Change, Time, and Contradiction: Some Scenes from the History of a Problem

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Abstract

The problem of what happens at an instant of change is an old and venerable one. Many natural considerations push towards the thought that a contradiction is realised at the instant. The problem was discussed at length by Aristotle and Medieval Latin philosophers. It informs the views of motion of Hegel and Russell. It is part of the contemporary case for dialetheism. In this essay we will review a number of crucial episodes in the history of the topic.

* * *

1 Introduction

Given any two interdependent quantities, x and y (e.g., force and acceleration, colour and place, position and momentum), one can change with respect to the other. If these quantities have continuous magnitudes, the rate of change of the first with respect to the second is dx/dy. Change *simpliciter* is when the second of these is time, t. It is this which will concern us in what follows.

Clearly, an object can have one state (e.g., *being red*, *moving*) and change to having another (e.g., *not being red*, *being at rest*). There is then a transition, and the question arises as to what to say about the state of the object in the process of transition. A particularly thorny issue is what to say when the transition occurs instantaneously (Mortensen 2020). We will focus on this in what follows.

The issue has been a persistent one through the history of Western philosophy (and for all I know, Eastern philosophy too). There is no hope of a comprehensive history in an essay of this length. What I will do is track through some important episodes from Ancient Greece to the present day. Contradiction will be what binds the episodes together.

2 Aristotle

Let us start with Aristotle (384–322 BCE). The problem we are concerned with is posed by joining two Aristotelian claims (Kretzmann 1982: §1). The first is that:

- A change is a change between contradictories.

The point is made in *Physics* V: 1, though one has to disentangle this from the discussion. The thought is pretty obvious, though. If p describes the state in question, and there is a change, p has to cease to be the case. Assuming that there are no truth-value gaps, $\neg p$ must then hold.

Given this, the change must occur at an instant of time. For suppose that it occurred over a period of time, say between t_1 and t_2:

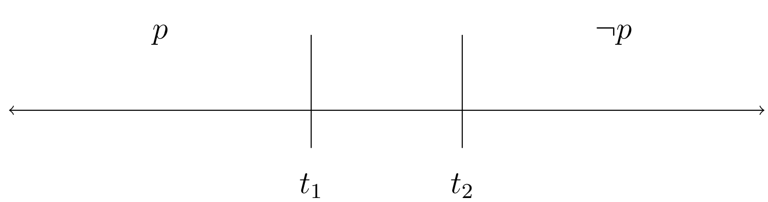

Fig. 1

Then p ceases to hold at t_1. So if change is between contradictories, $\neg p$ must hold thereafter. So t_1 is t_2. The transition can be no more than instantaneous.

The second claim is that:

- Time is continuous. Continua are not constituted by points (as would now be thought). A point is simply a cut in a continuum, and the difference between continuity and contiguity is that, in continuity, the boundary point is in both parts of the cut.

As Aristotle puts it, *Physics* 234^a5-10 (translations from Aristotle are from Barnes 1984):

> Now the now that is the extremity of both times must be one and the same; for if each extremity were different, then one could not be in succession of the other, because nothing continuous can be composed of things having no parts; and if the one is apart from the other, there will be time between them, because everything continuous is such that there is something between its limits described by the same name as itself.

Now, let us call the times at which p holds t_p, and the times at which $\neg p$ holds $t_{\neg p}$; then the point between them, τ, belongs to both. The situation, then, is as follows:

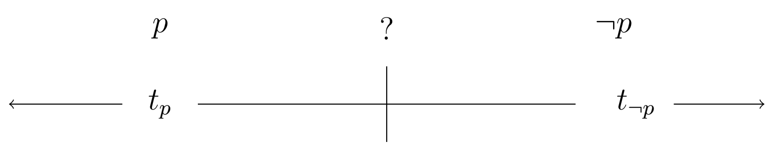

Fig. 2

But τ is in both t_p and $t_{\neg p}$. So p and $\neg p$ both hold at τ, contrary to the Principle of Non-Contradiction (PNC).

Aristotle cannot, of course, accept this. It would appear that he has, in fact, two solutions to the problem (Sorabji 1976, 1983; Strobach 1998: ch. 2). He distinguishes between what are, in effect, states that can hold instantaneously, and states that require a period of time (*Categories*, 5^a15–37). Examples of the first kind are *being a man*, *being white*; examples of the second kind are *being in motion*, *becoming white*.

The first solution (*Physics* 263^b9-15) appears to apply to changes between states of the first kind. The solution for these is to say that the instant of change belongs to only the posterior state:

> It is also plain that unless we hold that the point of time that divides earlier from later always belongs only to the later so far as the thing is concerned, we shall be involved in the consequence that the same thing at the same moment is and is not... It is true that the point is common to both times, the earlier as well as the latter, and that, while numerically one and the same, it is not so

> in definition, being the end of the one and the beginning of the other; but so far as the thing is concerned it always belongs to the later affection.

This solution fails to convince. If τ, the point of cut, is in both times, then, even if it can be characterised in two different ways, p and $\neg p$ will still both be true at it.

Aristotle's second solution (*Physics* 234^a24-234^b7) seems to apply to transitions between states of the second kind. For such transitions neither state holds at the instant of change:

> We will show that nothing can be in motion in a now... Neither can anything be at rest... inasmuch as it is the same now that belongs to both the times, and it is possible for a thing to be in motion throughout one time and to be at rest throughout the other, and that which is in motion or at rest for the whole of a time will be in motion or at rest in any part of it in which it is of such a nature as to be in motion or at rest: it will follow that the same thing can at the same time be at rest and in motion; for both of the times have the same extremity, viz. the now.

It is, to say the least, difficult to reconcile Aristotle's view that something is neither in motion nor at rest at an instant of change between the two, with his endorsement of the Principle of Excluded Middle (PEM) in Book 4 of *Metaphysics*.

3 Medieval Latin Philosophy

The problem was discussed at great length in the Latin Middle Ages, often in connection with inferences concerning the verbs *incipit* (begins) and *desinit* (ceases). The standard move was to assign τ to either t_p or $t_{\neg p}$, but not both. One finds this sort of solution in William of Sherwood (1190–1249), Peter of Spain (d. 1276?), William of Ockham (1285–1347), Richard Kilvington (1305-1361), Marsilius of Inghen (1340–1396). (Kretzmann 1976; Strobach 1998: ch. 3; Ciola 2017: 20 ff; Sorabji 1983). Some argued that it was in t_p; some that it was in $t_{\neg p}$; some that it might be in one or the other.

This makes something of a mess of Aristotle's account of continuity. Some sense had to be found to the thought that the cut (point of division) in a continuous magnitude was intrinsic to one side, but extrinsic to the other. But at least from a modern perspective, it seems to be moving in the right direction. Thus, consider a car starting from rest at $x = 0$, moving to the right with uniform acceleration. Then there is a last moment at which the car is at 0 and no first moment at which the car is at a position $x > 0$. Similarly, there is a last moment at which the velocity of the car, v, is 0, and no first moment at which its value is $v > 0$.

- Call a change where there is a last moment of the prior state, but no first moment of the posterior state a *type A change.*

On the other hand, suppose that the car comes in from the left and decelerates uniformly, coming to rest at $x = 0$. Then there is a first moment at which the car is at 0, and no last moment at which the car is at a position $x < 0$. Similarly, there is a first moment at which the velocity, v, is 0, and no last moment at which its value $v > 0$.

- Call a change where there is a first moment of the posterior state, but no last moment of the prior state a *type B change.*

However, there is now a new problem. Type A and B changes are natural when the changes are asymmetric. However, there are transitions where there seems to be perfect symmetry. For example, I walk through the door. At the instant that I (or better, my centre of gravity) is in the door frame (better, on the vertical plane going through the centre of gravity of the door frame), I would seem to be symmetrically poised between being in and not in, or out and not out. Another example: consider the instant of midnight between Monday 1st and Tuesday 2nd. Is this a time in Monday or a time in Tuesday? The instant seems perfectly symmetrically poised between the two. And if one wishes, one can add to the list of examples quantum transitions. An electron in an atom makes a transition from one quantum state ("orbit") to another. There is simply a discontinuity here, with no symmetry-breaking considerations.

In situations such as these, an asymmetric verdict concerning where the instant of change lies seems wrong. Of course, there are two symmetric possibilities: that the instant of change, τ, is in *neither* the prior nor the posterior state. Or that it is in *both* the prior and posterior state.

- Call a change where, at the instant of change, neither the prior nor the posterior state holds, a *type* Γ *change.*

- Call a change where, at the instant of change, both the prior and the posterior state hold, a *type Δ change.*

In a type Γ change p is neither true nor false at τ; in a type Δ change, p is both true and false at τ. Both sorts of change cause problems for those Medievals who subscribed to the PEM and PNC (that is, virtually all of them).

Are there ways of deciding whether a symmetric change is of type Γ or of type Δ? Certainly in some cases. Thus consider the instant of midnight. If this is of type Γ, then the instant is neither Monday nor Tuesday (and neither not Monday nor not Tuesday). But it is obviously not some other day, such as Friday 5th. So it seems to be outside time! This certainly seems wrong. Better to say that it is both Monday and Tuesday. The whole issue of symmetric boundaries, and the inconsistencies which they appear to deliver, is taken up by Weber and Cotnoir 2015. By analogy with what they say about space (§2.2), we might call a gap at the instant of midnight a rip in time.

4 Two Further Considerations

One might bolster these considerations by appealing to two further considerations. The first is a certain continuity principle endorsed by mathematicians such as Leibniz (1646–1716), L'Huilier (1750–1840), and others. One way to express it is thus (Priest 1982: §4):

- Whatever holds arbitrarily close to a limit holds at the limit.

The principle must be treated with great care (Priest 2006b: ch. 11). However, if this principle holds in the case at issue, since p holds arbitrarily close to τ (from the left), it holds at τ; and since $\neg p$ holds arbitrarily close to τ (from the right), it holds at τ. So we have a type Δ change.

The second consideration concerns states of change—or maybe better, states of *changing*. Any instant of change obviously realises a state of change in some sense: before, one thing; after, another. But it need not be a state of *changing*. Or to put it another way, there need be nothing intrinsic to an instant of change in this sense. Thus, suppose that we have an instantaneous change of type A; then there is no difference between the relevant state at τ and the states at times before it. The time realises a state of change only in virtue of what comes later. Symmetrically for a change of type B.

Now, are there states of *changing*, states that intrinsically constitute change? It is certainly counterintuitive to suppose that things can change without there being any changing. There would then be no change as such, simply a series of prior and posterior states. This conflicts with the natural thought that real change requires some sort of state of fluxation (Priest 2006b: 12.2). What could such a state be like?

The most obvious answer is that it is exactly the transition state in a type Γ or type Δ change. For these realise a state intrinsic to the instant, different from the prior and posterior states. Of the two, it is the type Δ change that is the most natural candidate. For in a type Γ change, the transition state is one where neither p nor $\neg p$ holds. Nothing in this requires $\neg p$ to happen. Indeed, there is nothing in principle to stop the situation returning immediately to the prior state. Compare this with a type Δ change. In this, $\neg p$ has already started. (So even if the system immediately reverted to being in just the state p, $\neg p$ would still have obtained—if only for an instant.)

5 Instants of Nature

A few Medieval Latin philosophers held a quite different view of the instants of change, one which accommodates symmetric changes. This was endorsed by Hugh of Newcastle (d. 1322), John Baconthorpe (1290–1347), and Landulf Caraccioli/Caracciolo (1280/1285–1351). (Knuuttila and Lehtinen 1979; Kretzmann 1982, §§2, 3; Spade 1982.) Drawing on a passages in Aristotle where he talks about simultaneity and priority in nature (*Categories* 12^b11–13, 15^a8–11) and possibly influenced by the theory of instants of nature developed by Duns Scotus (1265/66–1308) (Spade 1982 §5; Kretzmann 1982: 277) they suggested that at any *one* time, there could be instants of nature at which different things are true. So, in particular, at time τ, there could be different instants of nature ν_p and $\nu_{\neg p}$ at which p and $\neg p$ are (respectively) true. As Baconthorpe puts it (Kretzmann 1982: 277):

> The termini of a change are separated from each other only as much as the duration of the change that mediates between the termini, but an instantaneous change does not endure except for an instant alone; therefore its termini are separated not in accordance with the parts of a duration, but solely in accordance with the order of nature... the being and the not-being that are the

> termini of any such change occur at the same instant, although in the same instant there is the order of nature.

It must be said that the notion of an instant of nature is an obscure one. More than that, it would appear that to appeal to it is a patently *ad hoc* attempt to save the PNC.

Nor is it clear that it does save it. For a start, are the instants of nature at τ ordered? This is certainly suggested by talk of priority. And if we suppose so, then we may consider the instants of nature under that ordering. Presumably, the ordering of times imposes an ordering on instants of nature, so that if t_1 is before t_2, any instant of nature at t_1 is before any instant of nature at t_2. Moreover, if p holds at any time before τ, it presumably holds at all instants of nature for that time—and similarly for $\neg p$. Finally, it would be bizarre to suppose that ν_p came after $\nu_{\neg p}$ in this ordering, for then, the change would have to double back on itself. Given these things, we can run exactly the same problem with respect to the sequence of instants of nature, instead of instants of time. Nothing has been gained (Spade (1982), §5).

And even if instants of time are not ordered, the problem has not been avoided, merely hidden. We still talk of what is the case at a particular time. Now what is the situation at time τ? Given that p and $\neg p$ are contradictories, one or other must hold, and by symmetry, it must be both.

6 Infinitesimals

For our next episodes we move forward a few hundred years to events in what is usually called the scientific revolution. A number of things then happened which are relevant.

The first was the arithmetization of the continuum: real numbers could be assigned to points in a continuum. That made it possible to think of a continuum as composed of such points. The thought is not entirely unproblematic, but it did make it easier to understand how a boundary point could be intrinsic to (a member of) one side of a cut, but not the other.

The second was that, because of developments in physics, motion was the kind of change which took centre-stage—that is, velocity: change of place with respect to time; and the second-order quantity, acceleration: change of velocity with respect to time. However, since all change would seem to require motion of some kind, one might think of this as the most general kind of change.

The third was the development of a theory of infinitesimals to handle the mathematics of such change. According to this, given any point on the continuum, there is an infinitesimal displacement from it. So the velocity of an object in motion at a point, x, can be defined as the distance moved in such an infinitesimal time; and the acceleration can be defined as the amount the velocity changed in such a time (Boyer 1959: chs. 5, 6). (There is an obvious similarity between instants of nature and infinitesimals; but as far as I know, there is no historical connection between the former to the latter.)

Infinitessimals raised a certain problem. Let us suppose, for the sake of example, that an object is moving according to the equation $x = t^2$. Let δt be an infinitesimal displacement from t, and δx be the corresponding change in x. Then $x + \delta x = (t + \delta t)^2 = t^2 + 2t\delta t + (\delta t)^2$. So $\delta x = 2t\delta t + (\delta t)^2$; and dividing by, δt, ${}^{\delta x}/_{\delta t} = 2t + \delta t$. Now, δt is infinitesimally close to 0, so we can ignore it and thus obtain ${}^{\delta x}/_{\delta t} = 2t$. The problem is that we have assumed that δt is non-zero (to divide by it), and effectively that it is zero (at the end). This didn't seem to worry mathematicians much, who carried on regardless, though it did worry some philosophers. Berkeley, for example, ridiculed the use of infinitesimals, calling them 'ghosts of departed quantities' (*The Analyst; of a Discourse Addressed to an Infidel Mathematician*, 1734).

7 Hegel

One philosopher who did address the issue about infinitesimals squarely was Georg Hegel (1770–1831) (Priest 2006a). The continuous and the discrete are contradictory categories, but for such things there is the category which is their dialectical synthesis. This is a variable point: the infinitesimal. It has the property of being a point, so having zero extension, and being extended, so having nonzero extension (Haldane 1892 Vol. I: 268):

> To us there is no contradiction in the idea that the here of space and the now of time [i.e., variable points in a continuum] are considered as a continuity or length; but their notion is self contradictory. Self-identity or continuity is absolute cohesion, the destruction of all difference, of all negation, of all being for self; the point, on the contrary, is pure being-for-self, absolute self distinction and the destruction of all identity and all connection with what is different.

This understanding allowed Hegel a view of motion afforded by the infinitesimal calculus. To be in motion at an instant is precisely to move an infinitesimal amount. Thus (Miller 1970: 43):

> [When a body is moving] there are three different places: the present place, the place about to be occupied and the place which has just been vacated; the vanishing of the dimension of time is paralyzed. But at the same time there is only one place, a universal of these places, which remains unchanged throughout all the changes [i.e., the variable point]; it is duration existing immediately in accordance with its notion, and as such it is motion.

That is (Miller 1969: 440):

> External, sensible motion is itself ... [Contradiction's] immediate existence. Something moves not because it is here at one point of time and there at another, but because at one and the same point of time it is here and not here, and in this here both is and is not. We must grant the old dialecticians the contradictions which they prove in motion; but what follows is not that there is no motion, but rather that motion is existent Contradiction itself.

In other words, if an object is in motion at point p at a time t, it is both at p and not at p, since it hasn't quite got there yet, and it has also gone a little bit further. So suppose that we have an object moving uniformly from the left, which reaches the point p at time τ. Then we might depict the situation thus:

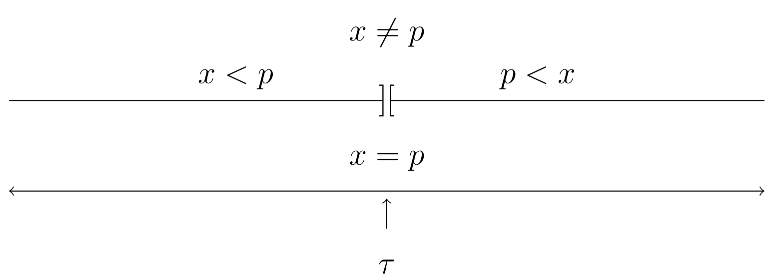

Fig. 3

The square brackets indicate that the intervals are closed, and so contain their endpoints, τ. And since $x < p$ and $p < x$ at τ, $x \neq p$ there (for two reasons).

In fact, the difference between an object at rest at point p at time t and one in motion at point p at time t is precisely that the state of the former is consistent: it is where it is and nowhere else. The state of the latter is inconsistent. It is both there and not there. So the state of motion is an intrinsic state of change, a state of *changing*.

Another feature of Hegel's account, as might be guessed from his reference to 'the old dialecticians', is that it provides him with a solution to some of Zeno's paradoxes. In this context, it is the Arrow which is most important. Consider an arrow fired towards a target. At each instant, the progress made on its journey is zero: it is where it is, and nowhere else. But the whole time is made up of such instants, so the progress made in the whole journey is the sum of the progresses made at each instant. But adding zero to zero, even infinitely many times, is zero. So the arrow never advances on its journey.

Hegel's solution should now be pretty obvious. The arrow *does* make an advance at an instant. It is where it is, but it has already gone a little bit further (and maybe hasn't quite got there yet). So the progress made at an instant is not zero.

8 Russell

Let us now move forward about another 100 years. As is well known, due to the work of Augustin-Louis Cauchy (1789–1857), Karl Weierstrass (1815–1897), and others, the use of infinitesimals in the differential and integral calculus was replaced towards the end of the 19th Century by the now familiar use of limits (Priest 1998). The possible significance of this fact was not lost on Bertrand Russell (1872–1970), who articulated what is perhaps the currently standard view of change/motion (Russell 1903: sect 447):

> Motion consists in the fact that, by the occupation of a place at a time, a correlation is established between places and times; when different times, throughout any period, however short, are correlated with different places, there is motion; when different times throughout some period, however short, are all correlated with the same place, there is rest.

Russell is actually inconsistent since, after giving this definition, he allows that something may be momentarily at rest if its positional derivative with respect to time is zero at that instant. This is quite compatible with its being in motion in the official sense. But let this pass.

Given this account, there is no such thing as an intrinsic state of motion. Whether or not something is in motion at time t depends on what happens in a neighbourhood of that time. Russell, in fact, points out that there is no such thing as an intrinsic state of change, and even revels in it (Russell 1903 pp. 351, 350, xxxiii. I have spliced the quotations together without, I think, doing an injustice to Russell):

> [Zeno's arrow argument] denies that there is such a thing as a state of motion...
>
> This has usually been thought so monstrous a paradox as scarcely to deserve serious attention. To my mind, I confess, it seems a very plain statement of a very elementary fact, and its neglect has, I think, caused the quagmire in which the philosophy of change has long been immersed...
>
> Change does not involve a state of change.

The claim that things can change whilst there is nothing changing is obviously counter-intuitive enough. But Russell's view leaves us bereft of a solution to the Arrow Paradox. It is agreed that progress made at any instant is zero, but somehow progress made in the sum of them is non-zero. All one can do is cite standard measure theory, according to which the union of an uncountably infinite collection of sets, each of length (measure) zero, can be non-zero. But that is not a solution: it is just a restatement of what needs to be explained.

9 The Paraconsistent Present

Moreover, all this notwithstanding, it remains the case, as we saw in §3, that there are instants of change which are apparently symmetric, and so where the change would appear to be of type Δ. And if so, such change is dialetheic. Since Russell subscribed to the PNC, he could not have admitted this. Indeed, the idea cannot be accommodated in "classical"—aka Frege-Russell—logic. However, it can be accommodated in a paraconsistent logic (Priest 2006b: 11.3). (A paraconsistent logic is one which does not validate the principle of Explosion: for all A and B: $A, \neg A \vdash B$.) Let us now see how.

There are many paraconsistent logics. The semantics of a simple paraconsistent tense logic is given in the appendix to this paper. In what follows, we may take the set of times, W, to be the real numbers $\mathbb{R}$, and R to be the standard ordering on these, $<$.

The following is an interpretation with a type Γ transition point. The transition point is $\tau \in W$. p is assigned true (only) at all points $t < \tau$, false (only) at all points $t > \tau$, and true and false at τ:

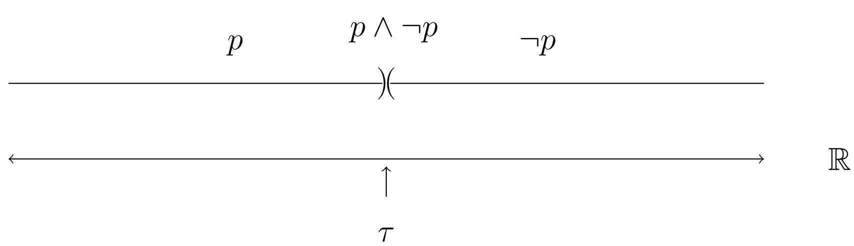

Fig. 4

The round brackets indicate that the intervals are open, and so do not contain their endpoints, τ.

In a more expressive (first-order) language we can express the Hegelian understanding of motion represented in Fig. 3. Suppose we have a point-particle, a, in motion. Let Px express the fact that a is at point x. Then if at time τ, a is at x_τ, it is also at points x_t for all t in some small interval around τ, θ_τ. And since it is at each x_t, it is not at any other. Hence at τ, $Px_t \wedge \neg Px_t$ for all $t \in \theta_\tau$:

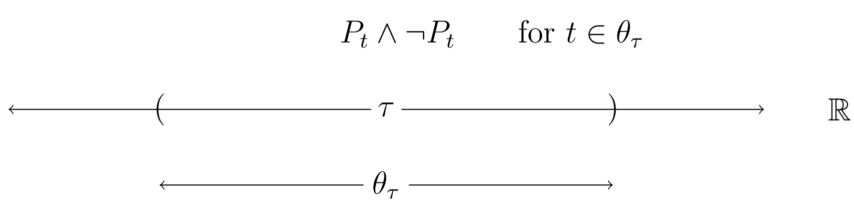

Fig. 5

This doesn't quite depict a Hegelian account of motion yet. If a is moving continuously, every time is a point of change, that is, a τ. So the situation depicted holds at every point of time. That is just a matter of complicating the diagram.

If we want to build infinitesimals into the picture (as did Hegel), we have to take W to be, not $\mathbb{R}$, but the non-standard real line, ${}^*\mathbb{R}$, as delivered by non-standard analysis (Bell 2022: sect. 7). θ_τ can then be the infinitesimal monad around τ—that is, the set of points an infinitesimal distance from τ.

There is nothing in the paraconsistent semantics which *requires* contradictory states of change to be instantaneous. The following depicts a perfectly legitimate interpretation:

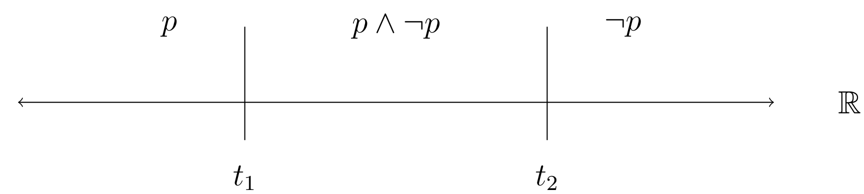

Fig. 6

Indeed, in Fig. 5, if θ_τ is the infinitesimal monad around τ, and every point is subject to the same treatment as τ, the same contradictions will hold at every point in θ_τ, since every point in it has the same monad.

One might reasonably ask whether, in the diagram of Fig. 6, there is a transition state between p and $p \wedge \neg p$ at t_1 (and symmetrically at t_2). There is. A transition state is one in which both prior and posterior states obtain. So it is the state where $p \wedge (p \wedge \neg p)$ holds. This, of course, is just $p \wedge \neg p$ itself. To be transitioning into a transition state is already to be in a transition state!

One might suggest that this is not fair. The change is between p being true only and it being both true and false (Littmann 2022; Priest 2017). To express this thought we need the language to contain a truth predicate, T, and a name-forming device $\langle . \rangle$. We then have the situation illustrated as follows:

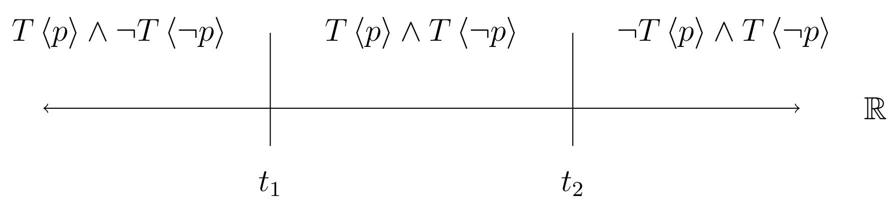

Fig. 7

Here, if t_1 is a state of transition between $T\langle p\rangle \wedge \neg T\langle \neg p\rangle$ and $T\langle p\rangle \wedge T\langle \neg p\rangle$, then at t_1 we have $T\langle p\rangle \wedge (T\langle \neg p\rangle \wedge \neg T\langle \neg p\rangle)$, and so a "higher order" contradiction. Symmetrically for t_2.

Note that there is nothing in these semantics which *requires* there to be a contradiction holding at some time. There are interpretations at which what holds at each time is perfectly classical. However, there are constraints which enforce contradiction under certain conditions. One of these is the Leibniz Continuity Condition. In the semantics, the Condition can be formulated in various different ways. Here is a natural one (Priest 2006b: 168):

- For every propositional parameter, p, and every x, $z \in W$, if $1 \in \nu_y(p)$ for every $x < y < z$ then $1 \in \nu_x(p)$ and $1 \in \nu_z(p)$.

And the same for 0. If an interpretation satisfies this condition, then abutting intervals where p holds and $\neg p$ holds ensure that both hold at the point of abutment. Thus in Fig. 4, $p \wedge \neg p$ must hold at τ, given what holds at either side.

It may seem rather arbitrary to impose this condition on only propositional parameters. But one should not expect it to hold for arbitrary sentences, or one can prove that I will live forever! Suppose that my life is finite, and that I die at time t. Take any time of my life before that. Then 'I am alive' is true then. Since time is dense, 'It will be the case that I am alive' is also true then. By the continuity condition, it is true at t. So I am alive after t!

10 Conclusion

The view that change delivers contradiction has been something of a minority view in Western philosophy, dominated as this has been by an Aristotelian *horror contradictionis*. However, as we have seen, it has a habit of punctuating this orthodoxy persistently. The techniques of contemporary paraconsistent logic now give us the ability to formulate the idea in precise mathematical terms. Perhaps, this time, the genie will not go back into the bottle quietly.

11 Appendix: A Paraconsistent Tense Logic

In this appendix, I will give the semantics of the tense logic deployed in §9, based on the paraconsistent logic LP. (Full details can be found in Priest 1982.)

The propositional language contains the connectives $\wedge$, $\vee$, and $\neg$, and the tense operators $\mathbf{F}$ and $\mathbf{P}$. An interpretation is a structure $\langle W, R, \nu \rangle$. W is a non-empty set of times (worlds), R is a binary relation on W, and ν is a map from each propositional parameter, p, and world, w, to a non-empty subset, $\nu_w(p)$, of $\{0, 1\}$. (If we drop the condition that the subset is non-empty, we get the logic FDE. If we add the condition that $\nu_w(p) \neq \{0, 1\}$, we get classical logic.)

We define truth, $\Vdash^+$, and falsity, $\Vdash^-$, at a time (world), w, as follows:

- $w \Vdash^+ p$ iff $1 \in \nu_w(p)$
- $w \Vdash^- p$ iff $0 \in \nu_w(p)$
- $w \Vdash^+ \neg A$ iff $w \Vdash^- A$
- $w \Vdash^- \neg A$ iff $w \Vdash^+ A$
- $w \Vdash^+ A \wedge B$ iff $w \Vdash^+ A$ and $w \Vdash^+ B$
- $w \Vdash^- A \wedge B$ iff $w \Vdash^- A$ or $w \Vdash^- B$
- $w \Vdash^+ A \vee B$ iff $w \Vdash^+ A$ or $w \Vdash^+ B$
- $w \Vdash^- A \vee B$ iff $w \Vdash^- A$ and $w \Vdash^- B$
- $w \Vdash^+ \mathbf{F}A$ iff for some w' such that wRw', $w' \Vdash^+ A$
- $w \Vdash^- \mathbf{F}A$ iff for all w' such that wRw', $w' \Vdash^- A$
- $w \Vdash^+ \mathbf{P}A$ iff for some w' such that $w'Rw$, $w' \Vdash^+ A$
- $w \Vdash^- \mathbf{P}A$ iff for all w' such that $w'Rw$, $w' \Vdash^- A$

Validity is defined in terms of preservation of truth at all worlds of all interpretations:

- $\Sigma \models A$ iff for every interpretation, $\langle W, R, \nu \rangle$, and for every $w \in W$, if $w \Vdash^+ B$ for all $B \in \Sigma$, $w \Vdash^+ A$.

References

[1] Barnes, J. (ed.) (1984) *The Complete Works of Aristotle*, Princeton, NJ: Princeton University Press.

[2] Bell, J. (2022) "Continuity and Infinitesimals", in E. Zalta (ed.), *Stanford Encyclopedia of Philosophy*, https://plato.stanford.edu/entries/continuity/.

[3] Boyer, C. (1959) *The History of the Calculus and Its Conceptual Development*, New York, NY: Dover.

[4] Ciola, G. (2017) "Marsilius of Inghen on *Incipit* and *Desinit* in Consequentiae II, Chapters 4–5", *Vivarium* 55: 170–198.

[5] Haldane, E. (tr.) (1892) *Hegel's Lectures on the History of Philosophy*, London: Routledge and Kegan Paul.

[6] Knuuttila, S., and Lehtinen, A. (1979) "Continuity and Contradiction: a Fourteenth Century Controversy", *Synthese* 40: 189–207.

[7] Kretzmann, N. (1976) "Incipit/Desinit", pp. 101-36 of P. Machamer and R. Turnbull (eds.), *Motion and Time, Space and Matter: Interactions in the History of Philosophy and Science*, Columbus, OH: Ohio State University Press.

[8] Kretzmann, N. (1982) "Continuity, Contrariety, Contradiction, and Change", ch. 10 of N. Kretzmann, (ed.) *Infinity and Continuity in Ancient and Medieval Thought*, Ithaca, NY: Cornell University Press.

[9] Littmann, G. (2012) "Moments of Change", *Acta Analytica* 27: 29–44.

[10] Miller, A. (tr.) (1969) *Hegel's Science of Logic*, London: Allen and Unwin.

[11] Miller, A. (tr.) (1970) *Philosophy of Nature: Being Part Two of the Encyclopaedia of the Philosophical Sciences*, Oxford: Clarendon Press.

[12] Mortensen, C. (2020) "Change and Inconsistency', in E. Zalta (ed.), *Stanford Encyclopedia of Philosophy*, https://plato.stanford.edu/entries/change/.

[13] Priest, G. (1982) "To Be and Not to Be: Dialectical Tense Logic", *Studia Logica* 41: 249–268.

[14] Priest, G. (1998) "Number", in E. Craig (ed.), *Encyclopedia of Philosophy*, Vol. 7, pp. 47–54, London: Routledge.

[15] Priest, G. (2006a) "Motion", in D. Borchert (ed.), *Encyclopedia of Philosophy*, 2nd edn, Vol. 6, pp. 409–411, London: Macmillan, .

[16] Priest, G. (2006b) *In Contradiction*, 2nd edn, Oxford: Oxford University Press.

[17] Priest, G. (2017) "Contradiction and the Instant of Change Revisited", *Vivarium* 55: 217–226.

[18] Priest, G., Tanaka, K., and Weber, Z. (2022) "Paraconsistent Logic" in E. Zalta (ed.), *Stanford Encyclopedia of Philosophy*, https://plato.stanford.edu/entries/logic-paraconsistent/.

[19] Russell, B. (1903) *Principles of Mathematics*, Cambridge: Cambridge University Press.

[20] Sorabji, R. (1976) "Aristotle on the Instant of Change", *Proceedings of the Aristotelian Society, Supplementary Volumes* 50: 69–89.

[21] Sorabji, R. (1983) "Starting and Stopping", ch. 26 of *Time, Creation and the Continuum*, Chicago, IL: Chicago University Press.

[22] Spade, P. (1982), "Quasi-Aristotelianism", ch. 11 of N. Kretzmann (ed.), *Infinity and Continuity in Ancient and Medieval Thought*, Ithaca, NY: Cornell University Press.

[23] Strobach, N. (1998) *The Moment of Change: a Systematic History in the Philosophy of Space and Time*, Dordrecht: Kluwer Academic Publishers.

[24] Weber, Z., and Cotnoir, A. (2015), 'Inconsistent Boundaries', *Synthese* 192: 1267–1294.

Further Reading

- Mortensen (2020). [A general discussion of the area.]

- Sorabji, R. (1976). [On Aristotle's solutions to the problem of instantaneous change.]
- Strobach (1998). [On medieval discussions of instants of change.]
- Knuuttila and Lehtinen (1979). [On the theory of instants of nature.]
- Boyer (1959), chs. 5, 6. [On the infinitesimal calculus.]
- Priest, G. (2006a). [On the accounts of motion of Hegel and Russell.]
- Priest, G. (2006b), chs. 11, 12. [On paraconsistent accounts of change and motion.]