Chapter 10
Realism, Antirealism, and Paraconsistency

Graham Priest

10.1 Introduction

The debate between realists and antirealists (about various topics) has occasioned an enormous literature in the last 35 years.[1] Usually this is carried out in terms of the contrast between classical and intuitionist logic. Intuitionist logic is not, of course, the only non-classical logic.[2] Another important class of such logics comprises paraconsistent logics. How do they fit into the debate? This note answers the question. Paraconsistency, as such, is neutral to the debate, in the sense that there are paraconsistent logics that are as unfriendly to antirealism as classical logic; and there are paraconsistent logics that are as susceptible to an antirealist understanding as intuitionist logic. I will show this by considering just one family of paraconsistent logics: those that have a binary relational semantics.

10.2 Classical vs. Intuitionist Logic

The heart of the realist/antirealist debate about some matter concerns whether sentences about it are to be given truth conditions where the notion of truth in question is verification-transcendent. Thus, consider classical propositional logic. We may suppose that the language contains the connectives $\vee$, $\wedge$, $\supset$, and $\neg$. An evaluation is a function, ν, which assigns a truth value (1 or 0) to every propositional parameter. This is then extended to such a map for all formulas by the familiar conditions. For all formulas, A and B:

$$\nu(A \vee B) = 1 \text{ iff } \nu(A) = 1 \text{ or } \nu(B) = 1$$

[1] For a gentle introduction, see [11, chap. 8].

[2] For an introduction to non-classical logics, see Priest [6, 9].

G. Priest (✉)
Departments of Philosophy, University of Melbourne, Melbourne, VIC, Australia

University of St Andrews, St Andrews, UK

The Graduate Center, City University of New York, New York, NY, USA
e-mail: g.priest@unimelb.edu.au

S. Rahman et al. (eds.), *The Realism-Antirealism Debate in the Age of Alternative Logics*, Logic, Epistemology, and the Unity of Science 23,
DOI 10.1007/978-94-007-1923-1_10,

$\nu(A \wedge B) = 1$ iff $\nu(A) = 1$ and $\nu(B) = 1$
$\nu(A \supset B) = 1$ iff $\nu(A) = 0$ or $\nu(B) = 1$
$\nu(\neg A) = 1$ iff $\nu(A) = 0$

Validly is defined as terms of truth preservation in all evaluations.

The truth conditions for negation are such that $\neg A$ is true simply if A fails to be true. Generally speaking, we may have no way of telling of a sentence that it fails to be true. We would appear to have no way of knowing, for example, whether 'There are unicorn-like creatures at some place and time in the cosmos', fails to be true. The truth condition of negation are therefore verification-transcendent. More generally, the logic verifies the Law of Excluded Middle (LEM): for all A, $A \vee \neg A$ is valid. Yet for some As we may have no way of verifying either A or $\neg A$.

Compare this with intuitionist logic. This can be given a familiar Kripke-style semantics.[3] An interpretation is a structure $\langle W, R, \nu\rangle$. W is a non-empty set of worlds. These are to be thought of as states of information. Each contains the things that have been verified at a certain stage. That is:

$\nu_w(A) = 1$ iff A is verified at w

R is a binary accessibility relation on the worlds. $w_1 R w_2$ means that w_2 is a possible state of information obtained from w_1 by adding some number (possibly zero) of verifications. It is therefore reflexive and transitive. ν is a map which assigns a truth value, $\nu_w(p)$ (1 or 0), to each propositional parameter, p, at each world, w. There is a heredity constraint:

if $\nu_{w_1}(p)$ and $w_1 R w_2$ then $\nu_{w_2}(p)$

What is verified stays verified. (Given the truth conditions for the connectives, this extends to all formulas.) The truth conditions for the connectives are given as follows:

$\nu_w(A \vee B) = 1$ iff $\nu_w(A) = 1$ or $\nu_w(B) = 1$
$\nu_w(A \wedge B) = 1$ iff $\nu_w(A) = 1$ and $\nu_w(B) = 1$
$\nu_w(A \supset B) = 1$ iff for all w' such that wRw', $\nu_{w'}(A) \neq 1$ or $\nu_{w'}(B) = 1$
$\nu_w(\neg A) = 1$ iff for all w' such that wRw', $\nu_{w'}(A) = 0$

Validity is defined in terms of truth preservation at all worlds of all interpretations.

The truth conditions for negation say that $\neg A$ holds at a world if at no further worlds A holds. Intuitively, the only way for this to happen is for us to have a verification that A will never be verified. (If we have such a verification, A will never be verified. Conversely, if we have no such verification, then there is a possible

[3] See [6, §6.3].

future in which A is verified.) Hence, the semantics give a plausible representation of the fact that the truth conditions of negation may be understood in terms of verification.

Similarly, the truth conditions of the other connectives can be thought of in terms of verification. A conjunction is verified iff both conjuncts are. A disjunction is verified iff a disjunct is. The truth conditions for the conditional say that we will never have A without B. The only way for this to happen is for us to have a construction that turns verifications of A into verifications of B. (If we have such a construction, any verification of A may be turned straightforwardly into a verification of B. Conversely, if we have no such construction, then there is a possible future in which A is verified but not B.)

10.3 The Logic of Constructible Negation

Let us now consider the logic of constructible negation, N_3, first proposed by Nelson [3]. This is essentially intuitionist logic with a different account of negation. It can be given a Kripke-style semantics as follows. An interpretation is a structure $\langle W, R, \rho\rangle$. W and R are as for intuitionist logic. ρ is a world-indexed relation between propositional parameters and $\{1, 0\}$, subject to the constraint that for every propositional parameter, p, and world, w:

Exclusion: it is not the case that $p\rho_w 1$ and $p\rho_w 0$

Nothing is both true and false. (And given the truth/falsity conditions for the connectives, this extends to all formulas.) Since truth and falsity are not independent, two heredity conditions are required. For every propositional parameter, p:

if $p\rho_{w_1}1$ and $w_1 R w_2$ then $p\rho_{w_2}1$
if $p\rho_{w_1}0$ and $w_1 R w_2$ then $p\rho_{w_2}0$

Again, given the truth conditions for the connectives, this extends to all formulas. The truth/falsity conditions for the connective are as follows:

$(A \vee B)\rho_w 1$ iff $A\rho_w 1$ or $B\rho_w 1$
$(A \vee B)\rho_w 0$ iff $A\rho_w 0$ and $B\rho_w 0$

$(A \wedge B)\rho_w 1$ iff $A\rho_w 1$ and $B\rho_w 1$
$(A \wedge B)\rho_w 0$ iff $A\rho_w 0$ or $B\rho_w 0$

$(A \supset B)\rho_w 1$ iff for all w' such that wRw', it is not the case that $A\rho_{w'} 1$ or $B\rho_{w'} 1$
$(A \supset B)\rho_w 0$ iff $A\rho_w 1$ and $B\rho_w 0$

$\neg A\rho_w 1$ iff $A\rho_w 0$
$\neg A\rho_w 0$ iff $A\rho_w 1$

Validity is defined, as usual, in terms of truth preservation at all worlds of all interpretations.

As far as the positive logic goes, this is just the same as intuitionist logic. The semantics vary only in their handling of negation.[4] The question is whether this makes good antirealist sense. In particular, what are we to make of the fact that $A\rho_w 0$?

The obvious answer is that, just as $A\rho_w 1$ means that A is verified at w, so $A\rho_w 0$ means that A ia falsified. Does the antirealist have an independent notion of falsification? The answer is 'yes'. There are ways of showing that something is false directly. Perception can be such a method. One can see directly that something is *not* the case. For example, when you enter a room you can see that Pierre is not there. You do not have to see that the things in the room are a table, a chair, ... and reason: Pierre is not a table, Pierre is not a chair, ... therefore Pierre is not in the room.[5] Moreover, if one is not tied to intuitonist logic, one can give a *direct* proof of a negated sentence. And given the fact that verification *and* falsification make perfectly good independent sense, then the logic of constructible negation makes perfectly good antirealist sense. Modulo this change, we tell exactly the same story as for intuitionist logic.

Given the understanding of direct falsification, the falsity conditions for the connectives are straightforward, with the exception of those for the conditional. Generally speaking, it is less than clear when one should take a conditional to be false. The Nelson conditions say that a conditional, $A \supset B$, is false iff there is an actual counter-example, $A \wedge \neg B$. It is certainly plausible that this is a sufficient condition. It is less clear that this is necessary.

Unlike intuitionist logic, N_3—and all the logics in the same family that we will go on to note—verifies both halves of the Law of Double Negation, and especially $\neg\neg A \models A$. From the antirealist perspective that N_3 provides, the failure of this in intuitionist logic is entirely an artifact of its semantic one-sidedness. Contraposition, on the other hand, fails: $A \rightarrow B \not\models \neg B \rightarrow \neg A$. Truth preservation forward does not guarantee falsity preservation backwards.

Note that it is possible to have a second negation in the language, which behaves as does intuitionist negation. Thus, suppose that there is a constant, $\perp$, that is false everywhere: an intuitionist-like negation may be defined as $A \supset \perp$. Then $A \supset \perp$ is true at w if A is true at no accessible world, and $A \supset \perp$ is false at w if A is true at w. We may think of $\neg A$ as expressing a direct falsification, and $A \supset \perp$ as expressing an inferential one. $\neg A$ is stronger than $A \supset \perp$. If $\neg A$ holds at w, it holds at all accessibile worlds, by heredity. Hence A fails at all accessible worlds, that is, $A \supset \perp$ is true at w: $\neg A \models A \supset \perp$. But it is quite possible to have $A \supset \perp$ true at

[4] On all this, see [9, §9.7a].

[5] This and other examples are discussed in [8, chap. 3] (esp. 3.5).

w, that is, A true at no accessible world, without having $\neg A$ true at w ($\neg A$ may, in fact, fail at all accessible worlds): $A \supset \bot \nvDash \neg A$.[6]

10.4 Paraconsistency

N_3 is not a paraconsistent logic. In it, contradictions still entail everything. However, we get a paraconsistent logic, N_4, simply by dropping the Exclusion constraint.[7] In this, contradictions do not imply everything. Can we make sense of this liberalisation from an antirealist perspective?

In N_4, truth and falsity at a world, that is, verification and falsification, have *complete* independence. That one may be in a position to verify neither A nor $\neg A$, that is, in a position neither to verify nor to falsify A, is standard antirealist fare. The thought that one might be in a position to verify both A and $\neg A$, that is, in a position both to verify and to falsify A, is more radical. Yet it makes perfectly good sense. In some paradoxes, such as Berry's, for example, one can give a verification (direct proof) of some claim and a falsification (direct proof of the negation) of it. For another example, there are many terms in science that are multi-criterial; that is, for which we have more than one criterion for applying them. Obvious examples are temperature terms. That the fluid in some beaker has a temperature of 20°C can be verified by both a correctly functioning mercury thermometer and a correctly functioning electro-chemical thermometer. However, there is no reason a priori why these two criteria should hang together. It could be the case that, by one criterion, the fluid has a temperature of 20°C; yet by the other it has a temperature of 21°C, and so it does not have a temperature of 20°C. Arguably, there are places in the history of science where exactly this divergence of criteria has happened.[8] N_4, therefore, is a perfectly acceptable antirealist logic.[9]

These considerations show, incidentally, not only that there are paraconsistent logics that have antirealist interpretations, but that dialetheism is also quite compatible with antirealism. In the situation explained, A is both verified and falsified; that is, both A and $\neg A$ are true.

There is another logic with constructible negation in the vicinity of N_3 and N_4. Start with N_4, and augment this with the constraint which is the dual of Exclusion. For every propositional parameter, p, and world, w:

Exhaustion: either $p\rho_w 1$ or $p\rho_w 0$

---

[6] In N_4, which we shall meet in the next section, the inference in both directions fails. The fact that $\neg A$ is true at a world does not entail that A is not true there.

[7] N_4 was first proposed by Almukdad and Nelson [1]. N_3 and N_4 are discussed in [13], and also in [9, §9.7a], where they are called L_3 and L_4.

[8] The point is made in [4, chap. 1] and further discussed in [10, §2.II.i].

[9] Rumfitt [12] argues for treating truth and falsity even-handedly, in the way required by N_3 and N_4. He does so by analysing falsity in terms of a primitive notion of rejection. This will do for N_3, but not for N_4, which would require one the be able to simultaneously accept and reject something.

If all else remains the same, this does not guarantee that the condition carries over to all sentences. (The induction proof breaks down in the case for the conditional). To ensure that is does, we have to change the falsity conditions for conditional to:

$$(A \supset B)\rho_w 0 \text{ iff for some } w' \text{ such that } wRw',\ A\rho_{w'}1 \text{ and } B\rho_{w'}0$$

Note that with these falsity conditions, the heredity condition no longer holds for all formulas, though it does hold for positive (negation-free) formulas.[10] Call the resulting logic, M.

As may be seen, it verifies the LEM. M is a paraconsistent logic, but not one that is acceptable to an antirealist since it verifies the LEM.

It should also be noted that the $\vee$-$\wedge$-$\neg$ fragments of N_4, M, and N_3 are the well known many-valued logics FDE, LP, and K_3, respectively.[11] (In these, the world structure becomes, in fact, irrelevant.) The first two of these are paraconsistent, but the second is ruled out for antirealist purposes because it verifies the LEM. FDE, however, is a perfectly acceptable paraconsistent antirealist logic.

10.5 Quantified Intuitionist Logic

So far, we have considered only propositional logics. Do the considerations carry over once we add quantifiers?

A Kripke interpretation for first order intuitionist logic is a structure $\langle W, R, D, \nu\rangle$. W and R are as in the propositional case. For every $w \in W$, D_w is a set of objects, subject to the constraint that:

$$\text{if } w_1 R w_2 \text{ then } D_{w_1} \subseteq D_{w_2}$$

The domain contains all those things we have constructed at that stage. We may construct new objects later, but what has been constructed stays constructed. For every constant, c, in the language, $\nu(c) \in D_w$ for all $w \in W$; and for every n-place predicate, P, and $w \in W$, $\nu_w(P) \subseteq D_w^n$, subject to the constraint that:

$$\text{if } w_1 R w_2 \text{ then } \nu_{w_1}(P) \subseteq \nu_{w_2}(P)$$

which is now the appropriate form of the heredity constraint.

Truth values $(1, 0)$ at worlds are assigned to atomic formulas by the conditions:

$$\nu_w(Pa_1 \ldots a_n) = 1 \text{ iff } \langle \nu(a_1), \ldots, \nu(a_n)\rangle \in \nu_w(P)$$

---

[10] This means that the logic is not closed under uniform substitution. Closure can be regained by dropping the heredity condition for propositional parameters. This produces a system almost identical to that of [5, chap. 6]. The only difference is in the properties of the accessability relation.

[11] See [6, chaps. 7 and 8].

The truth conditions for the connectives are as in the propositional case. For the quantifiers:

$\nu_w(\exists x A) = 1$ iff for some $d \in D_w$, $\nu_w(A_x(k_d))$
$\nu_w(\forall x A) = 1$ iff for all w' such that wRw' and all $d \in D_{w'}$, $\nu_w(A_x(k_d))$

Here, we take the language to be augmented by a set of constants, k_d, such that k_d denotes d, and $A_x(c)$ is A with all free occurrences of x replaced by c. Note that the truth conditions for $\exists$ relate to just the instances at the world at issue, whilst those for $\forall$ relate to both it and all its future worlds.

As in the propositional case, one can show that:

if $w_1 R w_2$ and $\nu_{w_1}(A) = 1$ then $\nu_{w_2}(A) = 1$

Validity is defined, as in the propositional case, in terms of truth preservation at all worlds of all interpretations.[12]

These truth conditions naturally capture an appropriate antirealist understanding of the quantifiers. $\exists x A$ is verified at a stage just if some instance is. And $\forall x A$ is verified if every instance is verified whatever we go on to construct. Intuitively, this can happen only if we have a construction that applies to any object we come up with, d, to give a proof of $A_x(k_d)$. (If there is such a construction, then whatever object we construct at a later time, there will be a proof that it satisfies A. Conversely, if there is no such construction, then there is a possible development in which we find an object for which there is no proof.)

10.6 Quantified Logics of Constructible Negation

A first order version of N_4 is obtained from its propositional logic as for intuitionist logic. An interpretation is a structure $\langle W, R, D, \nu \rangle$. Interpretations are the same as for intuitionism, except that for every world, w, $\nu_w(P) = \langle E, A \rangle$, where $E, A \subseteq D_w^n$. (E and A are the extension and antiextension of P at w. I will write them as $\nu_w^+(P)$ and $\nu_w^-(P)$, respectively.) We now need a double heredity constraint:

if $w_1 R w_2$ then $\nu_{w_1}^+(P) \subseteq \nu_{w_2}^+(P)$
if $w_1 R w_2$ then $\nu_{w_1}^-(P) \subseteq \nu_{w_2}^-(P)$

And the relation ρ is defined in the natural way:

$Pa_1 \ldots a_n \rho_w 1$ iff $\langle \nu(a_1), \ldots, \nu(a_n) \rangle \in \nu_w^+(P)$
$Pa_1 \ldots a_n \rho_w 0$ iff $\langle \nu(a_1), \ldots, \nu(a_n) \rangle \in \nu_w^-(P)$

12 See [9, chap. 20].

The truth/falsity conditions for the connectives are as in the propositional case. For the quantifiers:

$\exists x A\rho_w 1$ iff for some $d \in D_w$, $A_x(k_d)\rho_w 1$
$\exists x A\rho_w 0$ iff for all w' such that wRw' and all $d \in D_{w'}$, $A_x(k_d)\rho_{w'} 0$
$\forall x A\rho_w 1$ iff for all w' such that wRw' and all $d \in D_{w'}$, $A_x(k_d)\rho_{w'} 1$
$\forall x A\rho_w 0$ iff for some $d \in D_w$, $A_x(k_d)\rho_w 0$

As in the propositional case, one can show that:

if $w_1 R w_2$ and $A\rho_{w_1} = 1$ then $A\rho_{w_2} = 1$
if $w_1 R w_2$ and $A\rho_{w_1} = 0$ then $A\rho_{w_2} = 0$

Validity is defined, as usual, in terms of truth preservation at all worlds of all interpretations.

Note that the falsity conditions for $\forall$ and $\exists$ are the reverse of what one might have expected. The quantifier $\exists$ relates to the world in question and all its future worlds; the quantifier $\forall$ relates just to the world in question. This is required to ensure that the heredity conditions hold for all formulas.

As for intuitionist logic, these truth/fasity conditions naturally capture an appropriate antirealist understanding of the quantifiers. $\exists x A$ is verified at a stage just if some instance is. It is falsified if every instance is falsified whatever we go on to construct. Intuitively, this can happen only if we have a construction that applies to any object we come up with, d, to give a proof of $\neg A_x(k_d)$. (If there is such a construction, then whatever object we construct at a later time, there will be a proof that it satisfies $\neg A$. Conversely, if there is no such construction, then there is a possible development in which we find an object for which there is no such proof.)

Dually, $\forall x A$ is falsified at a stage just if some instance is. It is verified if every instance is verified whatever we go on to construct. Intuitively, this can happen only if we have a construction that applies to any object we come up with, d, to give a proof of $A_x(k_d)$. (If there is such a construction, then whatever object we construct at a later time, there will be a proof that it satisfies A. Conversely, if there is no such construction, then there is a possible development in which we find an object for which there is no such proof.)

First-order versions of N_3 and M are obtained in exactly the same way. For N_3 we need the extra constraint that $\nu_w^+(P) \cap \nu_w^-(P) = \phi$. For M, the appropriate constraint is that $\nu_w^+(P) \cup \nu_w^-(P) = D_w^n$, and we modify the falsity conditions for the conditional as in the propositional case. It is not difficult to show that for N_3, no formula, A, is such that $A\rho_w 1$ and $A_{\rho_w} 0$; and for M, every formula, A, is such that either $A\rho_w 1$ or $A_{\rho_w} 0$.

Quantified N_3 is acceptable to an antirealist, but it is not paraconsistent. Quantified M is paraconsistent, but not acceptable to an antirealist because it validates the LEM. N_4 is both acceptable to an antirealist and paraconsistent.

Perhaps the most surprising thing about N_4 (and the other two logics) from the present perspective, is the following. Intuitionist logic verifies only the first three of the classical negation/quantifier exchange principles:

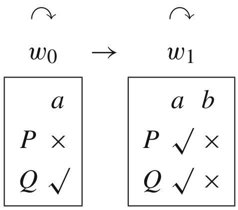
$$\forall x \neg Px \vDash \neg \exists x Px$$
$$\neg \exists x Px \vDash \forall x \neg Px$$
$$\exists x \neg Px \vDash \neg \forall x Px$$
$$\neg \forall x Px \vDash \exists x \neg Px$$

The fourth is invalid. The logics with relational semantics validate all four. Checking the first three is left as an exercise. Here is the fourth. Suppose that in world w of an interpretation $\neg \forall x Px \rho_w 1$. Then for some $d \in D_w$, $Pk_d \rho_w 0$. Hence, $\neg Pk_d \rho_w 1$, and $\exists x \neg Px \rho_w 1$.

It should be noted, though, that the intuitionistic invalidity:

$$\forall x (Pa \vee Qx) \nvDash Pa \vee \forall x Qx$$

still fails, since this does not involve negation at all. (Here is a diagram of a standard counter-model:



The boxes give the extensions of P and Q at each world. The anti-extensions are irrelevant.)

10.7 Conclusion

We have now seen, as promised, that there are paraconsistent logics that are antirealism-friendly, and paraconsistent logics that are not. The examples examined were logics that deploy a relational semantics for negation. The main feature of the these logics for present purposes is that they treat truth and falsity even-handedly. This results in the validity of the Law of Double Negation and all the classical negation/quantifier exchange principles. These are a striking divergence from standard intuitionist logic, but perfectly defensible from an antirealist perspective, as we have seen.

Of course, there are many other paraconsistent logics, of widely different kinds.[13] To determine on which side of the realism/antirealism fence each sits requires its own investigation. Sometimes this will be obvious. For example, if the logic verifies the LEM, it is not going to sit on the antirealist side. Sometimes it will not be obvious. For example, do the ternary relation and the * function standardly employed in the semantics of relevant logics sustain an antirealist interpretation? This is a hard question, if for no other reason than that it is not clear what to make of these notions quite generally.[14] However, the present paper suffices to establish the general neutrality of paraconsistency on the realism/antirealism issue.

Acknowledgements Versions of this paper were given at the Universities of Melbourne, St Andrews, and Lille in the second half of 2007. I am grateful to the audiences in those places for their helpful comments.

References

1. Almukdad, A., and D. Nelson. 1984. "Constructible Falsity and Inexact Predicates." *Journal of Symbolic Logic* 49:231–33.
2. Mares, E. 2004. *Relevant Logic: A Philosophical Interpretation*. Cambridge: Cambridge University Press.
3. Nelson, D. 1949. "Constructible Falsity." *Journal of Symbolic Logic* 14:14–26.
4. Papineau, D. 1979. *Theory and Meaning*. Oxford: Clarendon Press.
5. Priest, G. 1987. *In Contradiction*. Dordrecht: Martinus Nijhoff; 2nd (extended) Edition, Oxford: Oxford University Press, 2006.
6. Priest, G. 2001. *Introduction to Non-Classical Logic*. Cambridge, MA: Cambridge University Press; revised as Part 1 of G. Priest, *Introduction to Non-Classical Logic: From If to Is* (Cambridge: Cambridge University Press, 2008).
7. Priest, G. 2002. "Paraconsistent Logic." In *Handbook of Philosophical Logic*, edited by D. Gabbay and F. Guenthner (2nd Edition), vol. 6, 287–393. Dordrecht: Kluwer.
8. Priest, G. 2006. *Doubt Truth to Be a Liar*. Oxford: Oxford University Press.
9. Priest, G. 2008. *Introduction to Non-Classical Logic: From If to Is*. Cambridge: Cambridge University Press.
10. Priest, G., and R. Routley. 1989. "The Philosophical Significance and Inevitability of Paraconsistency." In *Paraconsistent Logic: Essays on the Inconsistent*, edited by G. Priest, R. Routley, and J. Norman, ch. 18. Munich: Philosophia Verlag.
11. Read, S. 1994. *Thinking About Logic: An Introduction to the Philosophy of Logic.* Oxford: Oxford University Press.
12. Rumfitt, I. 2000. " 'Yes' and 'No.' " *Mind* 109:781–823.
13. Wansing, H. 2001. "Negation." In *The Blackwell Guide to Philosophical Logic*, edited by L. Goble, ch. 19. Oxford: Blackwell.

[13] For a survey of paraconsistent logics, see [7].

[14] The * semantics for negation are very closely related to the relational semantics, and in simple cases are interdefinable with them. See [6, §9.6], and [9, §22.5]. One might therefore reasonably expect the considerations concerning the relational semantics to carry over to the * semantics. For one interpretation of the ternary relation in terms of information, and so broadly sympathetic to an anti-realist reading, see [2, chap. 3].