Jaśkowski and the Jains

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Abstract

In 1948 Jaśkowski introduced the first discussive logic. The main technical idea was to take what holds to be what is true at some possible world. Some 2,000 years earlier, Jain philosophers had advocated a similar idea, in their doctrine of *syādvāda*. Of course, these philosophers had no knowledge of contemporary logical notions; but the techniques pioneered by Jaśkowski can be deployed to make the Jain ideas mathematically precise. Moreover, Jain ideas suggest a new family of many-valued discussive logics. In this paper, I will explain all these matters.

Key Words: Stanisław Jaśkowski; Jain(a) philosophy; *saptabhaṇgī̄*; *syādvāda*; discussive/discursive logic; many-valued logic.

1 Introduction

As is widely known by Western logicians, in 1948, Stanisław Jaśkowski published the first influential paper on modern paraconsistent logic.[1] What is less well known—and what Jaśkowski himself could hardly have known—is

[1]The first actual paper was by Ivan Orlov in 1928, and arguably the second was by Grigore Moisil in 1942. However, neither of these papers was picked up at the time or had any substantial role in the development of the subject. On the first of these, see Došen (1992). On the second, see Drobyshevich, Odintsov, and Wansing (2022).

that similar ideas were held, and had been held for a couple of millennia, by Indian Jain philosophers.

Of course, these philosophers had no knowledge of the techniques of contemporary paraconsistent logic, but Jaśkowski's techniques can be used to give the Jain ideas a rigour to which they could not have aspired. In the other direction, the Jain ideas can be used to broaden Jaśkowski's techniques to deliver a much richer family of paraconsistent logics.

In this essay I will show all these things.

A word on terminology. In the literature one finds Jaśkowski's logic called both *discursive* and *discussive*. The Polish term used by Jaśkowski was *dyskusjyna*. This has a couple of different meanings, but the one clearly intended by Jaśkowski was *related to discussions*. In the French summary of his original paper he chose the term *discursive* as closest in meaning to what he had in mind (so a referee informs me). In English, the words *discussive* and *discursive* both have a couple of different meanings. *Discursive* can mean either *digressing from subject to subject* or *relating to discourse or modes of discourse. Discussive can* be used as meaning *related to discussion*, but the more standard meaning (apart from a medical use) is *doubt-dispelling* or *decisive*. So neither translation is great. In deference to advice from the referee, I will use *discussive*.

2 Jaśkowski

Jaśkowski's work is well known, but let me start with a brief summary of the relevant parts.[2]

[2] A referee wrote to me as follows; since relevant information about the origins of the paper are hard to find, I pass it on. 'Jaśkowski decided to publish his paper on discussive logic early in 1948, together with quite a number of other logical essays. Immediately after the end of the Second World War, he took up his position as professor of mathematics at the newly founded university [of Toruń]. The material difficulties are hardly imaginable from today's perspective. There were neither sufficient premises nor specialist literature. His teaching load and his tasks in organising everyday life at the university were enormous. The worries of daily life in Stalinist Poland were even more oppressive for him as a former landowner. There was no question of undisturbed, quiet research work. Apparently, there was strong pressure to quickly publish the research results, which were probably essentially produced during the war and without access to the usual scientific apparatus. I mention this background to the publication because it seems appropriate to keep the historical circumstances in mind when analysing the text.'

Jaśkowski was motivated by the thought that the information ("theses") available may be delivered by different sources ("participants in a discourse"), which may disagree with each other. Hence:[3]

> it would be proper to precede each thesis by the reservation: 'in accordance with the opinion of one of the participants in the discourse'.

A natural way to understand this prefix, Jaśkowski suggested, was as the modal operator, $\Diamond$:[4]

> if a thesis, A, is recorded in a discursive system, its intuitive sense ought to be interpreted so as if it were preceded by the symbol $\Diamond$, that is, the sense 'it is possible that A'.

Jaśkowski then goes on to suggest that the modal logic $S5$ may be used for this purpose. Possible-world semantics were not available to Jaśkowski at that time. But using these, we can put the propositional modal logic that Jaśkowski specified, D_2, as follows.[5]

An interpretation is a structure $\mathfrak{A} = \langle W, R, \nu \rangle$. W is a non-empty set of worlds. R is a binary relation on W, which is universal; that is, for all $x, y \in W$, xRy. ν maps every world and propositional parameter, p, to 1 or 0. Truth at a world is defined as follows:

- $w \Vdash p$ iff $\nu_w(p) = 1$
- $w \Vdash \neg A$ iff $w \nVdash A$
- $w \Vdash A \wedge B$ iff $w \Vdash A$ and $w \Vdash B$
- $w \Vdash A \vee B$ iff $w \Vdash A$ or $w \Vdash B$
- $w \Vdash \Diamond A$ iff for some $w' \in W$ such that wRw', $w' \Vdash A$
- $w \Vdash \Box A$ iff for all $w' \in W$ such that wRw', $w' \Vdash A$

[3] Jaśkowski (1969), p. 149.

[4] *Ibid.* Jaśkowski's notation was not that of contemporary logic, and I have updated it without comment in what follows.

[5] See Priest (2002), 4.2, 5.2.

$\rightarrow$ is defined in the usual way; and Jaśkowski also has a "discussive" conditional, $A \rightarrow_d B$, defined as $\Diamond A \rightarrow B$.

In a standard modal logic, one would define validity in terms of truth preservation at all worlds of all interpretations. But here is where Jaśkowski diverges. Say that A holds in $\mathfrak{A}$, $\mathfrak{A} \rhd A$, iff for some $w \in W$, $w \Vdash A$. Logical consequence, $\Sigma \models A$, is then defined as follows:

- for all $\mathfrak{A}$, if $\mathfrak{A} \rhd B$ for all $B \in \Sigma$, $\mathfrak{A} \rhd A$

That is, $\Diamond\Sigma \models_{S5} \Diamond A$, where $\Diamond\Sigma = \{\Diamond A : A \in \Sigma\}$. Let us call this notion of validity the *Jaśkowski move*.[6]

As is easy to see, $A, \neg A \not\models B$, though $A \wedge \neg A \models B$. It follows that Adjunction fails: $A, B \not\models A \wedge B$. For these reasons, Jaśkowski's logic is standardly termed a non-adjunctive paraconsistent logic.[7]

3 The Jains

Let us now turn to Jain thought. Jainism is an Indian philosophy of antiquity comparable to that of Buddhism. It appears to have been founded by Mahāvīra, who flourished some time in the 5th or 6th Century BCE.[8]

3.1 *Anekānta-Vāda*

The Jains endorsed the very distinctive view of *anekānta-vāda*; that is, the doctrine of non-onesidededness, as it is sometimes translated (*ekānta* = one-sided). They held that truth was not the prerogative of any one school. The views of Buddhists and Hindus, for example, may disagree about crucial matters, such as the existence of an individual soul; each has, nonetheless, an element of truth in it. This can be so because reality itself is multi-faceted. Thus, the doctrine of *anekānta-vāda* is sometimes glossed as the

[6]In his paper, as is common at that time, Jaśkowski defines logical truth, but not a consequence relation. However, given the material conditional it is, of course, always possible to define one in terms of logical truth (at least for finite sets of premises). $A_1,...A_n \models B$ iff $\models (A_1 \wedge ... \wedge A_n) \rightarrow B$.

[7]In a paper a year later (published in English as Jaśkowski (1999)) he introduced a discussive conjunction, $A \wedge_d B$, defined as $A \wedge \Diamond B$. As is not difficult to check, $A, B \models A \wedge_d B$.

[8]Some of what follows comes from Priest (2008a). For further discussions of Jain philosophy and its logic and metaphysics, see Gorisse (2023), Bharucha and Kamat (1984), Ganeri (2002), Hautamäki (1983), and Sarkar (1992).

doctrine of 'the many-sided nature of reality'. Reality is a complex, with a multitude of aspects; and each of the competing views provides a perspective, or standpoint (*naya*), which latches on to one such aspect. On its own, each standpoint is right enough, but incomplete. To grasp the complete picture, if indeed this is possible, one needs to have all the perspectives together—like seeing a cube from all six sides at once.

It follows that any statement to the effect that reality is thus and such, if taken categorically, will be, if not false, then certainly misleading. Better to express the view with an explicit reminder that it is correct from a certain perspective. This was the function with which Jaina logicians employed the word '*syāt*'. In the vernacular, this means something like 'it may be that', 'perhaps', or 'arguably'; but in the technical sense in which the Jain logicians used it, it may be best thought of as something like 'In a certain way...' or 'From a certain perspective...'. So instead of saying 'an individual soul exists', it is better to say '*syāt* an individual soul exists'.

The similarities between Jaśkowski's motivation and the Jain motivation is clear. Someone putting forward a view in a discussion can clearly be thought of as providing a perspective (*naya*) on the matter. And *Syāt* and Jaśkowski's $\Diamond$ can naturally be expected to behave in the same way: both are a way of garnering a certain set of views whilst at the same time separating them from others, so that one cannot automatically put them together. But before we exploit this fact, more needs to be said about Jain thought.

3.2 The Theory of Sevenfold Predication

In particular, we should look at the Jain theory of seven-fold division (*saptabhaṇgī*). A sentence may have one of seven truth values; or, as it is often put, there are seven predicates that may describe its semantic status. The matter is explained by the 12th century theorist, Vādideva Sūri, as follows:[9]

> The seven predicate theory consists in the use of seven claims about sentences, each preceded by 'arguably' or 'conditionally' (*syāt*) [all] concerning a single object and its particular properties, composed of assertions and denials, either simultaneously or successively, and without contradiction. They are as follows:

[9] *Pramāṅa-naya-tattvālokālaṁkāra*, ch. 4, vv. 15-21. Translation from Battacharya (1967).

(1) Arguably, it (i.e., some object) exists (*syād esty eva*). The first predicate pertains to an assertion.

(2) Arguably, it does not exist (*syād nāsty eva*). The second predicate pertains to a denial.

(3) Arguably, it exists; arguably it does not exist (*syād esty eva syād nāsty eva*). The third predicate pertains to successive assertion and denial.

(4) Arguably, it is non-assertable (*syād avaktavyam eva*). The fourth predicate pertains to a simultaneous assertion and denial.

(5) Arguably, it exists; arguably it is non-assertable (*syād esty eva syād avaktavyam eva*). The fifth predicate pertains to an assertion and a simultaneous assertion and denial.

(6) Arguably, it does not exist; arguably it is non-assertable (*syād nāsty eva syād avaktavyam eva*). The sixth predicate pertains to a denial and a simultaneous assertion and denial.

(7) Arguably, it exists; arguably it doesn't exist; arguably it is non-assertable (*syād esty eva syād nāsty eva syād avaktavyam eva*). The seventh predicate pertains to a successive assertion and denial and a simultaneous assertion and denial.

A perusal of the seven possibilities indicates that there are three basic ones, (1), (2), and (4). (1) says that the statement in question (that something exists) holds from a certain perspective. (2) says that from a certain perspective, it does not. (4) says that from a certain perspective, it has another status, non-assertable. The other four cases are the possible combinations of these three: every pair, and the triple.

But what is the third value? A natural thought is that it is *both true and false*. That is essentially how Vādideva Sūri glosses case (4) in the quotation above. Unfortunately, he also glosses it as *unassertable*. This is more like *neither true nor false*.

How, exactly, the third value is to be understand in Jain logic is a moot point. Some writers on Jainism go one way; some the other. I will leave the matter for scholars to argue about. In what follows, we will accommodate both possibilities.

4 Jain Discussive Logic

Given this background, we can now ask how one might formulate the Jain view in contemporary terms. Of course this is anachronistic in a certain sense. But applying new ideas to old is often a fruitful process.

4.1 A Formal Semantics

The obvious parallel between *syāt* and a modal operator of possibility makes a modal logic very natural. Contemporary modal logics are usually based on "classical", two-valued, logic. However, the Jains obviously envisage three values. So we have some kind of many-valued modal logic.[10] The Jains said little about how their values interact with logical operators as far as I know; so we are on our own here. The simplest possibility is to take the relevant 3-valued logic to be K_3 (if the third value is *neither true nor false*) or LP (if the third value is *both true and false*). In fact, we can handle both possibilities by starting with the more general FDE.

An interpretation is a structure $\mathfrak{A} = \langle W, R, \nu \rangle$. W and R are as before. But this time ν maps each world and propositional parameter to a subset of $\{1, 0\}$. Truth ($\Vdash^+$) and falsity ($\Vdash^-$) at a world are defined as follows:

- $w \Vdash^+ p$ iff $1 \in \nu_w(p)$
- $w \Vdash^- p$ iff $0 \in \nu_w(p)$
- $w \Vdash^+ \neg A$ iff $w \Vdash^- A$
- $w \Vdash^- \neg A$ iff $w \Vdash^+ A$
- $w \Vdash^+ A \wedge B$ iff $w \Vdash^+ A$ and $w \Vdash^+ B$
- $w \Vdash^- A \wedge B$ iff $w \Vdash^- A$ or $w \Vdash^- B$
- $w \Vdash^+ A \vee B$ iff $w \Vdash^+ A$ or $w \Vdash^+ B$
- $w \Vdash^- A \vee B$ iff $w \Vdash^- A$ and $w \Vdash^- B$
- $w \Vdash^+ \lozenge A$ iff for some $w' \in W$ such that wRw', $w' \Vdash^+ A$
- $w \Vdash^- \lozenge A$ iff for all $w' \in W$ such that wRw', $w' \Vdash^- A$

[10] On which, see Priest (2008b), ch. 11a.

- $w \Vdash^+ \Box A$ iff for all $w' \in W$ such that wRw', $w' \Vdash^+ A$
- $w \Vdash^- \Box A$ iff for some $w' \in W$ such that wRw', $w' \Vdash^- A$

$\to$ is defined in the usual way; one can also define $A \to_d B$ as $\Diamond A \to B$ if one wishes.

We could define validity in the usual modal way, as truth preservation in all worlds. This would give us the consequence relation of modal FDE, $\models_{FDE}$.[11] But let us use instead the Jaśkowski move. Say that A holds in $\mathfrak{A}$, $\mathfrak{A} \rhd A$, iff for some $w \in W$, $w \Vdash^+ A$. Then $\Sigma \models A$ iff:

- for all $\mathfrak{A}$, if $\mathfrak{A} \rhd B$ for all $B \in \Sigma$, $\mathfrak{A} \rhd A$

That is, $\Diamond\Sigma \models_{FDE} \Diamond A$.

Note that the discussive definition of validity is highly appropriate in the Jain case. Each world encodes one of the facets of reality. And each of these is equally correct. So to hold in the overall structure is simply to hold at some facet/world.

Let us now move to the 3-valued case. We can change the underlying many-valued logic to LP or K_3 simply by adding the condition that for all p, $\nu_w(p) \neq \emptyset$ (for LP) or $\nu_w(p) \neq \{1, 0\}$ (for K_3). A simple induction then shows that for any A:

- $\models^+ A$ or $\models^+ \neg A$ (for LP)
- $\not\models^+ A$ or $\not\models^+ \neg A$ (for K_3)

Given a formula, A, and a world, w, say that the value of A at w is:

- t if $w \Vdash^+ A$ and $w \nVdash^- A$
- f if $w \nVdash^+ A$ and $w \Vdash^- A$
- b if $w \Vdash^+ A$ and $w \Vdash^- A$
- n if $w \nVdash^+ A$ and $w \nVdash^- A$

[11] See Priest (2008b), 11a.4. The system treated there is the FDE analogue of the modal logic K (not $S5$) where R is arbitrary, but the changes necessary for $S5$ are obvious.

Let us call the third value, whatever it is (b or n), ξ. Then given any interpretation, there are seven possibilities for any given A:

	t at some worlds	f at some worlds	ξ at some worlds
1	$\surd$	$\surd$	$\surd$
2	$\surd$	$\surd$	$\times$
3	$\surd$	$\times$	$\surd$
4	$\surd$	$\times$	$\times$
5	$\times$	$\surd$	$\surd$
6	$\times$	$\surd$	$\times$
7	$\times$	$\times$	$\surd$

The 8th possibility:

- t no worlds, f at no worlds, and ξ at no worlds

cannot arise, since there is at least one world. Hence the seven cases of the *saptabhaṅgī* fall into place. Of course, had we done a similar thing using the 4-valued FDE, we would have had 15 cases. (I leave it to scholars to translate that into Sanskrit.)

Note that if $A \models B$ in modal K_3 or LP, $\Diamond A \models \Diamond B$, so the inference is also valid in the corresponding Jain versions. The converse does not hold. In K_3 and LP, $\Diamond\Diamond A \models \Diamond A$, so $\Diamond A \models A$ in the corresponding Jain logics. However, that inference does not hold in the modal logics themselves.

Even the one-way relationship between the modal logic and its Jain form fails in the multi-premise case, however. In the modal logics $A, B \models A \wedge B$, but $\Diamond A, \Diamond B \not\models \Diamond(A \wedge B)$. So the inference fails in the Jain logics.

The inference $\Diamond A, \Diamond\neg A \models \Diamond B$ fails in the modal logics. So $A, \neg A \models B$ fails in the Jain logics, as then does Adjunction: $A, B \models A \wedge B$.

The inference $\Diamond(A \wedge \neg A) \models \Diamond B$ holds in modal K_3. So $A \wedge \neg A \models B$ holds in Jain K_3. But $\Diamond(A \wedge \neg A) \not\models \Diamond B$ in modal LP. So $A \wedge \neg A \not\models B$ in Jain LP.

4.2 Tableaux

Appropriate tableau systems for our logics are easy to construct from those of the corresponding modal logics.[12] Lines are of the form $A, +i$ or $A, -i$, where i is a natural number. To determine the validity of the inference $\Sigma \vdash A$ (for

[12]In the same way, it is easy to construct semantical tableaux for Jaśkowski's original system from those for $S5$. (As in Priest (2008), ch. 3.)

finite Σ) we start with lines of the form $\Diamond B, +0$ for $B \in \Sigma$, and $\Diamond A, -0$. The rules are as follows:

$$\begin{array}{c} A \wedge B, +i \\ \downarrow \\ A, +i \\ B, +i \end{array} \qquad \begin{array}{c} A \wedge B, -i \\ \swarrow \quad \searrow \\ A, -i \quad B, -i \end{array}$$

$$\begin{array}{c} A \vee B, +i \\ \swarrow \quad \searrow \\ A, +i \quad B, +i \end{array} \qquad \begin{array}{c} A \vee B, -i \\ \downarrow \\ A, -i \\ B, -i \end{array}$$

$$\begin{array}{c} \neg(A \vee B), \pm i \\ \downarrow \\ \neg A \wedge \neg B, \pm i \end{array} \qquad \begin{array}{c} \neg(A \wedge B), \pm i \\ \downarrow \\ \neg A \vee \neg B, \pm i \end{array} \qquad \begin{array}{c} \neg\neg A, \pm i \\ \downarrow \\ A, \pm i \end{array}$$

$$\begin{array}{c} \neg\Box A, \pm i \\ \downarrow \\ \Diamond\neg A, \pm i \end{array} \qquad \begin{array}{c} \neg\Diamond A, \pm i \\ \downarrow \\ \Box\neg A, \pm i \end{array}$$

$$\begin{array}{c} \Box A, +i \\ \downarrow \\ A, +j \end{array} \qquad \begin{array}{c} \Box A, -i \\ \downarrow \\ A, -j \end{array} \qquad \begin{array}{c} \Diamond A, +i \\ \downarrow \\ A, +j \end{array} \qquad \begin{array}{c} \Diamond A, -i \\ \downarrow \\ A, -i \end{array}$$

In the middle two rules of the last quartet, j is new to the branch. In the two end rules, j is any index that appears on the branch.

A branch closes if things of the form $A, +i$ and $A, -i$ appear on it. A tableau closes if every branch closes.

This gives the system where the many-valued logic is FDE. For K_3 we add an extra closure rule: a branch closes if $A, +i$ and $\neg A, +i$ occur on it. For LP we add a different extra closure rule: a branch closes if $A, -i$ and $\neg A, -i$ occur on it. Soundness and completeness proofs are straightforward.[13]

As an example, let us show that $p, p \rightarrow_d q \vdash q$ where the many-valued

[13] See Priest (2008b), 11a.9. The proofs there are given for the modal logic K, not $S5$. But the changes are obvious.

logic is K_3:

$$\begin{array}{cc}
\multicolumn{2}{c}{\Diamond p, +0} \\
\multicolumn{2}{c}{\Diamond(\neg\Diamond p \vee q), +0} \\
\multicolumn{2}{c}{\Diamond q, -0} \\
\multicolumn{2}{c}{p, +1} \\
\multicolumn{2}{c}{\neg\Diamond p \vee q, +2} \\
\swarrow & \searrow \\
\neg\Diamond p, +2 & q, +2 \\
q, -0 & q, -0 \\
q, -1 & q, -1 \\
q, -2 & q, -2 \\
\Box\neg p, +2 & \times \\
\neg p, +0 & \\
\neg p, +1 & \\
\neg p, +2 & \\
\times &
\end{array}$$

The tableau does not close in LP, since the left-hand branch is open. ($\rightarrow$ does not detach in LP.)

Countermodels can be read off from open branches as usual.[14] The countermodel given by the left branch of the above tableau when the many-valued logic is LP can be depicted thus:

$$\begin{array}{ccc}
w_0 & w_1 & w_2 \\
-q & -q & -q \\
+p & & \\
+\neg p & +\neg p & +\neg p
\end{array}$$

At this point a full analysis of the Jain logics would be in order, and one might find worse places to start than establishing whether or not the three metatheorems proved by Jaśkowski in his paper carry over to the new logics. But that is a matter for future investigations; let us move on.

5 Jain-Jaśkowski Logics

As is well known, Jaśkowski's discussive logic can be generalised in many ways, for example, by changing the modal logic from $S5$ to some other normal

[14]For details, see Priest (2008b), 11a.5, 11a.6.

modal logic.[15] One can do exactly the same with the Jain versions. However, I will not pursue that matter here. I want to look at a different kind of generalisation. I will do this in three stages, pointing out the nature of the generalisation as we go along.

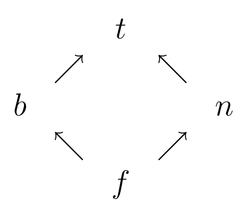
5.1 Stage 1: Many-Valued Logic

A semantic interpretation for a many-valued logic is a structure $M = \langle \mathcal{V}, \mathcal{D}, \{f_c : c \in C\} \rangle$, where $\mathcal{V}$ is the set of values, $\mathcal{D} \subseteq \mathcal{V}$ is the set of designated values, C is the set of connectives in the language, and if $c \in C$, f_c is the corresponding truth function. (That is, if c is an n-place connective, f_c is a function from $\mathcal{V}^n$ to $\mathcal{V}$.) Given an evaluation, ν, of the propositional parameters, this is extended to all formulas by the appropriate recursive clauses. Thus, if c is an n-place connective, and $c(A_1, ..., A_n)$ is a formula:

- $\nu(c(A_1, ..., A_n)) = f_c(\nu(A_1), ..., \nu(A_n))$

$\Sigma \models A$ iff for every evaluation, ν, if $\nu(B) \in \mathcal{D}$, for every $B \in \Sigma$, $\nu(A) \in \mathcal{D}$.

As is well known, the logics FDE, LP, and K_3 can be formulated as many-valued logics in this way.[16] The semantic values are represented by the familiar diamond lattice:



Negation maps t to f, and vice versa; b and n are fixed points. Conjunction is greatest lower bound, and disjunction is least upper bound.

FDE uses the full lattice. LP uses the left hand side. K_3 uses the right hand side. The designated values are t and b (when it is present).

5.2 Stage 2: Many-Valued Modal Logic

Given a many-valued logic, M, and a complete partial order on $\mathcal{V}$, $\leqslant$, an interpretation for its modal extension is a structure $\mathfrak{A} = \langle W, R, \nu \rangle$, where W

[15] See, for example, Kotas and da Costa (1977), Błaszczuk (1984), da Costa and Doria (1995), Dunin-Kęplicz, Powała, and Szałas (2008), Mruczek-Nasieniewska and Nasieniewski (2019).

[16] See, e.g., Priest (2008), ch. 8.

is a non-empty set of worlds, and R is a binary relation on W—perhaps satisfying certain constraints. For every propositional parameter, p, and world, w, $\nu_w(p) \in \mathcal{V}$. At each world, the value of a truth functional formula is evaluated according to M. For the modal operators:

- $\nu_w(\Box A) = glb\{\nu_{w'}(A) : wRw'\}$
- $\nu_w(\Diamond A) = lub\{\nu_{w'}(A) : wRw'\}$

where lub and glb are the last upper bound an greatest lower bound in the ordering $\leqslant$.

Validity, $\models$, is defined in terms of the preservation of designated values at all worlds:

- $\Sigma \models A$ iff in every interpretation and world, w, if $\nu_w(B) \in \mathcal{D}$ for all $B \in \mathcal{D}$, then $\nu_w(A) \in \mathcal{D}$

It is not difficult to show that these semantics produce the (non-discussive) modal extensions of FDE, LP, and K_3, which we met in 4.1.[17]

5.3 Jain-Jaśkowski Logics

The discussive version of these logics is obtained using the Jaśkowski move. Say that A holds in $\mathfrak{A}$, $\mathfrak{A} \rhd A$, iff for some $w \in W$, $\nu_w(A) \in \mathcal{D}$. Then $\Sigma \models A$ iff:

- for all $\mathfrak{A}$, if $\mathfrak{A} \rhd B$ for all $B \in \Sigma$, $\mathfrak{A} \rhd A$

That is, $\Diamond\Sigma \models \Diamond A$ in the corresponding many-valued modal logic.

The discussive logics of Section 4 are special cases of these logics, where the underlying many-valued logic is FDE, LP, or K_3, and the accessibility relation, R, is universal.

Call this family of logics *Jain-Jaśkowski discussive logics*—a class of logics not previously isolated as far as I am aware.[18] Naturally, the the logics in this

[17]See Priest (2008b), ch. 11a.

[18]A referee drew my attention to D'Ottaviano and da Costa (1970) and Kotas and da Costa (1978). The logics in these papers are simple many-valued logics. However, following an idea Łukasiewicz, they take possibility to be a truth function. This makes it behave very strangely, *qua* possibility operator. (See Priest (2008b), 7.10.) Moreover, since Jain-Jaśkowski logics handle the possibity operator with a world-semantics, none of these logics is in the Jain-Jaśkowski family. In particular, these are adjunctive systems.

family invite a careful investigation of their properties and relations; but that is best left for another occasion. The point of this essay was not to establish lots of interesting new theorems, but to lay bare an interesting analogy, and show it to be fruitfully applicable to both of its sides.

6 Conclusion

What we have seen in this essay is that Jaśkowski's discussive logic and the Jain *saptabhaṇgī* are driven by similar ideas. Information or truth is delivered by a number of different sources or perspectives. These can be marked out by the use of an operator, $\Diamond$ or *syāt*. Moreover, to hold *simpliciter* is to hold in (any) one of these sources or perspectives.

The engagement of Jaśkowski's techniques and the Jain ideas is fruitful in both directions. Jaśkowski's techniques may be used to deliver a precise formal account of the Jain ideas.[19] In the other direction, applying the Jain ideas—and in particular, the deployment of more than two truth values—opens the door to a whole new family of logics: Jain-Jaśkowski discussive logics.[20]

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[19] Another way of doing this is to use plurivalent logics. See Priest (2018), 6.9.

[20] Versions of this paper were given at the Sixth World Congress on Paraconsistency, University of Toruń, September 2022, the online Southern Summer Logic Day, organised by the Australasian Association for Logic, January 2023, and the conference Fitting at 80, CUNY Graduate Center, January 2023. I am grateful to the participants of those events for their helpful comments and suggestions. Thanks also go to two anonymous referees of the Journal.

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