Perspectives on the Universe

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Abstract

Joel Hamkins has advanced a well known view to the effect that there is no unique universe of sets. There is simply a plurality of such universes. We have, then, a pluriverse. A natural objection to this view is that there is still a single universe: the totality, V, in which all the members of the pluriverse find themselves. In this paper I consider a reply to the objection, to the effect that there is no such thing as V in itself. Rather, each member of the pluriverse simply gives a different perspective on what V is like. This view is then generalised in the light of mathematical pluralism. What emerges is a vastly expanded and logic-neutral view of the pluriverse.

Dedication: It gives me enormous pleasure to dedicate this essay to Alan Weir. Though we have never spent a lot of time together, our paths have crossed on many occasions, both in person and in print. Alan's work is always imaginative and insightful, and he is never afraid to beat paths in novel directions. As well as being a great philosopher, when it comes to giving talks, he is undoubtedly the funniest person I know. It is impossible to leave a talk by Alan without a smile on one's face. I have no doubt that he would have some words of wry humour about the following thoughts!

1 Introduction

Perspectivalism about some situation is, roughly, the view that there is no fact of the matter about it, *simpliciter*. There are just the views afforded by different perspectives (literal or metaphorical), all of them equally correct. It is common enough to endorse perspectivalism about whether a joke is funny or a dish is tasty. Such things are the case only with respect to a perspective afforded by a sense of humour or a gastronomic palate, respectively. It is less common to apply it to areas where we would standardly take it that there is indeed an objective fact of the matter. The point of this essay is to do just this. In what follows, I will apply the view to certain aspects of mathematics—an area normally taken to be a paradigm of objectivity.

The essay falls into two (related) parts. The first concerns the matter of whether there is a single universe of sets or a plurality thereof. The discussion here will naturally take us into the second half of the essay, where the issue will be subjected to concerns drawn from mathematical pluralism, the view that there are different kinds of mathematics which are all equally correct.

2 The Pluriverse

2.1 The Set-Theoretic Universe

A fairly orthodox view—at least until the last couple of decades—is that there is a unique universe of sets, V, and that this is characterised, at least partially, by the axioms of Zermelo Fraenkel set theory (with Choice), ZFC. (Of course, V is not itself a set in the totality, but in some undeniable sense, it is the totality of all sets.) The axioms of ZFC are insufficient to settle fundamental questions about sets, such as the Continuum Hypotheses (CH); but the answers to these questions are determinately true or false, none the less.

In the last dozen years or so, a rival view has emerged. There is no unique such universe. Instead, there is a plurality of universes. Each of them may satisfy the axioms of ZFC, but the CH (say) is true in some and false in others. As Joel Hamkins, one of the main defenders of this picture, puts it:[1]

> There are diverse distinct concepts of set, each instantiated in a corresponding set-theoretic universe, which exhibit diverse set-theoretic truths. Each such universe exists independently in the same Platonic sense that proponents of the universe regard their universe to exist.

[1] Hamkins (2012), p. 416 f. See this essay for a defence of the view.

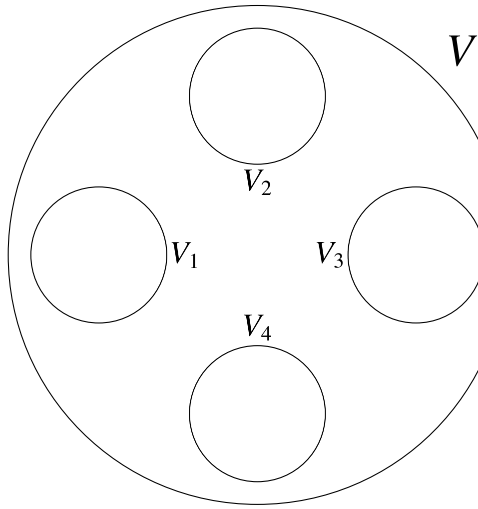
Hamkins calls this totality of universes the *multiverse*.[2] I prefer to use the term *pluriverse*. (As we will see, it's going to get much bigger!)

The pluriverse view is driven by the fact that attempts to determine the answers to the questions left open by *ZFC* in any satisfactory way have proven largely unsuccessful—at least as judged by any substantial consensus. On the other hand, set-theorists have a large experience of constructing and investigating different models of *ZFC* and the relationships between them. It is natural enough, then, to think of each such model as delivering its own universe; and each seems to be an equally valid realisation of the structure of sets. Hamkins calls these different *concept(ion)s* of sets. It might be better to say that we have but a single conception of set here (the *ZFC* conception), and that these are different *realisations* of it—in the same way that different mathematical groups deliver different realisation of one and the same notion of *group*. However, I will return to the topic of different conceptions of sethood in due course.

Natural as this picture of sets may seem in the light of developments in set-theory, it appears to face an obvious objection. The different denizens of the pluriverse would seem to inhabit the totality of *all* such universes, which, therefore, has a claim to be *the* universe of sets. A defender of the more traditional view will point out that all the models of *ZFC* that have been constructed appear to live within this, the cumulative hierarchy, V, as usually conceived.[3] Indeed, when the set-theorist is constructing and establishing the relations between the models of *ZFC*, they actually appear to be working *within* this (hyper-)universe. The situation, then, would appear to be this:

[2] As is clear, Hamkins subscribes to a Platonism concerning the universes. However, the pluriverse view is not, in itself, committed to platonism, and can be endorsed by various kinds of non-platonists as well.

[3] Though of course one cannot *prove* that there are such models in *ZFC* without adding extra axioms, such as large cardinal axioms.



V is the unique (hyper-)universe. Each of the V_is is simply some sub-collection which happens to validate all the axioms of ZFC. The pluriverse view, then presupposes that there is a unique universe, and so seems to be self-refuting.

2.2 Enter Perspectivalism

There is, however, a radical but intriguing reply. There is no (hyper-)universe as such: each member of the pluriverse has a universe, and these all provide equally valid perspectives on what the totality of all sets is like. This is perspectivalism.[4]

Perspectivalism about some matter is the view that there is no perspective-independent truth about it; we simply have a bunch of different perspectives, each of which is equally legitimate. As I observed, perspectivalism is the natural view about the funniness of a joke or the tastiness of a food. There is no truth *simpliciter* about such a matter. Truth is relative to a perspective. So the claim 'such and such a joke is funny' makes no more sense that 'this is to the right of that'. The latter makes sense only with respect to a visual perspective; the former only with

[4]The view was suggested to me by Joel Hamkins in conversation. As far as I know, he has not endorsed it explicitly in print, though he gestures towards it here (Hamkins (2012), p. 419): 'On the universe view, of course, forcing extensions of V are deemed illusory, for V is already everything, while the multiverse perspective regards V as a relative concept, referring to whichever universe is currently under consideration, without there being any absolute background universe. On the multiverse view, the use of the symbol V to mean "the universe" is something like an introduced constant that might refer to any of the universes in the multiverse, and for each of these the corresponding forcing extensions $V[G]$ are fully real.'

respect to a sense of humour. What *are* true or false *simpliciter* are things of the form 'with respect to such and such a spatial position this is to the right of that' and 'with respect to a certain sense of humour this a is funny joke'. Note that such perspectival truths are not subjective, at least in one sense, since different people can occupy the same spatial position (at different times), and different people can have the same sense of humour.

Perspectivalism about spatial orientation and humour are a forms of local perspectivalism. They apply only to certain properties of certain things. But global perspectivalism, according to which *every* claim is true or false only with respect to a perspective, is also well known in philosophy. It is a view often attributed to Nietzsche.[5] Interpreting Nietzsche is always a contentious matter. However, it is not contentious that Indian Jain philosophers held a global perspectivalism. For the Jains, reality was like a cut diamond with many facets. Each facet encodes a different aspect of reality, all of equal reality, and any claim is true or false only with respect to one facet.[6]

Of course, given perspectivalism about something, it is a fair question to ask what it is that the perspectival properties are properties *of*. For a local perspectivalism about, e.g., jokes, it is easy enough to answer this. They are all perspectives of the story told, the picture shown, etc. In the case of global perspectivalism, things are more problematic. To say anything about the object in question is but to provide a perspective of it. One can say nothing of the thing itself. It thus becomes a *ding an sic*h, a "something I know not what".

A further question concerns the relationship of perspectivalism to fictionalism. Fictionalist accounts are of many kinds.[7] However, the basic idea of fictionalism about some topic is that there are no truths *simpliciter* about it. All truths are of the form 'According to fiction F, such and such'. Clearly, 'According to fiction F, such and such' and 'According to perspective P, such and such' at least have a similar syntactic form. There may, in fact, be no formal difference between the two operators. However, the intents of fictionalism and perspectivalism are quite distinct. Fictionalism is an anti-realist view: the objects in question are, after all, *fictions*. Thus, if one is a fictionalist about numbers, it is not true that $1 + 1 = 2$. What is true is that in the "fiction" about numbers $1 + 1 = 2$. And that delivers no commitment to the existence the number 1, any more than Conan Doyle's stories deliver a commitment to the existence of Sherlock Holmes. By contrast,

[5] See Anderson (2017), esp. §6.2.

[6] See, e.g., Priest (2008).

[7] See Eklund (2019).

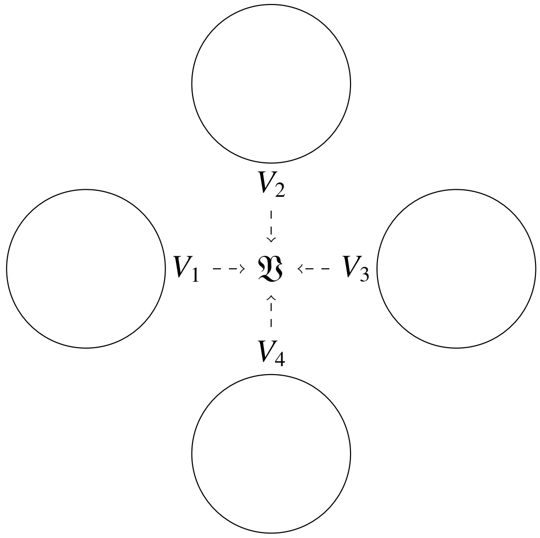
perspectivalism is not an anti-realist view. If a joke is funny from a perspective, or an apple is tasty from a perspective, there is no suggestion that the joke or the apple does not exist—quite the contrary.

2.3 Perspectivalism and the Pluriverse

Let us apply a perspectivalist view to the multiverse. All claims about V are relative to a certain perspective, and each perspective is perfectly legitimate. In particular, each member of the pluriverse, i, has its own take on universe of sets, V_i, and each of these provides an equally correct perspective on the object of the perspective. Of course, this raises the question of what that thing is. Matters are not so straightforward as in the case of a humour. But since this is not a global perspectivalism, we are not reduced to taking it to be an ineffable *ding an sich*. We may take the thing to be that mathematical object which is the totality of all collections, though this has no intrinsic structure. Supposing it had such a structure would be to take us back to a determinate-universe view, which is exactly what perspectivalism was invoked to avoid.[8]

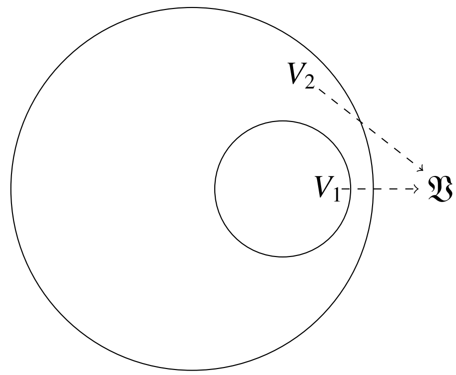
Let us write the target of the perspectives as $\mathfrak{V}$. The V of each member of the pluriverse gives a perspective on this. The situation may be depicted as follows:

[8] Another possibility is to adopt a conceptualist position, bringing matters of representation to bear. The object in question is the *concept* of the totality of all collections. One may take this to be vague, in that it can be precisified in many different ways, one for each perspective. But, again, it is crucial that the concept does not deliver any determinate structure. Such must remain a matter of perspective.



The different perspectives may well agree on many of the properties of V. For example, according to each of them, it satisfies the axioms of ZFC. Thus, when a set-theorist proves things using the axioms of ZFC, they establish something common to all perspectives. But other properties, such as whether the CH holds, may vary from perspective to perspective—the perspectives afforded by each of member of the pluriverse, as encoded in its own take on matters. Each of these perspectives is equally legitimate: there is no hyper-universe as such—in particular, no hyper-universe which contains all the others.

The differences in perspective can be more radical than so far indicated. Let us call something *absolute* if it holds in all universes/perspectives. Then that the totality of sets is uncountable is absolute. But suppose, as can happen, that we have two members of the pluriverse, V_1 and V_2, such that V_1 is a substructure of V_2. V_1 takes itself to be uncountable. But according to V_2, V_1 may be countable. We might depict this as follows:



From the perspective of V_2, V_1's perspective of $\mathfrak{V}$ will be but partial.

Such, then is a perspectivalist view about the universe of sets. If one endorses the pluriverse, and if perspectivalism does provide a solid reply to the obvious objection, this will speak in its favour. However, the aim of this paper is not to defend perspectivalism about the cosmos of sets as such. The aim of the present essays is much more modest: simply to explore the idea, and see where else it may take us.

2.4 Changing the Logic, Not the Subject

One further step is natural and obvious to those familiar with paraconsistent set theory.

Given the pluriverse understanding of set-theory, there is nothing sacrosanct about ZFC. We can take the pluriverse to contain things that do not validate all of the axioms of ZFC as Hamkins notes:[9]

> The background idea of the multiverse, of course, is that there should be a large collection of universes, each a model of (some kind of) set theory. There seems to be no reason to restrict inclusion only to ZFC models, as we can include models of weaker theories ZF, KP^-, KP, and so on, perhaps even down to second-order number theory, as this is set-theoretic in a sense. At the same time, there is no reason to consider all universes in the multiverse equally, and we may be simply more interested in the parts of the multiverse consisting of universes satisfying very strong theories, such as ZF plus large cardinals.

[9] Hamkins (2012), p. 436.

We can indeed consider stronger theories as well, and not just those of the kind that Hamkins notes. Using various model-theoretic techniques, one can construct set-theoretic models which validate all the theorems of *ZFC*, together with the naive comprehension principle (*CP*):[10]

- $\exists x \forall y (y \in x \equiv A(y))$

In such models, in particular, the universal set, V, will actually be a member of the totality. That is, $V \in V$. Obviously, the usual models of *ZFC* do not verify this. The reason, of course, is that *CP* is inconsistent with *ZFC*. Indeed, the set of things true in each of the models in question *is* inconsistent, though not trivial. This is because the underlying logic of these models is not classical logic, but the paraconsistent logic, *LP*.—Actually, the trivial model is one of these; but, as Hamkins notes, there is no reason why we have to find all members of the pluriverse (equally) interesting.[11]

Moving to this paraconsistent logic actually *expands* the pluriverse, since all classical models are *LP* models. It still contains all the classical universes verifying *ZFC*, but it now also contains all the *LP ZFC* universes. Indeed, we may throw in as well, for good measure, all the *LP* models of naive set theory which do not verify the theorems of *ZFC*. As Hamkins notes, we do not have to fetishize *ZFC*.

Nor is there any reason why the pluriverse *should* be constrained by the straightjacket of classical logic. The non-classical constructions deliver models of set theory which have clear mathematical interest.[12] And each provides a perspective on the totality of sets—indeed, an interesting and novel kind of perspective!

Of course, it might well be suggested that these paraconsistent perspectives on V are not legitimate because they use the wrong logic. We need not go into this here, however, since the matter is finessed by a consideration of mathematical pluralism, as we will now see.

[10] For these models, see Priest (2006), §18.4, and Priest (2017), §11.

[11] Naturally, one may ask in what theory these models are constructed. The answer is that they are constructed in *ZFC*. Clearly, there is an element of boot-strapping here. But boot-strapping is everywhere in set theory. Thus, for example, to prove that the Gödel's constructible hierarchy, *L*, is a model of the axioms of *ZFC* requires the very axioms of *ZFC*.

[12] The relationships between these and the classical models is also a topic which requires much further investigation.

3 ... and Mathematical Pluralism

3.1 Mathematics (Pl.)

A common enough assumption—again until relatively recently—is that there is one correct kind of mathematics, and that this is based on so called classical logic—the logic developed by Frege, Russell, and others around the turn of the 20th Century to do justice to the reasoning in the mathematics of their time.

The classical pluriverse does not, as such, threaten the picture. Set theory has to be reconceptualised as a theory like ring theory or topology, rather than like the theory of the natural numbers. The notion of a ring or a topological space is not meant to be categorical: it can have multiple realisations. By contrast, the theory of the natural numbers is standardly taken to have a unique intended realisation.[13]

Nor need the move to the paraconsistent logic *LP* threaten the claim that there is a uniquely correct logic for mathematics. This may not be classical logic, but is more general. (Classical logic can be obtained from *LP* by adding the principle of Explosion; and this may be taken to be a "contingent" feature of some mathematical structures.)

However, one thing that recent developments in mathematics has taught us is that interesting mathematical structures may be formed on the basis of a number of different non-classical logics. Thus, there are not only the various paraconsistent theories, such as set theory and topology, but intuitionist theories such as smooth infinitesimal analysis and Heyting Arithmetic plus Church's Thesis, which would collapse into triviality if classical logic were used. Hence, we have seen a number of authors defending the notion of mathematical pluralism.[14] There are many kinds of pure mathematics. In particular, there are those based on various non-classical logics. This is not to say that all such mathematics are equally interesting, rich, applicable, etc—just that they are all equally valid pure mathematical structures.[15]

[13]However, because of the supposed foundational role of set theory in mathematics, the plurality of set-theoretic universes challenges the supposed categorical nature of other structures, such as the natural numbers, as Hamkins (2012), §5, notes. See also Gitman and Hamkins (2010).

[14]See, e.g., Shapiro (2014), Priest (2013), and Priest (2019). As the last of these argues, the pluralism seems to dispose of any last vestiges of foundationalism in mathematics—in the sense that there is some ur-mathematics, which encompasses all of its branches—and so of set-theoretic reductionism.

[15]This does not, note, imply logical pluralism. The logical structures, and so the mathematical structures built on them, have clear mathematical interest, even if the logics are not the "one true logic". See Priest (202+).

It should be stressed that what is at issue here is *pure mathematics*. All of these pure mathematical structures are equally legitimate. Matters are different when it comes to applied mathematics. When a piece of mathematics is applied for some purpose (in physics, economics, linguistics, or whatever), one must find the mathematical domain whose structure is isomorphic (or near enough isomorphic) to the "real world" system in question, so that establishing results about the mathematical structure tells us about this as well.[16] The structure of the real-world system, hence, imposes constraints on the choice of the pure mathematical structure in question; all are not, therefore, equally legitimate.

3.2 The Real Pluriverse

Given mathematical pluralism, and the fact that we have already expanded the classical pluriverse once to obtain one in which there are universes whose underlying logic is—of necessity—one particular non-classical logic, it is natural to expand it further to universes of sets whose underlying logics are other non-classical logics.

Thus, one might have a universe of predicatively definable sets; a universe of sets with the set theoretic axioms of ZFC, but the underlying logic of which is intuitionist logic; or a set-theory based on the intuitionistic notion of a spread; or a fuzzy set-theory based on, say, Łukasiewicz continuum-valued logic; or a set-theory based on a relevant logic; or one based on quantum logic.[17] Alan Weir, of course, has his own version of set theory with the naive comprehension principle.[18] This would fit snugly into this framework—though to what extent he would accept the picture presented here, I leave it to him to say.

I think it fair to say that at least some of these theories of sets *are* different conceptions of sethood. But once one is a mathematical pluralist, there would appear to be no reason why the set-theoretic pluriverse should not contain all of them. Indeed, once one has given up the thought that there is a unique universe of sets, the very spirit of the enterprise is one of pluralism; and one should not put arbitrary limits on the plurality.

Of course, the pluriverse is now amazingly diverse, and in practice one might expect mathematicians to be interested in only fragments of it, e.g., those uni-

[16] See Priest (2016), §7.8.

[17] See, e.g., Feferman (1964), Crosilla (2019), Van Atten (2017), §3, Hájek and Haniková (2003), Weber (2012), respectively.

[18] See Weir (1998). The underlying logic has been something of a work in progress. For the most recent account of which I am aware, see Weir (2014).

verses based on classical logic, or those universes verifying the CP. But no mathematician is interested in *all* of mathematics.

3.3 ~~The~~ Structure of the Pluriverse

So we now have a large plurality of universes. Each universe, $\mathfrak{U}$, will have its own logic, $L_{\mathfrak{U}}$. Different things may hold in each $\mathfrak{U}$; so let us write $\mathfrak{U} \Vdash A$ to mean that the sentence A holds in $\mathfrak{U}$.[19] Then the set, $\Sigma_{\mathfrak{U}} = \{A : \mathfrak{U} \Vdash A\}$ will be closed under $L_{\mathfrak{U}}$. Set theorists may investigate each $\Sigma_{\mathfrak{U}}$; but of course, they will want to do more than that: they will want to investigate the relationships between the different universes.[20] For such an investigation one needs an account of the the properties of, and relations between, different universes. In other words, one needs a set theoretic structure in which the universes are themselves objects. The question is: which one?

Hamkins himself notes that his multiverse view provides a natural take on this matter. He says:[21]

> The multiverse perspective ultimately provides what I view as an enlargement of the theory/metatheory distinction. There are not merely two sides for this distinction, the object theory and the metatheory, but rather there is a vast hierarchy of metatheories. Every set-theoretic context, after all, provides in effect a meta-theoretic background for the models and theories that exist in that context, a model theory for the models and theories one finds there. Every model of set theory provides an interpretation of second-order logic, for example, using the sets and predicates existing there. Yet, a given model of set theory M may itself be a model inside a larger model of set theory N, and so what was previously the absolute set-theoretic background, for the people living inside M, becomes just one of the possible models of set theory, from the perspective of the larger model N. Each meta-theoretic context becomes just another model at the higher level. In this way, we have theory, metatheory, metametatheory and so on, a vast hierarchy of possible set-theoretic backgrounds.

[19] If this is a statement of set theory, and A itself is such a statement, this is really of the form $\mathfrak{U} \Vdash \langle A \rangle$, where angle brackets are whatever form of quotation is deployed in the set theory. As is standard practice in logic, I simply abuse notation here.

[20] As noted, e. g., by Shapiro (2014), p. 171.

[21] Hamkins (2020), ch. 8.

Of course, this view is not committed, *per se*, to the view that all universes are equally good for the purpose at hand. Thus, one might suppose, e.g., that the bigger the universe the better.

However, perspectivalism endorses just this thought. We may take any universe which includes the universes to be investigated, and investigate what happens within this structure. Note that there is no reason why the universes to be investigated must have the same internal logic as the universe in which they are embedded. Thus, a classical universe (one whose truths are closed under classical logic) may appear in a paraconsistent universe. Or an intuitionistic universe (one whose truths are closed only under intuitionist logic) may occur in a classical universe. As Hamkins notes, each hyper-universe with its internal logic provides, in effect, a metatheory for investigating the different universes. Classical logic, note, has no difficulty in making sense of a Kripke model for an intuitionist theory; and this structure may be investigated in a classical metalanguage.

The different hyper-universes in which the universes are embedded may well establish somewhat different relationships between them. Indeed, they may well deliver different properties of the relation $\mathfrak{U} \Vdash A$. Thus, Kripke models for intutitionist logic are complete, given a classical metalanguage; but not, given an intuitionist metalanguage.[22] So the different hyper-universes are liable to deliver quite different perspectives on matters.[23] But given perspectivalism, all perspectives are equally legitimate.

In his *Varieties of Logic*, Stewart Shapiro discusses the question of whether one should suppose there to be a unique foundation for mathematics; that is, whether one should suppose there to be a unique mathematical theory—and so logic—which can be used to formulate and compare all mathematical theories or structures.[24] This is somewhat more general than the matter I have been discussing. A hyper-universe provides, in effect, a very specific kind of metatheory for its sub-universes. (There can be other kinds of metatheories—for example, category theoretic ones.) However, Shapiro provides a pluralist answer to the question; and it is much the same as the one at which we have arrived above. He says:[25]

> To put the matter in general terms, if we become interested in re-

[22]See, McCarty (1991). For further discussion of this sort of variability, see Shapiro (2014), ch. 7.

[23]I note also that not all universes have to occur in all meta-universes. As I have already noted, a perspective can be partial.

[24]Shapiro (2014), pp. 177-81.

[25]Shapiro (2014), p. 180 f.

> lations among particular theories or systems, $S_1, S_2, ..., S_n$, then we need a perspective—theory, structure—from which we can refer to $S_1, S_2, ..., S_n$, at once, so to speak, and we need some account of the relations between theories of systems $S_1, S_2, ..., S_n$. But we do not need a single foundational theory that accommodates every possible mathematical theory, or every possible logic...

Indeed we do not. Nor should one expect one, given the pluriverse and mathematical pluralism.

3.4 Relativism

There remains one issue to be discussed: relativism. The truth of many claims is relative. Whether something is funny is relative to one's sense of humour. Whether something is surprising is relative to what one knows (or believes). This is not problematic. A form of relativism is problematic when its articulation appears to presuppose the non-relativeness of a claim that falls within the scope of the relativism in question. This certainly happens with global relativism: the claim that the truth of any claim is relative. The articulation of this appears to be a non-relative claim. One way to bring this out is to imagine (as does Plato in the *Theaetetus*, 161c, ff.) a confrontation between a relativist of this kind and a non-relativist. On pain of self-refutation, the relativist must admit that the view of the non-relativist, relative to their own view, is just as correct as that of the relativist, relative to their view. Hence, they can no longer defeat their opponent in debate.[26]

Now, mathematical pluralism itself is not self-defeating. It claims that all mathematical structures, based on whatever logic, are equally valid. But this is a claim of philosophy, not of mathematics, and so does not fall within its own scope; any problematic self-reference does not, then, arise.[27]

However, we are not concerned here with mathematical pluralism as such, but with the pluriverse in particular. As we noted, this does appear to be self-refuting, since it appears to presuppose a unique hyperverse. Perspectivalism was exactly a move to avoid this problem. However, perspectivalism may have relativist problems of its own. Certainly the Jain global perspectivalism does, since it is committed to the thought that the Jain view, the Hindu view, the Buddhist view, etc, are all equally correct relative to the appropriate perspective. Hence, a

[26]For a discussion of relativism in general, and Plato in particular, see Baghramian and Carter (2015).

[27]As Shapiro (2014), p. 183, notes.

Jain can no longer claim that their view is superior to a rival's. Let us not pursue this matter here, however.[28] The question is whether a perspectival view of V has similar problems.

It might well be thought to do so, since, one might suppose, the claim that there in no absolute universe of sets, but only different perspectives thereupon, appears to be giving a non-relative perspective on the matter. However, the perspectivalism in question is to the effect that any mathematical claim about V must be interpreted as relative to some universe. Since any claim in the language of set theory has its quantifiers relativised to V (at least implicitly) this is equivalent to the thought that any claim of set theory must be relativised to some universe. It is hence akin to the thought that any claim about a group is true or false only with respect to some group or other. As such, it is a claim *about* mathematics, not *of* mathematics. Hence, it does not fall within its own scope, and so is no more self-refuting than mathematical pluralism itself.

4 Conclusion: a Perspective on this Paper

This short paper has been concerned with the pluriverse of sets. We have seen that there are good reasons to generalise Hamkin's classical pluriverse to a pluriverse containing universes based on non-classical logics. We have also seen that taking each universe to have its own perspective on the totality of sets, each of which is equally legitimate, resolves the most pressing philosophical objection to the pluriverse. We have seen, moreover, that such perspectivalism provides an answer to the question of what metalogic should be used to investigate the totality of universes itself. Each member of the pluriverse gives us a perspective on the matter, and that perspective will be articulated with the internal logic of that universe. Finally, as we saw, this does not engender a pernicious relativism.

The pluriverse of sets in vast and diverse. Each universe in the pluriverse has a take on what it is like, and there is no sense in which any of them gets it objectively right. As far as the universe of sets goes, perspectives are all we have.[29]

[28] It is pursued in Priest (2008), §9.

[29] For helpful comments on an earlier draft of this paper, I'm grateful to Hartry Field, Joel Hamkins, Stewart Shapiro, and Alan Weir himself. The material was presented to a couple of seminars in Hartry Field's course on the philosophy of mathematics in the spring semester of 2020. I'm also grateful to a number of students in the class for their comments.

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