GRAHAM PRIEST

SEMANTIC CLOSURE, DESCRIPTIONS AND NON-TRIVIALITY

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ABSTRACT. It is known that a semantically closed theory with description may well be trivial if the principles concerning denotation and descriptions are formulated in certain ways, even if the underlying logic is paraconsistent. This paper establishes the non-triviality of a semantically closed theory with a natural, but non-extensional, description operator.

KEY WORDS: denotation, descriptions, non-extensionality, non-triviality, paraconsistency, semantic closure

1. Introduction: Semantic Closure and Triviality

Semantic closure is well known to lead to inconsistency. Specifically, suppose a language contains semantic predicates such as 'is true', 'satisfies', 'denotes'; suppose that a theory in that language contains the intuitively correct axioms governing these predicates, a modicum of self-reference, and is based on a logic containing a few simple logical principles. Then that theory is inconsistent. Provided that we choose a suitable paraconsistent logic as the underlying logic of the theory, however, it may be non-trivial. That is, its inconsistencies are localised, and do not spread everywhere.[1]

It is to be hoped, especially if one is of a dialetheic persuasion, that the same is true once descriptions are added to the machinery. This is a sensitive question, though. If one adds descriptions in a simple-minded way, triviality ensues. Fortunately, if one adds descriptions in a less simple-minded – but still natural – way, non-trivially is maintained. The point of this paper is to prove this.[2]

I will start by rehearsing some of the difficulties that descriptions cause. This will lead to a discussion of denotation-failure. After specifying a suitable free logic to handle such failure, I will then give the non-triviality argument.

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2. Denotation Failure

To illustrate the problems caused by descriptions, suppose that our theory is one of arithmetic. Let f be any 1-place arithmetic function. Consider the term 'f of the denotation of this very term'. Call this term τ, and let d be its denotation. τ denotes d, but it also denotes fd. Hence, $d = fd$. Now let f be the function that maps 0 to 1, and everything else to 0. Whatever d is, fd, is either 1 or 0. If $fd = 1,\ d = 1$, so $fd = 0$. If $fd = 0,\ d = 0$, so $fd = 1$. In either case, $0 = 1$, and triviality is not far behind.

The precise details of the above argument are spelled out in Priest (1997) and Priest (1998). The formalised argument uses a few principles of first order logic (with identity), some principles concerning descriptions, and the following principle concerning denotation:

$$\Delta\langle t\rangle s \dashv\vdash t = s. \qquad \textit{(Denotation)}$$

Here, Δxy means, intuitively, 'x denotes y', $\langle t\rangle$ is a name for the term t, and $\alpha \dashv\vdash \beta$ means $\alpha \vdash \beta$ and $\beta \vdash \alpha$.[3]

The natural objection to the above argument is that it fails, since the term τ may, in fact, have no denotation. For the case in question, it would certainly appear not to have one. This suggests that non-triviality may be achieved if we take the underlying logic of the theory to be a free paraconsistent logic, which allows for denotation-failure.[4] This is the suggestion we will pursue. I will take the underlying logic to be LP.[5] Standardly, this is not a free logic, but it can easily be turned into one. Doing so poses several choices, though. A crucial one is how to evaluate atomic sentences that contain non-denoting terms. The policy adopted here will be to evaluate such sentences as uniformly false – and not also true. (This does not, of course, mean that *all* sentences containing non-denoting terms are false.) The policy is a very natural one for extensional predicates[6] – the only kind at issue here. In the present context, it also has very happy consequences. For it renders identities of the form $t = t$ untrue if t does not denote. Now, $t = t$, together with *Denotation*, entails $D\langle t\rangle t$, and hence $\exists x\Delta\langle t\rangle x$: – everything denotes. In a free logic, this should obviously not be forthcoming. Without $t = t$, it is not.[7]

In the following section I will give the formal details of the logic. Next, I will describe an appropriate treatment of descriptions in the logic. I will then be in a position to give the required non-triviality proof.

3. Free LP

Let L be a first order language with identity and function symbols. Terms and formulas are defined as usual. I will reserve the word 'sentence' for formulas without free variables. A *free LP interpretation* is a triple, $A = \langle D, I, \delta \rangle$, where D is a non-empty domain of objects; δ is a partial function from the set of constants into D; for every n-place function symbol, f, $I(f)$ is a (total) function from D^n to D, and for every n-place predicate, P, $I(P)$ is a pair $\langle P^+, P^- \rangle$, such that $P^+ \cup P^- = D^n$. $=^+$ is the set $\{\langle x, x \rangle; x \in D\}$. I will write $\delta(t) = \infty$ to mean that $\delta(t)$ is undefined. To cut a few inessential corners, I will assume that for every $d \in D$, there is in the language a constant, $\mathbf{d}$, that denotes it. Thus, $\delta(\mathbf{d}) = d$. (Such constants can always be added if they are not already present.)

Given an interpretation, the denotation function, δ, is extended to a partial function evaluating all closed terms of the language by the (gap-in/gap-out) recursive condition: if for some $1 \leq i \leq n$, $\delta(t_i) = \infty$, $\delta(ft_1 \dots t_n) = \infty$. Otherwise, $\delta(ft_1 \dots t_n) = I(f)(\delta(t_1), \dots, \delta(t_n))$.

An *evaluation* for A is a relation, ρ_A, between the set of sentences and $\{1, 0\}$, satisfying the following conditions:

$Pt_1 \dots t_n \rho_A 1$ iff for all $1 \leq i \leq n$, $\delta(t_i) \neq \infty$, and $\langle \delta(t_1), \dots, \delta(t_n) \rangle \in P^+$,

$Pt_1 \dots t_n \rho_A 0$ iff for some $1 \leq i \leq n$, $\delta(t_i) = \infty$, or (for all $1 \leq i \leq n$, $\delta(t_i) \neq \infty$, and $\langle \delta(t_1), \dots, \delta(t_n) \rangle \in P^-$).

Thus, if a term in an atomic sentence fails to denote, the sentence is simply false (and not also true). The truth/falsity conditions for $\vee, \wedge, \neg$, are as usual. For the quantifiers:

$\exists x \alpha \rho_A 1$ iff for some $d \in D$, $\alpha(x/\mathbf{d}) \rho_A 1$,

$\exists x \alpha \rho_A 0$ iff for all $d \in D$, $\alpha(x/\mathbf{d}) \rho_A 0$.

The conditions for $\forall$ are the obvious dual ones.

Finally, validity: if $\Sigma \cup \{\alpha\}$ is a set of sentences, $\Sigma \models \alpha$ iff for all A, if $\beta, \rho_A 1$ for all $\beta \in \Sigma$, $\alpha \rho_A 1$.

It is clear that the propositional part of the logic is *LP*. The quantifier part is, however, a free logic. In particular, the inference schema $\alpha(x/t) \vdash \exists x \alpha$ is invalid. To see this, take α to be $\neg Px$, and consider an interpretation in which $P^- = \phi$, but $\delta(t) = \infty$. Then it is easy to check that $\neg Pt\rho 1$, but it is not the case that $\exists x \neg Px\rho 1$. Let us write Ey for $y = y$. Then it is clear that in any interpretation, $Et\rho 1$ iff $\delta(t) \neq \infty$. In particular, then, we have:

$E(t), \alpha(x/t) \models \exists x\alpha$. Similar considerations apply to the dual inference: $\forall x\alpha \vdash \alpha(x/t)$. This is not valid, but we have: $\forall x\alpha$, $Et \models \alpha(x/t)$. Finally, for identity: it is easy to check that $s = t, \alpha(x/t) \models \alpha(x/t)$. For if $s = t$ is true in an interpretation, $\delta(s) \neq \infty$, $\delta(t) \neq \infty$, and s and t denote the same thing. As we have already observed, the logical validity of the law of identity, in the form $t = t$, fails; but we do have it in the form $\forall xx = x$.

The general shape of the proof-theory for the logic is, then, clear. It is an interesting project to provide a formal characterisation of that theory, and to prove it sound and complete. However, this is not required for present purposes, so I shall not pursue the matter here.

One other fact about free *LP* that will be useful in what follows is a familiar monotonicity condition. Let A and B be interpretations. I will write $A \preceq B$ to mean that A and B have the some domain, and for every atomic formula, α:

$$\alpha\rho_A 1 \Rightarrow \alpha\rho_B 1,$$

$$\alpha\rho_A 0 \Rightarrow \alpha\rho_B 0.$$

It is easy to show that if $A \preceq B$, the displayed condition holds for all formulas. The proof is by a familiar induction, and I omit details.[8]

4. Descriptions

The other machinery involved in the triviality argument is a description operator. I will now spell out one suitable for the non-triviality argument.[9] An (indefinite) description operator, ε, is added to the language. Terms and formulas are defined in the usual way.

An interpretation for the new language is a quadruple, $\langle D, I, \delta, \Phi\rangle$, where D, I, and δ are as before, and Φ is a map from formulas to choice functions on the power-set of D. Specifically, for every formula, α, and for every non-empty $X \subseteq D$, $\Phi^\alpha X \in X$. The denotation of closed terms, and the truth/falsity values of sentences are defined by a joint recursion. All details are as before, except for the clause for descriptions, which is as follows. If α is a formula with at most one free variable, x, let us write $\overline{\alpha}$ for $\{d \in D;\ \alpha(x/\mathbf{d})\rho 1\}$. Then if $\overline{\alpha} = \phi$, $\delta(\varepsilon x\alpha) = \infty$; otherwise:

$$\delta(\varepsilon x\alpha) = \Phi^\alpha\overline{\alpha}.$$

This theory of descriptions is slightly unusual. It would be more normal to take Φ itself to be a choice function, and define $\delta(\varepsilon x\alpha)$ simply as $\Phi\overline{\alpha}$ when $\overline{\alpha} \neq \phi$. Such a way of handling descriptions verifies extensionality: if, in an interpretation, $\overline{\alpha} = \overline{\beta} \neq \phi$, $\varepsilon x\alpha = \varepsilon x\beta$ is true in it. However, there seems no intuitive reason as to why one should expect extensionality to hold: if all and only the men in the room are the rich people in the room, why should one suppose that 'a man in the room' refers to the same thing as 'a rich person in the room'?[10] In the present treatment, the defining condition of the description *and* its extension are taken into account in determining the description's denotation.

Non-extensionality has some other consequences for the logic of descriptions which should be noted. Most importantly, the inference $\exists x\alpha \vdash \alpha(x/\varepsilon x\alpha)$ is not, in general, valid. Certainly, if $\exists x\alpha$ is true in an interpretation, then for some d in the domain both $\alpha(x/\mathbf{d})$ and $\mathbf{d} = \varepsilon x\alpha$ are true. But the substitutivity of identicals is not, in general, valid. (For example, suppose that $\mathbf{d}_1 = \mathbf{d}_2$ is true in an interpretation; it does not follow that $\varepsilon x Px\mathbf{d}_1 = \varepsilon x Px\mathbf{d}_2$, since there is no guarantee that $\Phi^{Px\mathbf{d}_1} = \Phi^{Px\mathbf{d}_2}$.) However, substitutivity of identicals is valid if substitution is not into the scope of an ε-term, as a simple induction demonstrates. It follows that we do have $\exists x\alpha \vDash \alpha(x/\varepsilon x\alpha)$, provided that x does not occur within the scope of an ε in α.[11]

Let me make two final comments on the theory of descriptions here. The first is that the failure of the triviality argument of Section 2, as demonstrated by the following non-triviality proof, is in no way due to the non-extensionality of descriptions. This argument contains no substitution into (or quantification into) ε-terms.[12] The second comment is that, notwithstanding this, the non-extensionality of descriptions is necessary for the following non-triviality proof to work, as I will point out; and I see no way of modifying it to verify the properties of an extensional description operator. (There may, of course, be other non-triviality proofs for extensional description operators.)

5. The Non-Triviality Proof

Now to the non-triviality proof. Fix L as a language for semantically closed arithmetic. Specifically, L contains a numeral, $\mathbf{n}$, for every number, n, and a bunch of function symbols, including ones for successor, addition and multiplication. There are just two binary predicates: identity and the denotation predicate, Δ.[13] We assume some fixed arithmetic coding of formulas of L, and will let $\langle t\rangle$ be the numeral of the code of term t.

We are going to construct an interpretation that models *Denotation*.[14] In addition, all sentences true in the standard model of arithmetic will be true in the interpretation. In particular, the theory will contain the self-referential powers that arithmetic gives. Hence, the set of sentences true in the interpretation will be a free *LP* theory containing arithmetic, descriptions and *Denotation*. As it will be easy to see, not everything holds in the model. Hence, the non-triviality of this theory is established.

The model is defined by a fixed-point construction of the familiar Kripkean kind. We define a sequence of interpretations of L, $A_i = \langle N, I_i, \delta_i, \Phi_i \rangle$, $i \in On$, by transfinite induction. For every interpretation in the sequence, the domain is the set of natural numbers, N; I assigns every arithmetic function symbol its appropriate function; δ assigns every numeral the appropriate number, and $=^-$ and Δ^- are both N^2. Hence the only features of the interpretations that change as we ascend the ordinals are the Φ_i and the extension of Δ. I will write the extension of Δ in A_i as Δ_i^+. I will write ρ_{A_i} simply as ρ_i, and $\overline{\alpha_i}$ for $\{n \in N; \alpha(x/\mathbf{n})\rho_i 1\}$.

Let me give an informal description of the definition, which may make it easier to grasp the import of the following details. Δ_i^+ starts off as the empty relation when $i = 0$. As i increases, if $t = \mathbf{n}$ ever becomes true, $\langle\langle t \rangle, n\rangle$ is thrown into the extension of Δ, and remains there subsequently. For any α, Φ_0^α is an arbitrary choice function. For $i > 0$, $\Phi_i^\alpha X$ defaults to $\Phi_0^\alpha X$; but if, at any stage, $\overline{\alpha_i}$ becomes non-empty, then for all subsequent j, $\Phi_j^\alpha \overline{\alpha_j}$ is a fixed member of $\overline{\alpha_i}$ (and so, as we shall see, of $\overline{\alpha_j}$).

The precise definitions are as follows. For Δ^+:

$$\langle m, n\rangle \in \Delta_i^+ \text{ iff for some } j < i,\ t = \mathbf{n}\rho_j 1,$$
$$\text{where } m \text{ is the code of } t.$$

For Φ: Φ_0^α, is an arbitrary choice function; and if $i > 0$, Φ_i^α is the same as Φ_0^α except that if α has at most one free variable, and for some $j < i$, $\overline{\alpha}_j \neq \infty$:

$$\Phi_i^\alpha \overline{\alpha}_i = \Phi_j^\alpha \overline{\alpha}_j, \quad \text{for the least such } j.$$

It is not immediately obvious that these definitions do succeed in specifying an interpretation. Specifically, it is not clear that each Φ_i^α is a choice function. The next job is to prove this; we will also prove a few other useful lemmas along the way.

LEMMA 1. *If* $i \leq j$ *then* $\Delta_i^+ \subseteq \Delta_j^+$.

Proof. Suppose that $\langle m, n\rangle \in \Delta_i^+$. Then for some $k < i$, $t = \mathbf{n}\rho_k 1$, where m is the code of t. Hence, for some $k < j$, $t = \mathbf{n}\rho_k 1$. That is, $\langle m, n\rangle \in \Delta_j^+$. □

LEMMA 2. *If* $i \leq j$ *then*:

(i) *for every (closed) term, t, if* $\delta_i(t) = n \neq \infty$, $\delta_j(t) = n$,
(ii) *for every sentence,* α, $\alpha\rho_i 1 \Rightarrow \alpha\rho_j 1$ *and* $\alpha\rho_i 0 \Rightarrow \alpha\rho_j 0$.

Proof. Define the *depth* of a term, t, or formula, α, to be the length of the longest chain of nested ε-terms it contains. (So if α contains no ε-terms, its depth is 0, and the depth of $\varepsilon x P x$ is 1.) The proof is by induction on depth. Suppose the result holds for all terms and formulas of depth $< n$.

For (i): The terms of depth n are made up from constants and ε-terms of depth n, by the application of function symbols. The result is obvious if t is a constant. Let t be an ε-term of depth n. If $\delta_i(\varepsilon x\alpha) \neq \infty$, then $\overline{\alpha_i} \neq \phi$, and $\delta_i(\varepsilon x\alpha) = \Phi_i^\alpha \overline{\alpha_i} = \Phi_k^\alpha \overline{\alpha_k}$, where k is the least ordinal less than i, such that $\overline{\alpha_k} \neq \phi$. But since the depth of α is less than n, $\overline{\alpha_i} \subseteq \overline{\alpha_j}$, by induction hypothesis. It follows that $\overline{\alpha_j} \neq \phi$, and so that $\delta_j(\varepsilon x\alpha) = \Phi_j^\alpha \overline{\alpha_j} = \Phi_k^\alpha \overline{\alpha_k}$, as required. Since the interpretations of all function symbols are the same in A_i and A_j, the result follows.

For (ii): the formulas of depth n are generated from atomic formulas of depth n by means of connectives and quantifiers. Since these behave monotonically, it suffices to show the result for atomic sentences. If $s = t\rho_i 1$ then $\delta_i(s) = n \neq \infty$, $\delta_i(t) = m \neq \infty$, and $\langle n, m\rangle \in =^+$. But s and t have depth $\leq n$; so by induction hypothesis and (i): $\delta_j(s) = n$, $\delta_j(t) = m$; hence, $s = t\rho_j 1$. The case for 0 is trivial, since $s = t\rho_j 0$ regardless of whether $\delta_j(s)$ and $\delta_j(t)$ are defined. If $\Delta st\rho_i 1$ then $\delta_i(s) = n \neq \infty$, $\delta_i(t) = m \neq \infty$, and $\langle n, m\rangle \in \Delta_i^+$. Hence, $\delta_j(s) = n$, $\delta_j(t) = m$, by induction hypothesis and (i); and $\langle n, m\rangle \in \Delta_j^+$ (by Lemma 1); so $\Delta st\rho_j 1$. The case for 0 is, again, trivial. □

LEMMA 3. *For all i, if* $\overline{\alpha_i} \neq \phi$, *then* $\Phi_i^\alpha \overline{\alpha_i} \in \overline{\alpha_i}$.

Proof. The proof is by transfinite induction on i. If $\Phi_i^\alpha = \Phi_0^\alpha$, the result follows. So suppose that α has at most one free variable, and for some $j < i$, $\overline{\alpha_j} \neq \phi$. Then $\Phi_i^\alpha \overline{\alpha_i} = \Phi_j^\alpha \overline{\alpha_j}$, for the least such j. By induction hypothesis, $\Phi_j^\alpha \overline{\alpha_j} \in \overline{\alpha_j} \subseteq \overline{\alpha_i}$, by Lemma 2(ii). □

Lemma 3 assures us that each A_i is a well defined interpretation. By Lemma 1 and the usual cardinality considerations, there must be some ordinal, $*$, such that $\Delta_*^+ = \Delta_{*+1}^+$. A_* is our required interpretation. It not difficult to check that it verifies *Denotation*.

Note, first, that since $\langle t\rangle$ is a numeral $\delta_*(\langle t\rangle)$ is always defined. Let m be the code of t. Then $\delta_*(\langle t\rangle) = m$. Thus:

$$\begin{aligned}
\Delta\langle t\rangle s\rho_* 1 &\Rightarrow \delta_*(s) = n \text{ and } \langle m, n\rangle \in \Delta_*^+ \\
&\Rightarrow \delta_*(s) = n \text{ and } t = \mathbf{n}\rho_i 1 \text{ for some } i < * \\
&\Rightarrow \delta_*(s) = n \text{ and } t = \mathbf{n}\rho_* 1 \quad \text{by Lemma 2(ii)} \\
&\Rightarrow t = s\rho_* 1.
\end{aligned}$$

Conversely:

$$\begin{aligned} t = s\rho_* 1 &\Rightarrow \delta_*(s) = n \neq \infty \text{ and } t = \mathbf{n}\rho_* 1 \\ &\Rightarrow \delta_*(s) = n \text{ and } \langle m, n \rangle \in \Delta^+_{*+1} \\ &\Rightarrow \delta_*(s) = n \text{ and } \langle m, n \rangle \in \Delta^+_* \\ &\Rightarrow \Delta\langle t \rangle s\rho_* 1. \end{aligned}$$

Finally, it is not difficult to see that everything true in the standard model of arithmetic is true in A_*. For as far as the purely arithmetic language (i.e., the language not containing Δ or ε) goes, if B is the standard model of arithmetic $B \preceq A_*$. The result follows by monotonicity.[15]

Thus, the set of sentences true in A_* is a free *LP* theory with descriptions, containing arithmetic, and closed under *Denotation*. But, as is clear, $\mathbf{0} = \mathbf{1}$ is not true in A_*. Hence, this theory is non-trivial.

6. Conclusions

Let me finish with a few observations about the proof. The core of the construction is that required to obtain the monotonicity lemma, Lemma 2. Two devices secure this. One guarantees that the denotation of an ε-term, $\varepsilon x\alpha$, once obtained, remains fixed. This is secured by letting $\Phi^\alpha\overline{\alpha}$ be an arbitrary member of $\overline{\alpha}$, the first time that this set becomes non-empty, and then persisting with this selection for subsequent $\Phi^\alpha\overline{\alpha}$. It is this that requires the failure of extensionality for descriptions. For $\overline{\alpha}$ and $\overline{\beta}$ may come to be identical, by which time their selected members are already fixed and different.

The other device ensures that truth values are never lost as we ascend the ordinals. This is secured by requiring the antiextension of both $=$ and Δ to be D^2. Hence, atomic sentences that start off as false due to a non-denoting term, continue to be so, even once the term obtains a denotation (though they may then become true as well). A consequence of this is that the non-triviality result is a fairly crude one, in a certain sense; namely, it does not show that any negated atomic sentences is not provable in the theory. Hopefully, a more refined proof is possible to show that this is not the case.

The second comment concerns a detachable conditional connective, $\rightarrow$. The language of the theory proved non-trivial above does not contain such a connective. A natural question is, therefore: can the non-triviality proof be extended to one for a language which does, and in which *Denotation* is formulated as a biconditional? There are techniques, due to Brady, which give such an extension in the description-free case.[16] I think it likely that they can be applied in the present case too, though I have not worked through the details.

One consequence of such an extension of the proof would be the following. In the language with a conditional connective, one may define definite description terms, $\iota x\varphi(x)$, as: $\varepsilon x(\varphi(x) \wedge \forall y(\varphi(y) \rightarrow x = y)$. It is clear that at most one thing can satisfy the condition of such a description. Such terms are therefore, perforce, extensional.[17] And in a language whose only descriptive terms are ι-terms, terms would not have the more unusual features of non-extensional descriptions that I noted in Section 4.[18]

NOTES

[1] All this is documented in Priest (199+), Section 8.

[2] A non-triviality proof for a semantically closed theory with descriptions is given in Priest (1998). As is pointed out there (Section 8), though, this construction does not provide everything that might be desired. In particular, the logic of identity in the theory is highly non-standard, failing transitivity of identity.

[3] In Priest (1997), the argument is formulated in terms of a denotation *function*, and *Denotation* is formulated as a biconditional. But this is an inessential difference, as Priest (1998) shows. In particular, a denotation function, $\delta(x)$, can be defined as $\varepsilon y\Delta xy$, where ε is an appropriate description operator.

[4] With classical logic, allowing for denotation failure is of no help. This is because, even if some descriptions fail to denote, one can define a kind of description that always does so ('a thing that satisfies α, if there is such a thing, or 0 otherwise'). The above argument can be run with this kind of description. This version of the argument does use the disjunctive syllogism, however, and so fails with a paraconsistent logic. See Priest (1997).

[5] See, e.g., Priest (1987), Ch. 5.

[6] This claim is defended in Priest (1979).

[7] Rejecting $t = t$ is not mandatory for blocking this result, though. We might endorse $t = t$, and rely on the failure of existential generalisation in free logic to block it.

[8] See Priest (1998), Section 6.4.

[9] This is essentially the theory of descriptions given in Priest (1979).

[10] This is argued further in Priest (1979).

[11] It is also worth noting that universal instantiation (and its dual, existential generalisation) may also break down if quantifying into the scope of an ε-term, even when the term being substituted denotes. For example, it is easy enough to construct an interpretation in which $\forall x(\mathbf{e} = \varepsilon yPyx)$ is true: for every $d \in D$, simply arrange for $\Phi^{Px\mathbf{d}}\overline{Px\mathbf{d}}$ to be e. If t is any other term, however, $\Phi^{Pxt}\overline{Pxt}$ may well be distinct from e.

[12] As can be seen by checking the details in Priest (1998), Section 3.

[13] A truth predicate can be defined in terms of Δ and ε. See Priest (1998), Section 6.

[14] If terms fail to denote, one might want to restrict *Denotation* itself to those terms, t, that denote; but this is not necessary in the present approach.

[15] It is easy to check that the model also verifies the inference $\exists x\alpha \vdash \varepsilon x\alpha = \varepsilon x\alpha$. It quickly follows that it also verifies the inference $\exists x\alpha \vdash \exists y\Delta\langle\varepsilon x\alpha\rangle y$. This inference is

used in the extended version of the triviality argument described in fn. 4. See Priest (1997), Section 7.

[16] See Priest (199+), 8.2.

[17] Note that terms of the form $\iota x \Delta \langle t \rangle x$, would always have a denotation if t does, due to (the conditional form of) *Denotation*.

[18] I would like to thank a referee of the journal for some helpful comments on the presentation of the non-triviality proof.

REFERENCES

Priest, G. (1979): Indefinite descriptions, *Logique et Anal.* **85**(6): 5–21.

Priest, G. (1987): *In Contradiction*, Martinus Nijhoff, the Hague.

Priest, G. (1997): On a Paradox of Hilbert and Bernays, *J. Philos. Logic* **26**: 45–56.

Priest, G. (1998): The trivial object, and the non-triviality of a semantically closed theory with descriptions, *J. Appl. Non-Classical Logics* **8**: 171–83.

Priest, G. (199+): Paraconsistent logic. In: D. Gabbay and F. Guenthner (eds.), *Handbook of Philosophical Logic*, 2nd. edn, Kluwer Acad. Publ., Dordrecht, forthcoming.

The University of Queensland,
Philosophy Department,
Brisbane QLD 4072,
Australia.
E-mail: ggp@ants.uq.edu.au