On a version of one of Zeno's paradoxes

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I want to discuss a version of one of Zeno's paradoxes. It is a particularly ingenious version given by José Benardete, which seems to have gone largely unnoticed. The paradox can be put in many ways. Here is a particularly striking version of it, in Benardete's own words:

> A man decides to walk one mile from A to B. A god waits in readiness to throw up a wall blocking the man's further advance when the man has travelled 1/2 mile. A second god (unbeknown to the first) waits in readiness to throw up a wall of his own blocking the man's further advance when the man has travelled 1/4 mile. A third god ... &c. *ad infinitum*. It is clear that this infinite sequence of mere intentions (assuming the contrary-to-fact conditional that each god would succeed in executing his intentions if given the opportunity) logically entails the consequence that the man will be arrested at point A; he will not be able to pass beyond it, even though not a single wall will in fact be thrown down in his path. The ... [effect] will be described by the man as a strange field of force blocking has passage forward.[1]

Note that Benardete's paradox is not susceptible to resolutions of the kind standardly directed to Zenonian paradoxes of this kind. These point out that one can, in fact, do an infinite number of things in a finite time, provided only that the time interval between the actions decreases in a suitable way. Such an observation would seem to be completely irrelevant here.

I don't think that Benardete has wrung out the full force of the paradox, though. In the rest of this article I want to do just that. To nail it down, I will formalize it. Suppose that an object, a, (the man) is sliding in from $-\infty$ (left to right) along a frictionless plane (or as near friction-free as makes no difference). The 'force field' begins at the point A, $x = 0$. Before a reaches that point there are no obstacles to motion of any kind on the plane.

The formalization piggvy-backs on the theory of real numbers, and uses the additional predicates: Rx, 'the (leading edge) of a reaches point x'; Bx, 'a barrier (wall) is created at point x whilst a is to its left'.

Three general features of motion are given by the following conditions:

(1) $Rx \;\&\; y < x \rightarrow Ry$

[1] Bernadete, J. *Infinity: an Essay in Metaphysics*. Oxford: Clarendon Press, 1964, p. 259.

ANALYSIS 59.1, January 1999, pp. 1–2. © Graham Priest

(2) $By \mathbin{\&} y<x \rightarrow \neg Rx$
(3) $\neg \exists x(x<y \mathbin{\&} Bx) \rightarrow Ry$

(1) says that the motion is continuous. The object a cannot reach any point unless it traverses every point to its left. (2) just spells out the meaning of 'barrier'. If a barrier is created between where the object is and some point to its right, the object will never reach there. (3) is a version of Newton's first law of motion. An object will travel in a straight line unless impeded by something. In particular, a will reach any point to its right unless some barrier prevents it.

The final postulates we need characterize the condition concerning barriers:

(4) $x \leq 0 \rightarrow \neg Bx$
(5) $x>0 \rightarrow (Bx \leftrightarrow Rx/2)$

(4) says that there the area before $x=0$ is barrier-free. (5) says that to the right of $x=0$, a barrier will be created at point x if, but only if, the object reaches point $x/2$. (The barriers can be created by a god, or in any other suitable way.)

Given (1) to (5) we can infer that the a will advance no further than $x=0$. For suppose that $x>0$ and that Rx. Then since $x/4<x$, $Rx/4$, by (1). But then $Bx/2$, by (5) (since $x/2>0$). But $x/2<x$. Hence $\neg Rx$ by (2). Hence, by reductio, $x>0 \rightarrow \neg Rx$. The object a stops at A.

But then, no barriers are created: $x>0 \rightarrow x/2>0 \rightarrow \neg Rx/2 \rightarrow \neg Bx$ by (5). Hence, by (4), $\forall x \neg Bx$. Whence, for any y, $\forall x(x<y \rightarrow \neg By)$, i.e., $\neg \exists x(x<y \mathbin{\&} By)$. Thus by (3), Ry for every y. In particular, $y>0 \rightarrow Ry$: a advances to any point beyond A.

(1) to (5) are inconsistent. Now (1) to (3) are features of continuous motion, and are true in this world. (4) and (5) are not, presumably, true in this world. There are no parts of the world where passing certain spots brings barriers spontaneously into existence. But there appears to be no reason why there should not be areas like this. For example, there could be a demon. The demon is bound by the laws of physics, and so, in particular, cannot suspend the laws of motion (1) to (3). Yet it might 'mine' an area of space in accordance with (4) and (5). Alternatively, to use Bernadete's example, there might be an infinitude of such demons, each of which has the appropriate intention. In such a world, motion produces contradiction. Otherwise put: in our world, motion could be contradictory.[2]

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[2] Thanks to Mike Tooley for pointing out Bernadete's argument out to me, and for enjoyable discussions on the subject.