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## Logic as applied Mathematics — with Particular Application to the Notion of Logical Form

**Abstract.** The word ‘logic’ has many senses. Here we will understand it as meaning an account of what follows from what and why. With contemporary methodology, logic in this sense — though it may not always have been thought of in this way — is a branch of applied mathematics. This has various implications for how one understands a number of issues concerning validity. In this paper I will explain this perspective of logic, and explore some of its consequences with respect to the notion of logical form.

**Keywords:** pure mathematics; applied mathematics; pure logic; canonical application; logical constant; logical form; formal validity; material validity

### 1. Introduction

The word ‘logic’ has many meanings. Perhaps the most standard meaning amongst modern logicians is *what follows from what, and why*. That, at any rate, is how I will understand it in what follows.<sup>1</sup> Though it may not always be thought of in this way, the modern study of logic is a branch of applied mathematics.<sup>2</sup> The study of logic has not normally been thought of in this way. Thus, in particular, it almost inverts a logicist view of mathematics, which takes pure mathematics to be (part of) logic.<sup>3</sup> Unsurprisingly, then, looking at logic in this way puts a

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<sup>1</sup> For further discussion of the matter, see (Priest, 2014).

<sup>2</sup> In fact, I think it has always been this, though the sophistication of modern mathematics and its application to logic have made the matter patent. Medieval European logic provides an especially interesting case in this context, since technical Latin had been stylized to such a point that it had almost become a formal language.

<sup>3</sup> I owe this observation to a referee for this journal.

number of issues in the philosophy of logic in a distinctive perspective. In this essay I will explain the view that logic is a branch of applied mathematics, and explore some of its consequences with respect to the notion of logical form and related matters. I shall be concerned here with deductive logic only. Much (though not all) of what I will have to say carries over to non-deductive logic. However, a discussion of this is appropriate for another occasion.

## 2. Applying Mathematics

### 2.1. Pure and Applied Mathematics

To start with, let us reflect on applied mathematics itself.<sup>4</sup> Pure mathematics is the study of a variety of mathematical structures in and of themselves: topologies, geometries, number systems, and so on. In contemporary mathematics, these would normally be specified axiomatically, though, outside of geometry, the axiomatic method is a relatively modern methodology.

Applied mathematics is the use of a pure mathematical structure to investigate some non-mathematical topic, in physics, biology, economics, and so on. Of course, the developments of some pure mathematical structures were entangled with particular applications. Thus, the development of Euclidean geometry was integrally connected with what we might call its canonical application: the structure of space; and the infinitesimal calculus was integrally connected (at least in Newton) with its application to physical change. Indeed, these integrations were so intimate that it was hard to make the conceptual distinction between the mathematical structure and its application. But Kant notwithstanding, we now know better. Though Euclidean geometry is a perfectly fine pure mathematical structure, we now take it that it is not the correct one for the canonical application of geometry. The correct geometry for analysing the structure of space(-time) is not even one of constant curvature. Of course, a pure mathematical structure may have many applications, and Euclidean geometry may still be the correct mathematical structure for other applications — for example, local surveying. And some pure mathematical structures may have no applications at all: the theory of large infinities and the theory of surreal numbers are cases

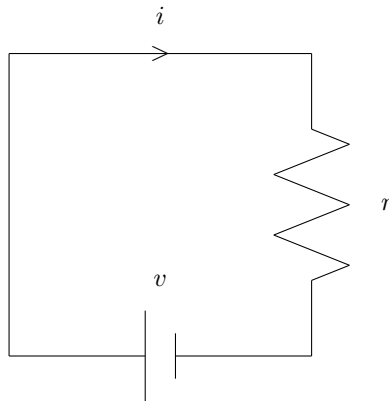
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<sup>4</sup> For more on the following, see (Priest, 2022).

in point. At least, they have no applications at present: some pure mathematical structures were investigated independently of any application, and were found to have one only later: for example, complex analysis in the theory of electricity, and group theory in Special Relativity. In the end, whether a pure mathematical structure has an application, and what the correct pure mathematical structure for any application is, are a matter of *a posteriori* discovery.

## 2.2. Applying Mathematics: an Example

So how does one apply a pure mathematical structure? Let us look at an example from physics. This uses Ohm's law to determine the current in a circuit. Suppose we have a simple electrical circuit with a battery and a resistor.



If the voltage,  $v$ , produced by the battery is 6 volts, and the resistance,  $r$ , of the resistor is 2 ohms. What current,  $i$  (in amps), flows? Ohm's law,  $v = ri$ , tells us that it is 3. What is going on here?

First, we start with a state of affairs in the physical world. This will involve some wires and other bits of electrical paraphernalia. Understanding what is going on, and making predictions about it, will involve the following steps. (The sequence here is not a temporal one.) First, we need to invoke three (theoretical) physical quantities: the current flowing,  $I$ , the resistance,  $R$ , and the voltage in the circuit,  $V$ . Call the set of physical quantities,  $\mathbb{P}$ . Next, these have to be assigned some mathematical values. Hence, there are three functional expressions,  $\mu_i$ ,  $\mu_r$ ,  $\mu_v$ , such that  $\mu_i$  means 'the value in amps of',  $\mu_r$  means 'the value in

ohms of’, and  $\mu_v$  means ‘the value in volts of’. (I will omit the subscripts on ‘ $\mu$ ’ if they are clear from the context.) In our case, the mathematical values are real numbers, members of  $\mathbb{R}$ . So the denotation of each  $\mu$  is a map from  $\mathbb{P}$  to  $\mathbb{R}$ . We can now enunciate Ohm’s Law:

- $\forall V, R, I(\mathcal{F}(V, R, I) \rightarrow \mu(V) = \mu(R) \times \mu(I))$

where  $\mathcal{F}(V, R, I)$  states that  $V$ ,  $R$ , and  $I$  are the quantities in an electrical circuit. Finally, we have to determine exactly how the mathematical entities and the operations on them work. In the case at hand, this is provided by the mathematical structure of the (classical) reals,  $\mathfrak{R}$ ,  $\langle \mathbb{R}, +, \times, 0, 1, < \rangle$ .

Now, in our present example, we have three particular quantities,  $V_0$ ,  $R_0$ , and  $I_0$ , such that  $\mathcal{F}(V_0, R_0, I_0)$ . Hence applying Ohm’s Law, we have  $\mu(V_0) = \mu(R_0) \times \mu(I_0)$ . We also have  $\mu(V_0) = 6$  and  $\mu(R_0) = 2$ . If we now choose new terms,  $v$ ,  $r$ , and  $i$ , for  $\mu(V_0)$ ,  $\mu(R_0)$ , and  $\mu(I_0)$ , respectively, we have the equations:

- $v = r \times i$
- $v = 6$
- $r = 2$

Moving to the pure mathematical level, if these statements hold in  $\mathfrak{R}$  then so does  $i = 3$ . So, moving back to the empirical level again,  $\mu(I_0) = 3$ . That is, the current in the circuit is 3 ohms.

### 2.3. The General Picture

Bearing our example in mind, the general schema for applying a pure mathematical structure is as follows.

Let us call the topic to which we are applying mathematics, for want of a better phrase, the “real world”. The real-world state of affairs will concern various real-world entities. The situation describing these and the laws governing them can be expressed by a set of statements,  $D$ . Pure mathematical statements,  $D'$ , concerning some structure,  $\mathfrak{A}$ , are abstracted from these statements, ignoring the “real world” interpretation of the mathematical quantities. Using what we know about  $\mathfrak{A}$ , we can infer some other statements,  $E'$ , that hold in the structure. These can be “de-abstracted”, bringing the “real world” interpretation back into the picture, to deliver some descriptions of the real-world situation,  $E$ . We may depict this as in Figure 1.

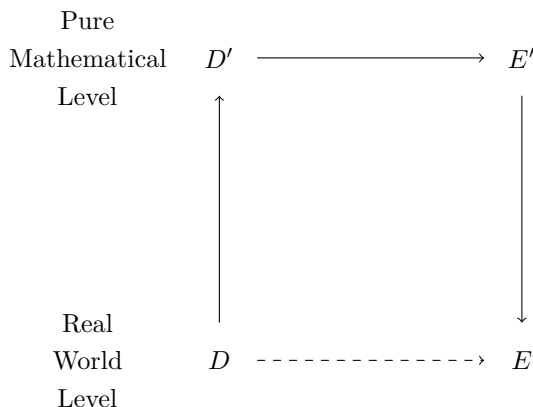


Figure 1.

What takes us from  $D'$  to  $E'$  in this procedure is pure mathematics: proofs concerning  $\mathfrak{A}$ . The rest of the picture is a matter of *a posteriori* discovery. That this is so in the case of the scientific laws involved is, of course, clear. But the finding of an appropriate mathematical structure,  $\mathfrak{A}$ , is, in principle, equally an *a posteriori* matter. This is exactly what the situation concerning the replacement of Euclidean geometry by Riemannian geometry in the theory of space(-time) showed us.

The mathematical structure applied,  $\mathfrak{A}$ , provides us, in effect, with a theory about the “real world”. Thus, Euclidean geometry and a Riemannian geometry, when given their canonical application, deliver theories of the structure of space(-time). Of course, such a theory may be interpreted in a number of different ways. One may interpret it realistically, in a certain sense. That is, the real world entities have a structure which is tracked by the mathematical structure. One may take it purely instrumentally. That is, the mathematics is no more than a convenient computation device, which seems to give the right results. Or — somewhere in between these two — one may think of the mathematics as providing a model of the real-world situation: one which tracks certain features of the real-world situation, but which ignores others, maybe in the cause of simplicity and tractability. How best to understand the theory delivered by a pure mathematical structure will depend on one’s philosophical proclivities and case by case consideration.

### 3. Logic as Applied Mathematics

#### 3.1. Pure and Applied Logic

Let us now turn to logic. A pure logic is a mathematical structure,  $\mathfrak{L}$ . There is a multitude of such logics (classical, intuitionist, paraconsistent, etc). Each comprises a formal language,  $\mathcal{L}$ , a consequence relation,  $\vdash$ , defined on sentences of  $\mathcal{L}$ , and possibly other machinery. In general  $\vdash \subseteq \wp(\mathcal{L}) \times \wp(\mathcal{L})$ , but for our purposes we need be concerned only with consequence relations where the first component is a finite set, and the second is a singleton.  $\mathfrak{L}$  may be specified proof theoretically, model theoretically, algebraically, or in some other way. The pure mathematical structure may establish the equivalence between different characterisations, and other important properties of  $\vdash$ , such as decidability or compactness. *Qua* pure mathematical structure, each logic, like each geometry, is perfectly in order as it is.

A pure logic may have many applications. It may be applied to simplify electrical circuits (as with Boolean logic), to parse sentences (as with the Lambek Calculus). But just as with geometry, logic has always had what one might call a canonical application: the analysis of arguments. These are arguments expressed in a vernacular language. When people argue, be they lawyers, politicians, historians, scientists, or wot not, they do not do so in a formal language. And, note, this is just as true of mathematicians. If you open the pages of a mathematics journal or text book, you will not find the argument presented in *Principia*ese, or any other formal language. People argue in a natural language (though some of the vocabulary used may be of a technical nature). The canonical application of logic is to evaluate such arguments. That is what it was originally invented for.

#### 3.2. The Canonical Application of a Pure Logic

How is this done? Suppose we have an argument phrased in a vernacular language,  $\mathcal{L}_V$ . Let this have premises  $\mathcal{P}_1, \dots, \mathcal{P}_n$ , and conclusion  $\mathcal{C}$ . We form the sentence:

- $\mathcal{A}_V$  : the inference from  $\mathcal{P}_1, \dots, \mathcal{P}_n$  to  $\mathcal{C}$  is valid.

That is:  $\mathcal{P}_1, \dots, \mathcal{P}_n$  entail  $\mathcal{C}$ .

The sentences of the language are transformed into sentences  $\mu(\mathcal{P}_1), \dots, \mu(\mathcal{P}_n), \mu(\mathcal{C})$ ; and  $\mathcal{A}_V$  is transformed into the sentence:

- $\mathcal{A}$ :  $\mu(\mathcal{P}_1), \dots, \mu(\mathcal{P}_n) \vdash \mu(\mathcal{C})$

The translation of vernacular sentences into  $\mathcal{L}$  is done by a process that is usually informal, but teachers of elementary logic courses will normally spend a considerable amount of time developing the required skills in their students. The appropriate translation may on some occasions itself be a matter of theoretical contention. (Thus, for example, the standard translation of a definite description is as a term of  $\mathcal{L}$ ; but according to Russell's theory of definite descriptions, the whole sentence in which it occurs is translated into a sentence which contains no corresponding noun-phrase.) Next, the mathematical machinery of  $\mathfrak{L}$  is applied to determine whether  $\mathcal{A}$  holds in the pure mathematical structure. The sentence is then “de-abstracted” back to the real world level, to tell us whether  $\mathcal{A}_V$  holds.

This procedure is an instance of the general schema of 2.3. The real-world level comprises vernacular arguments.  $D$  is a statement of validity for such an argument.  $D'$  is a corresponding mathematical statement of  $\mathfrak{L}$  to be proven or refuted, and  $E'$  is the result.  $E$  is then the verdict for the original argument.

### 3.3. Pure Logic as Theory

As in general for applied mathematics, a pure logic,  $\mathfrak{L}$  constitutes, in effect, a theory: one of the validity of inferences in the vernacular language (or the relevant fragment of it) — though of course, how one should interpret the theory (realistically, instrumentally, as a model, etc.) will be subject to the same considerations as before.

Many such theories of validity have been proposed, accepted, and/or rejected in the history of Western logic: Aristotelian syllogistic, Medieval (and contemporary) connexive logic, Medieval supposition theory, “classical” logic, intuitionist logic, paraconsistent logics — to name but a few of the most obvious ones. And of course, different theories may give different verdicts on various inferences. Thus, if  $\mathcal{A}_V$  concerns the inference:

- Donald Trump is corrupt and Donald Trump is not corrupt, so  $\pi$  is irrational

and  $\mathfrak{L}$  is classical logic, then it will return the verdict *valid*. But if  $\mathfrak{L}$  is a paraconsistent logic, it will return the verdict *invalid*.

Given a collection of different theories, the question — one which has played a major role in contemporary philosophy of logic — then arises as to which of them is rationally preferable. Primary amongst the considerations for determining the answer is one of adequacy to the data. In empirical cases, the data is provided by sensory observation and experimentation. In the case of logic it is provided by judgments about the validity or otherwise of particular inferences.<sup>5</sup> Thus:

- (1) Mary is wearing a red dress and red shoes; so Mary is wearing a red dress

strikes us as valid, but:

- (2) Mary is wearing either a red dress or red shoes; so Mary is wearing a red dress

strikes us as invalid. Getting these data points right is a mark in favour of a theory; getting them wrong is a mark against the theory.

Of course, as in the empirical sciences, data is not infallible. It can be wrong, and can be shown to be so by an otherwise good theory. Thus, an aberrant measurement in physical geometry may be taken to show that our measuring device, or our theory of how it works, is incorrect. Similarly, our naive judgments about the validity or otherwise of certain inferences may be wrong. ‘Mary’s dress is red, so Mary’s dress is coloured’ will strike most of us as valid. But standard logic says that it is not. What is valid is the inference with the extra premise ‘Whatever is red is coloured’. Of course, simply writing off an aberrant data point is bad methodology. Some independent explanation needs to be found. In the case of the logic example, a natural such explanation is that we frequently do not mention obvious premises (such as that all red things are coloured) because they *are* obvious, and life is short.

In situations of any theoretical complexity, adequacy or otherwise to the data will not settle the matter. For a start, theories may be equally adequate or inadequate. In practice, other criteria are also important, such as simplicity, unifying power, non-reliance on *ad hoc* hypotheses, and so on. So it is in logic as well.

In logical investigations of any sophistication, judgments of validity are embedded in a complex theory of validity, and validity is defined

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<sup>5</sup> Not, *note bene*, forms of inference. These are always some kind of theoretical generalisations.



in terms of truth, meaning, probability, or something else. The choice between different theories of this kind is then certain to bring into play, not only data concerning these notions, but judgments concerning the other theoretical virtues and how one aggregates all these things; but we need not go into this matter further here. Suffice it to say that the theory it is most rational to accept is the one which performs best overall. What we have here is some kind of abductive inference.<sup>6</sup>

I note that, despite what might be thought, this does not prejudge the issue of logical monism *vs* logical pluralism. Logical pluralism can mean many things, but perhaps the most interesting is that there are different notions of logical consequence for the different domains about which we reason (see, further, [Priest, 2006](#), ch. 12). If this is indeed the case, we choose the theory which is the best for each domain at issue by the same procedure. The choice between logical pluralism and logical monism itself is theoretical choice of a higher-order kind, to be determined by essentially the same methodology (see, further, [Priest, 202x](#))

The important thing to observe at present is that rational choice of theory is a fallible and, in a certain sense, an *a posteriori* one. It is fallible because the data against which a theory is measured is itself fallible; and, moreover, new and better theories may appear at any time. It is *a posteriori* because its acceptability is to be judged in the light of data and methodological criteria, not given by certain and infallible rational intuition.

This does not mean that one has to use sensory information. One can, of course, sometimes use such information to establish that an inference is invalid. For some inferences we may be able to see (literally) that the inference is invalid. For example, consider the inference:

- There are at least two people in the room; so there are a million people in the room.

where the inference concerns a certain room at a certain time. We may be able to see that the premise is true and the conclusion is not. But most cases will not be like this. One may judge that the inference (1) is valid, merely by thinking about it. In a similar way — though the analogy is not to be pushed too far — a native English speaker can simply reflect

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<sup>6</sup> See, further, ([Priest, 2016, 2021](#)). This view is often now known as ‘anti-exceptionalism’. I find this both ugly and potentially misleading. I prefer ‘abductivism’.

on the string of words ‘The 45th President of the USA was corrupt’, to see that it is grammatical, though that this is so is of course an *a posteriori* fact about English.

As with the application of a pure mathematical structure in any other area, then, in our diagram in Figure 1, what takes us from  $D'$  to  $E'$  is *a priori*: proof. But finding the right pure mathematical structure to apply is an *a posteriori* matter.

## 4. Logical Form and Related Matters

Let us now turn to how the notion of logical form and related matters appear from this perspective on logic.

### 4.1. Logical Constants

Let us start with the notion of a logical constant. The term ‘logical constant’ – in Latin, *syncategoremata* (lit.: something to be combined with something (else) with a self-standing meaning) – is a term of logician’s art. The place to find a precise characterisation is, then, in theory; that is, as applying to notions in a formal language.

How, exactly the syncategorematic notions are picked out will depend on the theory of validity employed. If the theory is a model-theoretic one, they will be the parts of the language that have dedicated clauses in the recursive truth conditions, as in:<sup>7</sup>

- $A \wedge B$  is true in a structure  $\mathcal{S}$  iff  $A$  is true in  $\mathcal{S}$  and  $B$  is true in  $\mathcal{S}$
- $\forall xA(x)$  is true in  $\mathcal{S}$  iff for all  $a$ ,  $A(a)$  is true in  $\mathcal{S}$
- $a = b$  is true in  $\mathcal{S}$  iff the denotation of  $a$  in  $\mathcal{S}$  is the same as that of  $b$

The other expressions – the *categorematic* ones – will have only schematic parametric clauses, the denotations of the parameters being provided by the structure. If the theory is a proof-theoretic one – say in terms of natural deduction – the syncategorematic notions are the ones that have specific rules of proof, such as:

$$\frac{A \quad B}{A \wedge B} \qquad \frac{\forall xA}{A_x(a)} \qquad \frac{a = b}{A_x(a)} \qquad \frac{A_x(a)}{A_x(b)}$$

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<sup>7</sup> The truth conditions for the universal quantifier here assume that every object in the domain has a name. I make this assumption to keep matters simple.

The categorematic terms have no specific rules. Of course, whether one should prefer a model-theoretic account of validity, a proof-theoretic account, or some other kind, is a matter of having determined the best theory already.

The notion of a logical constant delivered in this way carries over naturally to notions from the vernacular language. These are those notions which are represented by the logical constants in the formal language. That is, those notions which translate into formal logical constants (as determined by the function  $\mu$  of 3.2), in the way that ‘and’, ‘all’ and ‘is identical to’ are normally taken to translate into  $\wedge$ ,  $\forall$ , and  $=$ . (Though of course, the formal languages in question may have been designed with an eye on the fact that these vernacular notions appear to play a ubiquitous role in reasoning.)

Crucially, what is a logical constant is not to be determined *a priori*, but is read off from the logical theory of validity, which is determined *a posteriori*, in the way explained. As is to be expected, certain notions are likely to come out as logical constants under most plausible theories. There is not going to be much dispute about ‘and’, ‘all’, and nowadays ‘it must be the case that’. ‘is identical to’, ‘it will be the case that’, ‘is true’, ‘is a natural number’ are, in contemporary logic, more contentious.

## 4.2. Logical Form

Let us now turn to the matter of logical form itself. As far as the formal language,  $\mathcal{L}$ , goes, the matter is relatively straightforward. The logical form of a sentence is the sentence itself with each non-(logical constant) replaced (uniformly) by a parameter, that is, a variable of the appropriate category. Given most usual theories of validity (model-theoretic, proof-theoretic), parameters play no role in the validity of an inference. Hence, any two inferences of the same logical form will be equi-valid.

Matters for a vernacular sentence are slightly less straightforward, however. Given a logical theory,  $\mathfrak{L}$ , we may define the logical form of a vernacular sentence,  $A_v$ , to be the logical form of the sentence,  $A$ , of  $\mathcal{L}$ , into which it is translated when applying  $\mathfrak{L}$ . Clearly, this logical form is relative to  $\mathfrak{L}$ . But it might be felt that the logical form should be more determinate than this. To see why, just consider the inference: ‘John is happy, so someone is happy’. If  $\mathfrak{L}$  is classical propositional logic, the inference is of the form  $p \vdash q$ , which is invalid. But if  $\mathfrak{L}'$  is classical

first-order logic, the inference is of the form  $Hj \vdash \exists xHx$ , which is valid. It might well be felt that the sentence of first-order logic better captures the logical form of the vernacular sentence than does the sentence of propositional logic.

If so, one may define an absolute notion of logical form for a vernacular sentence as follows. Let us suppose that for every formal language,  $\mathcal{L}$ , the correct  $\mathfrak{L}$  is settled. And suppose, as one would expect, that if  $\mathcal{L}'$  extends  $\mathcal{L}$  then  $\mathfrak{L}'$  and  $\mathfrak{L}$  agree on the validity of inferences couched in  $\mathcal{L}$ . One may define the (absolute) logical form of a vernacular sentence to be the logical form of the sentence of  $\mathcal{L}$  into which it is translated, where  $\mathcal{L}$  is a language such that there is no  $\mathcal{L}'$  which extends  $\mathcal{L}$  and which changes the validity or otherwise of inferences formed of sentences in  $\mathcal{L}$ .<sup>8</sup>  $\mathcal{L}$  is, so to speak, the maximal (relevant) level of analysis of the sentence. Again, all this is *a posteriori*, being dependent, as it is, on an appropriate theory of validity.

### 4.3. Formal Validity

Let us move from logical form to formal validity.

Given a theory of validity provided by a mathematical structure,  $\mathfrak{L}$ , let us say that the notion of validity delivered is *formal* iff any two inferences with the same logical form are equi-valid. That is, if the forms of  $\mathcal{I}_1$  and  $\mathcal{I}_2$  are the same, both are valid or neither is.

Say that a validity relation is *closed under uniform substitution* if whenever  $C$  is a constant of some grammatical category,  $A$  is something of that category, and  $\mathcal{I}(C)$  is a valid inference, then so is  $\mathcal{I}(A)$ . If an inference relation is closed under uniform substitution, it is formal. For if  $\mathcal{I}(C_1)$  and  $\mathcal{I}(C_2)$  have the same logical form,  $\mathcal{I}(X)$  — supposing for the sake of simplicity that there is only one categorical term — then each can be obtained by uniform substitution from the other. The converse does not hold, however, since  $\mathcal{I}(C)$  and  $\mathcal{I}(A)$  may have different logical forms.

Notwithstanding, as observed, usual model theoretic and proof theoretic accounts of deductive validity deliver notions of validity closed

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<sup>8</sup> If there are two such  $\mathcal{L}$ s,  $\mathcal{L}_i$  and  $\mathcal{L}_j$ , there must be agreement about the validity of inferences in the relevant vocabulary. For we can form the language,  $\mathcal{L}_k$ , which is the language whose vocabulary is the union of those of  $\mathcal{L}_i$  and  $\mathcal{L}_j$ . *Ex hypothesi*,  $\mathfrak{L}_i$  and  $\mathfrak{L}_j$  agree with  $\mathfrak{L}_k$  on the validity of inferences in the relevant vocabulary, and so with each other.

under uniform substitution, and so are formal. This is by no means required of an account of deductive validity, though. Thus, suppose that validity is defined as necessary truth preservation. That is, the inference from  $\mathcal{P}_1, \dots, \mathcal{P}_n$  to  $\mathcal{C}$  is valid iff  $\Box((\mathcal{P}_1 \wedge \dots \wedge \mathcal{P}_n) \rightarrow \mathcal{C})$ . Given what are usually taken to be logical constants, the inferences:

- Dana is English, so Dana is British
- Dana is English, so Dana is happy

have the same logical form, but the first is valid and the second is not.

In fact, there are many inferences that appear to be deductively valid but are not formally valid, such as:

- (3)  $a$  is red; so  $a$  is coloured
- (4)  $a$  is red; so  $a$  is not blue

The Medieval description of such inferences was *materially valid*.

If one wants to hold that such inferences can be accounted for in terms of formal validity there are at least two options. The first, and most standard, is to take them to be enthymematically valid, with suppressed premises, such as *all red things are coloured* and *nothing is red and blue*.

However, such inferences can be taken to be formally valid provided we take the appropriate terms to be logical constants. What this means is that in a proof-theoretic theory of validity, they have their own rules of inference, such as:

$$\frac{a \text{ is red}}{a \text{ is not blue}} \qquad \frac{a \text{ is red}}{a \text{ is coloured}}$$

In a model-theoretic account, an interpretation will assign these predicates interpretation-independent denotations connected by structural constraints (in a way that a modal semantics puts constraints on properties of the accessibility relation). Thus, if *red* and *coloured* are represented in the language  $\mathcal{L}$  by the predicate constants  $R$  and  $C$  then the semantic recursive clauses will be:

- $Ra$  is true in  $\mathcal{S}$  iff the denotation of  $a$  is red
- $Ca$  is true in  $\mathcal{S}$  iff the denotation of  $a$  is coloured

and  $\mathcal{S}$  will be required to satisfy the constraint that the extension of  $R$  is a subset of the extension of  $C$ . Normally, these constraints will be taken as “meaning postulates”.

If one decides to eschew a notion of material validity, which of these two approaches to adopt will depend on the theoretical virtues of each account.<sup>9</sup>

## 5. Conclusion

In this essay I have shown how it is that logic is a branch of applied mathematics. A pure logic can be applied to evaluate vernacular inferences in the same way that any other pure mathematical structure can be applied to deliver an account of some “real world” phenomenon. When thus applied, it delivers a theory of what follows from what and — if the mathematical structure is sufficiently generous — why. The best such theory is then to be determined on the basis of the usual criteria of theory choice.

Such an account provides a distinctive perspective on a number of issues in the philosophy of logic; notably, as we have seen, on the nature of logical constants and logical form. In particular, an understanding of both notions is delivered by our best such theory, whatever, in the end, that turns out to be.

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<sup>9</sup> And perhaps, if there is nothing to choose between the two approaches, one might well conclude that the matter is entirely one of convention.

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