Buddhism, Emptiness, and Paradox

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Abstract

In Mahāyāna Buddhism, and following Nāgārjuna, there is a familiar paradox, which may be put, following Candrakīrti, as: *all things have one nature, that is, no nature*. In the first half of the paper, I will explain the paradox, endorsing the thought that there is a genuine contradiction here. In the second half, I will provide a formal model of this in the paraconsistent logic second-order LP, showing that the contradictions do not spread into more mundane areas.

All things have one nature, that is, no nature. *Aṣṭasāhasrikā Prajñāpārāmitā Sūtra*

1 Introduction

When opening texts of the Buddhist philosophical canon, it is not uncommon to find contradictory statements. These can be performing many different functions: they can be parts of a *reductio* argument, a piece of poetic license, something said to engender reflection. But sometimes the contradictions are asserted and intended to understood as true.[1]

[1]See, further, Deguchi, Garfield, and Priest (2008).

One of the subjects which standardly elicits contradictions of this kind is the ultimate nature of reality (in Sanskrit, *paramārtha satya*). It does so in the thought of Nāgārjuna (fl. 1st or 2nd c. CE);[2] but such was Nāgārjuna's influence that the matter runs through all subsequent schools of Māhāyana Buddhism.[3]

In fact, ultimate reality threatens contradiction for several reasons. It is only one of these which will concern us here. This is to the effect that all there is empty, and so has no nature; but that, however, is its nature.[4]

In the first part of this essay, I will explain how and why the contradiction arises. Those who have inherited Aristotle's *horror contradictionis*, may well take the contradiction to show that the view is senseless and incoherent. To establish that it is not, in the second part of the essay I will show how it can be accommodated and made precise, using a second-order paraconsistent logic. A brief interlude between the two parts explains the central ideas of the technical machinery employed, for those who are not familiar with them. I will end the essay with a few methodological reflections. A technical appendix spells out the formal matters in full technical detail.

2 The Paradox of Emptiness

The paradox we will be concerned with is about emptiness.[5] To understand what it is, it will help to know something of the philosophical tradition before Nāgārjuna—the Abhidharma tradition.[6]

According to Abhidharma, reality is ultimately composed of certain objects termed *dharmas*. Exactly what these are was a matter of some debate, but they were standardly taken to be what are now called *tropes*. The exact details need not concern us here. The important point is that the *dharmas* were taken to have *svabhāva*. How best to translate the term is somewhat moot. Literally, *sva/bhāva* means self/being. The word is often translated as *essence*, though this is somewhat problematic because of its Aristotelian associations. In the present context, *nature* seems as good a translation as

[2]On Nāgārjuna, see Westerhoff (2022).

[3]This is tracked in East Asian Buddhist philosophy in Deguchi, Garfield, Priest, and Sharf (2021).

[4]Another is the closely connected contradiction that ultimate reality is both ineffable and effable. See Garfield and Priest (2003), esp. 16.7 of the reprint.

[5]Garfield and Priest (2003) term it *Nāgārjuna's Paradox.*

[6]On which, see Ronkin (2022).

any. The point is that each *dharma* was taken to be what it was, in and of itself, and so independently of anything else. One might say that each was a metaphysical atom.

Spinning off a whole new slate of sūtras, the *Prajñāpārāmitā* (Perfection of Wisdom) *Sūtras*, Nāgārjuna launched an attack on the Abhidharma picture in his *Mūlamadhyamakakārikā* (*Fundamental Verses of the Middle Way*).[7] In this, he argued that everything was ultimately empty (*śūnya*); and what it was empty *of* was *svabhāva*. Every thing is what it is, not in and of itself, but in virtue of its relationships to other things.[8] For him, then, every thing has no nature. How successful Nāgārjuna's arguments were, we need not discuss. They were certainly contentious, as can be inferred from the fact that he wrote another text, *Vigrahavyāvartanī* (*Dispeller of Disputes*)[9] to answer his critics. However, his view was integrated into all subsequent Mahāyāna Buddhisms.

So according to Nāgārjuna, things have no nature. The rub is that emptiness (*śūnyatā*) is the very essence of things, their nature. So objects *do* have a nature. As Garfield and I put it:[10]

> ...since all things are empty, all things lack any ultimate nature; and this is a characterisation of what things are like from the ultimate perspective. Thus, ultimately, things are empty. But emptiness is, by definition, the lack of any essence or ultimate nature. Nature, or essence, is just what empty things are empty of. Hence, ultimately, things must lack emptiness. To be ultimately empty is, ultimately, to lack emptiness. In other words, emptiness is the nature of all things; in virtue of this, they have no nature, not even emptiness.

It might be suggested that emptiness is a property of things, but not one which gives their nature. But things do not simply *happen* to be empty, as some things happen to be painted blue. Nāgārjuna's arguments are designed to show that all things cannot but be empty, that there is no other mode of existence of which they are capable. It is part of the very nature of

[7] See Garfield (1995) and Priest (2013).

[8] Notably, its parts, its causes—and maybe effects—and concepts; again, this need not concern us here.

[9] Bhattacarya, Johnston, and Kunst (1978).

[10] Garfield and Priest (2003), 16.7 of reprint.

phenomena *per se*. As Candrakīrti (600-650),[11] one of the most influential commentators on Nāgārjuna, puts it:[12]

> As it is said in the great *Ratnakūta Sūtra*, 'Things are not empty because of emptiness; to be a thing is to be empty. Things are not without defining characteristics through characteristiclessness; to be a thing is to be without a defining characteristic ... whoever understands things in this way, Kāśyapa, will understand perfectly how everything has been explained to be in the middle path'.

Emptiness is, then, a nature.[13]

So things are empty of all nature; and that is their nature. In fact, the claim that things are thus contradictory is to be found in the earliest Mahāyāna sūtras. Thus, we have in the *Aṣṭasāhasrikā Prajñāpārāmitā Sūtra*:[14]

> By their nature, the things are not a determinate entity. Their nature is a non-nature; it is their non-nature that is their nature. For they have only one nature, i.e., no nature.

Here, then, is our contradiction: *all things have one nature, that is, no nature.*

3 Interlude on Paraconsistent Logic

In the next part of the essay, I want to show how one may understand this contradiction and its truth using some standard tools of paraconsistent logic. Since these may be unfamiliar to some people—indeed the very tools of formal logic may be unfamiliar—let me try to explain matters gently. (A formal specification of the semantics can be found in the Appendix to this paper.) The logic I shall describe is *LP* (with an added conditional, making

[11] On Candrakīrti, see Hayes (2023), §5.

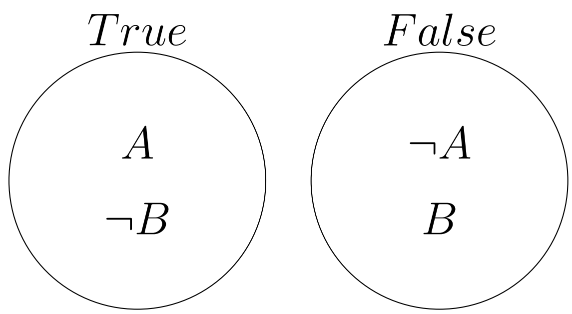
[12] *Prasannapadā*, ch. 13; trans. by Garfield from 83b-84a of the Tibetan Canon. A looser translation is given by Sprung (1979), p. 248. The *Ratnakūta Sūtra* is actually a compendium of 49 separate early Mahāyāna sūtras.

[13] Sometimes *svabhāva* is translated as *intrinsic nature*, that is, a property that something would have even if there were nothing else. It can also be shown that emptiness is an intrinsic nature in this sense. See Priest (2014), 13.7.

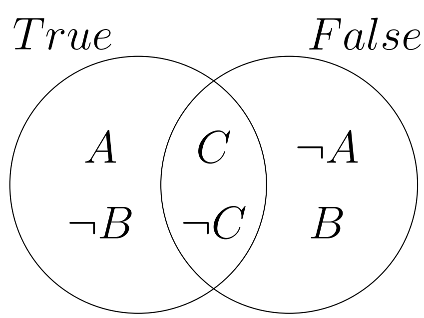
[14] Bhattacharya, Johnston, and Kunst (1978), p. 23.

it RM_3). There are several other paraconsistent logics which could be used, but LP is arguably the simplest and most natural paraconsistent logic.

In so-called classical logic (that is, the logic invented by Frege and Russell at the turn of the 20th Century), every situation partitions sentences into two sets which are exclusive and exhaustive: those that are true (in the situation) and those that are false (in the situation). Negation toggles a sentence between the two: if A is true, $\neg A$ is false; and if B is false, $\neg B$ is true. So we have:

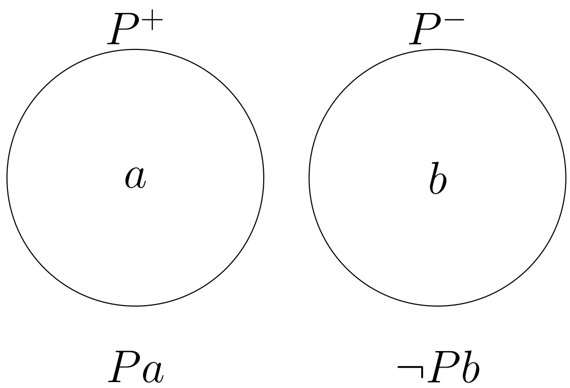


A paraconsistent logic allows for some contradictions to be true (in a situation). That is, it allows for some things to be both true and false. In fact, things are exactly the same as in classical logic, except that in some situations, truth and falsity may overlap, thus:

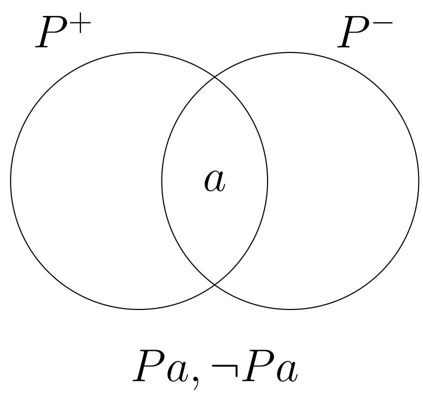


If C is true and false (and so in the lens-shaped overlap), then $\neg C$ is false and true; that is, in the same lens shape.

The simplest sentences of the language have the form of a predication, Pa (a is P). A predicate, P, has an extension, P^+, and an anti-extension, P^-. The first of these is the set of things of which the predicate is true. The second is the set of things of which it is false. In classical logic, the extension and anti-extension are exclusive and exhaustive, thus:



But as one would expect, in a paraconsistent logic, in some situations the extension and anti-extension overlap:



Turning to quantifiers, these work the same way in classical logic and LP:

- $\exists x Px$ is true if P is true of some object in the domain.
- $\exists x Px$ is false if P is false of all objects in the domain
- $\forall x Px$ is true if P is true of all objects in the domain
- $\forall x Px$ is false if P is false of some object in the domain

Finally, since this is to be the second-order version of the, we have two domains of quantification, $\mathcal{D}_1$ and $\mathcal{D}_2$. $\mathcal{D}_1$ is the range of lower case variables, and is the domain of objects. $\mathcal{D}_2$ is the range of upper case variables, and is the domain of properties. The members of $\mathcal{D}_2$ have an extension and and anti-extension, each of which is a subset of $\mathcal{D}_1$. In classical logic, these two are exclusive and exhaustive. In a paraconsistent logic they may overlap.

This, I hope, is enough to provide some understanding of the technical construction of the next section.

4 Articulating the Paradox of Emptiness

In this section I will give a simple logical interpretation which verifies the paradox of emptiness (but by no means all contradictions). I do this informally. Full technical details can be found in the Appendix to the paper.

To express the paradox, we need the following non-logical vocabulary. Natures are properties (though not all properties are natures). So we need:

- A first-order predicate, Ex, expressing the fact that x is empty
- A second-order predicate, $\mathcal{N}Y$, expressing the fact that Y is a nature

These must satisfy the paradoxical conditions:

[1] $\forall x(Ex \leftrightarrow \neg\exists Y(\mathcal{N}Y \wedge Yx))$ [To be empty is to have no nature]

[2] $\forall x(\mathcal{N}E \wedge Ex)$ [Emptiness is a nature of all things]

[3] $\forall Y\forall x((\mathcal{N}Y \wedge Yx) \rightarrow Y = E)$ [And their only nature]

[4] $\neg\mathcal{N}E$ [Emptiness is not a nature.]

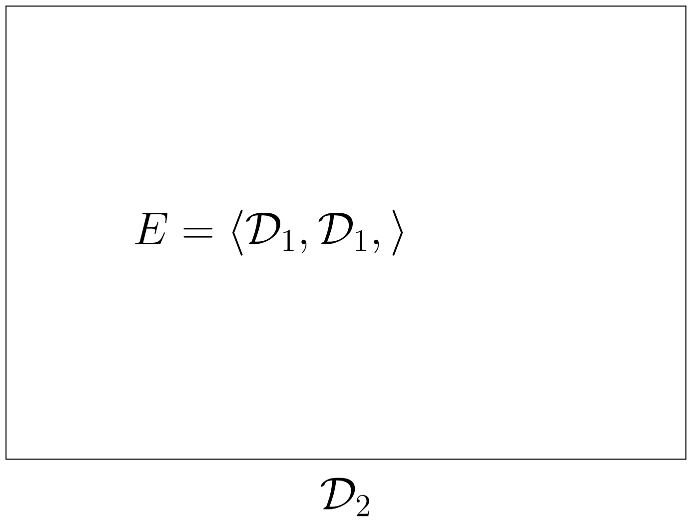
[2] and [4] are our contradiction. But note that [1] and [2] already deliver contradiction. By [2], for any x, $\mathcal{N}E \wedge Ex$, so $\exists Y(\mathcal{N}Y \wedge Yx)$. That is, by [1], $\neg Ex$, contradicting [2].

The model conditions we require are as follows.

- $E^+ = E^- = \mathcal{D}_1$
- $E \in \mathcal{N}^+ \cap \mathcal{N}^-$
- if $D \in \mathcal{N}^+$ then $D = E$ or $D^+ = \emptyset$

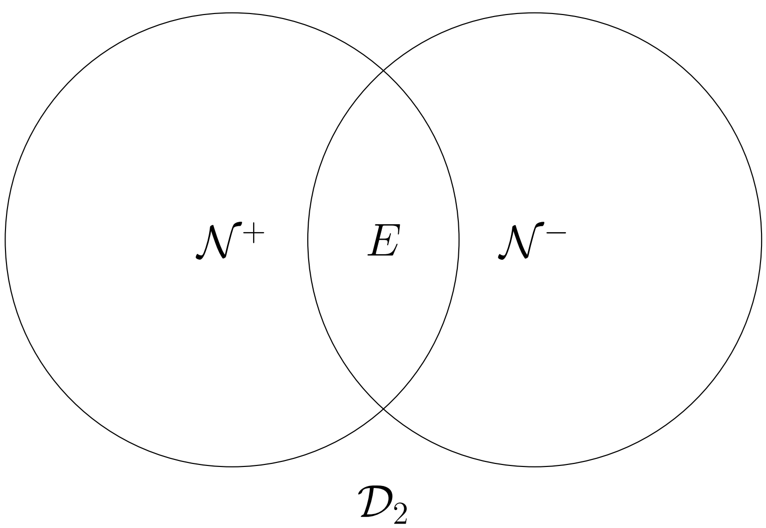
These may be illustrated as follows:

$E^+ = E^- = \mathcal{D}_1$:



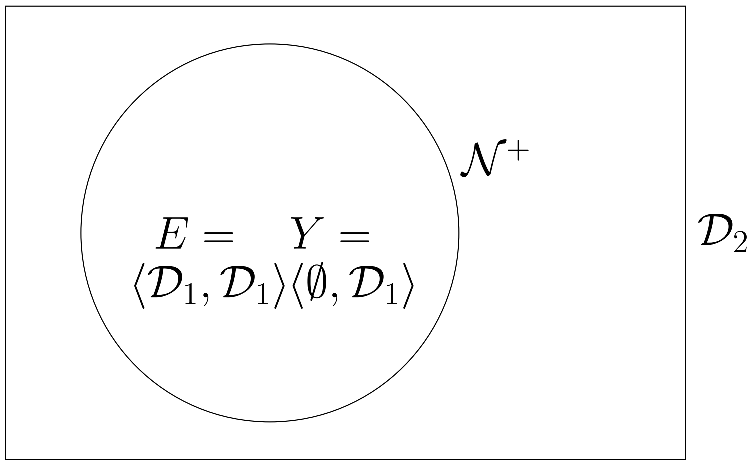
All objects are empty and not empty.

$E \in \mathcal{N}^+ \cap \mathcal{N}^-$:



Emptiness both is and is not a nature.

If $D \in \mathcal{N}^+$ then $D = E$ or $D^+ = \emptyset$:



If something is a nature, it is either E or something which nothing satisfies.

It is not difficult to show that these conditions verify all of [1]–[4]. However, I leave the details to the technical Appendix of the paper.

Note that any predicate other than $\mathcal{N}$ and E may be a classical predicate. (That is, its extension and anti-extension are disjoint.) So true contradictions do not have to spread beyond the reach of $\mathcal{N}$ and E. In particular, if all the other predicates are classical, any purely first-order sentence not containing E behaves consistently.

5 Conclusion: Some Methodological Reflections

The model of the previous section shows that the paradox of emptiness is mathematically as sensible and coherent as it can be. Of course, that does not show that the contradiction is actually true. To do that, one would have to engage with the arguments of Nāgārjuna which deliver it.

Naturally, the reconstruction of the paradox is anachronistic. Nothing like the logical tools I have used were in the repertoire of the philosophers we have met (or of any other at the time). However, that does not make their use illicit, any more than using the tools of 19th century mathematics to articulate Newton's 17th century theory of gravity and dynamics; or does using the tools of modern logic to analyse the ontological arguments for the existence of God put forward by Medieval and early Modern philosophers. We now just have better mathematical tools than these thinkers—or our Buddhist philosophers—did. Our Mahāyāna philosophers had no hesitation in using new ideas to articulate what they took to be the insights of the tradition that they inherited. There can be no principled objection to later generations doing the same.

The use of the formal machinery *would* be objectionably anachronistic if its deployment actually deformed the views in question, twisting them into something entirely different. But I see no good reason to suppose that it does so. The formal machinery used ($\mathcal{N}$, E, $\neg$, etc.) just makes precise the notions employed by our Buddhist philosophers. In exactly the same way, the contemporary machinery of formal arithmetic provides a tighter understanding of the counting machinery they used. Indeed, just as the apparatus of formal arithmetic gives us a better understanding of this counting machinery, the apparatus of formal logic can give us a better understanding of the conceptual machinery deployed in this Buddhist philosophical discourse.

If Nāgārjuna were reborn in the 21st century and learned the techniques of contemporary logic, I have no doubt that he would be delighted with them. They would provide a powerful tool in his armory to be used against his critics—both then and now.

6 Appendix: Formal Details

In this appendix I give the full formal details of the construction for those who may be interested.

6.1 The Logic LP/RM_3

Let us start with the logic of second-order LP/RM_3 and its semantics[15] The language is the standard language of second-order logic, though we will

[15]For LP, see Priest (1987), ch. 5, and for the second-order case, Priest (2002), 7.2. For RM_3, see Priest (2008), 7.4. A more realistic conditional (e.g., a relevant conditional) could be used in the present case, though it would make the semantics more complicated. The present conditional is adquate for our purposes.

assume (as is often not the case) that there are second-order predicates, including identity. To keep matters simple, there are no function symbols, and all predicates other than the identity predicate are monadic. All second-order variables are also monadic.

An interpretation for the language is a structure, $\langle \mathcal{D}_1, \mathcal{D}_2, \mathcal{D}_3, \delta, +, - \rangle$, where $\mathcal{D}_1$, $\mathcal{D}_2$, and $\mathcal{D}_3$ are non-empty sets (of objects, properties of objects, and properties of properties). $+$ and $-$ are functions such that:

- if $D \in \mathcal{D}_2$, $D = \langle D^+, D^- \rangle$, where $D^+ \cup D^- = \mathcal{D}_1$
- if $D \in \mathcal{D}_3$, $D = \langle D^+, D^- \rangle$, where $D^+ \cup D^- = \mathcal{D}_2$

D^+ and D^- are the extension and anti-extension of D. For δ:

- $\delta(t) \in \mathcal{D}_1$, for any term, t
- $\delta(P) \in \mathcal{D}_2$, for any first-order predicate, P
- $\delta(\mathcal{P}) \in \mathcal{D}_3$, for any (monadic) second-order predicate, $\mathcal{P}$

If we write $\Vdash^+$ for truth, and $\Vdash^-$ for falsity, the truth and falsity conditions are:

- $\Vdash^+ Pt$ iff $\delta(t) \in (\delta(P))^+$
- $\Vdash^- Pt$ iff $\delta(t) \in (\delta(P))^-$

- $\Vdash^+ \mathcal{P}P$ iff $\delta(P) \in (\delta(\mathcal{P}))^+$
- $\Vdash^- \mathcal{P}P$ iff $\delta(P) \in (\delta(\mathcal{P}))^-$

- $\Vdash^+ P = Q$ iff $\delta(P) = \delta(Q)$
- $\Vdash^- P = Q$ iff $\delta(P) \neq \delta(Q)$

- $\Vdash^+ \neg A$ iff $\Vdash^- A$
- $\Vdash^- \neg A$ iff $\Vdash^+ A$

- $\Vdash^+ A \wedge B$ iff $\Vdash^+ A$ and $\Vdash^+ B$
- $\Vdash^- A \wedge B$ iff $\Vdash^- A$ or $\Vdash^- B$

- $\Vdash^+ A \vee B$ iff $\Vdash^+ A$ or $\Vdash^+ B$
- $\Vdash^- A \vee B$ iff $\Vdash^- A$ and $\Vdash^- B$

For the conditional:

- $\Vdash^+ A \to B$ iff (if $\Vdash^+ A$ then $\Vdash^+ B$) and (if $\Vdash^- B$ then $\Vdash^- A$)
- $\Vdash^- A \to B$ iff $\Vdash^+ A$ and $\Vdash^- B$

For quantifiers, to keep matters simple, we take every member of $\mathcal{D}_1$ and $\mathcal{D}_2$ to be names of themselves. $A_x(d)$ is A with every free occurrence of x replaced by d. The comments apply *mutatis mutandis* to second-order matters.

- $\Vdash^+ \forall x A$ iff, for all $d \in \mathcal{D}_1$, $\Vdash^+ A_x(d)$
- $\Vdash^- \forall x A$ iff, for some $d \in \mathcal{D}_1$, $\Vdash^- A_x(d)$

- $\Vdash^+ \exists x A$ iff, for some $d \in \mathcal{D}_1$, $\Vdash^+ A_x(d)$
- $\Vdash^- \exists x A$ iff, for all $d \in \mathcal{D}_1$, $\Vdash^- A_x(d)$

- $\Vdash^+ \forall X A$ iff, for all $D \in \mathcal{D}_2$, $\Vdash^+ A_X(D)$
- $\Vdash^- \forall X A$ iff, for some $D \in \mathcal{D}_2$, $\Vdash^- A_X(D)$

- $\Vdash^+ \exists X A$ iff, for some $D \in \mathcal{D}_2$, $\Vdash^+ A_X(D)$
- $\Vdash^- \exists X A$ iff, for all $D \in \mathcal{D}_2$, $\Vdash^- A_X(D)$

Note that there are no third-order quantifiers.

Validity is defined in the standard way, as truth preservation in all interpretations:

- $\Sigma \models A$ iff for every interpretation, if $\Vdash^+ B$ for every $B \in \Sigma$, $\Vdash^+ A$

6.2 The Model

Our conditions to be modelled are:

[1] $\forall x(Ex \leftrightarrow \neg\exists Y(\mathcal{N}Y \wedge Yx))$

[2] $\forall x(\mathcal{N}E \wedge Ex)$

[3] $\forall Y\forall x((\mathcal{N}Y \wedge Yx) \rightarrow Y = E)$

[4] $\neg\mathcal{N}E$

To model these conditions, we choose an interpretation such that:

- $(\delta(E))^+ = (\delta(E))^- = \mathcal{D}_1$
- $\delta(E) \in (\delta(\mathcal{N}))^+ \cap (\delta(\mathcal{N}))^-$
- if $D \in (\delta(\mathcal{N}))^+$ then $D = \delta(E)$ or $D^+ = \emptyset$ (so $D^- = \mathcal{D}_1$)

Other information can be filled in as one wishes.

In verifying the conditions, [2], and [4] are immediate.

For [1]: for any $d \in \mathcal{D}_1$, $\Vdash^+ Ed$ and $\Vdash^- Ed$, so both truth and falsity are preserved from right to left. We show that for any $d \in \mathcal{D}_1$:

[5] $\Vdash^+ \exists Y(\mathcal{N}Y \wedge Yd))$

[6] $\Vdash^- \exists Y(\mathcal{N}Y \wedge Yd))$

So truth and falsity are also preserved from left to right. $\Vdash^+ \mathcal{N}E \wedge Ed$, so [5] holds. For any $D \in \mathcal{D}_2$, $\Vdash^+ \mathcal{N}D$ or $\Vdash^- \mathcal{N}D$. In the first case, $\Vdash^- Dd$. So in either case, $\Vdash^- \mathcal{N}D \wedge Dd$. So [6] holds.

For [3]: suppose that for some $d \in \mathcal{D}_1$, $D \in \mathcal{D}_2$, $\Vdash^+ \mathcal{N}D$ and $\Vdash^+ Dd$, then $D = \delta(E)$. Hence truth is preserved forward. But, for any $D \in \mathcal{D}_2$, $\Vdash^- \mathcal{N}D$ or $\Vdash^+ \mathcal{N}D$. In the second case, for any $d \in \mathcal{D}_1$, $\Vdash^- Dd$. So in both cases $\Vdash^- \mathcal{N}D \wedge Dd$. So falsity is preserved backwards.

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