wish to contribute.

Happy paraconsistent *Reasoner*!

Federica Russo
Philosophy, Louvain & Kent

# §2

## FEATURES

## Graham Priest and Diderik Batens interview each other

Graham Priest is Boyce Gibson Professor of Philosophy at the University of Melbourne and Diderik Batens is Professor of Logic at the Centre for Logic and Philosophy of Science, University of Ghent. They have both produced an impressive amount of work in the area of paraconsistent logic. As their interviews show, Graham's and Diderik's disagreements on specific points or on foundational aspects become a fertile field for paraconsistency to develop further. I am very pleased to introduce Graham Priest and Diderik Batens.

DIDERIK BATENS INTERVIEWS GRAHAM PRIEST

Diderik Batens: Most arguments you offered for dialetheism derive from conceptual considerations: language, arithmetic, set theory, and the like. Do you think that true contradictions are unavoidable because of properties of humans, rather than because of properties of nature in general?

Graham Priest (pictured): Any statement is part of a language; and language, with its meaning, is a human product. But statements describe reality; and, in general, if they are true, they are so in virtue of that reality as well. Hence, any truth is liable to be a product of both of these factors. Dialetheias are no different in this regard. Of course, the reality which a language describes may itself be a human product, but often it is not. Thus, natural objects, such as a planet, are not. If a natural object in motion generates dialetheias, as I hold, then the truth of these will be partly a function of a human product (language and its meanings), and partly a function of nature and its doings.

DB: Over the years, you have elaborated an impressive technical as well as philosophical underpinning for a monolithic and dialetheist conception of human knowledge. The construction will not be complete and the arguments will not be final until there is a paraconsistent set theory that allows for the formulation of a fully fledged and coherent metatheory. Are there hopes for this to be realized soon?

GP: I agree that a paraconsistent set theory, and a paraconsistent metatheory within this, are absolutely essential. Perhaps the most natural way of obtaining them is to have an axiomatic system based on the naive principles:

COMPREHENSION $\exists x \forall y (y \in x \leftrightarrow \alpha)$ for every formula, $\alpha$

EXTENSIONALITY $\forall x \forall y (\forall z (z \in x \leftrightarrow z \in y) \rightarrow x = y)$

from which one can deduce, via the appropriate paraconsistent logic, standard results of set theory (including the paradoxes!) and also those of metatheory—e.g., appropriate soundness and completeness results. Such we do not have at the moment. To avoid triviality, the logic must not endorse various principles which are used in the orthodox proofs of these results (such as Contraction). The nearest we have come to this so far is in the work of my student, Zach Weber, who has shown how to prove most of the results of standard set theory without the problematic principles. A different way to go (described in detail in ch. 18 of the second edition of *In Contradiction*) is not axiomatic, but model-theoretic. The structure of the universe of sets is very rich: it contains the cumulative hierarchy as a (consistent) part, but also many other sets as well (non-well-founded, inconsistent, etc.) One can show that there are structures of this kind that are models not only of the appropriate naive set-theory but also of the theorems of Zermelo Fraenkel set theory. One may assume that the universe of sets is such a structure, in which case anything provable is *ZF* (including standard metatheoretic results) holds in the universe, and so is acceptable from a paraconsistent perspective too.

DB: In *Contradiction* you introduce denial as a propositional attitude. You also state that a dialetheist may commit himself to the falsehood of a statement $A$ by stating that $A$ *relevantly* implies everything (formally $A \rightarrow \perp$ *with* $\perp \rightarrow B$). Is there a difference between denying $A$ and asserting $A \rightarrow \perp$?

GP: I take assertion and denial to be different speech acts. Essentially, to assert something is to show that you accept it; to deny something is to show that you reject it. The same syntactic string can be uttered with different illocutionary forces. (So an utterance of 'The door is open' could be an assertion, a question, a command.) The utterance of a string of the form $\neg A$ can be a denial of $A$: it often is. But sometimes it can simply be an assertion of $\neg A$. For example, if someone accepts $A$ and $\neg A$—because they think it is a dialetheia, or for some other reason—an assertion of $\neg A$ can function in this way. If someone accepts $A$ and $A \rightarrow \perp$ then they are committed to everything. An utterance of $A \rightarrow \perp$ will normally, therefore, function as a denial of $A$. But in the

mouth of a trivialist (a person who accepts everything), it will simply be an assertion of $A \rightarrow \bot$.

DB: One of your central arguments may be paraphrased as follows. "If classical negation (or material implication) is sensible, there are true classical contradictions, and hence all statements are true. But this is not the case. So classical negation is not sensible." Suppose we restrict the formation rules of natural language: a statement is not well-formed if its well-formedness entails a classical contradiction. Why would such a restriction be less acceptable than declaring classical negation and material implication nonsensical?

GP: The grammatical (formation) rules of a natural language are what they are, and we have no control over them. We could, of course, change those rules, and so produce a new language. The test suggested will not, as it stands, deliver classical consistency. This is because $A$ and $B$ may each, on its own, pass the test, even though $A$ and $B$ together deliver classical inconsistency. But even assuming that some more holistic test could be devised, the strategy is still problematic. There would be no way to tell whether a sentence of the new language is grammatical (since there is no decision procedure for inconsistency). A language such that one cannot effectively tell whether a string is a grammatical cannot be used. And in any case, we are still faced with the problem of giving an account of the semantics of the original (our) language.

Graham Priest interviews Diderik Batens

Graham Priest: We are both known as paraconsistent logicians. I am a dialetheist (believing that some contradictions are actually true). You have always been hesitant about adopting the title, but I think that you are as much a dialetheist as I am (albeit of a somewhat different kind). Are you a dialetheist?

Diderik Batens (pictured): You described different kinds of true contradictions. In some papers, you argue that some languages, combined with a sensible understanding of truth, have true sentences of the form $A$-and-not-$A$. I cannot see how a competent person could deny this. I even argued for the following stronger position: the world may be thus that its best description in a humanly manageable language contains true contradictions.

This being said, I disagree with many of your arguments for dialetheism. You often presuppose an ontology that makes no sense to me. For example, you take the English language to form a system that is similar to a formal language.

Central to our differences is that I am a contextualist—see my answer to the next ques-

tion. I am also convinced that one should try to replace inconsistent theories by consistent ones, even if this may be impossible for all of them. My reasons are that, if the replacement is possible without loss of empirical adequacy or conceptual clarity, we obtain a gain. Often, however, problems with the empirical adequacy or coherence or elegance of our knowledge may be more urgent than its consistency.

GP: I am a logical monist, and hold that there is essentially one correct deductive logic. You have always been a pluralist, holding that different logics are appropriate for different contexts. Your position could be thought of as some kind of logical instrumentalism: logics are just tools, and on any occasion one can use whichever one gives the best answer; there is no further consideration to which a logic must answer. Are you an instrumentalist?

DB: A deductive logic fixes the meaning of a fragment of a language. Languages are not God-given but are complex social constructions. We (try to) modify them in view of what we (think to) learn about the world. Such conceptual changes occur frequently in the languages of the sciences and, with some delay, in natural languages as well. Which languages are most adequate to handle certain aspects of the world cannot be settled a priori. Few will balk at this for 'referring terms' such as "phlogiston" or "mass". I claim it also holds for logical terms. The language of quantum mechanics offers a nice illustration. You yourself gave empirical arguments for dialetheism—whether I agree with them is not the point here. So my view is this: logicians develop logics just like one invents instruments, but nature (as knowable by us) determines which are the good instruments.

This qualifies your phrase "gives the best answer". Moreover, a logical instrumentalist has to *justify* that a specific language is used to tackle a specific problem. The Ghent group has contributed to the solution of this difficult but fascinating problem.

My reasons for this brand of instrumentalism derive from my view on knowledge. Now and presumably forever, our best knowledge will not form a monolithic set of statements. Our knowledge systems consist of chunks that are more or less internally coherent, but need not be and often cannot be mutually coherent. So a unique language cannot be used in all contexts, for languages involve presuppositions. A further argument regarding language derives from the way in which humans tackle problems. Let a context be a problem solving situation in the broadest sense of the term. In a context, we rely on the best relevant (unquestioned) *part of* our knowledge. The parts we rely on in different contexts need not to be coherent. In one context we may try to figure out the nature of heat. In a different context we assert statements that presuppose our present view on heat. So the meaning of the language elements varies
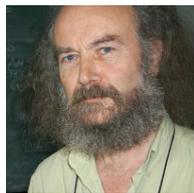
with the context. This is why I consider it sensible to use classical logic in one context and a paraconsistent logic in a different context. Your objections to classical logic typically presuppose a knowledge system and a language that are both universal and monolithic. For me these are just two fictions of Western philosophy.

GP: You are perhaps best known for your adaptive logics, an invention that I admire very much. Adaptive logics are just one kind of non-monotonic logic, however. Do you think that there is anything very special about adaptive logics within that general class?

DB: Many adaptive logics, including inconsistency-adaptive logics, are indeed non-monotonic, but others are monotonic. The aim of the enterprise was to characterize all forms of defeasible reasoning by an adaptive logic in standard format, which is a specific and strict structure. The standard format offers the proof theory and semantics as well as most of the metatheoretic properties, including soundness and completeness. The proof theory for defeasible logics is remarkable. It allows one to explicate human reasoning. Incidentally, defeasible logics do not concern deduction, but the formal characterization of *methods*. They are instruments according to everyone's view. And they are numerous, as desired.

Today, adaptive logics form the most elegant unifying frame that I know of (all known first order defeasible reasoning forms are characterized). If the future offers a better unifying frame, so be it.

Inconsistency deserves a separate comment. Inconsistency-adaptive logics are useful instruments for trying to restore consistency wherever possible. They locate the inconsistencies and interpret theories as consistently as possible; other adaptive logics guide one to remove inconsistencies. If one does not try to remove inconsistencies where possible, as is the case for you, one will still consider most classical reasoning as correct because *most* contradictions are false. You made this point: adaptive logics offer a way to systematically recapture most classical reasoning. So inconsistency-adaptive logics are useful instruments for everyone, from the classical logician to the hard dialetheist.

## Paraconsistent set theory

The concept of a *set* is simple to state: A set is any collection of objects that is itself an object, and its identity is completely determined by its members. In first order logic this concept is captured in a pair of axioms, which look like the definitions of identity and predication, respectively:

$$x = y \leftrightarrow (\forall z)(z \in x \leftrightarrow z \in y),$$
$$\Phi(x) \leftrightarrow x \in \{z : \Phi(z)\}.$$

Frege stated the set concept in a single axiom,

$$\{z : \Phi\} = \{z : \Psi\} \leftrightarrow (\forall z)(\Phi(z) \leftrightarrow \Psi(z)).$$

Frege's axiom looks like a tautology: *The set of $\Phi$s is identical to the set of $\Psi$s exactly when the $\Phi$s are all and only the $\Psi$s.* That is obvious to the point of banality. Sets and concepts, or properties, or predicates-in-extension, are all much the same thing. Let us call this the *naive set concept*.

As is well known, the set concept has inconsistent consequences. The inconsistency is not an accident, nor is it unimportant. These inconsistencies—paradoxes, since they are contradictions hiding inside of a tautology—all arise from a *diagonal* construction, the most famous of which is Russell's from 1902, arising in the concept of membership itself. Where $R = \{z : z \notin z\}$,

$$R \in R \leftrightarrow R \notin R.$$

Then by the law of excluded middle,

$$R \in R \land R \notin R,$$

a contradiction.

Diagonals have been and continue to be a very fertile source of information. One prominent attitude takes discoveries like Russell's to show that our intuitions are "bankrupt," because contradiction is the worst thing that can happen—worse, say, than abandoning hope of a precise theory of sets. *Paraconsistency* can be taken as the doctrine that a contradiction is not the worst thing that can happen. In fact, since contradiction does seem to be the sort of thing that happens, it is rather unhelpful to panic when they do. Any logic is paraconsistent when the inference from $\Phi, \neg\Phi$ to $\Psi$ for arbitrary $\Psi$, called *explosion*, is invalid. In this way, paradoxes can be accommodated. For important philosophical concepts like sets, paraconsistent reasoning should be used.

To develop a paraconsistent set theory, some otherwise familiar inferences beyond explosion are not truth preserving and so cannot be used. The disjunctive syllogism is the most famous example; contraction is another, due to Curry's paradox. Depending on choices, there are others, just because the set concept is very powerful and cannot be used without care. The hard work for the naive set theorist is to prove core theorems by purely paraconsistent arguments, which in many cases cannot follow the proofs found in standard texts.

The hard work is worthwhile, though. More than just being accommodated by a paraconsistent theory, the diagonal paradoxes that arise naturally in our naive concepts can actually be shown to be very fecund. The details will depend on exactly which paraconsistent logic is being used; there are many such logics, e.g., da Costa's $C_1^=$ or Priest's *LP*, and these have been variously