Naming the Nameless
Comments on Fujikawa

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Abstract

Some things appear to generate a paradox of ineffabity, to the effect that they are both effable and ineffable. In 'Ineffability and Nonobjecthood', Naoya Fujikawa formulates a 'strengthened version of the ineffability paradox': that the thing in question is ineffable, is both true and false and neither true nor false. He then goes on to discuss a number ways in which one might address the paradox. What follows is an attempt to think through the issues. First, the paper analyses the reasoning that goes into the strengthened paradox. It then comments on the possible responses discussed by Fujikawa.

1 Introduction

Ineffability and its features are things that arise in many philosophical traditions—East and West. One obvious feature is that the matter appears to give rise to contradiction. The traditions talk about whatever it is that is supposed to be ineffable, showing that it is not ineffable. One may try to wriggle out of the contradiction in a number of ways; but my own view is that if there really is something that is ineffable and talked about, one should just accept the contradiction that it is both effable and ineffable.

In his insightful essay 'Ineffability and Nonobjecthood',[1] Naoya Fujikawa formulates a 'strengthened version of the ineffability paradox': that the thing in question is ineffable, is both true and false and neither true nor false. He then goes on to discuss a number ways in which one might address the paradox. What follows is an attempt to think through the issues. First, I will analyse the reasoning that goes into the strengthened paradox. I will then comment on the possible responses discussed by Fujikawa.[2]

2 Objects that are Not Objects

Let us start with the notion of an object. To be an object is to be something. (I do not say 'something existent.') How exactly to understand this is not immediately clear: the word 'is' in English (and its translations in other languages) is ambiguous.[3] Two obvious ways of representing 'x is something' in formalese are:

[1] $\exists y\, y = x$

[2] $\exists P\, Px$

As emphasised, the quantifiers here simply mean *some*, not *some existent*. Given some very natural assumptions, [1] and [2] are equivalent. If $\exists y\, y = x$ then $\lambda z(\exists y\, y = z)x$, so $\exists P\, Px$. Conversely, $x = x$, so $\exists y\, y = x$. In particular, this is a logical consequence of $\exists P\, Px$. Since [1] and [2] are equivalent, and following Fujikawa, we may work with the first of these.

Again given some natural assumptions, [1] is equivalent to $x = x$. As we noted, $\exists y\, y = x$ follows from $x = x$; and the latter, being a logical truth follows from the former. Moreover, the equivalence holds in contraposed form as well. If $\neg\exists y\, y = x$ then $\forall y\; y \neq x$, so $x \neq x$. But suppose that $x \neq x$. Then for any y, $x = y \vee x \neq y$. In the first case $x \neq y$ by the substitutivity of identicals. In both cases, then, $x \neq y$. So $\forall y\, x \neq y$.That is $\neg\exists y\, x = y$. Hence we may simply take 'x is an object' to be $x = x$ and 'x is not an object' as $x \neq x$.

Are there things that are not objects (as well, of course, as being objects); that is, is it the case that $\exists x\, x \neq x$? Arguably *yes*. In *One*,[4] I argued that

[1] Fujikawa (202+).

[2] Many thanks go to Fujikawa for his comments on a first draft of this essay.

[3] See Priest (2016), ch. 17.

[4] Priest (2014a).

any partite object must have a part which welds all the others into a unity. This is its gluon, g; and for reasons that we do not need to go into here, $g \neq g$.

3 Nothing

A quite different reason for there being objects that are not objects concerns the very particular object **nothing**. As Fujikawa notes, the word 'nothing' is ambiguous. It can be a quantifier phrase or a noun phrase. We are concerned here with the noun phrase. I shall boldface it to emphasise this.

Nothing is clearly a paradoxical notion. It is something. (You can think about it: you are now. Your thought is not contentless: you are thinking about something.) But it is, well, nothing, no thing. So, it is and is not an object. **Nothing** has played a very significant role in the metaphysics of many thinkers, East and West: Wang Bi, Meister Eckhart, Hegel, Heidegger, Nishida, Sartre. This is not the place to go into the history.[5] However, let me briefly comment on one strand of it.

Nothing (無; Chin: *wu*; Jap: *mu*) plays a central role in Sino-Japanese Buddhism, where it is used as an appellation for ultimate reality, as opposed to the beings (有; Chin: *you*; Jap: *yu*) of conventional reality—our phenomenological world.

For example, Jizang (吉藏, 549–623) was a major thinker in the *Sanlun* (Three Treaties, 三論) School of Buddhism—a Chinese version of the Indian Madhyamaka school. He constructs a sophisticated hierarchy of levels of the conventional and the ultimate, each of which is transcended by a more profound level.[6] At the first level, he says, ordinary people take conventional reality (有) at face value. Wise people know that it is empty (無). Then:[7]

> Next comes the second stage, which explains that both being (有) and nonbeing (無) belong to worldly truth, whereas non-duality (neither being nor non-being) belongs to absolute truth. It shows that being and non-being are two extremes, being the one and non-being the other... Because the absolute [truth of non-being] and the worldly [truth of being]... are both two extremes,

[5]Priest (202+b), Part 3.
[6]See Priest (2018), ch. 7.
[7]Chan (1963), p. 360. Interpolations in square brackets are the translators.

> they therefore constitute worldly truth, and because neither-the-absolute-nor-the-worldly... are the Middle Path without duality, they constitute the highest truth.

There are several more levels; but we need not pursue the matter. Jizang's use of 無 to refer to ultimate reality is clear.

That **nothing** both is and is not comes out particularly clearly in the thought of the Japanese philosopher Nishida Kitarō (西田幾多郎, 1870–1945),[8] as Fujikawa notes. Nishida rarely mentioned Buddhism explicitly, but his whole thought is steeped in the thinking of Zen, a form of which he practiced. The relevant part of his thought here is his theory of *basho*. Each object is in one or more *basho* (場所, place, topos). The *basho* provides a framework for discourse about the object. Each *basho* is not an object within itself, and so is a nothingness with respect to that *basho*, a relative nothingness (相対無, *sōtai mu*). Each *basho* is nested within other *basho*—all except one, which contains all the others, and therefore provides a framework for the whole system. This is absolute nothingness (絶対無, *zettai mu*). *Zettai mu* is Nishida's take on ultimate reality or, what is the same for him (and Zen), enlightened consciousness.

Zettai mu is paradoxical: it both is and is not (an object). *Zettai mu* is no thing/being/object. It is not an object since it is the "negation" of beings. Nishida says:[9]

> We can first of all simply distinguish between the nothing that negates a certain thing, that is, relative nothing, and the nothing that negates all being, that is, absolute nothing.

However, it is an object, since we can think and talk about it. As he says:[10]

> Nothing [however] is also an object of thinking. It becomes a being by adding some kind of determination to it. In the sense that the species is included in the genus, being is implaced in nothing. Needless to say, [even] to think of it as nothing is to think of it as an already determined being.

For Nishida, then, 無, that is, **nothing**, both is and is not a object.

[8] See Maraldo (2019).

[9] Krummel and Nagamoto (2012), p. 72.

[10] Krummel and Nagamoto (2012), p. 85. Interpolations are the translators'.

One does not have to follow Buddhism and Nishida down their paths to hold that **nothing** is some thing and no thing. One may give a mereological account of what **nothing** is, viz., the mereological fusion of no things—or equivalently, the absence (complement) of everything—from which these things follow. However, this is not the place to go into this matter.[11] Let us move on.

4 Denotation

The next topic for discussion is denotation. Semantic notions such as truth, satisfaction, and denotation appear to satisfy simple general principles. Thus, notoriously, truth satisfies the T-Schema:

[Tr] $\qquad T\langle A\rangle \Leftrightarrow A$

where Tx is 'x is true', A is any closed sentence (without indexicals), and $\langle . \rangle$ is a name-forming functor. $\Leftrightarrow$ is some kind of biconditional. We may leave its exact nature undetermined, though some of its properties will feature in what follows. The analogue for denotation is:

[Den] $\qquad \forall x(\langle t\rangle Dx \Leftrightarrow t = x)$

where yDx is 'y denotes x' and t is any closed term (without indexicals).

True, these naive principles have been challenged on the ground that they lead to paradoxes of self-reference, but I have defended them elsewhere,[12] and shall not do so here. Of more relevance is the question of whether the $\Leftrightarrow$ involved in these principles contraposes.[13] Deflationists about these semantic notions[14] hold that the left and right hand sides are synonymous, so the conditionals contrapose. I am inclined to the view that they do not.[15]

[11] See Priest (2014), 6.13, and (202+b), chs. 5, 6.

[12] Priest (1987), ch. 1

[13] The corresponding principle for satisfaction (of a formula with one free variable) is:

[Sat] $\qquad \forall x(xS\langle B_y\rangle \Leftrightarrow B_y(x))$

Here, xSy is 'x satisfies y', and B_y is any formula with at most one free variable, y. $B_y(x)$ is B with any free occurrence of y replaced by x (with any relabelling of bound variables necessary to avoid clashes). In fact, [Tr] and [Den] can easily be obtained from [Sat] by appropriate definitions. $T\langle A\rangle$ is $\forall y\, yS\langle A\rangle$, and $\langle t\rangle Dx$ is $xS\langle y = t\rangle$. This at least suggests that the biconditionals of [Tr] and [Den] should be treated in the same way.

[14] E.g., Beall (2009), Field (2008).

[15] Priest (1987), 4.9.

Focussing on [Den], let 'n' be the name for any object that is not an object. Instantiating the D-Schema, we get:

- $\langle n \rangle\, Dn \leftrightarrow n = n$.

And since $n = n$, it follows that $\langle n \rangle\, Dn$. Hence:

[∃Left] $\quad \exists x\, xDn$

However, since n is not an object, $\neg\exists y\, y = n$. That is, $\forall y\, y \neq n$, and so for any term, t, $t \neq n$. Instantiating the D-Schema again we get:

- $\langle t \rangle\, Dn \leftrightarrow t = n$.

And assuming that this biconditional contraposes, we have:

- $\neg \langle t \rangle\, Dn$

Of course, if x is not a referring term at all, it is not a name of n. So n has no name (not even 'n'):

[¬∃Left] $\quad \neg\exists x\, xDn$.

Moreover, since n is not an object, $\forall x \neg x = n$. But again, contraposing [Den] tells us that $\neg \langle n \rangle\, Dx \Leftrightarrow x \neq n$. So $\forall x \neg \langle n \rangle\, Dx$. '$n$' is a non-referring term:

[¬∃Right] $\quad \neg\exists x\, \langle n \rangle\, Dx$

The last two arguments depend on the contraposability of [Den], and as I have said, one may balk at this. But for the special case where the n in question is **nothing**, one may arrive at all these things by arguments that do not presuppose [Den] at all. Let $\mathbf{n}$ be the name of **nothing**. Then by fiat, $\langle \mathbf{n} \rangle\, D\mathbf{n}$. So:

[∃Left] $\quad \exists x\, xD\mathbf{n}$

But **Nothing** is the absence of everything. So no name refers to it—not even $\mathbf{n}$. There is literally nothing there for it to refer to. Hence, **nothing** is ineffable:

[¬∃Left] $\quad \neg\exists x\, xD\mathbf{n}$

Again, '$\mathbf{n}$' refers to **nothing**. And given that the name is not ambiguous, it refers to nothing else. But since **nothing** is the absence of everything, there is literally nothing there for it to refer to. Hence:

[$\neg\exists$Right] $\quad \neg\exists x \langle \mathbf{n} \rangle Dx$

Noticing [$\neg\exists$Right] is, I think, the central (and important) insight of Fujikawa's paper. He uses this to argue that some claims about an n of our kind (or of $\mathbf{n}$ in particular) are neither true nor false—most notably, the claims that n is effable or ineffable. Let us see how.

5 Ineffability

What it means for something to be ineffable might be understood in different ways, but for present purposes, let us say that an object is effable if it can be characterised in some way or other; it is ineffable if this is not the case. If something is ineffable, this does not mean that one cannot refer to it. Simply to refer is not to characterise. But clearly, being able to refer to something is a necessary condition for characterising it. Plausibly, it is a sufficient condition also. For if one can refer to it, one can say, e.g., that it is self-identical, an object, and so on. So, following Fujikawa, we may define effability as having a name. Let us write Ex as meaning 'x is effable', and Ix as meaning that 'x is ineffable'. These may then be defined as follows:

- Ex: $\exists y\, yDx$
- Ix: $\neg Ex$, i.e., $\neg\exists y\, yDx$

Hence, by [$\exists$Left] and [$\neg\exists$Left] (at least in the special case when 'n' is '$\mathbf{n}$'):

- En
- $\neg En$

Then assuming [Tr], $T\langle En\rangle$ and $T\langle \neg En\rangle$. That is, En is both true and false. (Note that I use *false* to mean *having a true negation*, and do not assume that this means the same as *not being true*.)

Let us call this the *primary paradox*. Fujikawa articulates a derivative paradox. Let us call this the *secondary paradox*. This is to the effect that $\neg T\langle En\rangle$ and $\neg T\langle \neg En\rangle$: En is neither true nor false.

If [Tr] contraposes then truth commutes with negation. For:

- $T\langle \neg A\rangle \Leftrightarrow \neg A$
- $\neg A \Leftrightarrow \neg T\langle A\rangle$

The result holds by transitivity. So the first thing to note about the secondary paradox is that if [Tr] contraposes, the primary and secondary paradoxes are equivalent to each other (modulo double-negation). So if one accepts the contraposability of [Tr], given the primary paradox, one should just accept the secondary one.

Fujikawa's argument for the secondary paradox is not this, however. It rests on the claim that if $A(x)$ is any atomic condition, and t has no referent, then $A(t)$ has no truth value. That is, for such an $A(x)$:

[Neutral] $\neg\exists x \langle t\rangle Dx \Rightarrow (\neg T\langle A(t)\rangle \wedge \neg T\langle \neg A(t)\rangle)$

This condition is implemented in neutral free logics.[16] So assuming $\neg\exists$Right;

- $\neg T\langle A(n)\rangle \wedge \neg T\langle \neg A(n)\rangle$

(at least when n is $\mathbf{n}$).

Actually, to get the secondary paradox, since En is not an atomic sentence, [Neutral] needs to apply to some non-atomic sentences as well, as Fujikawa notes (fn. 8). In other words, we need to assume that if, whatever x is, the value of $A(x,t)$ is neither true nor false, so is the value of $\exists x A(x,t)$. This makes a certain assumption about how quantifiers work, but it is natural enough; so in the present context we may let pass without comment.

[Neutral] is Fregean orthodoxy—indeed a much stronger form, where $A(t)$ is an arbitrary formula, as Fujikawa notes. Frege would, of course have taken the consequent of [Neutral] to behave consistently. Clearly, we are not making this assumption here. More importantly, Frege's view is problematic, even for atomic sentences.[17] Notoriously, the account has a problem with negative existentials. We say truly that the greatest prime number does not exist. Or suppose that someone believes that the Pope in 100 BCE was a woman. I may say: the Pope in 100 BCE was not a woman: there was no Pope in 100 BCE.

A quite different (and arguably more adequate) strategy concerning non-denoting terms is simply to take atomic sentences containing them to be false, and so their negations to be true. That is, where $A(t)$ is atomic, we have:

[Negative] $\neg\exists x \langle t\rangle Dx \Rightarrow T\langle \neg A(t)\rangle$

[16] See Priest (2008), 13.4, 21.7.
[17] See Priest (2008), 7.8.

This is the strategy adopted in negative free logics.[18] A natural way of avoiding the secondary paradox is, therefore, to adopt this strategy concerning non-denoting terms (and to reject the transitivity of [Tr]), as I will show in the next section. Those who do not care for such matters can skip this, and simply take my word for it.

6 Formal Matters

To make matters as clear as possible, I will proceed in four stages.

6.1 Identity

Let $\mathcal{L}$ be the usual language of first order arithmetic. To keep matters simple, we can suppose that this is formulated in such a way that the only terms are numerals. Let $\mathcal{N}$ be the standard (classical) interpretation of $\mathcal{L}$. Since every classical interpretation is an LP interpretation, $\mathcal{N}$ is also an LP interpretation. Let $\mathcal{N}^0$ be the LP interpretation which is the same as $\mathcal{N}$, except that $\langle 0, 0\rangle$ is in the anti-extension of $=$. Then, as is well known, everything true in $\mathcal{N}$ is true in $\mathcal{N}^0$.[19] But $\mathcal{N}^0$ makes more things true, notably $\underline{0} \neq \underline{0}$. (I use underlining to indicate the numeral of a number.) As may be checked, the model verifies both $\exists x\, x = \underline{0}$ and $\forall x\, x \neq x$. In what follows, 0 will be our example of an object that is not an object.

6.2 Denotation

We now extend the model $\mathcal{N}^0$ to a model of [Den], $\mathcal{N}^0_D$. Well, not quite: LP does not have a detachable conditional, but we may interpret $\Rightarrow$ as truth preservation in $\mathcal{N}^0_D$. This means that one cannot formulate [Den] as universally quantified, but since every object in the domain of our model has a name, we can formulate it equivalently schematically:

[SDen] $\qquad \langle t\rangle\, D\underline{k} \Leftrightarrow t = \underline{k}$

where k is any natural number. So we extend $\mathcal{L}$ with a new two-place predicate, D. We assume any standard gödel coding, #, and take $\langle t\rangle$ to be $\underline{\#t}$.

[18]See Priest (2008), 21.7. It is advocated in a dialetheic context in Priest (1987), 4.7.
[19]Priest (2002), 4.6 and 6.4.

Let $den(t)$ be the denotation of t, $e_=$ be the extension of $=$, and $a_=$ be the antiextension of $=$ (all in $\mathcal{N}^0$, and so $\mathcal{N}^0_D$).

If we set the extension of D to be $\{\langle \#t, k\rangle : \langle den(t), k\rangle \in e_=\}$, then [SDen] holds in both directions. In particular, the model verifies $\exists x\, xD\underline{0}$. The contraposed form need not hold. For example, $\underline{0} \neq \underline{0}$ holds, though $\neg\underline{\#\underline{0}}D\underline{0}$ may not. However, if we set the antiextension of D to be $\{\langle \#t, k\rangle : \langle den(t), k\rangle \in a_=\}$, it does. In particular, the model verifies $\neg\exists x\, xD\underline{0}$ (though not $\neg\exists x\, xD\underline{k}$, if k is not 0). In particular, the model verifies both $E\underline{0}$ and $\neg E\underline{0}$.

6.3 Contraposible Truth

For truth, we build on $\mathcal{N}^0_D$, where [Den] contraposes. First, we add a monadic truth predicate, T. Consider the following model of the extended language. For vocabulary other than T, matters are as in $\mathcal{N}^0_D$. For T: if k is the code of a formula, it is in both the extension and antiextenion of T; otherwise, it is just the antiextension.

Using this as the ground model, we construct a fixed point for T, $\mathcal{N}^0_{DT}$, in the usual way, ascending through the ordinals.[20] As we ascend, everything remains the same, except for the extension and antiextension of T for codes of formulas. For successor ordinals, $\#A$ is in the extension of T if A is true at its predecessor; $\#A$ is in the antiextension of T if A is false at is predecessor. At limit ordinals, the extension and antiextension of T are just whatever stable value they have achieved at that point. Eventually, we arrive at a stage where the value of A is the same as the value of $T\underline{\#A}$. So interpreting $\Rightarrow$ as truth preservation in $\mathcal{N}^0_{DT}$ we obtain a model of [Tr] where $\Rightarrow$ contraposes. Since the construction varies the values only of those formulas containing T, since $\mathcal{N}^0_D$ verifies $E\underline{0}$ and $\neg E\underline{0}$, $\mathcal{N}^0_{DT}$ verfies these, as it does $T\langle E\underline{0}\rangle \wedge T\langle \neg E\underline{0}\rangle$, and so $\neg T\langle E\underline{0}\rangle \wedge \neg T\langle \neg E\underline{0}\rangle$.

6.4 Non-Contraposible Truth

Constructing an interpretation where [Tr] fails to contrapose is less standard, but one suitable way of doing it in the present context is as follows.[21] We construct the fixed point as before, with a small variation concerning the

[20] See Priest (2002), 8.1.

[21] This is a slight variation on the construction given in Priest (2010), Sect. 11.

extension and antiextension of T at successor ordinals. If A contains 'T', or is an atomic sentence which does not, the construction is as before. However, for any other sentence, A, if at the previous stage A is true (true only or both true and false), A is put in the extension of T, but not its antiextension.

Working at the fixed point: [Tr] now fails to contrapose. If A is any non-atomic T-free sentence which is both true and false, $T\langle A\rangle$ is true only. (If A is a sentence that contains 'T ', negation still communtes with truth.)

We now show that the model verifies [Negative]. If$\neg\exists x\ \langle t\rangle\, D\underline{k}$ holds then $k = 0$. The predicate of an atomic sentence is either T, D, or $=$. So any atomic sentnce, $A(\underline{0})$, is either $T\underline{0}$ or $\underline{0} = \underline{j}$ (or $\underline{j} = \underline{0}$), $\underline{0}D\underline{j}$ or $\underline{j}D\underline{0}$.

- Provided that 0 is not the gödel number of any formula (which we can ensured by an appropriate coding), $T\underline{0}$ is false only, so $\neg T\underline{0}$ is true only, and $T\langle\neg T\underline{0}\rangle$ is true.

- If the denotation of $\underline{j}$ is not 0, $\underline{0} = \underline{j}$ is false only, so $T\left\langle\neg\underline{0} = \underline{j}\right\rangle$; and if the denotation of $\underline{j}$ is 0, then $\underline{0} = \underline{j}$ is both true and false; so again $T\left\langle\neg\underline{0} = \underline{j}\right\rangle$.

- If 0 is not the gödel number of any term (which again can be ensured by appropriate coding), then $\underline{0}D\underline{j}$ is false only, so $T\left\langle\neg\underline{0}D\underline{j}\right\rangle$; and if $\underline{j}D\underline{0}$, then $j = 0$, in which case $\underline{j}D\underline{0}$ is both true and false, so $T\left\langle\neg\underline{j}D\underline{0}\right\rangle$.

So [Negative] is satisfied in all cases.

However, $E\underline{0}$ and $\neg E\underline{0}$ are both true and false at the ground model, and neither is atomic. Hence both $T\langle E\underline{0}\rangle$ and $T\langle\neg E\underline{0}\rangle$ are true only at the fixed point, and so $\neg T\langle E\underline{0}\rangle$ and $\neg T\langle\neg E\underline{0}\rangle$ are false only at the fixed point. So the secondary paradox fails.

For a minor variation which verifies $\neg T\langle E\underline{0}\rangle$ and $\neg T\langle\neg E\underline{0}\rangle$, we simply treat sentences containing 'D' in the way that we treated sentences containing 'T', when ascending the ordinals.

I note that the models of this and the last sub-section show that the behaviour of T in $\mathcal{N}^0_{DT}$ does not spread contradictions into the language of the ground model. The sentences without T which are verified are exactly those which hold in the ground model.

7 Taking Stock

The considerations thus far with respect to the secondary paradox have been somewhat complex; so before we proceed, let us take stock.

As we saw, given the argument which uses [Den], one can get off the train of thought which delivers the primary paradox straight away, by denying the contraposability of $\Leftrightarrow$. But we may be forced into the paradox for the special case of **n** in a way which does not deploy [Den] at all. And in any case, perhaps the major interest of Fujikawa's paper is the question of whether, given the primary paradox, there is a secondary one.

Given this, we have seen a couple of ways one may respond to the secondary paradox. We can simply accept the contraposability of [Tr], and so the equivalence of the primary and secondary paradoxes. Alternatively, we may reject [Neutral] in favour of [Negative]. Without the contraposability of [Tr], the secondary paradox is simply not forthcoming.

8 Fujikawa's Possible Responses

In the light of the preceding considerations let us consider the responses to the paradox which Fujikawa discusses. He has three.

8.1 Rejecting [Neutral]

The first is to reject [Neutral] in favour of [Negative], and so the secondary paradox. As I have discussed, this is a solution which has much to recommend it.

Fujikawa says that the thought is attractive to those who identify untruth with falsity, but suggests that it is not so for a dialetheist: if $A \wedge \neg A$ is a dialetheia, A is false, but not untrue. However, this is too fast. Those who adopt the contraposability of [Tr] explicitly accept the equivalence of $\neg T \langle A \rangle$ and $T \langle \neg A \rangle$, as we have noted. Moreover, even one who does not endorse this thought, though they do not identify untruth with falsity, may yet take the former to be a *sufficient condition* for the latter—so delivering Excluded Middle.[22]

[22]See Priest (1987), 4.8.

8.2 Restricting [Den] and the Principle of Identity

The second suggestion is to break the applicability of [Den] in the way required for the primary paradox.[23] This blocks the primary paradox, and so the secondary paradox which piggy-backs on it. As we have seen, rejecting the contraposability of [Den] will do exactly this.

Fujikawa suggests a number of ways in which one might break the applicability of [Den]. One may insist that the ns in question are such that $n = n$, but not $n \neq n$. This has the same effect as disallowing the contraposed form of [Den]. Another suggestion is to restrict [Den] to those x that are consistently objects. Thus, we cannot apply it to those x such that $x = x \wedge x \neq x$. This is not exactly the same as the rejection of contraposition, but has a similar effect. Unfortunately, this condition is hard to formulate in any effective way. Perhaps the most radical suggestion is that, for the ns in question, we do not even have $n = n$. This blocks the relevant uncontraposed applications of [Den]. However, as Fujikawa notes, this requires adopting a somewhat unusual account of identity.[24]

However, in many ways, these moves are beside the point, since, as we saw in §4, there are arguments for the primary paradox in the special case of **n**, which do not use [Den] at all.

However, rejecting the principle of identity, $\forall x\, x = x$, clearly has an impact on the considerations of §2 as well. Indeed, one can no longer define 'x is an object' as $\exists y\, y = x$, since some xs in the domain of quantification, $\mathcal{D}$, may not satisfy this. One might simply refuse to call such things objects (as Fujikawa suggests); but I think it better to take the things which do not satisfy $x = x$ simply to form a special class of objects. After all, the things in the domain are, well, things; that is, objects in some sense.

How, then, might one express the claim that x is an object, that is, is in the domain of quantification? In fact, if $\forall x A(x)$ is any logical truth, $A(x)$ will do the job. (It is just that, in this framework, $\forall x\, x = x$ is no longer a logical truth.) A minor rub will occur if the logic to be deployed has no logical truths (as is the case with FDE). We must then introduce a new

[23]Fujikawa also mentions the possibility of rejecting the premise that some things are both objects and not objects, as any classical logician would. That debate is not on the agenda here.

[24]Though it is easy enough to formalise. One just takes the extension of $=$, $e_=$, to be a proper subset of $\langle\{d, d\rangle\, \{d \in \mathcal{D}\}$, where $\mathcal{D}$ is the domain of quantification. The most natural candidate for the anti-extension of $=$, $a_=$, is then $\mathcal{D}^2 - e_=$. Though other things are also possible.

monadic predicate, G (for *Gegenstand*), thought of as a logical constant, whose extension is $\{d : d \in \mathcal{D}\}$. Of course, if it is true of some objects that they are also not objects, the antiextension of G will overlap this.

8.3 Allowing for the Possibility of Both and Neither

The third response considered by Fujikawa is to accept the paradoxical conclusion that is En and its like are both true and false and neither true nor false. That can, of course, only be a first move. One needs to understand how this is possible; and this requires an articulation of a semantics which allows for this possibility—and shows, moreover, that these contradictions are under control: contradiction does not spread to untoward places. Fujikawa states that this cannot can be done in LP, but can be done in a semantics with the values *both true and false* and *neither true nor false*, provided that the semantics allows for sentences to take both values in some way. Such constructions are well known and technically feasible.[25]

The claim that accepting such a status for En requires moving away from LP is false, however. The models of 6.3 and 6.4 show exactly how it is to be done. What makes this possible is, of course, the fact that the truth predicate is in the language itself. In other words, we are talking about *truth simpliciter*. In the constructions envisaged by Fujikawa, the truth predicate is in the metalanguage; and it is not *truth simpliciter* that we are dealing with, but *truth-in-an-interpretation*. This raises two questions: Is such a move necessary? Is it even coherent?

First, is it necessary? Interpretations are set-theoretic structures. Sets have nothing intrinsically to do with truth. Neither, for that matter, do interpretations. Their major function is in defining validity. Recall that Tarski's celebrated definition of truth[26] concerned *truth simpliciter*, and did not mention interpretations at all.

Moreover, all sorts of things can hold in an interpretation which have nothing to do with truth. For example, one can define an interpretation of the language of arithmetic in which '$\underline{0} + \underline{0} = \underline{16}$', '$\neg\exists x\, x > \underline{100}$', or wot not, are true. To be relevant to truth itself, the interpretation must be of a particular kind, namely, one that "gets things right". One might argue about what, exactly, that means, but a natural thought is that truth must coincide

[25] My own articulation of such a semantics is given in Priest (2014b).

[26] Tarski (1936).

with truth in that interpretation. That is, for an (interpreted) sentence of the language, the interpretation, $\mathcal{I}$, must be such that:

- $T\langle A\rangle \Leftrightarrow \langle A\rangle$ is true in $\mathcal{I}$

where the $\Leftrightarrow$ must contrapose, since the two notions are required to coincide.

We know that if the language is that of set theory, and interpretations are classical interpretations in ZFC, there is no such $\mathcal{I}$. However, there seem to be no reasons to believe this in the present case, where only the language of arithmetic in involved. But now we strike another issue. Since we are considering the possibility that, for some As, $T\langle A\rangle$ and $\neg T\langle A\rangle$, and this is not to be ruled out by fiat, it must be possible for it to be the case that one can have both of: $\langle A\rangle$ is true in $\mathcal{I}$, and it is not the case that $\langle A\rangle$ is true in $\mathcal{I}$. This means that our theory of interpretations cannot be based on classical logic: it must be based on a paraconsistent logic. This may indeed be possible, but it is fair to say that there is as yet no clear answer as to how best to do this.[27]

However, set these matters aside. The main point is that when we have to consider which interpretations are the ones that are relevant in the present case, we are thrown back to considering ones that are faithful to truth *simpliciter*. So we may as well just consider this in the first place, and ignore all the distractions.

Let us turn to the second question: is it coherent? Matters at hand concern truth, but they also concern denotation. And if it is truth-in-an-interpretation that is at issue, then it is denotation-in-an-interpretation that is at issue. If we are considering the truth of a sentence, $A(t)$, in an interpretation, $\mathcal{I}$, what t actually denotes is irrelevant; it is what t denotes in $\mathcal{I}$ that is relevant. The problem is now that the very sentences whose semantic status is at issue (such as En) themselves contain the denotation (*simpliciter*) predicate, D. Moreover, we cannot simply replace this with the notion of denotation in an interpretation. Sentences such as ' "t denotes x in interpretation $\mathcal{I}$" is true in interpretation $\mathcal{I}$' make no sense. The fact that we now have two notions of denotation in play is, at the very least, discordant.

Actually, it is worse than that. Incoherence threatens. We can not just be dealing with an arbitrary interpretation. Just as with truth, we need to be considering interpretations that "get things right". Thus, for example, there are interpretations of the language of arithmetic where $\underline{0}$ denotes 37 in

[27] For one discussion, see Priest (2020).

that interpretation. That gets things wrong. And the natural condition for the interpretation to get things right with respect to denotation is that for all terms, t:

$$[3] \qquad \forall x(\langle t\rangle\, Dx \Leftrightarrow \langle t\rangle \text{ denotes } x \text{ in } \mathcal{I})$$

(where the $\Leftrightarrow$ contraposes) and so, in particular:

$$[4] \qquad \langle n\rangle\, Dn \Leftrightarrow \langle n\rangle \text{ denotes } n \text{ in } \mathcal{I}$$

Given that the language as interpreted in $\mathcal{I}$ is talking about what it is supposed to be talking about, the right hand side must be true; but in the present context, the left hand side is both true and false, so the right hand side must be false as well. The contradictions in the object language bleed into the metalanguage. The construction must therefore be regimented with a paraconsistent logic; and as we noted with truth, how exactly this is to be achieved is still an open issue.

However, matters are worse than this. For the contradictions in the metalanguage are problematic. If the right hand side is false, this means that the language is being interpreted in such a way that it is not functioning as it is supposed to. So we find ourseselves in a self-referential position of the kind in which Wittgenstein finds himself at the end of the *Tractatus*: the conculsion drawn not only contradicts what is otherwise established, but undercuts its very own ground.[28]

Maybe there is some way of putting a happy spin on this matter, but it is not clear to me what this might be; and absent this, the move from truth (and denotation) to truth (and denotation) in an interpretation is not only unnecessary but incoherent.

9 Conclusion

So much for my analysis of Fujikawa's own suggestions, and with it my analysis of the several delicate matters his paper raises. The fact that I have disagreed with Fujikawa in a number of places does not indicate that I think ill of the paper. On the contrary. Often the most important contributions to philosophy are the discoveries of fruitful new problems/paradoxes. (One

[28]See Priest (2015), §III.

only has to think of the discovery of Russell's paradox to be reminded of this fact.) And this, Fujikawa's paper certainly does.

But, to conclude: it may fairly be asked: what is my own response to Fujikawa's secondary paradox? I think, given present considerations, the most plausible response to it is to endorse [Negative], and so the argument for the secondary paradox.

However, if there *is* a cogent argument of this conclusion (for example, deploying the contraposition of [Tr]), so be it. The contradiction is clearly under control; and we may safely say that this is another interesting fact about truth.

Let me end by returning to a Buddhist note. In a famous verse of the *Mūlamadhyamakakārikā*, one of the most important Buddhist philosophers, Nāgārjuna, says:[29]

> Everything is real and is not real,
> Both real and not real,
> Neither real nor not real.
> This is the Lord Buddha's teaching.

Perhaps we might say that '**nothing** is ineffable' is true, not true, true and not true, neither true nor not true. This is Fujikawa's teaching.

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