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The Logical Structure of Dialectic

Graham Priest^{a,b,c}

^aDepartment of Philosophy, Graduate Center, City University of New York, New York, NY, USA; ^bDepartment of Philosophy, University of Melbourne, Melbourne, Australia; ^cDepartment of Philosophy I, Ruhr University of Bochum, Bochum, Germany

ABSTRACT

I give a formal model of dialectical progression, as found in Hegel and Marx. The model is outlined in the first half of the paper, and deploys the tools of a formal paraconsistent logic. In the second half, I discuss a number of examples of dialectical progressions to be found in Hegel and Marx, showing how they fit the model.

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1. Introduction

Dialectic, in the sense of Hegel and Marx, is a dynamic process in which contradictions arise and are *aufgehoben*—an impossible word to translate into English, since it means both *removed* and *preserved*; a common translation is *sublated*, which seems as good as anything. Commentators disagree about how, exactly, the process is to be understood. But I take it that the contradictions involved are—at least sometimes—contradictions in the logician's sense (that is, things of the form $A \wedge \neg A$); and that dialectics requires dialetheism: the view that some contradictions hold (at least at some times).¹

Dialetheism says nothing about the dynamical aspect of dialectic, however. How, exactly, do the contradictions arise? And what exactly is sublation? The point of this paper is to provide a simple model of the matter. In the first main part I will outline the model. I will then illustrate it with some examples from Hegel and Marx. An appendix gives the precise formal details for those who wish them.

2. Dialetheism

But first, a word on dialetheism. Make no mistake that Hegel is a dialetheist—though commentators, in thrall to Aristotle's *horror contradictionis*, may say otherwise. He says so as plainly as anyone can. Thus we have in the *Logic*:²

[...] common experience [...] says that [...] there is a host of contradictory things, contradictory arrangements, whose contradiction exists not merely in external reflection, but in themselves.

CONTACT Graham Priest  priest.graham@gmail.com

¹ See Priest 1990, Hegel 1931, Priest 2019.

² Hegel 1969, 440.

And he asserts boldly a few lines later, taking on board Zeno's paradoxes:

External sensuous motion is contradiction's immediate existence. Something moves, not because at one moment it is here and at another there, but because at one and the same moment it is here and not here, because in this 'here', it at once is and is not.

Or again, Hegel's solution to the Liar paradox, given in his discussion of Eubulides in the *Lectures on the History of Philosophy* (part 1, chapter 2, C.1.b) is that the liar statement:³

[...] both lies and does not lie [...] For here we have a union of opposites, lying and truth, and their immediate contradiction [...]

He also berates the error of those who have tried, futilely, to give a 'one sided' answer to the question of the status of the liar statement, plumping—as most logicians have done over the ages—for one side of the contradiction or the other.⁴

Of course, the fact that Hegel was a dialetheist tells you nothing in itself about his dialectics—other than the fact that he need not shy away from dialetheias. And indeed, he does not: he makes central use of them. Let us now see how.

3. Outline of the Model

The model of dialectics that follows can be spelled out in precise logical terms. I do this in a technical appendix to the paper. Here I provide an informal description, more intelligible for non-logicians.

The construction deploys a formal paraconsistent logic—that is, a logic which allows for interpretations of the language (situations) where things may be both true and false. So, for some A s we may have A and $\neg A$, and so $A \wedge \neg A$ —though not everything is true. (The principle of inference called *Explosion*, $A \wedge \neg A \models B$, fails.) Most paraconsistent logics can be used for the purpose at hand, but for the sake of definiteness, I use the logic LP .⁵ In paraconsistent logics, a (monadic) predicate P , has an extension $\delta^+(P)$, and an anti-extension, $\delta^-(P)$. The former contains the things of which P is true; the latter contains the things of which P is false, that is, of which $\neg P$ holds.⁶

A dialectical progression is a sequence of situations (interpretations): S_1, S_2, S_3, \dots I will write the extension and anti-extension of a predicate, P , in S_i as $\delta_i^+(P)$ and $\delta_i^-(P)$, respectively. The progression goes in triples: the *root stage*, its *negation*, and the *negation of the negation*.⁷ Informally, in the negation stage, the situation concerning some object at the root stage becomes its opposite. In the negation of the negation, we return to the root stage, but at a 'higher' level.

Situation S_1 concerns (amongst other things) a particular predicate, P , and object, a . In this situation, Pa is true, and not false. That is, a is in $\delta_1^+(P)$, but not in $\delta_1^-(P)$. In situation 2, the negation, this situation is reversed, so Pa is false, but not true. That is, a is in $\delta_2^-(P)$,

³ Hegel 1955, 460.

⁴ See, further, Priest 1990, Priest 2019.

⁵ See, e.g. Priest 2006, ch. 5.

⁶ Of course, for fuller logical exegeses of Hegel, the formal language would need to contain not just monadic predicates, but binary predicates, and maybe even predicates of higher adicity. However, simple monadic predicates will suffice for our purposes.

⁷ Hegel refers to them thus, occasionally (e.g. *Lesser Logic*, para 95), though the terminology is, perhaps, more frequently associated with Engels. In Fichtean terminology, the stages are the thesis, antithesis, and synthesis.

but not in $\delta_2^+(P)$. In situation 3, the negation of the negation, Pa is both true and false; that is, a is in $\delta_3^+(P) \cap \delta_3^-(P)$: so $Pa \wedge \neg Pa$. I note that there is no suggestion here that the extension and anti-extension of P change only with respect to a .⁸

In fact, the connection between Pa and $\neg Pa$ at this stage is typically much tighter than mere conjunction: a 's being P is its being $\neg P$.⁹ To formalise this requires a more expressive language. The language needs to contain identity, $=$, and an operator, $\hat{}$, which turns a sentence into a noun-phrase. \hat{Pa} means *that Pa*, or *a's being P*. The connection between Pa and $\neg Pa$ can then be written as: $\hat{Pa} = \hat{\neg Pa}$ (though of course, we have $\hat{Pa} \neq \hat{\neg Pa}$ as well). This identity entails that $Pa \wedge \neg Pa$. For either Pa or $\neg Pa$. In the first case (the other is similar), \hat{Pa} is true. But then $\hat{\neg Pa}$ is true (by the substitutivity of identicals), so $\neg Pa$. So both Pa and $\neg Pa$.

So much for the first 3-cycle. At the end of this, another starts, the last stage of the first being essentially the first of the next. Taking this to be so in the most flat-footed way, would, however, ensure that the dialectic concerning P could not continue. For if we now repeat the process, the next three stages would give us, for some object, x : $Px \wedge \neg Px$, $\neg(Px \wedge \neg Px)$, $(Px \wedge \neg Px) \wedge \neg(Px \wedge \neg Px)$. But in LP , $Px \wedge \neg Px$ entails $\neg(Px \wedge \neg Px)$, so there could not be an object satisfying $Px \wedge \neg Px$ but not its negation, as required for the first step of a cycle.¹⁰

Something, then, is still missing from the picture: the emergence of a novel concept. In moving from stage 3 to 4, the language used to describe the situation is augmented with a new predicate, P^\dagger .¹¹ Things concerning the old vocabulary remain the same, but P^\dagger is true of just the things that both P and $\neg P$ are true of. That is:

- $\delta_4^+(P^\dagger) = \delta_3^+(P) \cap \delta_3^-(P)$

We may take the anti-extension of P^\dagger to be the complement of this, making P^\dagger a consistent predicate. That is:

- $\delta_4^-(P^\dagger) = D - \delta_4^+(P^\dagger)$

where D is the domain of all the objects in the situation.¹²

Call these these *aufheben conditions*. Because of them, in situation 4, for any object, b , $P^\dagger b$ is true iff $Pb \wedge \neg Pb$ is true; so in particular, $P^\dagger a$. But because of the anti-extension of P^\dagger , it is not the case that $\neg P^\dagger a$.

Given this augmentation of the language, a new triple then begins with respect to P^\dagger , and some object b (maybe a , maybe something different). The second 3-cycle takes us to stage 6. At this point, a new predicate $P^{\dagger\dagger}$ is added to the language, which relates to P^\dagger as P^\dagger relates to P . We then start the next 3-cycle. And so on.

⁸ In some applications of dialectics, it makes more sense to think of $\neg P$ as a contrary of P , rather than the contradictory. But in this case $Pa \wedge \neg Pa$ is still a contradiction. (*Red* and *green* are contraries; and if something is green, it is not red.)

⁹ See *Priest 1990*, sect. 8.

¹⁰ One could avoid this by using a different paraconsistent logic. However, a Hegelian dialectical progression requires a new concept to emerge here, so something like the next step is necessary anyway.

¹¹ Alternatively, if we have the operator $\hat{}$ in the language, we can simply take $P^\dagger x$ to be $\hat{Px} = \hat{\neg Px}$. $P^\dagger a$ then entails $Pa \wedge \neg Pa$, though it may not be entailed by it.

¹² Actually, this is not necessary for what follows. All we need is that $a \notin \delta_4^-(P^\dagger)$, but let us keep matters simple.

As we have seen, $P^\dagger a$ is consistently true at stage 4. However, contradiction has not been eliminated, for it is still true that $Pa \wedge \neg Pa$. Contradiction, then, has both been removed and is preserved—that is, it is *aufgehoben*. Moreover, even though P^\dagger changes its extension during the next cycle, P may not, so the contradiction may remain.

Friends of the Principle of Non-Contradiction who do not want to admit Hegel's dialecticism, are fond of saying that the contradictions that arise at a stage of the dialectic are removed at the next state (never to reappear). This fails to take account of the bicameral meaning of *aufheben*. Contradictions are *preserved* too. As Hegel says:¹³

'To *sublate*' has a twofold meaning in the language: on the one hand it means to preserve, to maintain, and equally it also means to cause to cease, to put an end to [...] Thus, what is sublated is at the same time preserved; it has only lost its immediacy [...]

The model at hand shows exactly how you can have both a removal and a preservation.

Let me end this section by emphasising that it is not the intention of this model to capture all the aspects of dialectics (epistemic, metaphysical, etc)—far from it. This is simply a model of the *logical* relations involved. These provide an essential part of the whole picture, however.

4. Applications: Hegel

So much for the abstract structure of dialectical progression. In what follows, I will illustrate it with some examples from Hegel and Marx. This means showing the particular a , P , and P^\dagger , involved in these cases. It should be said, straight way, that Hegel often had to stretch matters to be able to fit things into the dialectical pattern. Liberties were taken with the notions of contradiction and with the novel concepts. This is more so with Marx, and even more so again with Engels. But this is perhaps the fate of all who would try to fit such complex matters into a somewhat procrustean framework.

This warning having been made, it is easy enough to find paradigm examples in Hegel and Marx which clearly fit the model. Let us start with Hegel.¹⁴

A most clear example is the dialectic which begins Hegel's *Logic*. The *Logic* tells the story of the dialectical evolution of concepts, from the most basic, *being*, to the most complex, the *absolute idea*. Each step in the progression produces a concept more adequate to grasp reality. *Being* is the least adequate; the *absolute idea*, capturing as it does the absolute, is the most.

The *Logic* starts with a dialectical progression from *being*, to *nothing* (that is, *non-being*) to *becoming*, to *determinate being*.¹⁵ To see how it fits the above model, take a to be any object—or maybe reality as a whole. Px is 'x is'. At Stage 1, a is. But simply being is no different from not being anything at all, since there is no differentiation. So at the second

¹³ Hegel 1969, 107. Note that all italics in all quotations are the translator's.

¹⁴ For an excellent general exposition of Hegel's philosophy, see Taylor 1975.

¹⁵ Vol. I, Bk. 1, Sec. 1, Ch. 1. Hegel does, it is true, balk at using *non-being* instead of *nothing*. The reason is that the use of explicit negation would build in a formal element to the opposition; and Hegel wishes the opposition, for reasons that are not relevant here, to be a matter of content, not form. However, he points out that using *non-being* instead, gives exactly the same result: 'Should it be held more correct to oppose to being, *non-being* instead of nothing, there would be no objection to this so far as the result is concerned, for in *non-being* the relation to *being* is contained; both being and its negation are enunciated in a *single* term, nothing, as it is in becoming. But we are concerned first of all not with the form of opposition [...] but with the abstract immediate negation: nothing, purely on its own account, negation devoid of any relations [...]' (Hegel 1969, 83).

stage of the progression, $\neg Pa$, a is not. At the third stage, a is something that is and is not. $Pa \wedge \neg Pa$. This is *becoming*.¹⁶

Becoming is the unseparatedness of being and nothing, not the unity which abstracts from being and nothing; but as the unity of *being* and *nothing* it is this *determinate* unity, in which there *is* both being and nothing.

Why becoming? According to the dialectical account of change which Hegel endorses, it is precisely being in a contradictory state which differentiates a dynamic state from a static one. See his reference to Zeno, in Section 2 above.¹⁷

The new concept that occurs at this stage is *determinate being*, since if x is changing, it is *something*. That is, its being (*is*) has a determination (changing). $P^\dagger x$, then, is ‘ x has determinate being’.¹⁸

From becoming there issues determinate being, which is the simple oneness of being and nothing. Because of this oneness it has the form of *immediacy*. Its mediation, becoming, lies behind it; it has sublated itself and determinate being appears, therefore, as a first, as a starting point for the ensuing development. It is first of all in the one-sided determination of *being*; the other determination, *nothing*, will likewise display itself and in contrast to it.

So for any x , $P^\dagger x$ iff $Px \wedge \neg Px$. In particular then, $P^\dagger a$. We are then off on the next 3-cycle of the dialectic.

The dialectical progression of the *Logic* is not a temporal one. Its moments are the stages of the development of a series of increasingly complex concepts. However, for Hegel, the conceptual development is embedded in a development in the material world, and so in time. These developments in time track the conceptual development—though not necessarily in a strictly chronological order. This brings us to our second example—perhaps Hegel’s most famous. This is from the *Phenomenology*, and is the master/slave dialectic.¹⁹

The full story has many complexities.²⁰ However, at its simplest, it is this. True self-consciousness of a people requires mutual recognition; but at a certain stage of human development, no one is prepared to offer recognition to another. When two people face off, each therefore tries to force the other to provide that recognition. Eventually, one captures and enslaves the other. Let the master and the slave be m and s , respectively. The concept involved in the dialectic, Px , is ‘ x is has being-for-self’. At Stage 1, both s and m have this; in particular, Ps . As a result of the enslavement, the slave no longer has such being: they have being-for-another. That is, $\neg Ps$. Thus we have Stage 2:²¹

Both moments [sc. consciousness for itself, and consciousness for another] are essential, since, in the first instance, they are unlike and opposed, and their reflexion into unity has not yet come to light, they stand as two opposed forms of consciousness. The one is independent, and its essential nature is to be for itself; the other is dependent, and its essence is life or existence for another. The former is the Master, or Lord, the latter the Bondsman.

Because the master forces the slave to labour, the slave then acquires skills and powers they did not have before; so whilst still having being-for-another, they acquire a new

¹⁶ Hegel 1969, 105.

¹⁷ See, further, Priest 2006, ch. 12.

¹⁸ Hegel 1969, 109.

¹⁹ Pt. B, Ch. 4, Sec. A.

²⁰ See Taylor 1975, 153 ff.

²¹ Hegel 1931, 234.

being-for-self. Hence at Stage 3, the slave has both being-for-self, and being-for-other: $P_s \wedge \neg P_s$. Moreover, the master does not obtain what was sought, since the recognition of the slave is not freely given; but the master is forced to recognise the slave, since they depend on the slave. So, by a dialectical irony, it is the slave who obtains what was sought: the mutual recognition required for self-consciousness. So $P^\dagger x$ is ‘ x has self-consciousness’, and for any b , $P^\dagger b$ iff $Pb \wedge \neg Pb$. In particular, then, $P^\dagger s$:²²

Self-consciousness exists in itself and for itself, in that, and only by the fact that it exists for another self-consciousness; that is to say, it is only by being acknowledged or ‘recognized’ [...] [I]ts moments must on the one hand be strictly kept apart in detailed distinctness, and, on the other, in this distinction must, at the same time, also be taken as not distinguished, or must always be accepted and understood in their opposite sense. This double meaning of what is distinguished lies in the nature of self-consciousness.

5. Applications: Marx

Let us now turn to Marx. Marx employed Hegelian ideas, including the dialectics, in his social philosophy. Famously, he reinterpreted it, though: the dialectics no longer concerned the materially embodied developments of concepts, but concerned, instead, material conditions, notably of social and economic production themselves. He did not change the structure of dialectical progressions, however.

Marx was at his most Hegelian in his earlier writings, but there are clear applications of the dialectics in the later writings. A notable example of this is in Chs. 1–3 of Vol. I of *Capital*, where Marx deals with use value, exchange value, and the emergence of money.

In a subsistence economy, people produce economic goods to use. These are, in Marxian terms, a *use value*. But once a surplus is produced, these can be exchanged. Things exchanged are an *exchange value*. Things are either used or exchanged, not both. So use value and exchange value are contradictory (or at least contrary) concepts:²³

The same commodity cannot, therefore, simultaneously appear in both forms [sc. a use value and an exchange value] in the same expression of value. These forms exclude each other as polar opposites.

At a certain stage of production, exchange needs to become mediated by a common currency. This is something whose use value *is* its exchange value, and this is money. In the first instance this was, historically, gold.

So let g be gold. Let Px be ‘ x is a use value’. In Stage 1 of the dialectics, Pg . Gold is something that is used (as jewellery, etc.) At Stage 2, gold becomes an exchange value, $\neg Pg$: people trade it. At the third stage, it is both a use value and an exchange value. Indeed, its use value *is* its exchange value, $Pg \wedge \neg Pg$. In this case, $P^\dagger x$ is ‘ x is money’—a new social category. Gold is money, $P^\dagger g$:²⁴

[T]he exchange of commodities implies contradictory and mutually exclusive conditions. The further development of the commodity does not abolish these contradictions, but rather provides the form within which they have room to move. This is, in general, the way in which real contradictions are resolved.

²² Hegel 1931, 229.

²³ Marx 1976, 140.

²⁴ Marx 1976, 198.

Note that this is no *mere* conceptual development. For that something now satisfies the new concept is a novel fact realised by the world.

More famously, Marx often deploys the dialectics in describing large-scale social change. This will give a second example of Marxian dialectics to illustrate the model: his account of the transition from feudalism to capitalism. Again, the full story has many complexities,²⁵ but in essence it is as follows.

At one time, Europe was a feudal society. Labourers were bound to both the land and their feudal lords. With the breakdown of feudalism, these things disappeared; indeed the nascent capitalism drove people off the land to provide the required labour force. Labourers were then free of these bonds. However, now, to live, they had to sell themselves into ‘wage slavery’. Because of their very freedom, they had to bind themselves to capitalist employers who use their labour power.²⁶

So let l be some generic labourer (or maybe better, labour power itself). Let Px be ‘ x is bound’. At Stage 1 of the dialectics the labourer is bound by their feudal chains, Pl . At Stage 2 they are freed from these bonds, $\neg Pl$. As a free agent of their labour power, they then, of necessity, bind themselves into employment, $Pl \wedge \neg Pl$. They are both free to sell their labour and bound to do so:²⁷

The contract by which he [sc. the wage labourer] sold his labour power to the capitalist proved in black and white, so to speak, that he was free to dispose of himself. But when the transaction was concluded, it was discovered that he was no ‘free agent’, that the period of time for which he is forced to sell his labour power is the period of time for which he is forced to sell it.

This is Stage 3. The new condition that emerges is wage labour. So $P^\dagger x$ in this case, is ‘ x is a wage labourer’, and we have $P^\dagger l$.

6. Conclusion

There is, of course, much more to be said about the details of the examples from Hegel and Marx that I have chosen. Many other examples might also be discussed. I hope, however, that the examples serve to illustrate the rather abstract structure of the first part of the paper, and show that this structure really is a plausible formal analysis of the dialectics of Hegel and Marx.

I note that the model does not offer an account of *why* a dialectical progression occurs. That is, in fact, different for Marx and for Hegel. In a Hegelian dialectic, it is reflections on concepts which drives the dialectic. In a Marxian dialectic, it is natural laws of the kind one finds in physics which does so. Neither have I said much about the notion of negation itself. The model, in fact, makes very few assumptions about negation: only that it is paraconsistent. There is much more to be said about these and many other matters pertinent to dialectic.

The aim of the paper is, then, a very limited one. It is simply to give an account of the logical structure of dialectical progressions. That is not the end of a story about dialectics. It is just the beginning.²⁸

²⁵ See Katz 1993.

²⁶ The story is well told in Thompson 1980.

²⁷ Marx 1976, 415. Note that the freedom and bondage in question are not in different respects. See Priest 1991, 472 f.

²⁸ Versions of this paper were given at the conference *Logic and Politics*, at the University of Paderborn, December 2013, the conference *Hegel, Analytic Philosophy and Formal Logic*, Purdue University, October 2014, and in a seminar in the (online)

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Appendix. Technical Appendix

A.1 The Logic LP

In this appendix, I will give a precise specification of the progression informally described in Section 3. I start with the paraconsistent logic employed, LP.²⁹

The semantics of this logic applies to any standard first-order language, \mathcal{L} . This contains connectives for negation, conjunction, disjunction (\neg , \wedge , \vee), and the particular and universal quantifiers (\exists and \forall). ($A \supset B$ can be defined as $\neg A \vee B$.) For our purposes, we can assume that all predicates are monadic, and that there are no function symbols.

An interpretation for \mathcal{L} , \mathfrak{M} , is a pair $\langle D, \delta \rangle$, where D is the non-empty domain of quantification; and δ is a function such that for every constant, c , $\delta(c) \in D$, and for every predicate, P , $\delta(P) = \langle X, Y \rangle$, where $X \cup Y = D$. We will write X and Y as $\delta^+(P)$ and $\delta^-(P)$, respectively. These are the *extension* and *anti-extension* of P .³⁰

We now define what it is for a formula to be true, \Vdash^+ , or false, \Vdash^- , in \mathfrak{M} . For the quantifiers, we assume that \mathcal{L} has been extended with a bunch of new constants, one for every $d \in D$. For simplicity, I will use each object as its own name. $A_x(c)$ is the formula A with every free occurrence of the variable x replaced with the constant c .

For closed formulas in this language:

- $\Vdash^+ Pc$ iff $\delta(c) \in \delta^+(P)$
- $\Vdash^- Pc$ iff $\delta(c) \in \delta^-(P)$
- $\Vdash^+ \neg A$ iff $\Vdash^- A$
- $\Vdash^- \neg A$ iff $\Vdash^+ A$
- $\Vdash^+ A \wedge B$ iff $\Vdash^+ A$ and $\Vdash^+ B$
- $\Vdash^- A \wedge B$ iff $\Vdash^- A$ or $\Vdash^- B$
- $\Vdash^+ A \vee B$ iff $\Vdash^+ A$ or $\Vdash^+ B$
- $\Vdash^- A \vee B$ iff $\Vdash^- A$ and $\Vdash^- B$
- $\Vdash^+ \exists xA$ iff for some $d \in D$ $\Vdash^+ A_x(d)$
- $\Vdash^- \exists xA$ iff for all $d \in D$ $\Vdash^- A_x(d)$

series, *Hegel and Dialetheism*, November 2021. Many thanks go to the audiences for their helpful comments, and especially to Leon Geerdink and Stefan Schick.

²⁹ For details of this, see Priest 2006, ch. 5.

³⁰ If we drop the constraint that for all predicates, P , $\delta^+(P) \cup \delta^-(P) = D$, we have the logic of First Degree Entailment, FDE. If we add the constraint that for all predicates, P , $\delta^+(P) \cap \delta^-(P) = \emptyset$, we have classical logic, CL.

- $\Vdash^+ \forall x A$ iff for all $d \in D \Vdash^+ A_x(d)$
- $\Vdash^- \forall x A$ iff for some $d \in D \Vdash^- A_x(d)$

An interpretation is a *model* of a formula, A , iff $\Vdash^+ A$; it is a model of a set of formulas, Σ , if it is a model for every member of Σ . And an inference is valid, $\Sigma \vDash A$, iff every interpretation which is a model of Σ is a model of A .

A.2 The Dialectical Progression

Using these semantics, we can now give a formal definition of a dialectical progression.

A progression is a sequence of *LP* interpretations, \mathfrak{M}_n (with denotation function δ_n), objects, a_n , and predicates, P_n , for $n < \omega$,³¹ such that for every n :

- each interpretation has the same domain,³² D
- $a_n \in D$
- every constant has the same denotation in every interpretation
- $\mathfrak{M}_{3n}, \mathfrak{M}_{3n+1}, \mathfrak{M}_{3n+2}$ have the same language
- P_0 is in the language of \mathfrak{M}_0 ; and the language of \mathfrak{M}_{3n+3} is the language of \mathfrak{M}_{3n+2} together with the new predicate, P_{n+1}
- $a_n \in \delta_{3n}^+(P_n)$ and $a_n \notin \delta_{3n}^-(P_n)$
- $a_n \notin \delta_{3n+1}^+(P_n)$ and $a_n \in \delta_{3n+1}^-(P_n)$
- $a_n \in \delta_{3n+2}^+(P_n) \cap \delta_{3n+2}^-(P_n)$
- The extension and anti-extension of all the predicates in the language of \mathfrak{M}_{3n+2} are the same in \mathfrak{M}_{3n+2} and \mathfrak{M}_{3n+3} . For P_{n+1} :
 - $\delta_{3n+1}^+(P_{n+1}) = \delta_{3n}^+(P_n) \cap \delta_{3n}^-(P_n)$
 - $\delta_{3n+1}^-(P_{n+1}) = D - \delta_{3n+1}^+(P_{n+1})$

These are the *aufheben* conditions.

According to this definition, a dialectical progression has a first state, but the definitions would make just as much sense if the sequence were indexed by the integers (positive and negative), so that the progression stretched back indefinitely, as well as forward.

³¹ Or, if the dialectic is finite, for $n < 3m$, for some $m > 0$.

³² One can accommodate the fact that things come into and go out of existence by supposing that there is a monadic existence predicate whose extension/anti-extension varies with n .