument continues in the same fashion. Now since fx and $\bar{f} = f$, also $\bar{f}x$, so $fx \;\&\; \bar{f}x$. While this gives dialethism of a sort, it could be contended that it is only "predicate dialethism", which is compatible with non-paraconsistent positions, indeed with an extension of classical logic.[18] To reach dialethism proper it needs also to be granted

(ii) Among the suitable opposites is some pair, h and $\bar{h}$ say, such that $\bar{h}z$ iff $\sim hz$, i.e. for which predicate and sentence negation coincide (for suitable z). Then indeed dialethism proper follows: for $hz \;\&\; \sim hz$ holds.

The best evidence that the premisses, and indeed the conclusion, can be ascribed to Heraclitus derives perhaps from the "river fragments", where the opposites are those involving change (and measure and motion). Consider in particular, the proposition 'In the same river, we both step and do not step, we are and we are not' (Freeman, 1948, p. 28, #49a). The obvious symbolism—also easily disputed—is in each case hz & ~hz.

A standard argument for such contradictions (presumably known to Heraclitus but not explicit in Heraclitus' work) is that in change, for example in motion which is change of position, there is at each stage a moment where the changing item is both in a given state, because it has just reached that state, but also not in that state, because it is not stationary but moving through and beyond that state.[19] However, precisely the same sort of argument the Eleatics took as a *reductio ad absurdum* of the hypothesis that change occurred; for whatever yields a contradiction, such as motion does, is impossible.

Sharply opposed, then, to the Heraclitan position, with its unity of opposites, and account of change through identity of opposites, was the Eleatic position deriving from Parmenides, an extreme position from which, however, both the classical establishment positions of Plato and Aristotle and the contemporary establishment position (incorporating classical logic) may be seen to descend.[20] Appreciation of some features of the orthodox (op)position—features that appeared in an early state in the dialectic of paraconsistency versus consistency, of Heraclitus versus the Eleatics—is important in gaining a fuller grasp of what the paraconsistent enterprise is about and what it is up against. The central point is not that the Eleatic position with its themes that nothing changed, that motion was impossible, directly opposed Heraclitus' view that everything was in change,[21] that all was in flux, though this opposition is real enough; it is firstly that the Eleatic position was sustained by a series of hard arguments, most notoriously by the paradoxes of Zeno, a collection of generalized *reductio ad absurdum* arguments, all designed to show that the assumption that motion occurred led to contradiction, from which it was concluded, applying a consistency assumption, that motion could not occur; and secondly because the only way out appeared to be through rejection of the consistency assumption of "Eleatic logic". The impact of arguments that change implies contradiction—important for their fundamental place in the history of dialectic (and accordingly reserved for detailed presentation and discussion in the next chapter)—can thus be interpreted in two quite distinct ways; as a Parmenidean *modus tollens* against change or a Heraclitan *modus ponens* for dialethism. The latter became the view of Hegel; indeed Hegel was to take motion and change as affording paradigmatic examples of contradictions in nature.

Aristotle devoted a considerable part of his metaphysical theory to refuting would-be Greek paraconsistentists,[22] first of all by trying to show that consistency assumptions, and the connected Law of Non-Contradiction could not be given up, rationally, or indeed without foregoing discourse altogether (the ultimate failure of these very important arguments is discussed below, in 5.2), and secondly, by offering an analysis of change which appeared to avoid contradictions.[23] Key parts of this analysis involved the use of time to avoid contradiction—instead of saying that a changing thing was both in a given state and also not in that state, it was said that the thing was in that state at time t_1 but not in that state at a different time t_2—and the theory of potentiality—required to reunify these now temporarily isolated states as parts of the one (and same) change.[24] The appeal to different temporal quantifiers illustrates the *method of (alleged) equivocation* used since ancient times to avoid contradiction and reinforce consistency hypotheses; namely, where both A and ~A appear to hold, find a respect or factor or difference r such that it can be said that A holds in respect r_1 and ~A in respect r_2. It can then be said that a contradiction resulted only by equivocation on respect or factor r. Often however the method of alleged equivocation does not work in a convincing way, and it breaks down in an irrepairable way with the semantical paradoxes, as the Megarians were the first to realize.

In fact Parmenides and Zeno had, earlier on, to resort to the method of equivocation in order to escape contradictions their data seemed to deliver, that things both moved (as a matter of observation) and did not, that Achilles overtook the Tortoise and also could not do so. They drew a fundamental distinction of respects, between how things seem and how they are, between Appearance and Reality. Then it can be said, without contradiction, that Achilles overtakes the Tortoise as regards appearances, but not in reality. This crucial distinction undermined, however, another basic thesis of Parmenides, the One-world thesis, for it seemed to lead to a distinction of worlds, of the world of Appearance from the world of Reality. Plato and also Aristotle (not merely through his very un-Eleatic theory of potentiality) accordingly rejected Parmenides' world-monism and adopted instead "many-world" positions. Such theories need not of course lead in paraconsistent directions, because the many worlds can all be consistent ones (as in model theory and modal logic semantics).

Some of Heraclitus' aphorisms have been taken to suggest, or imply, an *extravagant* dialethism, that Heraclitus espoused not merely that *some* contradictions are true, but that *all* are true.[25] There is obviously a gross difference between these positions, a difference mainstream logics cannot adequately recognize. While the textual evidence available does not indict Heraclitus of extravagant paraconsistentism, so we have argued, some of the Sophists—who make up the next wave of Greek paraconsistentism—do not escape indictment easily. Protagoras, in particular, with his thesis that everything is true, seems to be committed to just such an extravagant position. For if every statement is true, then so is every contradiction; while conversely for any statement A, because A & ~A is true since contradictory, A is true, by simplification.

It is not difficult to see, in principle at least, why some of the Sophists should have arrived at a paraconsistent stance. For according to them, both sides of various (important) views can hold, or hold equally well. That is, in a given situation, perhaps even in the factual world T, both A and ~A hold (and are, for instance, supported by sound arguments) for some A. So long as not everything holds in the given situation, a paraconsistent position results. Such appears to have been the position of some Sophists (as we shall see). How was it that one of the first and most important of the Sophists, Protagoras, went further and adopted what is, at first sight, the trivial position that everything is true?[26]

Commonly an attempt is made to get Protagoras out of this fix (this stupidity, it is often thought) by having him say something different from what he did say, e.g. that everything is relative, that every proposition is supportable, etc. But supportability is not truth, and relativity does not on its own explain how Protagoras could have held what he did. Modern semantical theory can however explain the situation. For it is a theorem of universal semantics that *every statement is true according to its own lights.*[27] In short, one can so relativize the truth-determining framework, the model structure, as to bring out any given statement as true. Man—or at least men who have discovered the semantical trick—is the measure of truth and falsity, and so of what there is and is not; and Man can simultaneously affirm the themes of correspondence theory to the effect that the things that are are [so] and the things that are not . . . are not [so].[28] But, it will be protested, relativism is not paraconsistency; the two are very different.[29] And so they are; however such semantical relativism presupposes a paraconsistent stance. For in order that an isolated contradictory proposition is brought out as true (in its own lights) it is essential that there be included in the modelling involved situations where contradictions hold true. If, for example, only classical models were admitted it would be impossible to confirm Protagoras' themes, to refute logical laws or to conclusively support contradictory statements. More directly, it can simply be concluded from the semantical relativism[30] that everything is true according to its own lights (e.g. in its canonical model structure), that contradictory theories *can* be true.

Given this paraconsistent setting much of the rest of Protagoras' reported work—which is not after all an extravagant dialethism—falls into place. It is a relatively straightforward matter then to see how there can be arguments *against* what are in fact necessary truths and *for* what are in fact[31] logical falsehoods. Since there is little problem (on even a modal view) in seeing how contingent statements can be both supported and rejected (in different possible worlds), the setting brings out Protagoras' further theme that every statement is both supportable and refutable, that there are contradictory arguments about everything, [32]—a theme that persisted into the later scepticism of Pyrrho and Carneades.

What the semantical reconstruction may appear to throw into doubt is the claim of Diogenes Laertius that Protagoras' anticipated Antisthenes' argument that contradiction is impossible:

As we learn from Plato in the *Euthydemus*, he [Protagoras] was the first to use in discussion the argument of Antisthenes which strives to prove that contradiction is impossible, and the first to point out how to attack and refute any proposition laid down.[33]

But Protagoras could very well have used such an *argument*—it accords, after all, with his theme that every statement can be supported as well as refuted—without being committed to the position that contradiction is impossible. Indeed he could not coherently hold such a position, since he can bring contradictions also out as true and so as realizable.

The work of Antisthenes, to the small extent that we know it,[34] does however also admit of a—very different—paraconsistent construal. According to Antisthenes, it is impossible to speak falsely,[35] so whatever is said is true (as, in a sense, with Protagoras), so in particular contradictions (if uttered) are true. It is not too hard to see why Plato argued that the position was self-refuting.[36] First, the thesis that the thesis itself is false is true; so it is false. Secondly, if contradiction *is* impossible, then contradiction occurs truly in the judgement of impossibility, so contradiction is possible. But it is not according to Antisthenes. Of course Plato's clever arguments, as they involve consistency assumptions, are readily turned by a seasoned dialethician, especially the second.

But Antisthenes appears to have been a paraconsistentist only obliquely. His motivation and argumentation seem decidedly different from those that typically underlie paraconsistent approaches. He is said to have denied the possibility of contradiction or logical falsehood, but his arguments, were they to work, would show similar difficulties with falsehood; so it is not too surprising that it is common to attribute to him (as we have done) the (stronger) view that it is impossible to speak falsely. One of his arguments, reminiscent of Parmenides, started from the assumption that all statements are subject-predicate in form, saying of an object what it is. Now such statements can only fail to be true by referring to nothing at all or by referring to something different from the object which has the predicate. But neither case is possible. The first alternative is ruled out (by the strong Ontological Assumption): it is impossible to speak of what does not exist. And the second alternative is ruled out by an analogous assumption: it is impossible for one to speak of something different from what one intended to.[37] Both assumptions have seemed plausible to many philosophers; both are however false. The argument does not provide a solid basis for contending that contradictions are sometimes (in some situations) not false. The main argument for which Antisthenes is known begins by claiming further that all statements are, or are reducible to, apparent identities, specifically to statements of the form *x is y*. Thus Antisthenes is sometimes accused of confusing statements of identity and predication—a major tangle in ancient thinking, which Plato gets the credit for cleaning up satisfactorily.[38] But it is not so clear that Antisthenes was confused, or that Plato's clean-up was required (*or* was desirable). A shallow reason is that subject-predicate statements such as x is f appears to be equivalent to given identities like x is an f, i.e. x is an object which is f, which *is* of the form *x is y*, where y is an object which is f. A deeper reason is given by theories like Leśniewski's, which take as basic the undifferentiated *is*, symbolized 'ε'. With modern logical technology it is much harder to resist giving Antisthenes the first stage of his argument (but thereby even easier to resist granting further stages). His further contention is that no object can admit equation with an object differing in conception from it. While such combinations [statements] of the form *x is y*, would normally be merely written off as false (non-significant, etc., as the case may be), Antisthenes wants to reject them altogether as statements. Such combinations must be rejected 'because the conception of one is different from the conception of the other, and two things with different conceptions can never be . . . the same',[39] or, if the argument is to go through, even *said* to be the "same". Why would anyone want to say this? In the first place, a strong version of the Ontological Assumption, the Parmenidean form already seen in the initial argument, appears to be at work again. If x and y are not the same, then the sameness (of conception) the statement "x is y" is trying to ascribe to x and y does not exist, and so cannot be spoken of.[40] In the second place, because of the philosophical pay-off. One corollary of the argument was supposed to be a Parmenidean doctrine of the oneness of being; 'for if nothing can be predicated of anything else, . . . being can alone be predicated itself'.[41] Another corollary Antisthenes aimed at, according to Zeller,[42] was proof of the impossibility of speculative knowledge. Presumably such knowledge would require what cannot be said (or thought either?). At this point, as at others,[43] Antisthenes appears to anticipate some aspects of Wittgenstein's work.

Evidently, if Antisthenes was a dialethician, he was a decidedly extreme one. Antisthenes was at one time a pupil of Gorgias, another of the Sophists who deserves mention in a history of paraconsistent approaches. Gorgias can be seen as initiating yet another direction of paraconsistent thought, that which eventually found much fuller expression in the theory of objects (discussed in 5.1 below). Major themes of the theory of objects were instantiated in Gorgias' writings; notably, the rejection of the Ontological Assumption (and therewith the Reference Theory) are implied by Gorgias' thesis that no universals exist, and in particular that neither Being nor Non-Being exist.[44] It is evident that Gorgias thought we could think and argue perfectly well about what does not exist, without furthermore implying that it does exist in some way, and he proceeded to argue very skillfully in such a fashion. Moreover components of Meinong's freedom theses are clearly illustrated elsewhere in Gorgias' work, e.g., in the theme that 'conjecture is open to all in everything'.[45] While there appears to be little evidence that Gorgias would have adopted a dialethic position, a paraconsistent approach is required to accommodate in a direct way the components of the theory of objects and what Gorgias assumes concerning intensional functors such as those of belief and conjecture.

Isocrates, a member of the Establishment, was explicitly critical of Gorgias as well as of Protagoras and of Zeno and Melissus. In his *Helena* (an oration apparently intended, among other things, to send up Gorgias) he also attacked three contemporary "classes of triflers". The first of these triflers were presumably the Antistheneans; they comprised 'those who maintain that it is impossible to speak falsely, or to utter a contradiction or to "deliver two contradictory discourses" about the same matter' (*Helena*, § 1). The other classes of triflers are now taken to be Megarians and Academics (cf. Freese, 1894, p. 290), but they also included, it is reasonable to conjecture, the author of *Dissoi Logoi*. The pre-Aristotelian fragment, now called the *Dissoi Logoi*, is discussed as follows by the Kneales;[46]

> It is obviously part of the protracted debate on the possibility of falsehood and contradiction.... the author seems to be arguing that it is possible not only to make contradictory statements but even to maintain in a variety of contexts two plausible theses which contradict each other. To this end he sets out a series of antinomies, each one with thesis and antithesis.

Thus the *Dissoi Logoi* would certainly seem to be a dialethist tract. However the matter is not quite so clear. The thesis of the *Dissoi Logoi*, Taylor conjectures,[47] is

> to reinforce the Eleatic doctrine that ταπσλλά, the contents of the world of sensible experience, are unknowable, and that no belief about them is any truer than its contradictory.

Hence at this stage paraconsistency is beginning to merge with what was to become a major element in later classical thought, scepticism. Orthodox scepticism can be formulated without paraconsistency. However the sceptical positions of Carneades, Pyrrho and others appear, like the position of Protagoras, to require paraconsistent underpinning and the support of paraconsistent approaches; but, once again, as so much of the relevant work is lost, the matter must remain in considerable doubt.

Very strong evidence that dialethic positions were taken in classical antiquity comes not only from Heraclitus and the Sophists, but from the work of the Megarians, and in particular from their treatment of (semantical) paradoxes, especially the Liar, said to have been discovered or devised by the Megarian, Eubulides.[48] As remarked, according to some Megarians, the paradoxes yield propositions which are at once both false and true. The evidence may not be entirely conclusive, but it is not so flimsy as to justfy Mates' contention regarding the Liar, that 'We do not know how any of the competent logicians of antiquity attempted to solve antinomy'.[49] Admittedly it depends on who is counted competent, and some might doubt that paraconsistent logicians make the grade! In any case, Bocheński considers two important ancient efforts—distinct from the Megarian approach—at solving the Liar.[50] While there was subsequently much dispute over what these attempts amounted to, the shape of the attempts, by Aristotle and Chrysippus, is recorded.

In the *Sophistic Refutations*,[51] Aristotle specifically presents the paradox as 'the problem whether the same man can at the same time say what is both false and true'. Thus the Liar paradox was initially perceived in the obvious way. Aristotle's treatment of the problem was however less obvious and took the following rather cryptic form: 'There is, however, nothing to prevent it [the paradox statement] from being false absolutely, though true in some particular respect or relation, i.e. being true in some things though not true absolutely'.[52] This seems to fit exactly within the second "resolution" of the Liar proposed by Peirce[53] and discussed in more detail below since it seems to have paraconsistent possibilities. In fact it is more plausibly seen not as a paraconsistent resolution, but as a further application of the method of equivocation—though it tries to take some account of paraconsistent data through the notion of "truth in some respect".

The solution of Chrysippus, as recorded, has even less dialethic potential, but suggests a non-significance approach, that paradox-generating sentences lack meaning. It is important here, however, in that it explicitly rejects both Aristotelian *and* dialethic resolutions, thereby confirming the claim that a dialethic resolution had some currency. According to Chrysippus, so it is reported,

> the (fallacy) about the truth-speaker and similar ones are to be... (solved in a similar way). One should not say that they say true and (also) false; nor should one conjecture in another way, that the same (statement) is expressive of true and false simultaneously, but that they have no meaning at all.[54] And he rejects the aforementioned proposition and also the proposition that one can say true and false simultaneously and that in all such (matters) the sentence is sometimes simple, and sometimes expressive of more [as solutions].

What the Megarians and Stoics said about the semantical paradoxes should be integrated of course with what they said about truth and falsity, and also with what they said about implication. For these are *not* independent issues. Unfortunately again, as to the Megarians' views on such matters, comparatively little is known. Much more is known about the theory of truth in Stoic logic.[55] Whereas the Megarian theory appears to have been four-valued, with values true, false, neither true nor false, and both true and false, the Stoic theory, of the truth-values of propositions or *lecta* (the basic carriers of truth-values), was (at most) three-valued, lacking the value, *both.* However in their theory of the truth-values of *presentations* the Stoics did assume a four-value pattern: '... some presentations, are both true and false, and some are neither'.[56] Examples they gave of presentations which are both true and false include the following: first, "when a man imagines in his dreams that Dion is standing beside him (when Dion is alive)"; and secondly, the "image of Electra as a Fury visualized by Orestes in his madness."[57] By contrast, the example of the bent oar was not accepted as a presentation which is both true and false. Why not? An answer—obvious now in the light of intensional semantics—is that the latter example would have induced inconsistency in the factual world T, whereas the two accepted examples involve joint truth and falsity *off* T, in worlds picked out by intensional functors such as those concerning dreaming and insanity. Now it might be claimed that this shows little about propositional truth and falsity which however is what an argument to paraconsistency requires. But in fact the Stoics considered propositional truth as fundamental, all other types reducing to it.[58] So joint truth and falsity of presentations reflects back onto propositional terms. But whether it delivers a paraconsistent position depends upon how the reflection is effected. We are told that 'a presentation is true iff a proposition accurately describing it is true'.[59] And let us suppose, as is not unreasonable, that a similar scheme holds for the predicate 'false'. Let p be a presentation and p# a proposition that accurately describes it. Then where p is a presentation which is both true and false, a dialethic result is forced, contrary to other information we have as to Stoic views. The problem lies, not with the falsity reduction scheme, but with both schemes which require situational-relativization to yield the result that where p is both true and false there is some situation s_p where proposition p# is both true and false. Given that such situations are ones that we can reason about, as we have been doing and is in any case an outcome of the propositional ascriptions, the Stoic position which emerges is a paraconsistent one. For since not every proposition is true in situation s_p, s_p is appropriately inconsistent but not trivial. In sum, the Stoics were committed to contradictions holding off T, but not their holding at T.

The conclusion we are pushed towards is, then, that whereas the Megarians took a dialethic stance, some at least of the Stoics adopted (only) a weaker paraconsistent position. But it looks as if the Stoic ranks may have been split over the issue, as they were split over the closely related matter of the correct theory of implication and conditionals. Indeed there is further evidence from the Stoic theory of "connexive implication" that the Stoics (including Chrysippus) who held that theory were commited to a paraconsistent position. For under this theory, the implicational principle of *ex falso quodlibet* was rejected, since an arbitrary proposition q may not be connected to p & ~p.[60]

It may be that paraconsistent positions, of a Megarian or Stoic cast, persisted into late antiquity.[61] But 'with the end of the old Stoa there begins a period into which hardly any research has been done'.[62]

2. Elements of paraconsistent thinking in Eastern philosophy

In some ways the situation is at present even more vexed here than in the case of classical antiquity. For although there appears to be more data—less crucial material has been destroyed or lost—there are severe difficulties in obtaining a clear view of the situation. In part, but perhaps only in small part, this is due to the nature of the material itself. It is also due to the linguistic inaccessibility of much of the material, and the seriously distorting lenses of translators and commentators on the material that has been rendered more accessible. Often this distortion can be directly ascribed to a narrow training in Western analytic thinking and frequently uncritical assimilation of anti-paraconsistent assumptions.[63] This surfaces, for example, in incomprehension in commentators as to how philosophers, such as the Jains, can reject the Law of Non-Contradiction. It appears, differently, in descriptions of Nāgārjuna's negative dialectic in terms, and in a fashion, evidently borrowed from Hegel (thus e.g. Murti, 1955, pp. 127–8, and Streng, 1967, pp. 148–9 where the same account is used).

Furthermore, the way paraconsistency enters in Chinese thought, as in Indian, is not through a set of theses that can be simply pointed to as evidence of paraconsistent approaches, but in *the way contradictions are tolerated* and *used to illustrate* points.

This is particularly true of the Tao, which contains definite elements of paraconsistency: 'Laotse is full of paradoxes', e.g. 'Do nothing and everything is done'.[64] But a theory may contain paradoxes, or apparent contradictions, without necessarily containing any unresolved or unresolvable contradictions. Consider, for example, Meinong's 'There are things of which it is true that there are no such things' and Jesus's 'He who loses life shall find it'.[65] However there are at least consistent subtheories of the theory of objects and presumably of Christianity which can consistently assimilate the respective paradoxes. So additional evidence is required to show that Tao exhibits genuinely paraconsistent elements. Some of the main evidence for this is rather like some of that adduced in the case of Heraclitus; the relativity of opposites, and the levelling of all opposites, of all things, into one. All things are one. Unity is achieved through complements, or opposites. But also (more in the fashion of Parmenides) the One is eternal and unchanging. Thus the One certainly appears to exhibit inconsistent features and to violate classical logical principles. And according to Needham it was obvious to the Taoists that the Law of Contradiction was constantly being flaunted: 'The natural sciences were always in a position of having to say "It is and yet it isn't". Hence in due course the dialectical and many-valued logics of the post-Hegelian world. Hence the extraordinary influence of the traces of dialectical or dynamic logic in the ancient Chinese thinkers, including the Mohists...' [66]

Both the Mohists[67] and the Dialecticians have some claim to be accounted early paraconsistents. In particular Hiu Shih (?380–305 BC) who lived during the Warring States period, and belonged to a school of philosophers that was known as the 'Dialecticians' or the 'School of Forms and Names', considered contradictions to be the great insights of the world, the keys to the universe, as it were. (He is also said to have been bent on proving the impossible possible.) Although the many books of Hiu Shih have been lost, some of the paradoxes he propounded have been recorded in the *Chuang-Tzu*[68] (chapter 33). The sixth paradox there presented is 'The South has no limit and has a limit', which has the apparent form $p \& \sim p$.[69]

With Indian thought the position is rather clearer. At least two important positions had paraconsistent commitments: Jainism and Mādhyamika Buddhism. The Jains from

> a very early date flatly den[ied] the law of Contradiction. At a time when the battle raged between the founders of Buddhism and the Sānkhyas,[70] when the latter maintained that "everything is eternal", because Matter is eternal, and the former rejoined that "everything is non-eternal", because Matter is a fiction, the Jainas opposed both parties by maintaining that "everything is eternal and non-eternal simultaneously". According to this theory you could neither wholly affirm, nor wholly deny any attribute of its subject. Both affirmation and denial were untrue. The real relation was something half way between affirmation and denial. Like the doctrine of Anaxagoras in Greece, this denial seems directed much more against the law of Excluded Middle, than against the law of Contradiction. However in the problem of Universals and Particulars the Jainas adopted an attitude of a direct challenge to the law of Contradiction. They maintained that the concrete object was a particularized universal, a universal and a particular at the same time. Such is also the attitude of one of the earliest Buddhist sects, the sect of the Vāsiputrīyas.... They maintained that the Personality ... was ... something existing and non-existing at the same time.[71]

Challenging the law of Contradiction as the Jains do does not necessarily commit them to a paraconsistent position: it depends further on how negation is interpreted. The situation is complicated because the Jains apparently talk about contradictions in two different ways.[72] On the one hand there is a cancellation way: the joint assertion of Yes and No is (it is said) ineffable in language, for 'yes' and 'no' being in perfect equilibrium, will automatically cancel each other. On the other hand they allow joint assertion of Yes and No, which is expressible in language, and hence on more classical construals must be somehow resolvable through some indigenous device. The device is that of pluralism (which *shades* into a certain relativism).

The Jains held as a central theme the non-one-sidedness of truth:—'The Jainas contend that one should try to understand the particular point of view of each disputing party if one wishes to grasp completely the truth of the situation. The *total* truth . . . may be derived from the integration of all different viewpoints'.[73] In this respect the Jains anticipate contemporary discussive logic, initiated by Jaśkowski, and they may similarly be interpreted in terms of integration of different worlds, or positions, reflecting partial truth (see 5.5, p. 44). Naturally such a theory risks trivialization unless some (cogent) restrictions are imposed on the parties admitted as having obtained partial truth—restrictions of a type that might well be applied to block amalgamations leading to violations of Non-Contradiction.

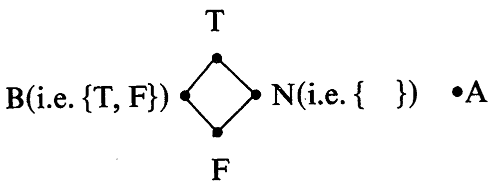
Unlike the Jains, the Mādhyamikas apparently affirmed the law of Contradiction.[74] But this did not prevent a certain unity of opposites, e.g. in the negative dialectic of Nāgārjuna, a concept, such as Being, can become indistinguishable from its opposite, Non-Being.[75]

The negative, or destructive, dialectic was based on a four-valued scheme, apparently deriving from Buddha. There were the fundamental metaphysical questions on which Buddha was mysteriously silent. Why? There were well-known questions which the Buddha declared to be *avyākrta*, by which it is commonly (but likely erroneously) said he meant the answers were inexpressible. For the moment we shall simply say they have value A (for alternative or *avyākrta*). Among the questions concerned were the following:

(1) Whether the world is (a) eternal (b) or not, (c) or both, (d) or neither
(2) Whether the world is (a) finite, (b) infinite, (c) or both, (d) or neither
(3) Whether the Tathāgata (a) exists after death, (b) or does not, (c) or both (d) or neither.[76]

The four alternatives, or values, formed the *basis* of the tetralemma (Cataṣkoti) of Nāgārjuna's dialectic. The values are, in effect, true, false, both true and false, and neither true nor false; e.g. it is either true that the world is eternal, or false that it is, or both or neither (using the T-scheme).

For the four alternatives yield four values, one for each answer, and conversely the four-valuedness transfers to items as follows. Consider a statement such as "d exists" which may have any of the four values. Then (by T-schemes) there are four cases; d exists and does not, d neither exists nor does not, d exists, d does not exist. Thus the four-values may be represented on the logical lattice (L_4)[77], shown on the left,



—to which both the Buddha and Nāgārjuna in effect added the further value A, for The Rest, for everything that did not fit into the too neat and clean logical lattice.

Buddha, incidentally, was perfectly clear that the classical two-valued barbell lattice T•——•F was quite inadequate: The 'yes' or 'no' answer to [the] fundamental questions could not do justice to truth. Buddha called such speculations mere 'ditthivada' (dogmatism), and refused to be drawn into them.[78] Whereas the two-valued position was accounted dogmatism, the four-valued was construed as rationalism. This too was inadequate, as Buddha had realized, and as Nāgārjuna proceeded to show by his destructive dialectic. Unfortunately for the rationalist picture, there were isolated exceptions where *none* of the four values or cases obtain, e.g. where d above is emptiness.[79] Representing the position of the Buddha appears to require (at least) a five-valued logic. But such a framework is as much grist to the paraconsistency mill as the four-valued rationalist picture from which it emerged.

Nāgārjuna's argumentative procedure was to draw out and expose the disconcerting or antinomic consequences of *each* of the four rationalist alternatives.

> The technique of the dialectic consisted in drawing out the implications of the view of the opponent on the basis of the principles accepted by himself and thus showing the self-contradictory character of that view. The opponent was hoisted with his own petard. He was reduced to the position of absurdity when the self-contradictory consequences of his own assumptions were revealed. The dialectic was thus a rejection of views by *reductio ad absurdum* argument. Technically this was known as *prasánga.*
>
> The purpose of the dialectic was to *disprove* the views advanced by others, not to prove any view of one's own. He who advances a view must necessarily prove it to others whom he wants to convince; he who has no view to advance is under no such necessity (Singh [n.d.]).

But although primarily negative in its thrust, the procedure yielded (like modern "no theses" positions) some positive insights.[80] Among those were suggestions as to the limits of reason or of expressibility—it is not entirely clear which (what the Buddha hinted at suggested one, and the rejection of rationalism suggests the other), but it had better not be either, at least in any straightforward way. For the structure of the dialectic and the details of the arguments are expressible without any serious loss, and the arguments conform to rational standards, and do not force us into any (imported) Kantian "super rationality".[81] This can be confirmed by examination of Nāgārjuna's application of his dialectic, for example to the notions of causality and of nirvana.[82]

The theory of Nāgārjuna is often said to represent the high point of historic buddhist philosophy. Thereafter the dialectic potentiality, at any rate, of that philosophy appears to decline.

3. Paraconsistent thought in Christendom

Returning to Western Europe and moving on in time, we find that orthodox philosophy in Christendom was strongly anti-paraconsistent. The reason for this is undoubtedly the prominent influence of Aristotle and his logic. Despite this, paraconsistent thought continued as a minor, and sometimes heretical, tradition, which is not surprising when one notes how close to the wind Christian theology sails.

For a start, there were isolated figures such as Peter Damiani (1007–1072). Damiani resolved the paradox of God's omnipotence by retaining the unrestricted omnipotence of God, but admitting violations of the Law of Non-Contradiction. God could presumably both produce stones too heavy for anyone to lift and lift such stones, tie a knot that could not be untied and untie such knots, and so on.

> Commenting on the words of the psalmist *Omnia quaecumque voluit fecit*, Damian claims an absolute omnipotence for God. The Almighty has not subjected nature to inviolable laws; and if He wishes He could bring about that that which happened in the past did not happen. Certainly such an assertion seems to violate the principle of contradiction; but this principle is valid only for our poor human reasoning ... and does not apply to the Majesty of God and sacred knowledge.[83]

Unfortunately, Damiani combined his rejection of Non-Contradiction with a rejection of dialectics (as a useless *superfluum*) and a disdain for philosophy (as worthless and impotent) and for reason.[84] Thus, his position is not entirely coherent. For as de Wulf goes on to point out, 'All the same he makes use of reasoning in his *De Divina Omnipotent*... in which he shows that the rules of human knowledge cannot be applied to God' (p. 155).

More important than such isolated figures, however, was the whole tradition of Neo-Platonism, which contained significant paraconsistent elements. It is usually thought that whilst Plato may have been willing to concede the inconsistency of the empirical world (*Parmenides* 129 B), he insisted upon the consistency of the world of forms. Although this is the standard interpretation, the theory of forms quickly leads to inconsistencies,[85] and there is textual evidence (especially in the *Parmenides*) to suggest that Plato may have thought that the One (the form of forms) had inconsistent properties. Though this is not the most plausible interpretation of Plato's thought (in virtue of Plato's discussion of the issue in the *Sophist*) it was certainly the way that a number of Neo-Platonists interpreted it.

For example, the founder of Neo-Platonism, Plotinus, was wont to attribute contradictory properties to the One, such as that it is everything and nothing, everywhere and nowhere, and so on.[86] It is sometimes suggested that Plotinus thought that the One was ineffable and, hence, that his attributions of contradictions to it are attempts to express this.[87] Certainly he did think that the One was ineffable. However, this does not resolve the contradictions but multiplies them. To call something ineffable and then write many pages describing it, explaining how it creates the empirical world, how it can come to be known, etc., is a contradiction of which Plotinus can hardly fail to have been aware. Moreover, Plotinus is ready enough to ascribe contradictions to the soul, which is quite effable.[88]

Many of the contradictory aspects of Plotinus' thought were taken up by later Neo-Platonists such as Proclus, Damascius,[89] Pseudo-Dionysius and John Scotus Erigena. These in turn (especially the last two) were very influential in the Christian, Mystical, Neo-Platonic revival of the late Middle Ages and early Renaissance which also had significant paraconsistent elements. Meister Eckhart, for example, asserted that God was being, and yet, beyond being, and, therefore, not being.[90] However, the apogee of this tradition was, with little doubt, Nicholas of Cusa. For Cusa, the One, identified—as by the other Christian Neo-Platonists—as God, is the reconciliation of all contradictories (*Of Learned Ignorance*, 1, XXII). All things are true (and false) of God. Thus:

... in no way do they [distinctions] exist in the absolute maximum [the One] ... The absolute maximum ... is all things and, whilst being all, it is none of them; in other words, it is at once the maximum and minimum of being (*Of Learned Ignorance*, 1, IV).

What is more, whilst God is Father, Son and Holy Spirit, Infinity, Truth and Substance, he is quite ineffable and beyond description (*Of Learned Ignorance*, 1, XXIV, XXVI). Quite consistently 'Cusanus criticized the Aristotelians for insisting on the principle of non-contradiction and stubbornly refusing to admit the compatibility of contradictions in reality'.[91] Many people influenced by Cusanus were to take up similar positions: Boehme, Fludd, Campanella, and the unfortunate Giordano Bruno.[92] Though the influence of Neo-Platonism waned after the Renaissance, it was to exact a profound influence on German idealism. Hegel's Absolute is the One.

Nowhere in medieval times, it seems, was a paraconsistent approach to *insolubilia*, i.e. semantical paradoxes, seriously considered—though it was commonly admitted that '*insolubilia*, as their name implies, cannot be solved without evident objection'.[93] The catalogues of "solutions" to insolubilia given in later scholastic writings do not include such an approach. It is true that the twelfth solution to the antinomies in scholastic times, on Bocheński's classification of these, looks a lot like a paraconsistent approach—and it is said to be 'commonly held by all today'!

... An insoluble proposition is a proposition which is supposed to be mentioned, and which, when it signifies precisely according to the circumstances supposed, yields the result that it is true and that it is false.[94]

But it is a matter of appearance only; for what is said goes on to avoid the evident conclusion that the proposition is both true and false—by a familiar strategem.

While dialethic (strongly paraconsistent) positions were uncommon in medieval and post-medieval logic, (weakly) paraconsistent positions were adopted, so it is now beginning to emerge. One striking later example is provided by the Cologne work of 1493, where an idea underlying recent semantics for relevant logics is in part anticipated. The argument of *Ex falso quodlibet* was broken, as in relevant logics, by rejection of Disjunctive Syllogism (in inferential form licensing inference from ~A and A v B to B) on the grounds that where both ~A and A are assumed ~A cannot *also* be used to rule out A in the disjunction A v B. The Cologne work, and other post-medieval work by de Soto, Javellus and others,[95] may in some measure reflect and build upon the earlier literature on *obligationes* only now beginning to be studied; for this work seems to reveal a much broader trend in the same direction of admitting situations other than the actual, where assertions both hold and not, that is, inconsistent situations.

According to the *obligationes*-literature, which appears to concern counterfactual reasoning among other things, one is sometimes explicitly allowed to reason from contradictory statements or impossibilities. In such cases the rule *ex falso quodlibet* was suspended; in short, a basic requirement for paraconsistency was met. However, only certain impossibilities were admitted (e.g. theological claims concerning the Trinity). As may be expected with medieval logic, there were competing theories of *obligationes*. Roger Swyneshed, for example, appears to have anticipated the recent non-adjunctive approach. In his theory, while one is sometimes required to concede two contradictory statements in a disputation that begins with a non-contradictory hypothesis, nonetheless, one must always deny the conjunction of these contradictories.

4. The modern revival: paraconsistent approaches through idealism and common sense

In the modern period, beginning with the Renaissance and Enlightenment (both so-conventionally-called), and running through until the beginning of the present century, two further[96] major philosophical positions emerged which were congenial to paraconsistent approaches and took paraconsistent shape in some of their elaborations, namely idealism, especially as elaborated by Hegel, and, very differently, the philosophy of common sense, especially as presented by Reid.

Idealism developed in two different forms, transcendental idealism, which was integrally linked with the revival of dialectic and adaptation of Greek idealism, and as a movement was centred in Germany, and non-transcendental idealism, which was typically coupled with empiricism (and took the form of phenomenalism) and had its main base in England. The first of these forms we consider in the next chapter when we take up the history of dialectic. The second form appears to have been given paraconsistent shape by some of its less-known practitioners, notably Collier.

In *Clavis Universalis*, published in 1713, Collier[97] argues that the external world is impossible. In doing so he anticipated the idealism of Berkeley and also the first two of the antinomies of Kant. Now idealism, though it can be congenial to paraconsistency, does not necessarily lead to it, and may well adhere to strong consistency assumptions. For example, the demonstration of the impossibility of a mind-independent external world may be taken just as a reductio argument, of classical form, and as showing no more than that there can exist no such world, certainly nothing to the effect that some contradictions hold true,[98] or may be or be considered true. Collier is however prepared to take substantial steps in dialethic directions; for example (on p. 61), he meets an objection that certain themes 'appear to be contradiction' with the response that they are indeed so but 'nevertheless true; nay, . . . I could easily show them a hundred such contradictions, which they themselves will acknowledge to be true'.

Collier's work owes what little currency it has to Reid who chanced upon one of the rare copies in a library. Reid in turn is important in the development of paraconsistent thinking for his repudiation of the traditional assumption that conception, and mental operations pretty generally, are restricted to what is possible, and that indeed conception provides a test of possibility. Reid argues, on the basis of common sense, that the objects of conception may be impossible, so the test is no test but a mistake concerning the logic of conception.[99] Suppose now we consider the deductive closure of what some person, Reid say, conceives at a stage at which he conceives some impossibility and so contradictory statements. Then in this situation which is deductively closed, both p and ~p hold for some p, but not every q since Reid is certainly not committed to the conception of everything (or every proposition). Thus the formalization of Reid's position would lead to at least a weakly paraconsistent theory, including non-trivial inconsistent situations, other than the actual situation. But by contrast with the apparently dialethic form of Collier's idealism, the common-sense philosophy of Reid took only a weakly paraconsistency direction.

There was a further live historical position from which a paraconsistent approach might have emerged—despite the heavy consistency and classical logic underpinnings of the position as customarily presented—namely pragmatism. For as with German idealism, so with American pragmatism, the classical underpinnings now typically infiltrated are dispensible, without sacrifice of main themes; indeed the positions are evidently more viable without classical handicap. It is easy to see that a position as flexible and adaptable to practice—including reasoning practice—as pragmatism is adjustable to absorb the advantages of paraconsistency. It has looked to some as if just such an adjustment was occurring in Peirce's pragmatism after 1868.[100]

There are grounds for claiming that Peirce proposed a dialethic solution of the Liar paradox, maintaining that

S1. This very proposition is false; and
S2. What is here written is not true

are both true and false.[101] Unfortunately—since Peirce would be a welcome addition to the dialethic band—the grounds shake under further investigation. Peirce appears to have tried two different resolutions of Liar paradoxes (such as S1 and S2), an *initial* solution (during 1864–5), with much in common with a no-proposition solution and a *revised* solution (after 1887) deriving from Paul of Venice. According to the initial solution, such statements as S1 and S2 'are about nothing'[102] and so are logically meaningless and not objects of logical laws. So far Peirce is in the camp of those who propose incompleteness or non-significance solutions to logical paradoxes; the resolution is not a paraconsistent one since it removes the paradoxes from the domain of logic. But he gives the resolution a strange twist, contending that, though logically meaningless, the statements are truth-valued and indeed both true and false! Statements which 'stand upon the boundary of the true and the false' are 'in both', much as the boundary of red and green regions 'is both red and green'. The severe semantical difficulties this initial solution leads into, with S2 both true and false and neither because meaningless, are avoided by the revised solution, according to which statements such as S2 are meaningful. Thus 'this proposition is false', far from being meaningless is self-contradictory. That is, it means two irreconcilable things. While it may look as if Peirce is certainly embarked on a paraconsistent course with the revised "solution", there are new considerations that throw this into doubt (apart from the fact that he makes no moves to amend classical logical principles he elsewhere adopts). Firstly, he adopts new accounts of truth and falsity, according to which a proposition is true only if it is true in all respects, in everything said, and false otherwise; and secondly, he insists that 'every proposition besides what it explicitly asserts, tacitly implies its own truth'. He then argues that what S1 strictly implies is false—as paraconsistent logic shows, it is also true—hence as S1 is not true in this respect, it is false, period. More generally, Liar paradoxical statements involve contradiction and are simply false and not also true. Peirce has obviously woven a tangled web here. Although there are clearly paraconsistent strands in it, it is not clear that any coherent theory, consistent or otherwise, can be extracted from it.

5. Contemporary paraconsistent development and approaches

We now turn to modern developments in paraconsistent logic. There is no strictly continuous development to be found here. This century, paraconsistency has been discovered by many different people working in isolation, who, only afterwards, if at all, became aware of the work of others and the tradition into which they fit.

Although formal investigation of explicitly paraconsistent logics apparently did not begin until after World War II, 1910 conveniently marks the beginning of contemporary work on paraconsistent theories. For 1910 saw three events of paraconsistent significance; publication of Łukasiewicz's (subsequently) seminal paper 'On the principle of contradiction in Aristotle', production of the second, revised edition of Meinong's *Über Annahmen*—the basic text on Meinong's theory of objects, enlarged among other things to meet Russell's objections that the theory violated the Principle of Non-Contradiction—and the appearance of the first of Vasil'év's short series of papers on nonclassical logic. None of these historically important approaches, of Meinong, Łukasiewicz, and Vasil'év, makes use of modern symbolic logic; all are set more or less within the framework of traditional logic of the period. Moreover, all these approaches, with the possible exception of Meinong, appear to be at best weakly paraconsistent; they allow at most for off-T inconsistency, i.e. for inconsistent situations beyond the factual world, T. And Łukasiewicz, though he clears the way for the repudiation of the Law of Non-Contradiction (LNC) and so opens one route to paraconsistency, does not himself take that route. Indeed, all these approaches, which we shall consider in turn, sacrifice significant grounds for and elements of paraconsistency.

5.1. A theory for contradictory objects: Meinong

Meinong is important in the development of paraconsistent approaches because of his theory of objects, a theory which included *inconsistent objects* and also, in its more comprehensive form, *defective objects* (like the Russell class) such as the logical and semantical paradoxes supply. Under the theory, which grows directly out of common sense, contradictory objects, along with other sorts of non-existent objects are genuine, perfectly good objects. They are objects of thought and other intensionsal attitudes, they are amenable to logical treatment, and they have features—including many of the features they seem to have.[103]

As a consequence of the theory then, there are objects, contradictory objects, which, in virtue of their nature, have contradictory features, but which are amenable to logical treatment. Plainly the usual logic is not adequate to the task (at least without considerable reorientation). Meinong was well aware of this:

B. Russell lays the real emphasis in the fact that by recognizing such objects the principle of contradiction would lose its unlimited validity. Naturally I can in no way avoid this consequence.... Indeed the principle of contradiction is directed by no one at anything other than the real and possible (1907, p. 16).

Contradictory objects, such as the round square, have contradictory properties; the round square is both round and also not round (because square). So the Principle of Non-Contradiction (LNC) in one form is violated, at least in that for some p, both p and also not-p. That this was taken by Meinong, as by Russell, to show that LNC had only limited validity, indicated clearly enough that Meinong was still operating in a consistency framework (something for which there is a good deal of independent evidence[104]). Otherwise he could simply have said that LNC was universally valid *and* that instances of its negation also held.

Although Meinong must have seen the theory of objects as set within some modification of traditional logic, he did not work out that logic to any conspicuous extent. Formal development of an appropriate logic for the theory of objects, paraconsistent logic, was to take another 40 years even to get under way. How such logics can, in turn, help in formalization of the theory of objects will be explained in a subsequent introduction.

Even though it did not yield a logic, Meinong's theory has an important conceptual role. His contradictory objects, taken to counterexample LNC, played an important role in Łukasiewicz's realization of the vulnerability of that fundamental principle and of the traditional arguments for it. And Łukasiewicz's conceptual work in turn motivated Jaśkowski's formal theory.

5.2. The overturning of conventional logical wisdom: Łukasiewicz

In his penetrating 1910 article, Łukasiewicz opens the way for paraconsistent enterprise, but does not follow the road opened, and at the end of the article veers away from such heresy. In his subsequent logical work Łukasiewicz became far removed from paraconsistent positions; for example, he accepted, and exploited, spread principles, he argued that material implication was an adequate medium for the formulation of Aristotle's theory of syllogistic, and even his many-valued logics (though subsequently refined for paraconsistent purposes) fail to meet conditions for paraconsistent logics. But then in 1910 the influence of Meinong was strong (see p. 488, p. 506 ff): subsequently that logically-liberating influence waned.

Łukasiewicz opens the way for paraconsistent enterprise by showing that arguments for the Principle of (Non-)Contradiction, LNC, all derived from Aristotle, are built on sand; that, in effect, nothing excludes the design of non-trivial contradictory theories and furthermore such theories may even be true. And he tentatively conjectures that, by analogy with the development of non-Euclidean geometries, 'a fundamental revision of the basic laws of

Aristotle's logic might perhaps lead to new non-Aristotelian systems of logic' (p. 486),[105] in particular, systems lacking, or admitting violation of, LNC.[106]

The bulk of the article (sections 1–17 inclusive) comprises 'historical-critical exposition' of Aristotle's formulation of, and attempts to establish, LNC. But the exposition is of much more than merely historical interest, since as Łukasiewicz says by way of introduction,

> Aristotle's intuitions regarding the principle of contradiction [LNC] are, for the most part and clear down to present day, the usual and traditional ones; and arguments for and against the principle can be found together in the Stagirite in greater completeness than in any one text book of logic (p. 487).

In short, the 'historical-critical exposition' also deals with 2000 years of (traditional) argument.

Łukasiewicz unravels three different formulations of LNC from Aristotle:[107] the ontological (or better *thing* or *object*) formulation; 'It is impossible that the same thing belong and not belong to the same thing at the same time and in the same respect'; the logical (or *semantical*) formulation; 'The most certain of all basic principles is that contradictory properties are not true together'; and the psychological (or *belief*) formulation; 'No one can believe that the same thing can (at the same time) be and not be' (p. 487). Although Aristotle equates the logical and ontological formulations, 'none of the[se] three formulations of the principle of contradiction is identical in meaning with the others...' (p. 489).

As Łukasiewicz shows in detail, Aristotle's attempt to prove the psychological form on the basis of the logical form fails. For the proof (summarized pp. 490–1) 'is incomplete because Aristotle did not demonstrate that acts of believing which correspond to contradictory properties are incompatible' (p. 491); and his further discussions of the point are 'inconclusive'—inevitably so, in virtue of counterexamples; 'there are sufficient examples in the history of philosophy where contradictions have been asserted at the same time and with full awareness' (p. 492). More generally, Łukasiewicz's criticisms of Aristotle's attempts to establish or entrench forms of LNC are conclusive against them, even though he imports several rather unnecessary, dubious or false assumptions in elaborating his points.[108]

'Although Aristotle proclaims the nondemonstrability of the [nonpsychological forms of the] principle of contradiction... he does not [attempt to] *prove this claim* [and]... he strives in spite of that to give demonstrations for the principle' (p. 494 with insert from p. 493). Whether this is in order or not,[109] the purported elenctic and *ad impossibile* demonstrations are either circular or else inadequate in one way or another and open to counterexamples (as Łukasiewicz shows in detail in the crucial central section 13 of his paper, pp. 498–9).

A 'specially . . . note-worthy . . . *shift of proof*' occurs in 'all of Aristotle's proofs *ad impossibile*' namely, he 'proves not that the mere *denial* of the principle of contradiction would lead to absurd consequences, rather he attempts to establish the impossibility of the assumption that *everything* is contradictory' (pp. 499–500, where several examples of this *some* to *all* shift in Aristotle are cited). 'However, he who denies the principle of contradiction or who demands a proof of it, surely does not need to accept that *everything* is contradictory . . .' (p. 499). Certainly not; yet a similar fallacious shift is subsequently made by Łukasiewicz, as will emerge.

Łukasiewicz claims, but does little to show, that there is 'good reason' for the shift 'in certain of Aristotle's positive convictions' (p. 500). Be that as it may, these alleged positive convictions are of the first importance for determining the *qualified status* the LNC is alleged to have in Aristotle's position. For Aristotle, like Meinong and Vasil'ev later, 'limits the range of validity of the principle of contradiction to actual existents only' (p. 501) and does *not* extend it to appearance (p. 502). The 'sensibly perceptible world, conceived as becoming and passing away, could *contain* contradictions' at least potentially (p. 501), but beyond this ephemeral world is 'another, eternal and non-ephemeral world of *substantial essences*, which remains intact and shielded from every contradiction' (p. 502).[110] That Aristotle did not here openly reveal his "true position" is part of his diplomacy in trying to enforce the LNC, diplomacy required 'to hold high the value of scientific research' against the flood of falsity which would have destroyed science in its infancy (p. 509). But the need for such diplomacy (or dishonesty) depends too on the fallacious shift of proof, confusing the admission of some falsity with 'open[ing] door and gate to every falsity' (p. 509).

Finally in his historical-critical exposition Łukasiewicz rejects the widespread view, which is Aristotle's view, that LNC is the most final and highest logical principle (p. 502–4). He argues firstly that this is not so even according to Aristotle, who recognized that 'the *principle of the syllogism* is independent of the principle of contradiction' (p. 503). The syllogism Łukasiewicz extracts from Aristotle, which remains valid when LNC does not, takes the form:

B is A

C, which is not-C, is B and not-B

∴ C is A[111]

Secondly he points out, correctly, that many other basic logical principles are independent of LNC (p. 504).

In the 'positive part' of his paper (p. 504 ff.) Łukasiewicz endeavours to show, more generally,[112] that there is no (logical) basis for adoption of the principle of (Non)Contradiction, that 'a *real* proof of the principle ... *cannot be carried out*'—summed up in the theme that 'the principle has ... no logical worth, since it is valid only as an assumption' (p. 508). But the argument which divides into two parts (sections 19 and 20) is not decisive. The first part, which aims to show that there is no justification of the principle, fails because, among other things, it does not exhaust the ways of attempting to establish LNC. For example, the principle can be shown valid semantically (as in the semantics for relevant logic). Granted such a proof does use semantical apparatus not available to Łukasiewicz, and does make use of notions under examination in the metatheory, so at least approaching circularity. The second part of the argument, which *interestingly* is based on 'the fact that there are *contradictory* objects' (p. 506), such as those Meinong pointed out, does not provide counterexamples to LNC in the fashion Łukasiewicz intends without appeal to a consistency assumption bound up with the LNC itself. Otherwise, even if a contradictory object does ensure that for some p, $p \wedge \sim p$, this does not prevent $\sim(p \wedge \sim p)$ from also holding.[113]

So though he has opened the way for the paraconsistent position, Łukasiewicz has not glimpsed the dialethic extension. And indeed he proceeds to indicate some of the time-honoured arguments against it, against 'a contradiction existing in reality', arguments that are no better than those for LNC that he has decisively dealt with in the negative part of this paper. His claim '*there is known to us no single case of a contradiction existing in reality*' (p. 507)—a very weak claim compared to those philosophers such as Hilbert in the Kantian tradition are inclined to make, since he goes on to contend that '*one will never be able to assert with full definiteness that actual objects contain no contradictions* (p. 508)[114]—depends firstly upon a type distinction between *abstract objects*, such as those of logic and mathematics, and *actual objects*. The former, which may well prove 'contradictory upon closer examination' and sometimes have (as with the Russell class), do not 'represent reality' and are somehow blocked from affecting it.[115] The argument for this weak consistency claim is, even so, not decisive because it depends on a dubious perceived/inferred distinction and because it omits important (inferred) cases such as micro-objects. Łukasiewicz argues that 'it is impossible to suppose that we might meet a contradiction in *perception*', on the "literalist" ground that 'the negation which inheres in contradictions is not at all perceptible' (p. 507). There need however be no such (picturing) correspondence as that presupposed, between what is perceived and how a description of the perceived items would be expanded to lay bare the contradiction. Perception of an impossible object, such as some of the items depicted in Escher drawings, need not, and does not, involve anything like "perception of a negation". Against inferred contradictions in reality, Łukasiewicz appeals, without any argument, to the traditional assumption that 'one will always find ways and means eventually to dismiss inferred contradictions' (p. 508), an assumption most commonly supported by the ancient method of manufacturing distinctions which then conveniently reveal equivocations in the inference, which applies equally against contradictions "in" any objects. Much of the subsequent dialectic (in the introductions) will be concerned, in one way or another, with exposing the serious weaknesses of the traditional assumption and methods underlying it. 'As a consequence' of its logical worthlessness, LNC 'acquires a *practical-ethical* value, which is all the more important', Łukasiewicz tries to persuade us (p. 508), using a conspicuous non-sequitur.[116] His brief pragmatic defence of the LNC—a defence he tried initially somewhat tentatively to transfer to Aristotle: 'it appears that even Aristotle at least sensed the practical-ethical worth of the principle'—is based on the mistaken theme that '*the principle of contradiction is the sole weapon against error and falsehood*' (p. 508).[117] The argument is that otherwise—should 'joint assertion and denial ... be possible'[118]—'we could not defend other propositions against false or deceitful propositions' (p. 508). For example, the falsely accused would find no way of proving his innocence, for without LNC the false accusation could not be removed! But here we have the same fallacious shift of proof as Łukasiewicz observed in Aristotle's arguments. It is enough to meet Łukasiewicz's argument[119] concerning the falsely accused that LNC holds in a range of ordinary circumstances, not that it holds as regards, say, the Russell paradox. The fallacious shift is repeated in Łukasiewicz's attempt to attribute a pragmatic argument for LNC, now as the ultimate lynch-pin for science, to Aristotle, said to issue, when Aristotle 'felt the weakness of his argument' for LNC, in his presentation of it as 'a final *axiom*, an unassailable *dogma*' (p. 509). 'Denial of the principle of contradiction would have opened door and gate to every falsity and nipped the young blossoming science in the bud' (p. 509). Not so: it *might* (depending on the given connection of negation and falsehood) have admitted *some* falsehood—also however true—but this would not spread everywhere unless erroneous spread principles were also supplied. If the protection of science were indeed the ground, Aristotle need hardly have turned against alternative theories of paraconsistent inclination with such 'internal fervour'.

The practical-ethical value of LNC was not evident to Meinong and subsequent exponents of the theory of objects; nor was it evident to Vasil'év.

5.3. The Russian forerunners: Vasil'év and Bochvar

With his 'imaginary logics' Vasil'év has been seen as anticipating many-valued logic, and as a forerunner of paraconsistent logic, but perhaps he is more accurately placed as one of the founders (along with McColl and Lewis) of intensional logics. As Arruda[120] points out, 'in no place does

Vasil'év speak of any other truth-values than true and false', and the notion of dimension, on which the claim that his logics are many-valued is based, has to do not with the number of different truth-values involved, but 'the number of . . . different qualities of a judgement, which is not only two as in traditional logic' (Arruda, 1977, p.x). In the narrower, correct, sense then in which many-valued logics have an intended semantical or matrix theory in terms of many values (usually finitely many values), Vasil'év proto-logics are not many-valued; the matrix method, for instance, and the many values on which it is based, are not glimpsed. However in the misleading wider sense, used by Łukasiewicz, 1951, for example, in which intensional logics are accounted many-valued, because, for example, they are not two-valued, and may be given representation in terms of many values (commonly infinitely many values), Vasil'év's proto-logics are many-valued, for they certainly seem to violate extensional two-valued requirements. Arruda's claim that Vasil'év is 'a forerunner of paraconsistent logic' (in 1977 and elsewhere) is, as will become evident, not that much more substantial.

Vasil'év's logical endeavours consist largely of a reworking of traditional Aristotelian logic. The traditional square of opposition, for example, though valid for judgements about facts, is said to break down for judgements about concepts, where an amended theory is required. The traditional theory of syllogism is said to be inadequate to accommodate judgement of neither affirmative nor negative but *indifferent* quality, and to require generalizing to allow for higher dimensions of quality. But the new theory of syllogism is not worked out properly. Reworkings and attempts to extend traditional logical theory were by no means uncommon through the nineteenth and twentieth century, but most of them lack much interest for alternative logics. What distinguishes Vasil'év's efforts from more orthodox run-of-the-mill ventures are three features:

(1) His rejection of the "law of contradiction"—as distinct from the semantical LNC, and
(2) His introduction and treatment of indifferent judgements, of the form "S is P and not P", both set within the framework of
(3) His Imaginary (non-Aristotelian) Logic.

The model and inspiration for Imaginary Logic is the Imaginary (i.e. non-Euclidean) geometry of Lobatchevski (and others). As Aristotelian logic, like Euclidean geometry, concerns the real world, so Imaginary Logic, like Hyperbolic geometry, concerns *imaginary worlds*, i.e. worlds mentally created or imagined.[121] The idea of worlds other than the real or factual—which is the genesis of intensional logic—runs right through the history of thought.[122] What is different, and exciting, in Vasil'év, is the idea that the logical laws may vary in such worlds. Both Vasil'év's main arguments in support of the feasibility and rationality of his Imaginary Logic consider changes in basic logical laws. Firstly, Vasil'év argues, rather like Łukasiewicz, that it is impossible to support, in a non-question-begging way, the uniqueness and immutability of the basic traditional laws of logic (Identity, Sufficient Reason, Contradiction, and Excluded Middle). In a way strangely reminiscent of Łukasiewicz's remarkable suggestion that Aristotle was aware that the Principle of Non-Contradiction could be denied but kept this quiet for political reasons and in order to undercut opponents, so Vasil'év contends that Excluded Middle 'appeared in Aristotle's mind in order to refute his adversaries, and not for logical reasons' and that his attempt to

> prove [the] law starting from his own definition of a judgement, which always affirms or denies, which is always true or false, so the middle term would neither be true nor false, and would not represent a judgement... contains a *petitio principii*, since the law of excluded middle is already subsumed in the definition of judgement (1911, p. 33).

Secondly, Vasil'év argues that the system of Aristotelian logic involves several axioms, and systems resulting by eliminating or replacing one of these axioms remain logic, and worthy of the name, just as the hyperbolic geometry counts as geometry.

The change of basic law of especial paraconsistent interest is that of the Law of Contradiction: it is precisely this that the Imaginary Logic abandons. But a first serious problem in getting clear about Vasil'év's achievement lies in determining what the "law of contradiction", LC, that Imaginary Logic does without, is. Vasil'év states LC in two forms, first, "A is not not-A" and, second, 'an object cannot have a predicate which contradicts it (or attribute which is incompatible with it)'. The first makes at best dubious sense where A is a term or object,[123] and the contrast made between LC and the semantical LNC strongly suggests that A is what it appears to be, a statement or judgement. In this case, the first might be formalized by either $A \leftrightarrow \sim\sim A$, LC0, or $\sim(A \leftrightarrow \sim A)$, LC1, for some suitable equivalence, $\leftrightarrow$. However neither of these seems to be what Vasil'év has in mind. (While the rejection of LC1 might have an important place in the removal of paradoxes, e.g. semantical ones, there is no evidence Vasil'év had paradoxes in view; his "counter-argument" to LC is of a very different cast.) For the second, allegedly equivalent, formulation of LC, also terse to the point of obscurity, appears different. The 'it' cannot strictly refer back to the object: what seems to be intended is that object a cannot both have a predicate f and this (i.e. fa) be contradicted by $\sim fa$, by its negation, (i.e. demodalizing and generalizing) $\sim(A \,\&\, \sim A)$, LC2 say.[124] The formulation of semantical LNC Vasil'év gives, in contradistinction to LC, suggests that LC2 is a correct representation; for what it is to be compared with is the principle, 'One and the same judgement cannot be simultaneously true and false', i.e. $\sim\Diamond(TA \,\&\, FA)$, whence $\sim(A \,\&\, FA)$, demodalizing and applying a truth (but not falsity) scheme.

According to Vasil'év, although LC is integral to Aristotelian logic and inevitable in the real world (where we do not have negative sense perception),[125] it can be rejected because it is a material (empirical, or real world only) principle. Semantical LNC is entirely different, and cannot be rejected, 'because if someone eliminates this law, he will be making a confusion between truth and falsity, and consequently he is not thinking logically' (1912, p. 217). Even if Vasil'év can be made to look like a paraconsistent logician, he is certainly no dialethic logician; and it looks as if he could be forced into the awkward, and ultimately incoherent, position—the position Arruda effectively places him in—of adopting a nonclassical and perhaps paraconsistent logic in combination with a classical metalogic and concomitantly rejecting the Tarski schema, at least the falsity scheme, $FA \leftrightarrow \sim A$.[126]

What is the evidence then that Vasil'év's position is a paraconsistent one? So far, rather slight. Rejection of any or all of LC0, LC1, and LC2 in a logic hardly establishes its paraconsistent character, since contradictory pairs of statements may still induce triviality. A simple example illustrates the point.[127] In the 3-valued Łukasiewicz logic $Ł_3$ both LC1 and LC2 are rejected. But spread principles such as $A \rightarrow . \sim A \rightarrow B$ remain in place. Arruda tends to assume that Vasil'év's removal of LC is enough to establish him as a forerunner of paraconsistent logic: it is not. To assess Vasil'év's claim to such a distinguished position requires further investigation both of his Imaginary Logic—of which unfortunately (for the task at hand) there are few further details at the sentential level—and of his imaginary worlds.

Vasil'év took it for granted that in the real world we do not have negative sense-perceptions, represented in judgements of the form "S is not P", but that these judgements are only obtained from affirmative judgements of the form "S is P" by inference (specifically, with a further major premiss of the form 'N is incompatible with M'). In imaginary worlds, however, we may have direct negative sense-perceptions yielding judgements in the same way as affirmative ones in the real world,[128] *independently* of affirmative ones. Accordingly it is not ruled out that both occur at once. That is, where α is ground for the affirmative judgement "S is P" and α^* ground for the negative judgement "S is not P", both α and α^* may obtain. In this case the *indifferent* judgement "S is P and not P" is true. What exactly this means is obscure. Vasil'év allows substitutions of colour predicates for P. Presumably he has in view situations where the light or the glass (S) is, for example, green and not green, because positive sense perceptions inform us it is green and negative ones it is not green. More familiar examples such as the bent oar help make such scenarios quite intelligible and even a little tempting: the oar is bent, so visual sense-perception informs us (or seems to), and is also not bent, so tactual perception informs us. The fairly accessible claim "The oar is bent and not bent" might be taken as a working example of Vasil'év's "S is P and not P". It is not difficult to see that admission that "The oar is bent and not bent", and so that "The oar is bent" and "The oar is not bent" are both true, whilst such statements as "the oar is at the South Pole" are false, forces some important logical changes from tradition and, when pushed, leads to paraconsistent logic.

However in treating Vasil'év as a forerunner of paraconsistent logic one must tread with great care, first in what his rejection of LC amounts to. For although one might in a logical reconstruction take "S is P and not P" ("S is P†") to be equivalent to "S is P and S is not P", Vasil'év maintains that there are cases where "S is P†" is true but both "S is P" and "S is not P" are false. This suggests that the negation in "S is P†" is merely predicate negation, and this would make his position quite compatible with classical sentential logic as we have seen in the case of Meinong. But even if it is something more like sentential negation that is involved (and hence the conjunction is non-standard) there are problems. For since, as we have seen, the Tarski falsity scheme ($FA \leftrightarrow \sim A$) is abandoned, it is no longer clear what negation amounts to. (Similar problems beset da Costa's paraconsistent logics as we shall see.) But most importantly, there are reasons to suppose that by "S is P†" Vasil'év often means something quite consistent in anyone's language (see further the alternative interpretations of indifferent judgements that Vasil'év offers, discussed in Arruda 1977). For he sometimes reads "S is P and not P" alternatively as "S may be P", which is certainly not contradictory; and what seems to lie behind the alternative reading is the following *generality interpretation*; "S is P" means that S is always P, "S is not P" that S is never P, that in all cases S is not P, and "S is P†" accordingly means that S is sometimes P and sometimes not P, i.e. that S may be P (in a familiar enough construal of the modal, adopted for instance by Russell). All of Vasil'év's proposed interpretations of his Imaginary Logic are of a similar generality type (including those in terms of similarity and difference and in terms of relative and absolute negation); all can be accommodated, more or less, in traditional logical theory; and *none* call for paraconsistent revision.

But for the sake of the argument let us suppose that Vasil'év has shown that LC2 is not valid in Imaginary Logic because the "logic" includes indifferent judgements. Do we then obtain a paraconsistent theory? It seems that we should; for there are situations when both C and $\sim C$ hold though not everything does (since e.g., the purely affirmative and negative judgements which sustain C and $\sim C$ are false). So a logic based on the theory should reject the rule $C, \sim C \vdash B$; and if it contains a proper implication, $\rightarrow$, truth-preserving over the situations the theory encompasses, then the spread principles $C \rightarrow . \sim C \rightarrow B$ and $C \& \sim C \rightarrow B$ should both be rejected. But this is speculative, because Vasil'év did not get around to considering such issues (nor given the historical setting of his work can he reasonably have been expected to). What he did try to show is that it is legitimate to operate logically with indifferent judgements, and how this leads to a revised theory of syllogism. But the underpinning theory must lead to *predicate* paraconsistency in the shape of the rejection of S is P and not P ⊢ B, and so *should* lead, given that indifferent judgements retain the intended LC refuting features, to an underlying paraconsistent logic. In this tenuous sense Vasil'év can be accounted a forerunner of paraconsistent logic.

The revolutionary thrust of the work of 1910 was not pursued, or widely perceived, and had little real impact. Vasil'év wrote no more logic, but turned to other things; Meinong was progressively diverted into value theory and, in any case, he died in 1920 leaving no dedicated disciples in object theory; only Łukasiewicz had a long continuing career in logic, but the influence of Meinong on his work soon waned, and he concentrated on other non-paraconsistent topics and his evident philosophical talents were not greatly exercised.

Work on paraconsistent theory did not begin again until the late Forties, apart, it seems, from the logical ideas of Bochvar; and from a progressive shift towards paraconsistency in the thought of Wittgenstein (but that thought has only recently been much publicly disseminated, and its influence is again, like that of the work of the 1910s, *now*).

Whether or not Vasil'év introduced many-valued logics and applied them in the analysis of contradictions, the idea of doing so appeared in Russia; it was proposed (again) by Bochvar in 1939 in his paper 'On a three-valued logical calculus and its application to the analysis of contradictions'.[129] Again however, it is not at all clear that Bochvar was proposing a strongly paraconsistent treatment of the contradictions said to be "analysed", namely the paradoxes of Russell and Grelling. *That* depends crucially on how the third value is interpreted and whether it is designated. The issue is not straightforward because Bochvar appears to suggest various interpretations. In the main interpretation proposed, the third-value, N, is read as 'nonsense', and is not designated. And this is how the 3-valued matrices turn out, with the matrices for connectives &, ∨, ∼ the classical significance ones. Accordingly, the treatment of the paradoxes is a non-significance one, not a dialethic one.[130] But Bochvar also proposes to construe N as 'undecidable' in the sense of 'having *some* element of undecidability about it', and as 'paradoxical'.[131] The classical significance matrices that Bochvar arrives at are not however appropriate for these interpretations.[132]

Although Bochvar does not then adopt a genuinely paraconsistent approach, in particular does not seriously consider a theory in which the contradictions (i.e. logico-semantical paradoxes) may hold, still it may be that the logical system he presents is (weakly) paraconsistent. It is not. For the system contains theses of the form T(A & ∼A) ⊃ TB, i.e. there is an "external" implication connective conforming to detachment for which the spread principle A & ∼A → B holds.[133] For related reasons, that they contain spread principles, familiar many-valued systems, such as the systems of Łukasiewicz, are not paraconsistent logics.[134] Even so with the very weak "internal" subsystem of his larger system Bochvar perhaps offers us the first "logic of paradox" or "calculus of antinomies". Such logics were to be periodically rediscovered over the next 40 years, something that was (then) necessary, for they were continually being lost sight of.

As with Bochvar so with the Chinese logician Moh Shaw-Kwei: although steps towards paraconsistency are taken, a (genuinely) paraconsistent treatment of the paradoxes is not attained. Like Bochvar, Moh is mainly working the other, incompleteness (non-significance), side of the street, though some of his results are important for paraconsistent theory. For in his 'Logical paradoxes for many-valued systems' (of 1954) Moh extends Curry's paradox to apply against systems containing higher order rules of Absorption. The argument shows among other things that very many finite-valued logics, including all finite-valued Łukasiewicz logics, trivialize if an unrestricted abstraction axiom is added to them, and so are unsuitable for major paraconsistent purposes. Moh raises the important question as to 'whether we could develop the theory of sets with unrestricted abstraction from the system $L_{\aleph_0}$' (p. 39), i.e. from the infinite-valued Łukasiewicz logic? The answer is Yes, it has recently been shown, though the resulting *consistent* theory of sets has some serious drawbacks.[135] However, the logic is, once again, like finite-valued Łukasiewicz logics, not a paraconsistent one, since it has as a thesis the spread law $A \rightarrow . \sim A \rightarrow B$.[136] For similar reasons, Moh's final point, 'that Łukasiewicz's interpretation of the system $Ł_3$ is not satisfactory' and that the third value should be interpreted as *paradoxical*, where 'we define a paradoxical proposition as one equivalent to its own negation' (p. 40), does not lead to a paraconsistent system, though the interpretational idea can and was later to do so when designated values were appropriately adjusted to allow paradoxical assertions as designated. Moh had part of the right idea, that paradox-generating assertions are paradoxical, but with $Ł_3$ had the wrong logical framework for paraconsistent treatment of the paradoxes.

5.4. *An isolated figure in contemporary history: Wittgenstein*

Wittgenstein's position[137] changes substantially in the course of his life-time. As regards his treatment of matters of paraconsistent concern such as negation, contradiction and paradoxes, three distinct periods have to be distinguished (since there are decisive differences in his positions in these phases): *early*, *transitional* and *late*. In his early, Tractarian, period he was committed to a very restrictive classical logical theory which entirely excluded paraconsistent approaches.[138] Though in his transitional phase Wittgenstein moved outside the confines of this narrow position, and was already prepared to concede that contradictions could (to some extent)[139] be allowed to arise in a theory (cf. 1975, p. 345), he still thought that paradoxes, logical antinomies in particular, *need* to be resolved, and could be resolved by removing ambiguities and equivocations through analysis of meanings of the expressions used in their formulation.

> ... the antinomies did not arise in the calculus but in our ordinary language, precisely because we use words ambiguously. Hence the resolution of the antinomies consists in replacing the hazy way of expressing ourselves by a precise one (by recalling the real meaning of our words). Thus the antinomies vanish by means of an *analysis*, not by means of a *proof*... A proof cannot dispel the fog, [the] *unclarity*. (McGuinness 1979, p. 122).

That was in 1930. But by 1939 his position had changed markedly. The requirement of analysis, and that the antinomies be excluded, are both abandoned. He implies that contradictions like the Liar don't matter; 'it is of no use; it is just a useless language-game' (Wittgenstein 1976, p. 207). And similarly: 'contradictions. Whether we're to say they have a meaning I don't know—but it's clear they don't have a use' (Wittgenstein 1976, p. 223, causing tension for the "meaning is use" equation). The themes that the Liar and other paradoxes are unusable, and that since they are unusable they can stand without removal and without harm in a language-game or calculus occur frequently (e.g. Wittgenstein 1964, p. 51). There is no need for the theory of types then, for with the Liar 'nothing has been done wrong' (Wittgenstein 1976, p. 207).[140] Wittgenstein indeed sometimes rejects distinction-of-meaning ways out of the paradoxes (1964, p. 102) and the type theory approach of the *Tractatus* (1964, p. 182). Sentences violating type theory, such as 'the class of liars is not a liar' are 'proper sentences' and 'there are language-game(s) with [such] sentence(s) too' (1964, p. 182).

Wittgenstein even asserts sometimes that he makes a statement with the Liar sentence, for instance (1976, p. 207), though two pages later he offers instead the option: either 'you may say that it's not a statement. Or you may say that it *is* a statement, but a useless one' (1976, p. 209). It makes a *considerable* logical difference—much more than Wittgenstein seems to realize—which of these choices is made. The first option that the sentences do not yield statements, leads to some sort of significance theory, especially given that Wittgenstein equates making a statement or having content with making sense (e.g. 1964, p. 171), which filters out paradox-generating sentences as statement-incapable or as not making sense. The second option leads however to either an incompleteness (many-valued) approach or, very differently, to an inconsistency (paraconsistency) approach. (While there are limited intermappings between these three approaches, they are *quite* distinct.)

Insofar as he mostly adopts the first option, Wittgenstein is not an exponent of paraconsistent logic or in any way prepared to sanction true contradictions (though he is clearly aware that motion *could* be said to involve a contradiction). Thus, for the most part, Wittgenstein took it that there are several sentences, which do not yield propositions, both paradoxes such as '"heterological" is heterological' (1964, p. 178; cf. also 1964, p. 102) and the Liar sentence (1964, pp. 130–131) and quasi-paradoxes such as analogous of the 'Gödel proposition' (1964, pp. 176–7). Such sentences have 'the *form* of a proposition', 'a propositional-pattern', but they do not express propositions, and Wittgenstein often says that they lack, or don't make, *sense* (1964, p. 177, and also pp. 117–18). Thus Wittgenstein wants to explain away paradoxes like that of heterologicality to give only a different, and benign, sense to 'The contradiction is true', in place of the obvious one[141]: namely, that a certain proposition is a contradiction and that that proposition is true. According to Wittgenstein what the expression means is: 'this really is a contradiction, and so you cannot use the word "'h'" as an argument in '$\xi \in$ h'.... "'*h*'" is one of those words which do not yield a proposition when inserted into '$\xi \in h$'.' (1964, p. 178) Indeed only 3 pages after he offers an option as to what to say, Wittgenstein asserts that 'in a sense [p & ~p] is bosh' (1976, p. 213).

A double inconsistency emerges in Wittgenstein's later position (which makes *it* a fit object for paraconsistent investigation). Firstly, if paradox-generating sentences do not make sense, then, since on the face of it they do make sense, some sort of meaning analysis is called for, at least to explain why appearances are misleading. But this contradicts the claim that *no* meaning analysis is required. (It does not conflict with the assertion that type theory is not needed, for this only offers a rather special meaning analysis.) In fact the theme that certain contradictions[142] do not express propositions, and so despite appearances are neither true nor false, receives little of the explanation or support it obviously requires in Wittgenstein's work. Secondly, Wittgenstein is simply inconsistent as to whether contradictions such as the Liar and Russell paradoxes do make sense. For example, as well as saying that they don't make sense, or at least allowing that one can say this, he also says: 'There is *one* mistake to avoid: one thinks that a contradiction *must* be senseless: that is to say, if e.g. we use the signs 'p', '~', '.' consistently, then 'p.~p' cannot say anything'. (1964, p. 171). Similarly he both allows that language-games can contain contradictions and says that 'a language-game can lose its sense through a contradiction, can lose the character of a language-game' (1984, p. 103).

There are then competing inconsistent strands in Wittgenstein's later work. And Wittgenstein has not really decided which option to take, though *insofar as he usually tends to fall back on a nonsignificance approach*, to the paradoxes at least, *his later position is definitely not a paraconsistent one.* It is worth pursuing however the strand that is beginning to emerge in Wittgenstein's later work which does have much in common with a paraconsistent approach. According to this strand, contradictions, including paradoxes, do, as we have seen, make sense; they are, or express, propositions. They can occur in and do not destroy language-games. They need not get one into any trouble (1976, p. 212). But in the on-going dialogue Wittgenstein is soon forced, by Turing's example of a bridge falling owing to a calculation in an inconsistent logic, to say, inconsistently with the no-trouble theme, that a contradiction may lead into trouble but that it is not more likely to do so than anything else (1976, p. 219).

Wittgenstein gives *no* examples of paraconsistent logics or calculi. He seems to think that no such examples are needed: 'our task is, not to discover calculi, but to describe the *present* situation' (1964, p. 104). While this is undoubtedly *part* of the business, our task is by no means confined to this. Moreover his remark leads to conflict with his insistence elsewhere on an adequate diet of examples, including revealing games. The examples Wittgenstein does select of inconsistent calculi *are* problematic—as paraconsistent theories are not—because they are trivial. An example he frequently considers is 'Frege's calculus, contradiction and all. But the contradiction is not presented as a disease' (e.g. 1964, p. 104). The trouble with calculating with this calculus is that it will lead to anything at all. It would lead to calculations under which, if applied, bridges *would* collapse, since coefficients could be arbitrarily, and so unsatisfactorily, determined. There is no control, unless implicit restrictions on what is done with it are somewhere or somehow imposed. Wittgenstein appears in fact to be operating with the idea that implicit restrictions are in force and that though contradictions *can* trivialize (1976, p. 224) they do not trivialize Frege's inconsistent calculus. There is considerable evidence for these claims. Firstly, it isn't true that with Frege's logic 'people went through doors into places from which they could go any damn where ... if they did this Frege's logic would be no good, would provide no guide. But it does provide a guide. People don't get into these troubles' (1976, p. 228). The reason is that people like Frege are controlled by 'normal rules of logic'; Frege was 'led also by our normal use of words' and so stayed out of trouble. Unfortunately the "normal rules" suggested resemble those of type theory, which is considerably removed from ordinary usage (see Goddard and Routley 1973); and worse, the rules would delete the contradiction which is supposed, in some sense, to hold, though it remains pretty inaccessible. Secondly, Wittgenstein takes it that Frege's calculus is a usable calculus, but 'if we allow contradictions in such a way that *anything* follows, then we would no longer get a calculus, or we'd get a useless thing resembling a calculus' (1976, p. 243, cf. also p. 228). Thirdly, he says (what is false) that 'no one draws conclusions from the Liar' (1964, p. 170; cf. 1976, p. 213). He considers the situation where a contradiction, like Russell's, has been found but we are 'not excited about it and had settled e.g. that no conclusions were to be drawn from it' (1964, p. 170). Such a stance *may* be alright, depending on how the restriction is applied. Mostly the restrictions Wittgenstein suggests are *quite inadequate* to make a calculus workable. For instance, drawing conclusions from certain steps that lead to contradictions has also to be excluded to avoid triviality; for explicit contradictions can normally be bypassed. Wittgenstein is made keenly aware of this problem by Turing, who pointed out that the rule not to 'draw any conclusions from a contradiction' is not 'enough. For . . . one could get around it and get any conclusion which one liked without actually going through the contradiction' (1976, p. 220). Subsequently Wittgenstein was worried by the problem of 'how to avoid *going through* the contradiction unawares' (1976, p. 227); but he offered no solution to it.

Much the same set of points applies to the other main example in Wittgenstein's meagre diet, division by (n − n) (perhaps best developed in 1964, p. 168–9). It too trivializes *unless* classical operations are restricted, unless it is observed that they are only valid for a given region. So it is also with naive set theory and inconsistent arithmetic: 'if a contradiction were now actually found in arithmetic—that would prove that an arithmetic with *such* a contradiction in it could render very good service' (1964, p. 181). But what would want modification is *not*, as Wittgenstein suggests 'our concept of the certainty required',[143] *but* the operations that could be applied in the vicinity of the contradiction *if* useability is not to be sacrificed; that is, the application of classical logic must be restricted.

Yet Wittgenstein, though he strictly formulated no paraconsistent logics as calculi (and apparently lacked any clear appreciation of non-classical logic, except for, what is equally unsuitable paraconsistently, intuitionistic theory), made considerable allowance for 'investigation of calculi containing contradictions' and predicted a time when 'people will actually be proud of having emancipated themselves even from consistency' (1975, p. 332; cf. also 1964, p. 312, 376). He outlines how a different attitude to contradictions could occur, where people *want* to produce a contradiction, a lot of people try, and at least *one* person succeeds (1964, p. 105). Wittgenstein does not himself succeed in producing a 'plausible purpose' for this behaviour: one (metaphysical) purpose would be to show, not as Wittgenstein suggests that 'everything in this world is uncertain', but that the world is inconsistent. Such people will 'be glad to lead their lives in the *neighbourhood* of a contradiction' (1964 p. 105), so to speak. He rightly rejects the ideas that a contradiction automatically destroys a calculus (1964, p. 170) or a livelihood; for contradictions can be 'sealed off' (cf. 1964, p. 104), and so allowed to stand (e.g. 1964, p. 168).

While his own efforts at furnishing interesting inconsistent calculi are more suggestive than satisfactory, he does include, and appreciate, some of the basic requirements for paraconsistent theories, e.g. as we have seen, rejection of the spread law $A \& \sim A \vdash B$ (1976, p. 209). But what he proposes instead is the ineffective and inadequate rule that no conclusions be drawn from a contradiction. The considerations he repeatedly adduces in favour of such a rule are

(I) that 'there is always time to deal with a contradiction when we get to it' (1976, p. 209; 1976, p. 210; 1964, p. 105; 1975, pp. 345–6)—as if calculi were always dynamic systems when often they are static—and

(II) 'when we get to it shouldn't we simply say, "This is no use—and we won't draw any conclusions from it"?' (1976, p. 209), but it can stand (e.g. 1975, pp. 345–6).

In like vein Wittgenstein wants to say 'something like, "Is it usefulness you are out for in your calculus? In that case you do not get any contradiction. And it you aren't out for usefulness—then it doesn't matter if you do get one"' (1964, p. 104). The approach through usefulness is wrong, in *several* respects. Plainly usefulness is not necessary for consistency, since many useless games are consistent. Nor is usefulness a guarantee of consistency. An inconsistent theory may be of considerable use, even in describing the world, as the infinitesimal calculus certainly seems to have been, and quantum theory (which is likely inconsistent) is. Similarly such theories may be used in making predictions, even if the explicit contradictions in them are not (cf. 1964, p. 52). Furthermore, it may matter if a contradiction is encountered, even where the objective is not usefulness: there are other objectives, such as (substantial) non-triviality, elegance, etc., that an inconsistent calculus may fail. Use, usefulness, useability are not the uniquely important tests Wittgenstein (cf. 1964, p. 105) and the pragmatists consider them to be.

Nor can contradictions simply be dealt with as they arise. By the time a paradox is found it may be too late, as with Frege's logic; the cancer may be beyond treatment. Characteristically contradictions, like cancers, do spread, even if their spread can often be, non-classically, contained. Wittgenstein's own proposal for a containment, stopping, is inadequate as we have seen: so is the basis for this proposal. He is obliged to say that paradoxes like the Liar are useless and lack an application, that 'Russell's "~f(f)" lacks above all application, and hence meaning' (1964 p. 166). For if such contradictions were technically useful we should want to do things with them, especially to draw out consequences. And we do. In fact Russell's paradox may have extremely important applications, e.g. if Arruda's conjecture is correct, in showing *within* set theory without special axioms of infinity that some numbers are inaccessible. Similarly semantical paradoxes and their analogues have important roles in establishing features of theories, e.g. of semantically closed arithmetic, in determining the viability of solutions, e.g. to the paradoxes themselves, and in obtaining limitative results, as to what can be proved in systems, what known, etc.

Although Wittgenstein is certain that the paradoxes are useless, he is quite uncertain, as remarked, as to what to say about their semantical status. He shrank from admitting that they could be true, and thus from a strongly paraconsistent position. He oscillated between sometimes allowing Russell's paradox a propositional role, more often rejecting it as not a proposition (e.g. 1964, p. 166), and occasionally embarking on the hopeless task of trying to find it an intermediate niche. Thus for instance, he considered how Russell's contradiction 'could be conceived as something supra-propositional, sometimes that towers above propositions and looks in both directions like a Janus head' (1964, p. 131). The unnecessary vacillation is in large measure because he failed to see that paradoxes could be sealed off logically—indeed in many ways—*without* use of devices like type theory which undermine their propositional status. If he had, then he could easily have adopted what in his later work he reaches for but never attains in a stable way, a weakly paraconsistent position, which allows for a variety of nontrivial theories (or language-games) distinct from the true theory (the true-false "game").

In this way he would have realized, in fair part, his stated aim of altering *attitudes* to contradiction and inconsistency, at least as regards the following:

(a) A calculus with a contradiction in it is in some way *essentially* defective.
(b) When a contradiction comes to light, some sort of remedial action is rationally demanded of us; we cannot coherently just let the thing be.
(c) There is such a thing as the *correct* logic, or set theory; and the paradoxes show that we have not found it. The problem of "solving" the paradoxes is a determinate one; it is that of finding the mistakes in the assumptions which lead to them.
(d) For any particular branch of mathematics, it is desirable that it be set up in such a way that contradictions can be avoided *mechanically*; that is, so that a slavish, unintelligent and totally aimless application of the rules of inference can never lead to any difficulty.
(e) Consistency-proofs are needed—or at least desirable. A system for which such a proof is missing, or unobtainable, is somehow insecure. 'Only the proof of consistency shows me that I can rely on the calculus' [1964 p. 107.]
(f) A hidden contradiction is just as bad as a revealed one. A system containing such a contradiction is totally spoiled by it. The contradiction is, as it were, a pervasive, general sickness of the system (Wright, 1980, pp. 296–7).

It has been claimed that conventionalism renders Wittgenstein's otherwise rather intractable position on contradiction coherent. For then mathematics and logic become games or like games, the rules of which are conventionally chosen, and games with inconsistent rules can still be interesting to play.[144] In terms of [the game] analogy, Wittgenstein's questioning of

some of the ordinary attitudes to contradictions which we listed are extremely easy to understand. There is, for example, no reason why a game with a contradiction, or some other flaw, in the rules *must* be regarded as essentially defective; nor is there any reason to insist that if the defect comes to light, some sort of remedial action is demanded of us (Wright, 1980, pp. 299–300).

But firstly, this sort of conventionalism (though it blends smoothly enough with paraconsistent positions) is neither necessary nor sufficient. It is not necessary because reconstruction of Wittgenstein's position as weakly paraconsistent will do as well. And it is not sufficient because this conventionalistic construal does not really avoid, or settle, the vexed issue of the semantical status of paradox-generating statements. Secondly, as Goldstein[145] argues, (1) mathematics and logic are not simply games—similarly Wright (p. 303) '... the assimilation of mathematics to a game ... seems a travesty'—and (2) Wittgenstein did not believe them to be such (1976, pp. 142–3; 1964, p. 163; 1974, pp. 289–95). As to (1), there are familiar applications of mathematics in engineering, architecture, areodynamics, etc.; the rules applied in the design of a bridge or a space shuttle are not a matter of conventional choice. 'Logic too, insofar as it is a codification of valid inference, cannot sustain a free choice of rules, for the rules we adopt must faithfully reflect our inferential practices' (Goldstein p. 2). Wittgenstein is said to assume the same (in 1978, p. 257, p. 397, and pp. 303–353): these practices,[146] which involve the making of inferences which are truth-preserving, belong to what Wittgenstein calls the 'true-false game' (in McGuinness 1979, p. 124).

Simple conventionalism alone will not render Wittgenstein's position coherent. What does *help* is Wittgenstein's larger picture of logic and mathematics as comprising (like a city of such suburbs) very many different sorts of games or calculi. In some of these, which may bear only a family resemblance to the true-false game, contradictions are not forbidden. This type of *many logics* or *many worlds* view in no way requires conventionalism however. Such a view has been incorporated in relevant (weakly paraconsistent) and paraconsistent positions. Such a view, open, with but little refinement of his position, to Wittgenstein, is moreover correct, inasmuch as there are very many different, and different sorts of, logics and "worlds",[147] including inconsistent ones.

A dialethic position supports most of the themes (quoted above) with respect to which Wittgenstein wants to change attitudes, again without appeal to conventionalism or the connected assimilation of mathematics to games. Commentators on Wittgenstein have claimed however that if the assimilation is not made, if mathematics does for example genuinely describe structures, then our "ordinary" 'attitudes to contradictions' are soundly based; thus, for example Wright (1980 p. 298) whom we shall take as our target. The problem with respect to inconsistent systems is, Wright alleges, twofold, as regards (1) their applicability and (2) their truth. As to (1), 'if a system is inconsistent, then the inferences permitted within it will not in general be truth-preserving when applied to contingent contexts' (p. 298). This is subsequently transformed (e.g. p. 303, p. 310) into the theme that inconsistent systems permit the derivation of false conclusions from true premisses. The application of relevant logics to inconsistent theories shows that this theme is mistaken. As the semantical analysis of relevant logics (developed e.g. in RLR) reveals, implication, and the grounds for derivation, remain truth-preserving even in inconsistent situations and theories.

As to (2), 'if we had thought of it as a systematic description of some abstract conceptual structure, then again, not all of its theorems can be regarded as correct descriptions of the intended structure' (p. 298). In fact this need not concern truth; for the structures correctly described may not be actual, but, e.g., purely hypothetical. But, in any case, it is again refuted by relevant theories; strong paraconsistency is not called for. It is enough that there are non-trivial inconsistent situations or structures, which inconsistent models will provide.[148]

Wright goes on to claim, in the ordinary way which takes no account of paraconsistency, that

> if ... the essential business of pure mathematical systems is to describe determinate conceptual structures and ... the notion of truth for ... theorems corresponds accordingly ... then it seems inescapable that contradiction *is* a total disaster, and demands remedy if there is to be any pure mathematics for the structure in question.

The work with relevant theories shows that that is not at all obvious and indeed simply assumes that even weaker paraconsistency is excluded. Why so?

> For not only does an inconsistent system not truly describe the intended structure; it does not *truly* describe anything at all (p. 298).

Firstly, the way this is put confuses strong and weak paraconsistency. To see this in sharp focus, replace 'truly' (twice) by 'adequately' in the preceding quote. Then the resulting claim is false. An inconsistent relevant theory may adequately describe the intended structure; soundness, and even completeness, theorems may be forthcoming. It is simply a mistake, a common enough but *serious* mistake, that such theories or systems do not describe anything at all. What they do not describe is a consistent structure; but an inconsistent structure is not nothing. But, secondly, even if the term 'truly' is taken literally (in the way that italicization of the second occurrence suggests), then the argument is not only incomplete, since it dogmatically suppresses the assumption that the real is consistent, but unsound since this assumption is false, as we will argue at greater length in the introduction to Part Four of this book.

Wright's contention 'that inconsistent systems are at best *useless*; that they can have no practical application' (p. 303) falls with his earlier claims. For it depends again on the mistaken assumptions that the rules of such systems will not be truth preserving, and that real *choice* of *right* theorems among the theorems yielded in the system will be required. He does throw in, in effect, the further point that the theorems of an inconsistent theory cannot be true under interpretation in an empirical domain. Whether this is so turns on whether empiricalness implies consistency or not.[149] If it does not, as many dialecticians have thought, the point fails.

Wittgenstein is an isolated figure in the development of paraconsistent thought. Although his work had, inevitably, a historical context, especially in fact the matter of the logical paradoxes and their repercussions, contemporaneous developments in the paraconsistent enterprise, and earlier ones (except for German idealism), appear to have had little or no influence on his work. Nor did he have much immediate impact on the paraconsistent enterprise, which evolved largely independently of his work. Only recently has he become an almost-establishment figure, to appeal to—in a rather qualified way—on paraconsistent themes, or, as often, someone to make good, in previously written-off parts of his "theory", by applying paraconsistent results.

As we have seen Wittgenstein did not seriously attempt to specify what a formal logic suitable for inconsistent situations and theories would be like, this was left to his contemporary, Jaśkowski.[150]

5.5. The Polish continuation: Jaśkowski

Jaśkowski introduces his fundamental (1948) paper by repeating some of the historical points Łukasiewicz had made, especially those concerning 'convincing reasoning which nevertheless yield[ed] two contradictory conclusions' (p. 143). Jaśkowski goes on to mention the logico-semantical paradoxes, and the heavy price exacted by restrictions that (appear to) restore consistency. He also remarks on how the levels-of-language theory (as it is now called)

> is at variance with the natural striving synthetically to formulate all the truths we know in a single language, and thus renders a synthesis of our knowledge more difficult (p. 144).

But that is all. He does *not* explicitly propose simple acceptance of the paradoxes as truths, established by sound arguments, and earlier indicates that theories admitting these paradoxes cannot now be considered as correct.

However subsequently he *does* consider representation of the Liar antinomy in the paraconsistent logic he arrives at, indicates that other paradoxes such as Russell's can be similarly treated, and remarks that ordinary procedures leading from inconsistency to triviality fail. He goes on to the important observation that the apparent breakdown of such proof procedures does not establish the non-triviality of inconsistent systems representing paradoxes such as the Liar. In fact he sketches in barest outline, probably for the first time for inconsistent theories, the problem of proof of non-triviality.

Jaśkowski also dismisses vagueness of terms, which 'can result in a contradiction of sentences' 'in every-day usage'; for with increased precision the inconsistency is removed. Finally Jaśkowski mentions what again he considers as a transient feature, inconsistencies in working hypotheses at given stages in the evolution of sciences such as physics. This too he pushes back, to the following:

> ... In some cases we have to do with a system of hypotheses which, if subjected to a too consistent analysis, would result in a contradiction between themselves or with a certain accepted law, but which we use in a way that is restricted so as not to yield a self-evident falsehood (p. 144).

That is to put the matter almost as the less hostile among the enemies of paraconsistency might. It is not highly sympathetic to paraconsistency; it is at best a very weak paraconsistent position and definitely not a dialethic position: the possibilities have not been seen.

Nonetheless Jaśkowski formulates, again apparently for the first time, the problem of determining the class of paraconsistent logics, at the sentential level. He clearly distinguishes two classically-conflated properties of systems; being *contradictory* (i.e. having theses A and ~A which contradict one another) and being *over-complete* (i.e. trivial). He then presents

> the problem of the logic of contradictory systems ... in the following manner; the task is to find a system of the sentential calculus which: 1) when applied to the contradictory systems would not always entail their overcompleteness, (2) would be rich enough to enable practical inference, (3) would have an intuitive justification. Obviously these conditions do not univocally determine the solutions since they may be satisfied in varying degrees, the satisfaction of condition (3) being rather difficult to appraise objectively (p. 145).

Much the same applies to condition (2), which is one reason why these conditions are not included in the definition of paraconsistency; another is their technical intractability (as well as inexactitude). In his subsequent practice Jaśkowski entirely ignores requirements (2) and (3). So begins the formal investigation of contradictory, or paraconsistent, systems.

Among "solutions" Jaśkowski mentions minimal logic, but concentrates upon what he calls *discursive logics.* Though minimal logic satisfies the letter of paraconsistency law, it violates the spirit, in virtue of the minimal thesis A→. ~A→~B. It is not evident, and Jaśkowski makes no effort to show, that minimal logic meets requirements (2) and (3). In fact these requirements are forgotten in all that Jaśkowski goes on to; so in fact Jaśkowski *is* concerned with paraconsistent logics as a whole. He does not, however, get far with their classification. Many-valued logics are tentatively set aside as not providing solutions. But, as it has turned out, it is only certain many-valued logics (in particular functionally complete ones) that do not afford solutions. Finite-valued relevant logics, for instance, do.[151] Jaśkowski really obtains one rather limited class of solutions to his problem—discursive logics—from among the many rich types there are, and almost all attention is rivetted on a single system D2 obtained by translation from modal system S5.

However the underlying ideas in reaching D2 are both interesting and more general, and link discursive logics not only to the long tradition of philosophical pluralism,[152] but also to other classes of logics very recently discerned, e.g. the non-monotonic logics of interest to computer scientists.[153] Discursive logic is intended as a formalization of a logic of *discourse*,[154] where different participants, e.g. in discourse, advance theses or pool opinions, all these being included as assertions in a single system. Jaśkowski's definition is vague; 'Let such a system which cannot be said to include theses that express opinions in agreement with one another, be termed a *discursive system*'. What Jaśkowski goes on to say helps, however, to clarify matters:

> To bring out the nature of the theses in such a system it would be proper to precede each thesis by the reservation; "in accordance with the opinion of one of the participants in the discourse" or "for a certain admissible meaning of the terms used". Hence the joining of a thesis to a discursive system has a different intuitive meaning than has assertion in an ordinary system. *Discursive assertion* includes an implicit reservation of the kind specified above, which

Jaśkowski adds in a remarkable, crucial, and unjustified, slide

> —out of the functions so far introduced in this paper—has its equivalent in possibility *Pos.* Accordingly, if a thesis A is recorded in a discursive system, its intuitive sense ought to be interpreted as if it were preceded by the symbol *Pos*, that is the sense: "it is possible that" (p. 147).

Jaśkowski's introduction supplies *one* basic component of discursive logic; or, put differently, a *first requirement that a logic* L, with an $\& - \vee - \sim$ sublogic and containing a functor $\Diamond$, which can be read "someone maintains that", *yields a discursive logic* DL, namely

DL1. A is a thesis of DL iff $\Diamond$A is a thesis of underlying logic L.[155] A logic DL so yielded, may lack fundamental logical operations, such as, to focus on the examples Jaśkowski takes as decisive, implication and equivalence functors. Hence, for example, the second requirement

DL2. There is definable in L a functor $\rightarrow_D$, of *discursive implication*, which demonstrably satisfies the requirements in DL for being an implication; in particular, it is closed under the *modus ponens* rule, i.e. A, $A \rightarrow_D B \twoheadrightarrow B$, and also maybe has theses such as $A \rightarrow_D A$.

The third requirement, for equivalence, in place of implication, is similar:

DL3. There is definable in L a further functor $\leftrightarrow_D$, of *discursive equivalence*, which demonstrably satisfies in DL at least the following conditions; A, $A \leftrightarrow_D B \twoheadrightarrow B$ and B, $A \leftrightarrow_D B \twoheadrightarrow A$.

DL is characterized as the logic with primitive connective set $\{\&, \vee, \sim, \rightarrow_D, \leftrightarrow_D\}$ i.e. where A and B are wff so are $\sim A$, $(A \& B)$, $(A \vee B)$, $(A \rightarrow_D B)$, $(A \leftrightarrow_D B)$, etc. DL tells us which wff are theses.

There are modal logics L whose own implication and equivalence functors satisfy requirements DL2 and DL3 and which accordingly furnish discursive logics, as so far defined, through DL. One such logic is Łukasiewicz's Ł-modal system (of 1951), for which it is provable, for instance, that $\Diamond(A \supset B) \& \Diamond A \supset . \Diamond B$, so guaranteeing *modus ponens* in DŁ. However DŁ does *not* satisfy the original motivating considerations in terms of which Jaśkowski introduced discursive systems in the first place. For DŁ is not paraconsistent. It will trivialize inconsistent additions, since it contains the spread law $\Diamond A \supset (\Diamond \sim A \supset \Diamond B)$.

A final requirement on a Jaśkowskian discursive logic is then

DL4. DL is paraconsistent.

Thus DŁ is not a Jaśkowskian discursive logic. In fact Jaśkowski does not strictly show that his main and only serious candidate system D2 is paraconsistent though he clearly assumes that it is, where he explains (p. 153) that even if material implication is added to the system the rule γ of Material Detachment is not valid.[156]

Where L is a modal logic, the Jaśkowskian discursive logic DL based on, or associated with, L is determined through the following definitions of $\rightarrow_D$ and $\leftrightarrow_D$:

Defn. 1. $A \rightarrow_D B =_{Df} \Diamond A \supset B$; and

Defn. 2. $A \leftrightarrow_D B =_{Df} (\Diamond A \supset B) \& (\Diamond B \supset \Diamond A)$.

Where L is system S5, DL is Jaśkowski's system D2. These definitions are not the only ones that serve to determine Jaśkowskian discursive logics; Jaśkowski mentions, but sets aside, another definition of $\rightarrow_D$,[157] and it is easy to see that there are alternatives to the asymmetrical definition of $\leftrightarrow_D$.

Apart from being the first explicitly paraconsistent logic, there is another important trend D2 helped to set, namely the rejection of Adjunction.

Jaśkowski puts it in terms of rejection of the wff $A \rightarrow_D . B \rightarrow_D . A \& B$, but the rejection, and the ground for it, is more far-reaching, and tells against the Adjunction Rule; A, B$\twoheadrightarrow$A & B:

> ... from the fact that a thesis A and a thesis B have been advanced in a discourse it does not follow that the thesis A & B has been advanced, because it may happen that A and B have been advanced by different persons. And from the formal point of view, from the fact that A is possible and B is possible it does not follow that A and B are possible simultaneously (p. 154).

Though the rejection of Adjunction leaves some strange gaps, (e.g. the rejection of equivalence decomposition, $(A \leftrightarrow_D B) \rightarrow_D (A \rightarrow_D B) \& (B \rightarrow_D A))$, it is essential to Jaśkowski's discursive logics. For it requires only a fairly minimal modal logic L to prove in DL both LNC, $\sim(A \& \sim A)$, and Conjunctive Spread, $A \& \sim A \rightarrow_D B$. In virture of LNC, discursive logics violate da Costa's conditions upon inconsistent logics, which are widely (but erroneously) insisted upon, namely that an inconsistent logic must reject LNC. But the real trouble with DL lies not with LNC but with the irrelevant Spread which forces the abandonment of Adjunction. Were Adjunction to hold, the sequence,

$$A, \sim A \twoheadrightarrow A \& \sim A$$
$$\twoheadrightarrow B$$

would show triviality of any contradictory extension of DL.

Not only then are Jaśkowski's discursive logics not Adjunctive; further, they are irrelevant, because, for example, of Conjunctive Spread. And these vices are connected. Given a choice of rejecting one or other of Adjunction and Conjunctive Spread to avoid paradoxes and catastrophic spread from an inconsistency, the rejection of Adjunction is the wrong choice.[158] For Adjunction itself spreads nothing, but merely assembles, conjoins, data already supplied. That is also why its rejection is unwarranted for leading intended applications. Consider, for instance, a paradox such as Russell's or the Liar, and let p_0 be the paradox-producing statement (e.g. $R \in R$). Then by the paradox arguments, and presumably in discursive logic both p_0 and $\sim p_0$. In *English* we can adjoin them (likewise in the metatheory we can conjoin them)—and reasonably enough, since both hold, both are intended to be true—but discursively we are not permitted to go on to $p_0 \& \sim p_0$. It is not, or not just, that & has departed from its normal interpretation as a conjunction, from meaning *and*,[159] but that discursive logic is not the right setting for capturing the way we do reason concerning the paradoxes. For what we say given p_0 and also $\sim p_0$ is: Yes, $p_0 \& \sim p_0$, but *that* is not a ground for going on to anything at all.

But doesn't the discursive interpretation compel the rejection of Adjunction? If it did, that would show that discursive logics offer an unsatisfactory approach to some of the problems they have been presented as handling, e.g. reasoning concerning paradoxes, and hence that discursive logics are only a quite proper class of paraconsistent logics. But in fact one can go either way, in discursive logic itself, on whether Adjunction holds. Even when DL is (inadequately) translated from L through a modal possibility functor, matters could go either way. For in system DŁ, Adjunction holds (and $A \rightarrow'_D . B \rightarrow'_D . A \& B$ is provable). Indeed Łukasiewicz is quite adamant that the ordinarily rejected principle $\Diamond A \& \Diamond B \rightarrow . \Diamond(A \& B)$ is nonetheless correct, and devotes some space, in his defence of the Ł modal system, to rebutting the usual counterexamples to it.[160] Appeal to the intended interpretation of the translation functor—which is not really ◇[161]—can also support Adjunction instead of telling against it. It depends on whether the pair A, B conjoined in Adjunction are seen as separately supplied and requiring joint validation in a single underlying framework, e.g. under one of the systems, in the opinion of one of the participants—in which case Adjunction does fail—or, though supplied by different frameworks, do not require joint validation in some one framework. Put differently, under the second construal, closure under certain logical operations is built in, in particular closure under Adjunction. In fact Jaśkowski builds in closure under *modus ponens*—even though opinions are commonly not closed under entailment, so there is a (first) perspective where *modus ponens* fails—but does not notice that the same thing can be done for Adjunction, that there is a second perspective where Adjunction holds.

Though the generation of discursive logics from modal logics is a clever and elegant formal idea, the intended interpretation does not sustain the system Jaśkowski studies or anything much like it. For logical possibility as encapsulated in S5 does not bear a good logical resemblance to the discursive operator. For instance, it is extremely doubtful that any discursive functor C, conforms to much in the way of modal reduction theses, certainly not those S5 and S4 supply, such as $C\sim CA \equiv \sim CA$ and $CCA \equiv CA$ respectively. Really, discursive logic presupposes a much weaker underlying logic than Jaśkowski allows. Moreover the interpretation of C has itself to be treated with more care than it has mostly received in discursive logics. If C merely reflects what a participant in discourse or a discussion asserts or what his or her opinion is, then C will not distribute in the requisite way over entailment. For a person does not assert all that his assertions entail. Thus C needs to be interpreted in terms of commitment or some similar notion which is initially closed under entailment. Thus C can be read, for instance 'Someone (in the discussion) is committed to (the statement that)' or 'Someone (participating in the discourse) is (logically) obliged to maintain that'. With C so interpreted, at least the $S2^0$ (and $S0.5^0$) principle $A \rightarrow B \Rightarrow CA \rightarrow CB$ is correct, and possibly the stronger $S3^0$ principle $A \rightarrow B \rightarrow .$

$CA \rightarrow CB$. But the principle converting $S2^0$ and $S3^0$ respectively to S2 and S3 does not seem to be correct, namely $A \rightarrow CA$. For there may be truths to which no one in the group is committed. Nor is the fix which Jaśkowski's strategy may suggest, namely $C(A \rightarrow CA)$, adequate, since no participant may be committed to it either. In any case $A \supset \Diamond A$ plays a crucial role in the straightforward proof of the principle of DL, $\Diamond[(\Diamond p \supset q) \supset . \Diamond p \supset \Diamond q]$[162] (and also in the alternative principle, $\Diamond[(\Diamond p \supset \Diamond q) \supset . \Diamond p \supset \Diamond q]$), that justifies *modus ponens* in DL. In view of its role, it might be thought worth a good deal of trouble to retain $A \rightarrow CA$, for example by the dubious ploy of requiring that every discussion includes an ideal participant committed to the truth. Such an *ideal participant* is also wanted on other grounds if a modal-type logic is to remain basic. Otherwise even the principle, CT where T is a tautology, may fail, since no participant may be committed to such principles as $A \vee \sim A$, for instance. The ideal participant can guarantee not only this principle but, given its commitment to the truth, the much stronger principle of "Necessitation", CA where A is a theorem, should that be required. Likewise, the ideal participant can (re)instate Adjunction, e.g. by underwriting $CA \supset . CB \supset C(A\&B)$. This principle is of course not demonstrable in usual modal logics, though it holds in the Ł modal logic. But it *can* be added to weaker modal logics, and at a cost appropriately modelled.[163]

It is evident that C can also easily be given alternative renditions, e.g. as 'It is rationally believed that'. It is in this way,[164] in particular, that discursive logics can be linked to other much more recent logical developments such as doxastic logics and non-monotonic logics. But this was not the direction in which discursive logics first led or the way the contemporary history of paraconsistent logic went. Rather Jaśkowski's seminal work on discursive logic as a paraconsistent logic served to bolster elaborations of paraconsistent logics in South America.

5.6. The Latin American development: da Costa's theories

The remarkable growth of paraconsistent logic and theory in South America, though it had roots in European thought, apparently began independently of movements in Poland and elsewhere. Only when the movement was already initiated were the more elaborate historical connections discovered (primarily by Arruda and da Costa).

In fact Asenjo's thesis of 1954[165] marks the beginning of paraconsistent logic in Latin America, though the thesis (which was not accessible) appears to have had little impact in South America, or elsewhere. The technical core of the thesis, which was a matrix calculus of antinomies, was published only much later (in Asenjo 1966), but the interesting philosophical motivation of the thesis was omitted.[166] What Asenjo's nicely motivated calculus of antinomies comes to (though Asenjo did not notice this) was almost what the Chinese logician Moh also proposed in 1954, a reinterpretation of Łukasiewicz's $Ł_3$ matrices with the third value as paradoxical or antinomic (see above). In short, the proposal is that of Bochvar but with superior matrices. Both Asenjo and Moh saw the third value as applying to paradoxical statements, and as being assigned to wff which are both true and false (or perhaps true iff false in Moh's case). But Asenjo's logic should differ from $Ł_3$, since he evidently intended (though he nowhere says) that the third value should be designated along with truth. *That* logic is not however derived;[167] the most that is offered in Asenjo (1966) is the claim that da Costa's system C_ω is sound with respect to the matrices; it is certainly far from complete, as is evident from Asenjo and Tamburino (1975, p. 21) where the logic is finally axiomatized.[168] In particular it contains, as well as an infinite sequence of explicit contradictions of the form $B_i \& \sim B_i$, theses incompatible with motivational arguments of the earlier work appealed to, notably LNC, $\sim(A \& \sim A)$.

With da Costa's work we arrive at something strikingly different from what had gone before, deliberately fashioned paraconsistent logical systems—not overtly matrix logics or translations of modal logics—designed to retain systemic strength, and throw out merely what is paraconsistently defective in classical logic.[169] What was (erroneously) thought to be mistaken was (as in Asenjo, 1966) the Law of Non-Contradiction in particular, but also *reductio*—so that the paraconsistent objective could be achieved simply by removal of the reduction scheme $A \to B \to . A \to \sim B \to . \sim A$ (responsible also for LNC) from Kleene's axiomatization of classical sentential logic. Simple and brilliant. So resulted system C_ω, a strong natural system for paraconsistent purposes, one might almost say. But the system, which amounted to Hilbert's positive logic (i.e. the positive part of intuitionism) supplemented by the negation axioms, $\sim\sim A \to A$ and $A \vee \sim A$, was evidently exceedingly weak in its "negation" part, and admitted of considerable further supplementation. Hence the C_n systems with $n \geqslant 1$ obtained by adding a curious sequence of negation postulates. Worse, however, the motivation rested on a mistake, that (as in Asenjo) what had to be got rid of were LNC and also *reductio*. This assumption runs right through da Costa's pioneering work.

Although da Costa's initial work on paraconsistent logic was, like Asenjo's, in the form of a thesis (1963a; again not generally accessible), it was almost immediately spun out in a series of papers, introducing in quick succession sentential logic for paraconsistent systems (in 1963), then—what appears again to have been entirely new—predicate logics, and predicate logics with equality for such systems, theories of descriptions for such systems and a set theory based on such a system (all in 1964).[170] Much of this development involved collaborative work. For shortly after the notes reporting on parts of his thesis, da Costa began what proved to be a long and fruitful collaboration with Arruda on paraconsistent logics and theories. Da Costa's work, and that with Arruda, undoubtedly represents the fullest early flowering of paraconsistent logic.

There are three basic groups of systems on which da Costa, Arruda, and their Brasilian school have built a wealth of superstructures (primarily quantification and set theories), namely the *C systems*, devised by da Costa, the *P systems*, due to da Costa and Arruda, and later certain *Jaśkowski systems* varying and generalizing Jaśkowski, and jointly investigated with Polish co-workers.[171]

The C systems, the best known and most fully investigated of these systems, constitute da Costa's main solution to the problem of constructing formal inconsistent systems at the sentential level. (These systems, $C_1, \ldots, C_n, \ldots, C_\omega$ are described in the Introduction to Part Two.) (Already in 1963 important conditions of adequacy were imposed on solutions of this problem. Such calculi should—to paraphrase da Costa—contain the most important theorems and rules of deduction of classical sentential logic, while satisfying also the following conditions.

I. In these calculi the principle of contradiction should not be generally valid;
II. From two contradictory statements it should not be possible in general to deduce any statement whatever;
III. The extension of these calculi to quantificational calculi should be immediate.[172]

These conditions, which are reminiscent of Jaśkowski's requirements,[173] persist, with minor variations, in da Costa's later work.

Thus in his important survey paper of 1974, 'On the theory of inconsistent formal systems', conditions I–III reappear in virtually the same form, except that III is (illicitly) specialized to the C_n systems ($1 \leq n \leq \omega$), and the lead-in is taken up in a further condition, IV, again specialized to C_n systems, but in appropriately more general form amounting to the following:

IV. These calculi should contain (for the most part) the schemata and rules of classical sentential logic so far as these do not interfere with the earlier conditions.

This condition, in particular, is, as da Costa remarks, 'vague'. Furthermore it is doubtful that many solutions to the general problem of design of inconsistent formal systems will satisfy it; only certain classically-maximal solutions will (and perhaps no C systems are among these). Nor is the condition desirable, unless classical logic is, as it were, correct apart from very minor deviations. But classical logic is no ideal and we should aim to keep only what is correct in it.

Of the remaining conditions, II is basic to the very characterization of paraconsistent logic; quantificational and *other* extensions of a suitable sort, as under III, are evidently desirable (though it is not necessary that they be immediately obtained, and there may well be debate, as with relevant logics, over the correctness or adequacy of certain postulates); but condition I is a mistake. It is certainly true that significant paraconsistent logics can be designed in which the principle of contradiction is generally valid, main relevant logics being of this sort. But, further, insistence upon condition I is, so we later argue, a hang-over from classical consistency assumptions, which is squarely among the things paraconsistent logics are concerned with removing.

Even if condition I were conceded (and also for that matter II), a variety of systems other than the C systems can satisfy it and the other conditions; extensions of the relevant system R − W provide just one class of examples.[174] Nor (we argue in the Introduction to Part Two) are the C systems a good choice. However they remain an important early choice, and are perhaps the best studied among properly paraconsistent logics.

Upon the C systems, in particular, a wealth of supersystems has been built: quantificational theories (the C* systems), description theories (the D systems) and various set theories paraconsistently varying classical set theories (e.g. the NF_n systems, the ZF_n systems, etc.).[175] The C systems do not however lend themselves to non-trivial extension by expected set theoretic axioms (e.g. relatively unimpeded comprehension) without a batch of rather ad hoc restrictions resembling those tacked onto classical set theories.[176] With C systems, that is, the liberation to be expected by going paraconsistent is not achieved.

The problem was recognized early on by Arruda and da Costa themselves. They remark that, as Moh Shaw-Kwei had shown, suitable unrestricted comprehension (i.e. abstraction) principles cannot be obtained in systems which contain both the rule of *modus ponens* and Absorption (i.e. Contraction) principles of some order.[177] Accordingly they began to investigate systems which escaped the difficulty, systems lacking *modus ponens* (namely the J systems already referenced) and the P systems, which are very weak relevant systems which lack all absorption principles.[178] The class of P systems has since been much extended. The systems have been shown to be of much interest for other purposes, and it has been proved that they can indeed provide the basis for non-trivial (if very weak) dialethic set theories with entirely unrestricted comprehension principles.[179]

5.7. *The position of relevant logics and the contrasting attitudes of their proponents*

Although some of the earliest systems of relevant logic (e.g. the 1912 logic of Lewis—see RLR 5.1—and the weak implicational logic of Church, 1951) were paraconsistent, the historically more important systems Π′ and Π″ of Ackermann (1956) were not. For they contained, as a primitive rule, the rule γ of Material Detachment (viz. ~A, A v B—▹B) from which the Spread Rule, A, ~A—▹B, followed. Hence the systems trivialize contradictory theories based upon them. For if a theory based on Π′ contains A_0 and $\sim A_0$ for some wff A_0, then by Spread, it contains every wff.

One of Anderson and Belnap's main achievements in reaxiomatizing Π′, to obtain the theoremwise-equivalent system E, was the removal of γ as a primitive rule.[180] But their motivation was far removed from anything to do with paraconsistency, and was based rather on a questionable normalization principle,[181] to the effect that to every primitive rule should correspond an entailment thesis. But this would require the dreaded Disjunctive Syllogism, $A \,\&\, (\sim A \vee B) \rightarrow B$, corresponding to γ as an entailment thesis (on standard extensional normalization of rules). Thus, given the normalization requirement, γ had to be removed as a primitive rule.

The evidence is, on the contrary, that Anderson would not have been sympathetic, at the time he was concerned with recasting Ackermann's systems, to a paraconsistent position or paraconsistent grounds for changing the systems. For instance, Anderson (in 1958) chastises Wittgenstein for, in effect, flirting with paraconsistency, for his '"so-what" attitude towards contradictions in mathematics' (alleged, p. 488, to be difficult to reconcile 'with his own view of language-games'),[182] for his recommendation that 'we stop playing the consistency-game altogether' (p. 489), for viewing 'as somehow perverse' 'the fact that avoidance of contradictions *is* held essential by mathematicians' (p. 488), Anderson's own view being, evidently, that the avoidance of contradictions is essential (see especially p. 489). Anderson's view is the mainstream classical view that contradiction renders a system or theory useless for intended or serious logical and mathematical purposes, even if for some bizarre purposes, such as aesthetic taste, contradictions may be turned out by a theory.

Nor does the work leading up to and embodied in Anderson and Belnap's monumental *Entailment* exhibit any softening toward paraconsistency. Instead a very considerable amount of effort was devoted to the recovery of the discarded rule γ, as an admissible rule of system E. In *Entailment* very little is in fact said about contradiction (it does not even rank a separate index listing).[183] But while the paraconsistent character of systems like R is recognized ('that an extension of R is negation inconsistent does not imply that it is Post inconsistent', i.e. trivial), still anything worth the name of "logic" that extends R is, when negation inconsistent, also trivial.[184] That R retains this power to properly reduce inconsistent "logics" to worthless triviality is seen as a virtue of R, a meritorious feature, not as the drawback it is.

Even now American proponents of relevant logic, such as Belnap and Dunn, are careful to insist that they are not proposing other than epistemic interpretations for their apparent assignments of joint truth and falsity to some statements in some situations (e.g. they can be *told* true and *told* false); they are *not,* they say, making the (absurd) suggestion that some statements are or may be literally both true and false (cf. Belnap, 1977 and Dunn, 1976).

The Australian approach through relevant logic has been very different. It has been semantically oriented almost from the outset in the late sixties, and almost from the beginning it has made allowance for inconsistent and incomplete worlds. There has been just one major shift, which began in the early seventies from a paraconsistent position towards a dialethic position, though not all Australian relevant logicians have made it.

5.8. The Australian movement

Initial Australian developments in the paraconsistency enterprise were sporadic and of a semantical cast. Goddard (in 1959) described ingenious situations where the Law of Non-Contradiction (LNC) failed, where for certain p, p & ~p held. The counterexamples to LNC proposed in fact bear a striking resemblance to those the dialecticians had used in showing that motion involved a contradiction (though Goddard made no such comparison); but whereas for the dialecticians the counterexamples held for the real world, for Goddard they held only for certain alternative worlds, strange discontinuous worlds remote from the real. What hold, however, in these inconsistent worlds are statements such as that (at the one time) a stone both 'is at B and is not at B' (p. 38). Since we can imagine and to some extent describe (pp. 38–9) a universe in which LNC fails, such "laws of thought" as LNC are not, so it is argued, laws of thought in the strong traditional sense; they hold conditionally upon certain requirements being satisfied, e.g. our universe having a certain continuous spatio-temporal structure (p. 39). There are more direct corollaries for paraconsistency: since there are universes where contradictions hold which are not trivial, a comprehensive logical theory must be a paraconsistent one. Goddard did not have the terminology to state the corollary in this way, but the idea is implicit.

In the early sixties Mackie got the idea (earlier tried, in one way or another, by Bochvar, Moh, Asenjo and others) that as well as true and false statements there were paradoxical statements, namely certain statements generated by the (semantical) paradoxes. Though this might easily have issued in a three-valued logic with values T, F and P—and is sometimes taken to have[185]—it did not appear to; and in fact Mackie remained a rather staunch devotee of classical two-valued logic.[186]

Elsewhere in Australia however, notably in New England (at Armidale) many-valued and intensional logic approaches to the paradoxes, and to much else, were soon under way in the early and mid sixties. Routley, who was interested in significance and had been researching paradoxes from a statement-incapability and incompleteness (and content loop failure) aspect teamed with Goddard, now in Armidale, who had been working on paradoxes from a significance angle; and jointly and, also separately, they produced a range of logics based on 3-valued (the third value being non-significance or statement-incapability) and 4-valued (the fourth value being incompleteness interpretations). Several of the systems that resulted were paraconsistent, as was observed (though in more old fashioned terminology) in Goddard and Routley (1973), which summed up (many years later) some of the work of the New England school. There the possibility of paraconsistent logics was proved, by matrix procedures (p. 285), and then several examples of dialethic logics were exhibited and discussed, including a paraconsistent connexive logic (see pp. 291–92). The theory developed was taken to confirm Wittgenstein's themes that inconsistency need by no means destroy a calculus.

A second line of research at New England also led in paraconsistent directions, namely the work on non-existent objects, including impossible objects, and quantificational theories that could include such objects in their domains. Reflection on the character of such objects resulted in the investigation of various Characterization Principles, and belated discovery of the studies of Meinong and the Graz school on the theory of objects. Furthermore application of expected Characterization Principles lead directly in turn to the need for paraconsistent logics.[187]

But it was a third and apparently (at the time) independent line of investigation which led to the distinctively Australian approach to paraconsistency through relevant logics. This was the logical, and especially semantical, study of paradox-free implication and conditionality brought to New England by Routley, who had been looking at the problems virtually since 1960. Some of the systems devised in New England, such as the I systems, though paraconsistent and paradox-free at the first degree, proved to be irrelevant at the higher degree.[188] Some of the systems, investigated jointly with Montgomery from New Zealand (with whom the rudiments of a general picture of implicational systems was being built up), have yet to be written up in the literature; many of the systems, such as the connexive systems studied, though lacking spread principles and sometimes paradox-free, were not really paraconsistent, but many were. Perhaps the most important paraconsistent directions in which the investigations tended were to the method of counterexamples to logical laws[189] and thence in the late sixties, by which time the New England group had dispersed again, to the semantical theory of inconsistent and incomplete worlds discerned by R. and V. Routley, in the first place for the first degree of logic of entailment.[190]

The semantical theory was extended in the early seventies by Routley and Meyer, to many sentential relevant logics,[191] which proved to be paraconsistent in an interesting semantical way, namely that the base worlds of the models initially arrived at, though generally complete, could well be inconsistent. Thus relevant logics could be straightforwardly and non-trivially rendered dialethic, an outcome which looked extremely promising in accounting logically for much philosophy in the dialectic tradition[192] as well as for paradoxes and their relatives. Soon after this work was initiated, the Australian and Brazilian groups discovered each others' work (through Makinson) and subsequently exchanges began.[193]

Closely connected with the logical paradoxes are such limitative results as Gödel's incompleteness theorems. In the early 70's these were investigated by Priest, who, being in Britain, was working in ignorance of paraconsistent research elsewhere. By 1972 he had come to the conclusion that a paraconsistent approach to problems in the area was required (see Priest, 1974, chapter 4); the results of this investigation, which arrived at full-blooded dialethism, were published in 1979. This paper was read in Canberra in 1976 when Priest moved to Australia, and Priest and Routley found, to their mutual astonishment, that they had been working along related lines. Since then their work has tended to converge, with their joint work so far culminating in the present volume.

Appendix: note on recent activity elsewhere

Apart from the main centres of research on paraconsistent theory in Australia, Latin America and Poland and more recently in Bulgaria,[194] investigation elsewhere has for the most part been by fairly isolated workers producing the occasional piece. And some of these pieces are marginal as regards belonging to or furthering the paraconsistent enterprise. This applies especially to work (allegedly) directed at clarifying or elaborating Hegel's logic or Marxist logic.

A worthwhile survey of recent activity on the paraconsistent front in Australia, Brasil, Poland, USA, Argentina, Belgium, Ecuador, Italy and Peru is given by Arruda.[195] There are however points that should be adjoined or amplified. In the first place, there is now a small group working on paraconsistent theories in Canada. Independently of S. Thomason (1974, which, though it had some circulation, was never published except in abstracted form[196]), Jennings and Schotch sometimes in collaboration with others, have begun detailed direct investigation of Non-adjunctive logics. Secondly, also in the Non-adjunctive tradition emanating from Jaśkowski, is Rescher and Brandom's recent text, *The Logic of Inconsistency*, 1980.

Thirdly, while paraconsistent studies are now a major component of logical activity in places in South America and Australia and at some centres in Poland and Bulgaria, they occupy such a privileged position nowhere else. Indeed, on the contrary, they are elsewhere very much a minority activity and commonly regarded with varying grades of scepticism. Particularly in the UK and USA, strong and unquestioned consistency assumptions impede all but peripheral paraconsistent investigations.

Notes

[1] According to Rescher, 1967, chapter 2.

[2] Such as are given in the Introduction to Part Two.

[3] In the case of Plato and Aristotle, examples are given in the Introduction to Part Four. Some of these inconsistencies are minor and would require little repair. Some however are major. It is perhaps worth making a start on outlining the contrast of the Classical Greek Establishment with Outsiders or Foreigners. Generally, the lifetime members of the Establishment were born in the right place, were citizens, were political conservatives, were propertied, did not need to work or teach for a living. But the Establishment had the usual more transient periphery of hangers-on; those who, while not meeting the criteria for direct membership curried favour with lifetime members and accepted main Establishment values and furthered its cause. Plato and his family, and Isocrates, for example, directly qualified as Establishment members (cf. Davies, 1971, on their wealth and holdings). Aristotle, however, occupied a peripheral position for the main part of his teaching life in Athens, a position he eventually lost. Plato and Aristotle, now seen as the intellectual giants of classical philosophy, in fact, represent rather the Classical Establishment—which strenuously opposed paraconsistent approaches. The Sophists, by contrast, were invariably outsiders, were not citizens, were not propertied, had to work for a living (and were looked down upon for so doing), and were not (ideologically or otherwise) part of the status quo or politically conservative. In other historical periods too there is a weak but significant correlation between political conservatism and the rejection of paraconsistency.

[4] The one classical inconsistent theory, the trivial one, is ruled out as a theory. The connexive theme concerning propositions—not so bizarre classically if propositions represent theories—is expressed in Aristotle's law $\sim(A \rightarrow \sim A)$, i.e. $A \circ A$, every wff A is self-consistent. The theme concerning theories is derivative because theories can be seen as proposition or conjunction of propositions.

[5] What Soviet logic does look like in formal detail is a much more sensitive matter; see the initial considerations in Routley and Meyer, 1976.

[6] So Diogenes Laertius reports, 1951, VIII, p. 309; and there seems no good reason to doubt his reliability on this occasion.

[7] Thus, e.g., Protagoras' works by the authorities of Athens. But it was not always at establishment hands by any means. Our "barbaric ancestors" who sacked and destroyed libraries of the ancient world have much to answer for also.

[8] Consider, for example, the distorted view we would have of Meinong (or even Frege) if we could read only Russell.

[9] The orthodox framework now takes the form of Anglo-American pragmatist-adjusted empiricism, which is fundamentally committed to classical logic. But since classical antiquity mainstream Western thought has subscribed to what arguably underlies empiricism, the Reference Theory (according to which, in

capsule form, truth is a function of reference), which does characteristically lead to consistency theses excluding paraconsistency (see further note 20 and for details see EMJB). Recent examples of attempts to squeeze Heraclitus into the consistency framework include Wheelwright, 1959 and Cleve, 1965, both authors bewitched by the Ontological Assumption.

[10] The centrality of this theme (not stated quite as such by Heraclitus) is generally agreed upon by commentators.

[11] Thus, e.g., Stokes, 1967, p. 478.

[12] Types of unity, in particular unity as continuity and unity as identity, were distinguished by Aristotle.

[13] See the case for construing harmony through connection made out in Stokes, 1967, p. 478.

[14] See Freeman, 1948, p. 28, #50.

[15] See, e.g., Freeman, 1948, p. 28, #51; also, differently, #55.

[16] This is called the 'First Law of Ecology' in Commoner, 1971; it has been much cited elsewhere. On this basis a case can be made out—no more far-fetched than many of the reconstructions of Heraclitus—for seeing Heraclitus as an early (if no doubt somewhat primitive) ecologist. There is to be found in Heraclitus a fragmented picture of an ecologically well-ordered universe; and it is not only the First Law that can be ascribed to Heraclitus. In his claim 'wisdom is . . . to act according to nature' (Freeman, 1948, p. 32, #112) Heraclitus captures the thrust of the 'Third Law', 'Nature knows best' (Commoner, 1971, p. 37). It can be argued that the Second Law, that 'Everything must go somewhere', which is essentially a conservation principle, is represented, even implied, by Heraclitus' exchange principle; 'there is an exchange: all things for Fire and Fire for all things, like goods for gold and gold for goods' (p. 90, cf. also #126).

[17] This is now often construed as the best approximation available to Heraclitus through which to say that the Logos is incorporeal or even abstract.

[18] See the classical representation of Meinong's theory of objects through predicate negation, in EMJB.

[19] This type of argument is examined in much greater detail in Priest, 1982, and 1985.

[20] The descent is traced in EMJB, chapter 1. The common component is, as there explained, the Reference Theory, according to which truth is a function of reference. This Theory yields the main elements of classical logical theory including the Ontological Assumption, to the effect that one can only speak truly of what exists, an Assumption that appears in the extreme form in Parmenides' thesis that what does not exist cannot be sensibly spoken about or discussed (cf. p. 11, ff.). Given that what exists is consistent (a common, though now disputed, assumption), it follows that the world as reflecting what is true and comprising the totality of what exists, must be consistent, and that truth cannot be contradictory.

[21] Thus Aristotle refers to 'the view of Heraclitus that all things are in motion' (*Topica* 1, 2, 104b, 20).

[22] It is unclear who, apart from Heraclitus (this on the basis of his "river" fragments), these people were. But Aristotle speaks, for example, of 'Those who . . . have arrived at their view (that contradictory and antithetically opposed characteristics can obtain simultaneously) on the basis of sensible perception in that they notice that from one and the same thing proceed contraries' (cited in Łukasiewicz, 1971, p. 401).

[23] On the problems with such analyses, see Priest, 1985.

[24] There is some evidence, adduced by Łukasiewicz and considered in 5.2 of this chapter, that Aristotle's theory of becoming led him into paradoxes, and effectively into a paraconsistent position as regards potentiality.

[25] To say that all contradictions contain an element of truth is a much less drastic thesis, and indeed should be no more controversial than a principle of excluded middle. For where C is a contradiction, C implies A & ~A for some A, and one of A and ~A is true.

[26] Protagoras held this extravagant theory according to Diogenes Laertius, 1951, p. 465, and also to Plato's *Theatetus* (152A sq.), to which Diogenes refers.

[27] See EMJB, p. 334. The matter is discussed in more detail in Routley, 1976.

[28] From Protagoras' *Truth*: see Freeman 1948, p. 125. The things Protagoras is speaking of, of which Man is the measure, appear to be truths (arguments and the like, i.e., primarily propositional items); but the theme holds derivatively for everything (for consider statements of the form 'a exists', 'b is').

[29] Relativism also differs significantly from pluralism; and though pluralism is often achieved by pinches of relativism, there are limits to how far this can go. *Pluralism,* like semantical relativism, *has a logical representation* within the framework of paraconsistent theory, *namely through discussive logics* (see especially the section on Jaśkowski below.)

[30] Semantical relativism fits in not only with popular relativism, that everyone is right in his or her own way, but also with what Lakatos terms Einstein's sarcastic insistence (against Bohr) that 'every theory is true provided one suitably associates its symbols with observed quantities' (quoted by Lakatos, 1970, p. 143).

[31] That is on the factual model; see EMJB, p. 34. Many such arguments are assembled in RLR.

[32] Freeman, 1948, p. 126; Diogenes Laertius, 1951.

[33] Diogenes Laertius, 1951, p. 485. The passage in Plato is *Euthydemus* 286c.

[34] On this see Guthrie 1969. Note that Guthrie sees Antisthenes as 'deeply involved in the argument about the use of language and the possibility of contradiction which formed part of the theoretical background of fifth-century rhetoric, and in which Protagoras played a leading part' (III, p. 304).

[35] Stronger and weaker versions of Antisthenes' position are in circulation; the weaker position is that it is impossible to speak inconsistently. Aristotle states it as 'the view that contradiction is impossible' (*Topica* 1, 2, 104b20), a formulation also given by Diogenes, 1951, ix, p. 465. In a rather different way Aristotle may have been committed to a related theme, to the connexive theme that every proposition is self-consistent, i.e., $A \circ A$.

[36] According to Diogenes Laertius, 1951.

[37] It may be that a correct principle can be extracted from this assumption, but the argument then requires *more* than that principle to succeed. How, in an Irish way, the given assumption rules out contradiction between discussants is nicely explained by Gillespie (quoted by Ross, 1924, p. 347, which compares the logic of Antisthenes with what is said to be 'Hobbes' similar nominalistic view'):

> A and B are supposed to be talking about the same thing ... A and B in their discussants make various assertions about the thing, which they no doubt call by the same name; but they do not necessarily attach the same or the right formula to the name. Still in no case can they be said to contradict each other; if both have in mind the right formula, they agree; if one has the right formula and the other a wrong one, they are speaking of different things; if both have wrong formulae in mind, neither is speaking of the thing at all.

[38] Kneales, 1962, p. 22.

[39] Zeller, 1877, p. 277. Zeller says the same argument was also presented by Stilpo. A main source for reconstructions like Zeller's is the following passage from Aristotle: 'Antisthenes foolishly claimed that nothing could be described except by its own conception—one predicate to one subject; from which it followed

that there could be no contradiction and almost that there could be no error' (*Metaphysics*, 5, 1024b).

[40] In case the reader thinks this is entirely stupid, he should consider Russell's celebrated third puzzle concerning denoting, which raises an analogous problem for *difference*: for a discussion of this puzzle, see Routley, 1980.

[41] Zeller, 1877 p. 277. Zeller tries to arrive at Antisthenes' theme by having us stop at names for (simple) things (p. 301) and ruling out predications, but such draconian measures are not required.

[42] Zeller, 1877, p. 296.

[43] Perhaps the most interesting anticipation is the rejection of essence. According to the Antisthenians 'it is impossible to define what a thing is (for the definition, they say, is a lengthy formula), but it is possible actually to teach others what a thing is like; e.g., we cannot say what silver is, but we can say that it is like tin' (Aristotle, *Metaphysics*, 8, 3, 1043b24). It is also, incidentally, worth noting that Antisthenes (though not a cynic but rather a 'precursor of cynicism') continues the ecological tradition of Greek alternative thinkers; what is known of his views on self-sufficiency is interesting in this regard.

[44] See *On Nature* included in Freeman, 1948, pp. 128-29.

[45] Freeman, 1948, p. 136.

[46] Kneales, 1962, p. 16.

[47] Taylor, 1911, p. 128.

[48] The ascription is explicit in Diogenes Laertius: see Bocheński, 1961, p. 131, where some of the impact of the paradox is outlined, including the remarkable anecdote concerning Philetas of Cos.

[49] Mates, 1953, p. 84.

[50] Bocheński, 1961, p. 132-33.

[51] *De Sophisticis Elenchis* 25, 180b 2-7.

[52] Ibid.

[53] There was, however, a prolonged medieval debate over what exactly Aristotle meant by what he said. Bocheński proceeds, without very much evidence, to ascribe a levels-solution to Aristotle, 1961, p. 132.

[54] According to Bocheński from whom this passage is quoted (1961, p. 135; the original source is Rüstow, 1910, p. 50), the Greek phrase is ambiguous and could just mean 'that whoever states the Liar attributes a false assertion to the proposition'.

[55] The main study of this work is Mates, 1953, p. 33ff.

[56] Mates, 1953, p. 34. For example truth value gaps are needed for generic objects. Generic Man is neither good nor barbarian, and so an incomplete object.

[57] This was said, too cryptically, to be true since caused by something that existed, viz. Electra, but false because the presentation was of a Fury. It is not difficult however to appreciate part of what is being required here.

[58] Thus Bocheński, 1961, p. 14; also Mates, 1953, pp. 34-35.

[59] Mates, 1953, p. 34.

[60] This point is elaborated in DRL. The fact that the Stoics, like Ackermann, appeared to accept principle γ of Material Detachment, shows only that this position was not a dialethic one, not that it could not be paraconsistent.

[61] In particular, the position of Boethius may well be paraconsistent, since a radically non-classical theory is required to accommodate logical principles espoused by Boethius, especially his law $A \rightarrow \sim B \leftrightarrow . \sim(A \rightarrow B)$.

[62] Bocheński, 1961, p. 134.

[63] Similar problems stand in the way of tracing the history of non-standard thought in other areas. For example, the dominance of the pervasive Reference Theory renders the tracking of thought that does not conform to it much more difficult

(cf. Routley, 1980). The predominance of classical logic makes the location of theories that repudiate parts of it so much harder, especially for those who (like us) commonly have to rely on secondary sources.

[64] See Yutang, 1948, pp. 48, 52 and 204.

[65] This example is given, along with others, in Yutang, 1948.

[66] Needham, 1969, p. 201.

[67] The Mohists were apparently, like the Taoists, anarchists, but of an interesting and perhaps *curious* sort, since they were specialists in military (violent) defence. As to what is, rather more relevant here, their logic, see Graham 1978.

[68] A text allegedly written by the Taoist Chuang Chou (?369–286 B.C.); for a translation, see Giles, 1926. See also Yu-Lan, 1952.

[69] However it could be reconstrued as a way of representing indeterminacy. The matter is further obscured by the historical setting, as the commentary in the *Chuang-Tzu* begins 'You cannot speak of ocean to a well-frog . . . '.

[70] Stcherbatsky subsequently suggests that Heraclitus' theory is akin to the Sānkhya position (p. 426), which seems to involve, in the thesis of ideality of cause and effect, a unity of opposites. In this event, the Sānkhya position is also open to paraconsistent construal.

[71] Stcherbatsky, 1962, p. 415.

[72] These different ways are in fact among those explained in Routley and Plumwood, 1983.

[73] Matilal, 1977.

[74] See, e.g. Robinson, 1957, Stcherbatsky, 1962, p. 425ff.

[75] Murti, 1955, pp. 127–128.

[76] Singh, (n.d.), p. 16.

[77] See further Routley, 1983.

[78] Singh (n.d.), p. 16.

[79] See Streng, 1967, p. 146.

[80] See Singh (n.d.), p. 19, for suggestions as to what these are.

[81] As Indian philosophers influenced, and over-impressed, by Western texts are inclined to suppose.

[82] For details see Singh (n.d.), p. 20ff.

[83] M. de Wulf, 1952, p. 155. This reference we owe to S. Haack, who goes on to conjecture that Descartes might also have thought that God could make it the case that the "laws of logic" are false (perhaps by way of an evil demon). Had Descartes thought this he would have been right at least as regards lesser lights than God (such as demons), as semantics now helps to show (see e.g. RLR).

[84] Not merely in the modern Marxist fashion for formal logic.

[85] See e.g. Griffin, 1984.

[86] See, for example, *Ennead V*, 2, 1, and also Gilson, 1972, p. 43ff.

[87] Wallis, 1972, pp. 57, 8.

[88] On which see Trouillard, 1970 p. 240.

[89] Trouillard, *op cit.*

[90] Smart, 1967, p. 450.

[91] Maurer, 1967, p. 497.

[92] For the references to the Neo-Platonists, we are indebted to P. V. Spade and, especially, Lorenzo Peña. A modern discussion and development of Neo-Platonist ideas from a paraconsistent perspective can be found in Peña, 1979 and 1989.

[93] Heytesbury, 1494, fol. 7rb.

[94] Bocheński, 1961, p. 244.

[95] See Ashworth, 1974. A much fuller account of the medieval and post-medieval anticipation of relevant logic is attempted in Routley and Norman, 1988.

[96] For the influence of medieval logic by no means vanished in the 16th and 17th centuries, contrary to popular assumption that scholastic logic was swept away with the advent of the Renaissance; see e.g. Ashworth, 1969.

[97] Arthur Collier (1680–1732) who wrote *Clavis Universalis*, 1909, should be clearly distinguished from his contemporary Anthony Collins (1676–1729) who wrote on free-thinking. An account of Collier's life and work by Leslie Stephens may be found in the *Dictionary of National Biography.*

[98] There are quite different positions that can be taken as to the consequences of true contradictions holding in the world, e.g. that contradictions, and such a world, exists, versus the (noneist) conclusion that no such proposition or world exists.

[99] Reid, 1895, p. 376. The matter is discussed in EMJB, p. 688.

[100] Certainly such a paraconsistent adaption (from a firmly classical meta-stance) is now occurring—more than a century later—in one strand of contemporary American pragmatism, notably in Rescher's thought; see especially Rescher and Brandom, 1980. The serious limitations of this work as a logic of inconsistency will be considered elsewhere. A fuller documentation of very recent paraconsistent developments would take some further account of Rescher's work. For although *The Logic of Inconsistency* represents rather an about-turn compared with much of Rescher's earlier work (e.g. that criticized in Routley and Meyer, 1976), still his generous philosophical framework can be readily adjusted to allow for paraconsistency, in non-adjunctive form for example, and some of his earlier suggestions, for example on truth-value gluts (an idea also entertained by relevant logic researchers at Pittsburgh), point in paraconsistent directions. Fortunately, then, Rescher's contributions have also been adequately presented elsewhere.

[101] Michael's exposition of Peirce, in 1975, which we follow, at several points suggests such a solution. All quotations from Peirce are taken from Michael.

[102] The argument that S2 is about nothing is that an attempt to find what it is about leads to an infinite regress. Peirce then anticipates Meinong's and Ryle's namely-rider location of self reference.

[103] The most accessible introduction by Meinong to this theory in his 'The theory of objects' in 1960. Recent presentations and elaborations of the theory of objects appear in EMJB, Plumwood and Routley, 1982, and Parsons, 1980.

[104] See EMJB, 'Three Meinongs', where it is also argued that what Meinong rejected was a predicate form of LNC. Through the predicate/sentence negation distinction a reconciliation of a sort between a theory of "contradictory objects" and classical logic can be effected; see e.g. Parsons, 1980. The same distinction of negations can correspondingly be put to work to effect a reconciliation, again of a sort, between dialectical and classical logic. An attempt is made to carry out this latter feat by Wald, 1975, p. 116 ff.

[105] As we shall see, Vasil'év independently makes a similar suggestion, which in contrast to Łukasiewicz, he tries to develop. As Arruda has remarked in 1977, there are striking parallels between Łukasiewicz and Vasil'év, including some Arruda does not record, such as the idea that perception is always of things positive, and negation is only inferred (cf. Łukasiewicz, 1971, p. 507). Also common ground is the point that traditionally LNC was much confused with other principles such as Double Negation (p. 493).

[106] It is sometimes said (e.g. by Wedin in introducing his 1971 translation of Łukasiewicz's article) that with the analogy Łukasiewicz may have already conceived the possibility of many-valued logics. This is not evident. What he had conceived is the possibility of logics which altered the basic laws of thought, a class of logics only properly overlapping many-valued logics. And what he does raise (p. 486) are several new and fundamental questions concerning

independence and interderivability of basic logical principles. Wedin's subsequent remarks (p. 486) about Łukasiewicz's amenability to altering his paper, effectively to make it more amenable to the friends of consistency, strike us as wrong. In particular, the non-triviality requirement is *not* 'something of a metalogical correlate to the logical principle of contradiction'.

[107] There are of course further formulations of LNC, e.g. modern syntactical formulations, other psychological forms concerning what can be conceived or imagined, and linguistic forms restricting descriptions of things. Indeed all forms can be seen as imposing restrictions of related sorts. Łukasiewicz goes on to formulate the versions extracted from Aristotle more precisely, p. 488.

[108] For example, the accusation of "psychologism", in 6(b) p. 491, is but poorly made out; the claims, in 7(a) and (b) p. 492, are dubious or unnecessary, for the principle concerned could be part of the logic of belief; the claim in (a′) p. 499 that the principle of contraposition presupposes the LNC is false. The contrast between the sharply delineated concepts of logic and the 'scraps of fluid and vague speech used in everyday life' is largely technologicians' prejudice.

[109] On the one hand Łukasiewicz suggests the procedure contains a contradiction (p. 495), on the other he says it is clear that Aristotle commits no contradiction (p. 496).

[110] Łukasiewicz's two-world interpretation brings Aristotle out as occupying a position very similar to Bradley, with a consistent world of reality behind the inconsistent world of appearances. Łukasiewicz's interpretation itself involves some *significant* shifts, e.g. not merely from how things are potentially to how they are in a world representing these potentialities, but that the latter world *is* a sensibly perceptible world (p. 502). Against Łukasiewicz it could be argued that Aristotle has not conceded that LNC, in object form, holds only for actual objects; rather that 'potentially, the same thing can have antithetically opposed characteristics at the same time, but not actually' (p. 501) commits Aristotle only to the potentiality theme $\Diamond fa \& \Diamond{\sim}fa$, not to (what Łukasiewicz's modal theory would imply) the theme $\Diamond(fa \& {\sim}fa)$ Łukasiewicz attributes to Aristotle.

[111] It is closely related to what we will find in Vasil'év.

[112] Aristotle notwithstanding as the negative part has indicated.

[113] Łukasiewicz shifts (p. 506 bottom) from "$x \in f \wedge x \notin f$" being certainly false to LNC's not being true except for 'objects . . . free from contradiction'. But the argument is garbled because the main italicized passage is grammatically deviant. Although the argument (strictly from instances of $x(\in \wedge \notin)f$ provided by contradictory objects) makes room for the predicate negation/sentence negation distinction that looms larger in Meinong, and can be important for a consistent treatment of inconsistent objects, that issue does not arise.

[114] The reason given for this scepticism is in part almost mystical; 'Man did not create the world and he is not in a position to penetrate its secrets; indeed he is not even lord and master of his own conceptual creations' (p. 508). The final qualification to 'conceptual' could be removed.

[115] Łukasiewicz does not try to explain the matter, which has puzzled many philosophers including several critics of Meinong's enterprise. An explanation is offered in EMJB, chapter 9.

[116] Even swifter is his other argument emphasizing the practical worth of the principle at the expense of its logical worth. We are supposed to 'see', from the role of LNC in combatting falsehood everywhere, 'that the necessity of recognizing the principle . . . is a *sign of the intellectual and ethical incompleteness of man*' (p. 508). This is rubbish (human chauvinist rubbish at that)—perhaps taken over uncritically from some more eminent philosopher—of a type fortunately not often encountered in Łukasiewicz.

[117] The LNC is hardly the sole weapon in any case, as operating with logics that lack it soon reveals. There are other tests of correctness, other fallacies, etc.

[118] The principle does not, however, directly rule out joint assertion and denial; it says (as distinct from *implies*, given further logical connections) nothing about assertion or denial.

[119] The argument is hardly conclusive: courts would simply have to resort to more elaborate procedures to determine perjury and guilt than the inconclusive more classical procedure now taken by logicians to be used. A discussion of when and how particular contradictions are rejected can be found in Priest, 1989.

[120] To Arruda we owe the main exposition of Vasil'év's work.

[121] The mentalistic restrictions, or reductions, that Vasil'év tries to impose are inessential. Still the idealist shift is commonplace (and evidently helpful) in attempts to break free from the empirical.

[122] But it is only recently that proper investigation of their logical and other properties has begun. Historically much more effort has been devoted to dispelling, or suppressing, the idea of alternative, and especially superior, worlds.

[123] Consider, e.g. "Man is not not-man" and "Meinong is not not-Meinong" and their various sentential and predicate formal representations.

[124] This, syntactical LNC, is one of the forms Arruda also gives of LC, the other being effectively a modalisation of this.

[125] Vasil'év's argument on these points (discussed by Arruda, 1977) is decidedly suspect, and, it seems, somewhat confused. One reason is that he thinks that the meaning of negation is determined by real world considerations and by the way our sense perceptions in fact operate, and that in order to amend Aristotelian (equated with: real world) logic we have to change the meaning of negation. However what he is groping for is clear enough; a non-classical negation rule.

[126] Or at least, like Meinong, an inconsistent-admitting predicate negation but a more classical (consistent) sentential negation.

[127] Less well-known relevant logics illustrate the point better.

[128] The difficulty in obtaining a workable positive/negative predicate distinction only compounds reservations properly felt about this way of trying to distinguish real and imaginary worlds. However the affirmative/negative distinction is not what matters, and may be dispensed with: what is important is the obtaining of incompatible facts.

[129] The paper, in Russian, is briefly reviewed by Church in 1939-40, and is discussed in more detail in Rescher, 1964, p. 294ff.

[130] But a kind of a dual thereof, non-significance treatments standing to dialethic ones somewhat as incompleteness stands to inconsistency.

[131] In Bochvar's three-valued logic of 1939, 'Bochvar proposes to construe (the third value) I as "undecidable" in the sense of "having some element of undecidability about it" (Rescher, 1968, p. 67). But Rescher goes on to say, what is puzzling, 'We are to think of I not as much as "intermediate" between truth and falsity but as *paradoxical* or even *meaningless*. We can think of such meaninglessness in terms of what is at issue in the classical semantical paradoxes . . .' (p. 67). For these are three very different interpretations of the third value—as undecidable, paradoxical, and meaningless—and characteristically go with different matrices. In fact the matrices ("truth tables") Bochvar presents are those that fit with the interpretation of the third value as meaningless, *not* as undecidable—or as *paradoxical.*

[132] Insofar as they are genuinely different from the non-significance interpretation. The point is argued in detail in Goddard and Routley, 1973, chapter 5. Consider, e.g. the conjunction A&B of A and B where A is false and B is undecidable: then A&B is not undecidable in truth value, for however B would be decided

A&B is false. The undecidability cannot be as to truth value but must indicate a semantic defect, i.e. the matter is one of significance after all.

[133] In the notation Rescher uses, the Principle $p \wedge \neg p \Rightarrow q$ is a thesis. The truth connective T, with matrix

	T	F	N
T	T	F	F

coincides with Bochvar's external operator A.

[134] The matter is considered in more detail shortly when Jaśkowski's work is studied.

[135] On both see White, 1979. Among the serious drawbacks are the unavailability of extensional axioms without inconsistency. *These* drawbacks are avoided in relevant sublogics of $L_{\aleph_0}$.

[136] This follows immediately from the first three axioms of Wajsberg's axiomatization of $L_{\aleph_0}$, namely from (1) $A \supset . B \supset A$, (2) $A \supset B \supset . B \supset C \supset . A \supset C$, (3) $\sim A \supset \sim B \supset . B \supset A$: cf. Rescher, 1968, p. 39.

[137] Naturally Wittgenstein's change of positions did not occur in a historical vacuum. His shift to a later paraconsistent-leaning position was influenced by German idealism for instance, as occasional examples in his work reveal. As to Wittgenstein's acquaintance with idealism and its influence on his work, see Toulmin and Janik, 1973.

[138] Though there is some doubt about this as the article by Goldstein in this volume shows.

[139] Qualification is necessary because Wittgenstein was also tempted to say, at this stage, that "contradictions" in pure calculi are not strictly contradictions, and should be differently represented, e.g. by a sign 'Z'. (The point is discussed in Appendix 2 of Routley and Plumwood, 1983, which complements the present discussion of Wittgenstein's position.) Subsequently Wittgenstein was quite clear that calculi containing contradictions (non-trivially) are still calculi and may be perfectly good parts of mathematics (e.g. 1964, p. 181).

Regrettably not enough of the transitional assumptions are expunged in later work; instead what tends to happen is that the assumptions or motivation for them are later obscured. A striking example concerns the matter of hidden contradictions which were to Wittgenstein something of an anathema. In 1930 Wittgenstein said roundly that 'it does not make sense to talk of *hidden contradictions*' (McGuinness, 1979, p. 174), the ground being an (indefensible) verification principle to the effect that where one doesn't have an effective method or criterion for something, talk of it doesn't make sense. In later work the underlying verification principle ruling "hidden contradictions" meaningless has disappeared from sight (though it is still operative in the intuitionistic bias of the work in philosophy of mathematics), and the theme as to hidden contradictions amended to this; as long as they are hidden they don't matter, and when they come into the open they can do no harm (1976, p. 219, for instance). As will be seen, this stand is *most* unsatisfactory; for contradictions can certainly matter when still hidden, and can do much harm even when visible.

[140] One has simply lost one's way. Containing a contradiction in a harmful way is like *not knowing one's way about* (1964, p. 104). This comparison breaks down on elaboration. One may know one's way about an elementary inconsistent calculus quite well, and be lost in a consistent one, etc.

[141] This also helps explain how Wittgenstein can talk of a 'true contradiction' (1964, pp. 178–9) and give the impression that it is alright, in certain cases, to assert contradictions.

[142] Of course a non-significance approach would normally endeavour to separate logico-semantical paradoxes from other (non-compulsory) contradictions. There is little going for the view that everyday contradictions don't make sense, except a confusion of *making sense* with *having content* construed connexively.

[143] Nor does the notion of *correctness*, or *objective determinacy*, require radical modification in the way some of Wittgenstein's remarks suggest. The discussion in the text may however be criticized, along the lines indicated in Wright (1980, pp. 310–1), for making an illicit assumption Wittgenstein would reject, to the effect that there are some underlying facts of the matter which determine correctness (including correctness of application), whereas 'for Wittgenstein, there is no Olympian standpoint from which it may be discerned who is giving the right account of the matter' (p. 311). The opposite view, that correctness can sometimes at least be discerned, is argued in EMJB chapter 11 and, for a range of logical principles, in RLR, chapter 2.

[144] Thus Wrigley, 1980; cf. also Wright, 1980, p. 310, discussed above. Note that although Wittgenstein's conventionalism is linked with construal of mathematics as a game, conventionalism as such does not entail that mathematics is a game.

[145] Goldstein 1981. We are much indebted to Goldstein for several references.

[146] But of course questions about the conventionality of the practices remain.

[147] Naturally not all worlds, and certainly not all logics, are on a par. The view is perfectly compatible with there being a unique factual world, and even One True Logic.

[148] Wright's whole discussion of inconsistent systems, such as an arithmetic containing a contradiction, turns on the erroneous assumption that such systems are trivial (see e.g. p. 306, top. p. 308, middle). He assumes without warrant that intuitive arithmetic is based on a logic with spread principles.

[149] A similar writing-in of a consistency assumption, on this occasion into the notion of *calculation*, is attempted on p. 307. This is illicit. Relevant and paraconsistent theories reveal quite straightforwardly that calculation is 'not . . . frustrated in an inconsistent system . . .'.

[150] Popper claims 'to have gone into this question . . . whether we can construct a system of logic in which contradictory statements do not entail every statement . . . and the answer is that such a system can be constructed. The system turns out, however, to be an extremely weak system. Very few of the ordinary rules of inference are left, not even the *modus ponens* . . .' (Popper, 1940, see p. 321). However Popper's claim is not only inaccurate but decidedly misleading. First, Popper's implicit claim that there is only one paraconsistent formal logic is false, as we shall see in the introduction to the next part of the book. Secondly, there are many quite strong paraconsistent systems in which *modus ponens* is valid, as we shall see there too. Thirdly, Popper's "negation" operator is not really a negation operator. It is a sort of dual to intuitionist "negation" for which $\vdash A, \sim A$ holds, but $A, \sim A \vdash A$ fails. It is therefore as Popper implicitly admits (1940, p. 323 top) a subcontrary forming operator, not a contradiction operator. To this extent, it is like da Costa's "negation" operator (see the introduction to the next part of the book, section 2.). Popper glimpsed the possibility of formal paraconsistent logic, but no more.

[151] See RLR 12.8.

[152] A tradition running from the Jains (see above) through Nozick, 1981, Introduction. The semantical analysis of discursive logics through possible worlds also enables an elegant representation of philosophical pluralism, a synthesis of different positions.

[153] See the 1981 issue of *Artificial Intelligence* devoted to non-monotonic logics, i.e. logics for which premiss augmentation fails.

[154] There are several different ways discourse can be represented logically. A non-telescopic method simply represents participants within the theory, as in the work of Hamblin and of McKenzie, for details of which see McKenzie, 1979.

[155] For base modal logics weaker than S5 the relation can be generalized to allow m-time iteration of $\Diamond$, thus DL^m. A is a thesis of DL^m iff $\Diamond^m A$ is a thesis of base logic L, where $\Diamond^m$ is a sequence of m occurrences of $\Diamond$, i.e. $\Diamond^m B = \Diamond(\Diamond^{m-1} B)$ and $\Diamond^1 B = \Diamond B$. Several results concerning such systems are summed up at the beginning of Kotas and da Costa, 1977.

[156] If Material Detachment were valid, the rule $\Diamond A, \Diamond(A \supset B) \twoheadrightarrow \Diamond B$ would be valid in S5 in which case $\Diamond(A \supset B) \,\&\, \Diamond A \supset. \Diamond B$ would be a thesis of S5, which as Jaśkowski points out (p. 150), it is not. To establish paraconsistency it is enough to find a falsifying assignment involved (in rejecting the non-thesis), which verifies $\Diamond A$ and $\Diamond{\sim}A$ and falsifies $\Diamond B$.

[157] Namely $A \rightarrow'_D B =_{df} \Diamond A \supset \Diamond B$. A problem with this is that it validates the principle $(A \rightarrow'_D B) \rightarrow'_{D}. \sim(A \rightarrow'_D B) \rightarrow'_D C$, which would violate the spirit of paraconsistency.

[158] This claim is further defended in the Introduction to Part Two. Naturally discursive logics can be based on relevant logics. Then they will reject Conjunctive Spread, but they may unnecessarily toss out Adjunction with it. Reinstating Adjunction requires a somewhat different approach to discursive logics.

[159] The departure from normality is rather like that of intuitionistic negation; just as discursive conjunction can be characterized in terms of & and $\Diamond$, so intuitionistic negation can be characterized, at least in some contexts, in terms of $\sim$ and $\Diamond$.

[160] See the end of his 1951. We do not concede that he is successful, but his arguments are of surprising weight, especially given the usual consensus opposing $\Diamond A \,\&\, \Diamond B \rightarrow \Diamond(A \,\&\, B)$. And the arguments have an interesting non-adjunctive character.

[161] For such possibility functors do not pick out the worlds of creatures' discourse, assertions, beliefs, positions, etc., in the way much literature on intensionality has supposed. The point is shortly elaborated in the text; and for much more detail, see EMJB 8.12.

[162] The following proof works in fairly minimal modal logics; since $q \supset \Diamond q$, $\Diamond q \supset \Diamond\Diamond q$ and, by the same principle, $\Box\Diamond p \supset \Diamond p$, $\Box\Diamond p \,\&\, {\sim}\Diamond\Diamond q \supset. \Diamond p \,\&\, {\sim}q$, whence, by the principle again $\Box\Diamond p \,\&\, {\sim}\Diamond\Diamond q \supset. \Diamond(\Diamond p \,\&\, {\sim}q)$. The rest of the argument turns primarily upon distribution of $\Diamond$ over $\vee$ using the equivalence $\Diamond(A \vee B) \equiv. \Diamond A \vee \Diamond B$. Since, by sentential logic, and eliminating $\Box$, $\Diamond{\sim}\Diamond p \vee \Diamond\Diamond q \vee \Diamond(\Diamond p \,\&\, {\sim}q)$, $\Diamond[{\sim}\Diamond p \vee \Diamond q \vee (\Diamond p \,\&\, {\sim}q)]$, i.e. $\Diamond[\Diamond p \supset q \supset. \Diamond p \supset \Diamond q]$.

[163] For example, the powerful condition, $TRH_1 \,\&\, TRH_2 \supset. H_1 = H_2$, i.e. there is only one world accessible from the base world T, will guarantee it, without collapsing modal logics to extensional. Obtaining an exact modelling is more difficult but can be accomplished, e.g. within the framework of relevant modal logic.

[164] There are other less direct linkages, e.g. through the dialogue-conversation picture where one of those involved (perhaps the ideal participant, who keeps a record of all decidable theorems) is a computer.

[165] Asenjo, 1954. See also Asenjo, 1966.

[166] Asenjo, 1966, perpetuates some common errors. In particular, he confuses *ex falso quodlibet* with *reductio*. A calculus of antinomies does not require, contrary to what Asenjo claims (p. 104) either the rejection of *reductio* or the rejection of Non-Contradiction.

[167] Though it is easy enough to obtain it, as in Goddard and Routley, 1973, chapter 6.

[168] Asenjo and Tamburino, 1975. This paper is largely a condensation of Tamburino 1972—largely, but not entirely, for the unexplained conditions on the comprehension axioms of the antinomic set theories given differ in perhaps significant ways.

[169] Thus da Costa's motivation compared directly with Anderson and Belnap's in arriving at the system E of entailment, where the objective was design of a system as strong as it could (reasonably) be within the classical confines, which satisfies the requirements of relevance and necessity. But Anderson and Belnap succeeded no more than da Costa did; see RLR 3.

[170] Again there is the assumption throughout that they constitute *the* logic of inconsistent systems.

[171] These are the main sets of systems, not the only ones. Arruda and da Costa also (like Popper earlier, and Fitch, 1952) studied systems where the rule of *modus ponens* fails, namely the J systems, for which see Arruda and da Costa, 1968, 1970, and 1974. An overview of da Costa's main work on paraconsistent logics and theories is given in Arruda, 1980. On da Costa's systems, see section 5.6.

[172] Da Costa, 1963a, our paraphrase. The main results of this note are recorded in da Costa, 1974.

[173] Da Costa was aware of this, since he refers to Jaśkowski when he introduces the hierarchy of systems based on C_1 as other possible solutions to the problem: there he says 'on the subject of this note see Jaśkowski...' 1963, p. 3792).

[174] R−W, and more generally relevant systems lacking Contraction and *reductio*, are examined later in the book.

[175] For details of most of these systems see Arruda, 1980.

[176] The main reason is the inclusion of full positive logic, especially the inclusion of Contraction principles, in the C systems. See especially the discussion of the Curry objection in the Introduction to Part Two.

[177] Arruda and da Costa, 1963, p. 83. The idea of a logic which permits unrestricted abstraction (almost implicit in Meinong) goes back technically at least to Moh, 1954 and ultimately to Frege. The idea, which is a good one, has since occurred to many others independently, including the authors.

[178] Neither J systems nor P systems by *any* means exhaust the types of approach they here represent.

[179] As to the first and second points see Routley and Loparic, 1978, and for elaboration, RLR 14. As to the third point, see Arruda and da Costa, 1984.

[180] Also Ackermann's rule δ of commutation (viz. $A \rightarrow . B \rightarrow C, B \twoheadrightarrow A \rightarrow C$) was removed, but *unlike* γ, δ is easily shown to be an admissible rule of E, by straightforward syntactical argument. The separation of E and Π' is (then) essentially through rule γ.

[181] The principle is challenged and rejected in RLR 3.7.

[182] It is not at all difficult to reconcile; see R. Routley and V. Plumwood, 1983. Indeed a neat synthesis can be obtained.

[183] This is so although at the same time as *Entailment* was being drafted at Pittsburgh, Asenjo's and Tamburino's work on inconsistent theories was being produced (in a different department however). There was apparently no cross-fertilization or noteworthy exchange between relevant and paraconsistent enterprises, though they were being undertaken in close proximity.

[184] See Anderson and Belnap, 1975 p. 461ff.

[185] For in dealing with Prior's family of semantical paradoxes, in 1961, Mackie worked with the assumption that assertions can be, as well as true (**T**) and false (**F**), either *paradoxical* (when **T** iff **F**, i.e. given his two-valued logical frame, when both **T** and **F**) or *vacuous* (when neither **T** nor **F**). The Liar-paradoxical assertion and assertions which logically reduce to it are paradoxical, while their

opposites, such as the Truth-teller (e.g. 'What I am now saying is true'), are vacuous or empty. But Mackie does not logically elaborate the theory indicated; he nowhere suggests what the theory immediately indicates, a 4-valued logic with values **T**, **F**, **P** or {**T**, **F**}, and **V** or { }, and would never have considered **P** a designated value. Although he contrasts "Logic", that is two-valued logic, with 'a wider sort of logic' needed 'to detect emptiness', a 'logic that pays attention to the method of deciding whether a given item is true or false (or obeyed or not obeyed, etc.) and sees whether this method is circular' (p. 241), he did not conceive of this logic as a formal logic. (Nor was such a logic seriously considered in his seminars when these ideas were developed and thrashed out, seminars in which one of the authors participated.)

And although he formulated 'some general principles that tell us when to expect emptiness and paradoxicalness' (p. 241), he took 'these principles as indicating regions in which emptiness and paradoxes are to be expected, and are to be guarded against or understood as the occassion demands' (p. 243). Any more formal guarantee against emptiness and paradox 'would need some kind of type-restriction ... but for most purposes this would be too sweeping' (p. 243). Thus he took paradoxes as items to be excluded or avoided as genuine statements, and the paradoxes as having solutions of a statement-incapability sort (an influential position at Oxford, which Mackie absorbed and helped import to Australasia). The wider logic 'has a bearing in the paradoxical cases too; for one way of expressing the solution of a paradox of the Liar type is to show that the paradoxical item is empty and therefore that the contradiction which would be generated if we took it as a genuine item does not matter' (p. 241). Although rudimentary elements of a paraconsistent position appeared, then, in Mackie's otherwise thoroughgoing empiricism, crucial elements of paraconsistency—specifically the taking of paradoxical statements to hold somewhere (to be designated in some situations)—were lacking, and were bound to be excluded by themes of empiricism (for reasons explained in EMJB 9.10).

[186] As is evident in his recent (English written and derivative) material on paradoxes included in his 1973.

[187] On both Characterization Postulates and the route therefrom to paraconsistency, see EMJB.

[188] On the I systems, see e.g. R. Routley, 1972, and for a much fuller exposition RLR chapter 7.

[189] See V. Routley, 1967, some of which was absorbed in RLR chapter 2.

[190] This was eventually published in shortened form as Routley and Routley, 1972.

[191] See the semantics of entailment series, initial papers of which appeared in the *Journal of Philosophical Logic* 1972-3 on. Detailed citations are given in RLR.

[192] Among the series of papers developing this material were Routley and Meyer, 1976, and R. Routley 1977, and 1979.

[193] Unfortunately the enormous costs of travelling between the countries has curtailed further visits.

[194] Apart from Petrov 1979, the main work from Bulgaria is indicated in papers included below.

[195] See her paper in this volume.

[196] This is in the tradition of "logic of paradoxes" and covers part of the same ground as that traversed in Priest, 1979.

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