The *Catuṣkoṭi*, the *Saptabhaṇgī* and "Non-Classical" Logic

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Abstract

The Principles of Excluded Middle and Non-Contradiction are highly orthodox in Western philosophy. They are much less so in Indian philosophy. Indeed there are logical/metaphysical positions that clearly violate them. One of these is the Buddhist *catuṣkoṭi;* another is the Jain *saptabhaṇgī.* Contemporary Western logicians have, however, investigated systems of "non-classical" logic in which these principles fail, and some of these bear important relationships to the *catuṣkoṭi* and the *saptabhaṇgī.* In this essay, we will look at these two principles, and see how these may inform and be informed by those systems.

Key words: *catuṣkoṭi*, *saptabhaṇgī,* Buddhism, Jainism, Nāgārjuna, Principle of Excluded Middle, Principle of Non-Contradiction, many-valued logic, plurivalent logic, *FDE*, *LP*, K_3.

1 Introduction

Aristotle enunciated and defended two important principles: the Principle of Excluded Middle (PEM), and the Principle of Non-Contradiction (PNC), which may be expressed as follows:

- PEM: every statement is either true or false
- PNC: no statement is both true and false

He does this in the *Metaphysics*, not the *Analytics*, which is where we find Aristotle's logical writings. However, these two principles have been cornerstones of Western logic ever since. True, there have been some who have balked at them. Oddly enough, Aristotle himself, in the somewhat notorious chapter 9 of *De Interpretatione*, at least appears to reject the PEM. And, though the interpretation is contentions, Hegel appears to reject the PNC in his *Logic*. However, those who have problematised the principles are lone historical voices.

It is fair to say that the PEM and PNC are still orthodox in contemporary logic. However, in the 20th Century, some Western logicians have certainly challenged these principles. (See, e.g., Priest (2008: 7.6-7.9).) Indeed, the century saw the development of formal (mathematical) logics which reject these principles: certain kinds of "non-classical" logics. These logics and their properties are now well understood. Nearly everything that Aristotle argued for has been rejected—or at least serious challenged in the two millenia since he wrote. Why the orthodoxy of Aristotle's views concerning the PEM and the PNC has lasted so long, is an interesting question, which we must leave for historians to ponder.

Matters in India are notably different. There have been defenders of the PEM and PNC, such as logicians in the (Hindu) Nyāyā School, and the Buddhist logicians Dignāga and his successor Dharmakīrti (fl. c. 6th c. CE). However, logical/metaphysical thinking which rejects both of these principles is much older. We find this in, for example, the Buddhist *catuṣkoṭi* and the Jain *saptabhaṇgī*. Of course, how to understand these ideas is a contentious scholarly matter. Moreover, these thinkers did not have the resources of contemporary mathematical logic at their disposal. However, there are contemporary mathematical logics which naturally do justice to such ideas—though those who invented them did not do so in response to anything in Indian thought.

In this essay, we will look at the *catuṣkoṭi* and *saptabhaṇgī*, and the formal logics in question, seeing how this meeting of minds works. Putting these two things together can benefit both. It shows that the Indian thinking can be put on a rigorous mathematical basis, and so give the lie to anyone who would take such thinking to be confused or irrational. Conversely, the Indian ideas can show that the logical systems are no mere formalism, but can be seen as encoding profound metaphysical views of the world.

2 Buddhism and the *Catuṣkoṭi*

2.1 Early Denials of the PEM and PNC

But let us start with some denials of the PEM and PNC in Indian thought which predates Buddhism and Jainism. The earliest Vedic text, the *Ṛg Veda* (? 1500-1200 BCE) contains what appear to be denials of the PEM. Thus, in describing the origins of the cosmos, it says (Koller and Koller (1991: 6)):

> There was neither non-existence nor existence then; there was neither the realm of space nor the sky which is beyond. What stirred? Where? In whose protection? Was there water, bottomless deep?

The denial is also to be found in one of the most famous parts of Hindu philosophy: *neti, neti* (literally: *not*, *not*), meaning *not this, not this*, or *neither this nor that.* Thus, in the *Bṛhadāraṇyaka Upaniṣad* (? c. 700 BCE) we read (Koller and Koller (1991: 22)):

> This Self is simply described as "Not, not". It is ungraspable. For it is not grasped. It is indestructible, for it is not destroyed. It has no attachment and is unfastened; it is not attached, and yet it is not unsteady. For it, immortal, passes beyond both these two states (in which one thinks) "For this reason I have done evil," "For this reason I have done good." It is not disturbed by good or evil things that are done or left undone; its heaven is not lost by any deed.

The passage is most naturally read as saying that one must reject all claims about the Self: it is neither this nor not this. So we have a denial of the PEM. But at the same time, it *does* endorse claims about the Self, for example, that it is immortal. So we have a denial of the PNC, at least implicitly.

A denial of the PNC also appears to have been found explicitly in the writings of the Hindu Ājīvika sect which flourished for a while after about the 5th Century BCE. Their texts are now lost, but in Abhayadeva's commentary on the *Samavayāṅga-Sūtra*, we find (Jayatilleke (1963: 155)):

> These Ājivikas are called Trairāṣikas. Why? The reason is that they entertain (*icchanti*) everything to be of a triple nature, viz. soul, non-soul, soul and non-soul; world, non-world, world and non-world; being, non-being, being and non-being, etc. Even in (*api*) considering standpoints they entertain a three-fold standpoint such as the substantial, the modal and the dual.

Finally the philosopher Sañjaya (dates uncertain, but probably 6th or 5th century BCE) was also known for denying the PEM and PNC—indeed for denying that anything is true, false, both, or neither, as a way of rejecting all views. (See Raju (1956: 694).)

Clearly, what is behind these various views are very distinctive metaphysical positions, to the effect that reality, or at least, parts of it, are either under- or over- determined. However, this is not the place to go into these matters.

2.2 The *Catuṣkoṭi*

With this background, let us now turn to the Buddhist *catuṣkoṭi* (four corners/points). According to this, a claim can be true, false, both, or neither. These are the four *koṭis* in question. So the principle is something like a "principle of excluded fifth".

That all four of these possibilities could be in play must have been a commonplace view by the time of the historical Buddha Siddhārtha Gautama (*Pāli*: Gotama) (fl. c. 6th c. BCE), since it appears to be taken for granted in a number of the *sūtras*—though there seems to be no connection between the third and fourth *koṭis* and specific Buddhist doctrines at this point.

Thus, in the *Agivacchagotta Sutta* we find the following (Ñāṇamoli and Bodhi (1995: 591)). Note that a *Tathāgata*—literally, (*one*) *thus gone*—is someone who has achieved enlightenment:

> "How is it, Master Gotama, does Master Gotama hold the view: 'After death a Tathāgata exists: only this is true, anything else is wrong'?"
>
> "Vaccha, I do not hold the view: 'After death a Tathāgata exists: only this is true, anything else is wrong.' "
>
> "How then, does Master Gotama hold the view: 'After death a Tathāgata does not exist: only this is true, anything else is wrong'?

> ” “Vaccha, I do not hold the view: ‘After death a Tathāgata does not exist: only this is true, anything else is wrong.’ ”
>
> “How is it, Master Gotama, does Master Gotama hold the view: ‘After death a Tathāgata both exists and does not exist: only this is true, anything else is wrong.’ ?”
>
> “Vaccha, I do not hold the view: ‘After death a Tathāgata both exists and does not exist: only this is true, anything else is wrong.’ ”
>
> “How then, does Master Gotama hold the view: ‘After death a Tathāgata neither exists nor does not exist: only this is true, anything else is wrong’ ?”
>
> “Vaccha, I do not hold the view: ‘After death a Tathagata neither exists nor does not exist: only this is true, anything else is wrong.’ ”

Vaccha asks about the status of an enlightened person after death, and runs through the four possibilities of the *catuṣkoṭi*. And both he and the Buddha take the partition for granted. Neither does the Buddha say ‘Don’t be silly, Vaccha. It makes no sense for something to be both true and false or neither true nor false’. Neither does he say ‘Aha, Vaccha, you are missing another possibility’. So they both seem to accept that this partition is exclusive and exhaustive.

It is true that the Buddha refuses to endorse any or the four *koṭis*. Why he does so is moot. Some *sūtras* have the Buddha continuing by saying something like: ‘Look, I’m telling you how to improve your life. You shouldn’t be worrying about all this metaphysical nonsense’. Others have him saying that none of the four possibilities ‘fits the case’. What this means is less than clear.

Some commentators (e.g., Siderits and Katsura (2013: 302)) suggest that since the person no longer exists we have a case of reference failure. The problem with this is that standard accounts of reference failure do not take us outside the *catuṣkoṭi*. Thus, for example, Frege takes such sentences to be neither true nor false, and Russell takes them to be simply false. These views are built into so called free logics of different kinds. (On these matters, see Priest (2008), 7.8 and ch. 13.)

Whatever the Buddha meant, the refusal to endorse any of the four *koṭis* certainly prefigures the later Buddhist view that there is a fifth possibility.

We will come to this in due course.

2.3 First Degree Entailment

So much for the Buddhist *catuṣkoṭi*. Let us now turn to an appropriate formal logic. This is a logic called ‘First Degree Entailment’ (FDE)—don’t ask.

Modern logic avoids all the irrelevant complexities of a natural language by dealing with formal languages: languages with a regular grammars and no ambiguity. A relation of logical consequence is defined on sentences of such a language. Let us stick with a simple propositional language. (It is easy to extend the techniques to more complex languages.) The simplest sentences of the language are called *propositional parameters*, and more complex sentences are constructed from these in an iterative process using sentence-connectives. In FDE, these are $\wedge$, $\vee$, and $\neg$. You can think of them as meaning ‘and’ (conjunction), ‘or’ (disjunction), and ‘it is not the case that’ (negation), respectively.

A standard way of defining a consequence relation is by giving the language a semantics. A semantics for FDE can be set up in a number of different ways. Here I describe a 4-valued semantics, where the connection with the *catuṣkoṭi* is at its most obvious. (See Priest (2008: ch. 8).) An interpretation for the language specifies values for every propositional parameter. In a 2-valued semantics there are just: *true and true only*, and *false and false only*. We can write these as t and f, respectively. In FDE, there are two more: *both true and false* and *neither true nor false*. We can write these as b and n, respectively. The four values form a structure that mathemticians call a lattice. A lattice can be depicted in the form of a diagram called a Hasse diagram. The diagram for the lattice we are dealing with here is as follows, and is often called the *Diamond Lattice*:

$$\begin{array}{ccccc} & & t & & \\ & \nearrow & & \nwarrow & \\ b & & & & n \\ & \nwarrow & & \nearrow & \\ & & f & & \end{array}$$

The *catuṣkoṭi* is evident.

Given an assignment of values to the propositional parameters, this is extented to an assignment of values to all sentences of the language recursively, by the following conditions.

If A has the value t, $\neg A$ has the value f, and vice versa. b and n are "fixed points" for negation. That is, if A has the value b, so does $\neg A$; and if A has the value n, so does $\neg A$.

The value of a conjunction is the greatest lower bound of the values of the conjuncts; that is, the greatest thing less than or equal to the values of both conjuncts. What that means is that one goes down the arrows of the Diamond Lattice till one gets to the first thing that is less that or equal to both of them. Thus:

- If A has the value t and B has the value b, $A \wedge B$ has the value b
- If A has the value b and B has the value n, $A \wedge B$ has the value f

For disjunctions one just goes *up* the arrows instead, giving the least upper bound. So:

- If A has the value t and B has the value b, $A \vee B$ has the value t
- If A has the value b and B has the value n, $A \vee B$ has the value t

Finally, the definition of validity, which we may write as $\models$: In a many-valued logic some of the values are said to be *designated*. (One may think of these as the values that represent some kind of truth.) An inference is *invalid* if it is possible that (i.e., there is an interpretation in which) the premises have a designated value and the conclusion does not. It is valid if it is not invalid. That is, whenever all the premises have a designated value, so does the conclusion. In FDE the designated values are t and b (since these are the two ways in which something can be true).

It is a relatively simply matter to determine whether particular inferences are valid or invalid. This can safely be left to the reader. I'll give some examples of valid inferences in the next section.

Let us just note that we do *not* have the following:

- $A \models B \vee \neg B$
- $A \wedge \neg A \models B$

The second inference is usually, now, called *Explosion*. The first has no standard name, but symmetry suggests *Implosion*.

For the failure of Implosion, take A to have the value t, and B to be have the value n—in which case, $B \vee \neg B$ has the value n. For the failure of Explosion, take B to have the value f, and A to be have the value b—in which case, $A \wedge \neg A$ has the value b.

Clearly, these two inferences are versions of the PEM and the PNC respectively. The first says that you have $B \vee \neg B$ *whatever else you have*. The second says that if you have $A \wedge \neg A$ you will have everything, including crazy things such as ‘the Moon is a cube’, ‘$1^2 = 17$’, etc. *So you can’t have* $A \wedge \neg A$. (One might think that the PNC would be expressed by the inference $A \models \neg(B \wedge \neg B)$, but it is not. If in an interpretation the value of B is b then $\neg(B \wedge \neg B)$ holds in the interpretation. Indeed $(B \wedge \neg B) \wedge \neg(B \wedge \neg B)$ holds!)

2.4 A Proof-Theoretic Characterisation

This relation of logical consequence can also be characterised by a set of rules of inference. (See Priest (2019).) FDE is characterised by (that is, is sound and complete with respect to) the following rules:

$$\frac{A \quad B}{A \wedge B} \qquad \frac{A \wedge B}{A} \qquad \frac{A \wedge B}{B}$$

$$\frac{A}{A \vee B} \qquad \frac{B}{A \vee B} \qquad \frac{A \vee B \quad \begin{array}{c}\overline{A}\\ \vdots \\ C\end{array} \quad \begin{array}{c}\overline{B}\\ \vdots \\ C\end{array}}{C}$$

$$\frac{A}{\overline{\neg\neg A}} \qquad \frac{\neg(A \vee B)}{\overline{\neg A \wedge \neg B}} \qquad \frac{\neg(A \wedge B)}{\overline{\neg A \vee \neg B}}$$

Premises are above the line; conclusions are below; a double line indicates that an inference goes both ways; and a line above a formula means that the rule discharges this assumption. That is, the final conclusion does not depend on this assumption. Thus, in the third rule for disjunction, A is used to deduce C, and B is used to deduce C. We then infer C depending on the premise $A \vee B$, but not on A and B themselves.

2.5 Nāgārjuna and The Buddha’s Silence

Let us return to the Buddha’s silence. This was picked up by Nāgārjuna (fl. 1st or 2nd c. CE). His text, the *Mulamadhyamakakārikā* (MMK, The

Fundamental Verses of the Middle Way) was the foundational text of all later (Mahāyāna) Buddhisms. (See Garfield (1995), Priest (2013), Siderits and Katsura (2013).)

In this, Nāgārjuna argues that there is nothing which exists with intrinsic nature. That is, everything is empty (*śūnya*) of *svabhāva* (self-being/nature). The *catuṣkoṭi* plays an important role in this. The arguments often start by assuming that something or other has *svabhāva*. It then divides the matter up into the four cases of the *catuṣkoṭi*, and shows that none of them can hold. Hence we have a four-way *reductio* of the assumption. The argument is one of *reductio ad absurdum*, not *reductio ad contradictionem*. It cannot be *ad contradictionem* because the third *koṭi* is one that explicitly allows for the possibility of a contradiction. But there are many things that are more absurd than some contradictions. The claim that you are a frog is more absurd than that the liar sentence ('this sentence is false') is both true and false. And Nāgārjuna has to show only that there is some consequence of the assumption that is absurd—or maybe just unacceptable to his opponents, since many of the argument are *ad hominem*.

Anyway, at MMK XXII: 11-12, Nāgārjuna picks up the Buddha's silence concerning the status of the enlightened person (*Tathāgata*) after death. There we have the following (translations from the MMK are from Garfield (1995)); note that the *catuṣkoṭi* is referred to by its Greek translation, *tetralemma*):

> 'Empty' should not be asserted.
>
> 'Nonempty' should not be asserted.
>
> Neither both nor neither should be asserted.
>
> They are used only nominally.
>
> How can the tetralemma of permanent and impermanent, etc.,
>
> Be true of the peaceful?
>
> How can the tetralemma of the finite, infinite, etc.,
>
> Be true of the peaceful?

Given that the *catuṣkoṭi* is an enumeration of all the things that can be said, the implication would appear to be that nothing can be said about the status of the *Tathāgata*. The state of affairs concerning the *Tathāgata* is simply ineffable.

In fact, we may apply the machinery of the *catuṣkoṭi* to states of affairs themselves. To do this, we must think of sentences, not as expressing propositions, but as referring to states of affairs. For each (possible) state of affairs, A, there is a corresponding negative state of affairs, $\neg A$. (So, corresponding the the state of affairs that the *Tathāgata* exists is the state of affairs that the *Tathāgata* does not exist.) Similarly, if A and B are states of affairs, there are conjunctive and disjunctive states of affairs $A \wedge B$ and $A \vee B$.

Now, states of affairs are not the kind of thing that are true or false, but the kind of thing that exist or do not. So we must now think of the four values of the *catuṣkoṭi* as follows:

- the value of A is t: A exists and $\neg A$ does not exist
- the value of A is f: A does not exist and $\neg A$ exists
- the value of A is b: both A and $\neg A$ exist
- the value of A is n: neither A nor $\neg A$ exists

And now we have a fifth possibility, ineffability. Call this value e. Clearly, it must be distinct from the others, since if we can say that A exists or does not, we can say something about it, and so it cannot be ineffable.

How does the value e work? It would seem that if A is ineffable so is $\neg A$, and so are $A \vee B$ and $A \wedge B$. So if the value of A is e, so is the value of anything of which it is a part.

The designated values are those that preserve existence, t and b. So e is not designated. (See Priest (2018), ch. 5.)

Hence, we have a 5-valued logic, which we may call $FDEe$. A system of rules of proof which are sound and complete with respect to these semantics is obtained by replacing the rules for $\vee$-introduction with the rules of weak $\vee$-introduction:

$$\frac{A \quad B^\dagger}{A \vee B} \qquad\qquad \frac{A^\dagger \quad B}{A \vee B}$$

where $C^\dagger$ is any formula which contains all the propositional parameters of C. (See Priest (2019).)

2.6 Paradox

We are not quite finished with the c*atuṣkoṭ*i yet. In Buddhist philosophy there is a standard distinction between conventional reality *(saṃvṛiti-satya*) and ultimate reality (*paramārtha-satya*). ('Satya' may be translated at *truth* or as *reality*. The former is the more usual translation.) Conventional reality is the world we are familiar with, our *Lebenswelt*. Ultimate reality is the way that things actually are behind these appearances. What, exactly, this is, is a contentious point of Buddhist philosophy; and Nāgārjuna is less than explicit on what he takes it to be, but at MMK XXII: 16a, b he tells us that:

> Whatever is the essence of the Tathāgata
>
> This is the essence of the world.

And it is clear that 'the world' is a reference to ultimate reality. It would seem, then, that this also is ineffable. This point is made explicitly at MMK XVIII: 9:

> Not dependent on another, peaceful and
>
> Not fabricated by mental fabrication,
>
> Not thought, without distinction.
>
> That is the character of reality.

Ultimate reality is beyond conceptualisation (without distinction); it is conventional reality that is created (fabricated) by conceptualisation. Indeed, that ultimate reality is ineffable is a common view in many later Buddhist schools, such as Yogācāra and Chan (Zen). (See Priest (2014b).)

But if ultimate reality is ineffable, there is an obvious issue. Nāgārjuna is himself talking about ultimate reality and so conceptualising it—as are those who follow him in this matter. So this reality would seem to be effable and ineffable.

The point troubled many Buddhist philosophers after the PNC had generally come to be accepted. Thus, take, for example, the Tibetan Māhāyana philosopher Gorampa (1429-1489). He is as clear as his Māhāyana predecessors that the ultimate is ineffable. He says in his *Synopsis of Madhyamaka* v. 75 (quoted in Kassor (2013)):

> The scriptures which negate proliferations of the four extremes [cf. of the *catuṣkoṭki*] refer to ultimate truth but not to the conventional, because the ultimate is devoid of conceptual proliferations, and the conventional is endowed with them.

But he also realizes that he is talking about it. Indeed, he does so in this very quote. Gorampa's response to the situation is to draw a distinction. Kassor describes matters thus (2013: 406):

> In the *Synopsis*, Gorampa divides ultimate truth into two: the nominal ultimate (*don dam rnam grags pa*) and the ultimate truth (*don dam bden pa*). While the ultimate truth... is free from conceptual proliferations, existing beyond the limits of thought, the nominal ultimate is simply a conceptual description of what the ultimate is like. Whenever ordinary persons talk about or conceptualize the ultimate, Gorampa argues that they are actually referring to the nominal ultimate. We cannot think or talk about the actual ultimate truth because it is beyond thoughts and language; any statement or thought about the ultimate is necessarily conceptual, and is, therefore, the nominal ultimate.

It does not take long to see that this hardly avoids contradiction. If all talk of the ultimate is about the nominal ultimate, then Gorampa's own talk of the ultimate is about the nominal ultimate. Since the nominal ultimate is effable, Gorampa's own claim that the ultimate is devoid of conceptual proliferations is just false.

So what does Nāgārjuna himself say about the matter? Nothing. Why? We can only conjecture. It cannot be because he failed to notice the situation. It would stare in the face of a much lesser philosopher. However, Nāgārjuna is working in the context of the *catuṣkoṭki*, the third *koṭki* of which explicitly allows for contradictions to be true. Perhaps, more wisely than some of his successors, he simply took the situation to provide a counterexample to the PNC. (See, further, Priest (2018).)

3 Jainism and the *Saptabhaṇgī*

3.1 The *Anekānta-Vāda*

Let us now move from Buddhism to Jainism. This is traditionally taken to have been founded by Mahāvīra (fl. 5th or 6th c. BCE), a rough contemporary of the Buddha.

Jainism has a very distinctive metaphysics, captured in the doctrine of *anekānta-vāda*—non-onesidededness, as it is sometimes translated. The

Jains believed that truth was not the prerogative of any one school. The views of Buddhists and Hindus, for example, may disagree about crucial matters, such as the existence of an individual soul; each has, nonetheless, an element of truth in it. This can be so because reality itself is multi-faceted. (See Ganeri (2001: 5.2).)

Reality is a complex, with a multitude of aspects; and each of the competing views provides a perspective, or standpoint (*naya*), which latches on to one such aspect. As Siddhasena (fl. 5th or 6th c. CE) puts it in the *Nyāyāvatāra*, v. 29 (Matilal (1981: 41)):

> Since a thing has manifold character, it is comprehended (only) by the omniscient. But a thing becomes the subject matter of a *naya*, when it is conceived from one particular standpoint.

On its own, each standpoint is right enough, but incomplete. To grasp the complete picture, if indeed this is possible, one needs to have all the perspectives together—like seeing a cube from all six sides at once.

It follows that any statement to the effect that reality is thus and such, if taken categorically, will be, if not false, then certainly misleading. Better to express the view with an explicit reminder that it is correct from a certain perspective. This was the function with which Jain logicians employed the word '*syāt*'. In the vernacular, this means something like 'it may be that', 'perhaps', or 'arguably'; but in the technical sense in which the Jain logicians used it, it may be best thought of as something like 'In a certain way...' or 'From a certain perspective...'. (Matilal (1981: 52), Ganeri (2001: 5.5).) So instead of saying 'An individual soul exists', it is better to say '*Syāt* an individual soul exists'. This is the Jain method of *syād-vāda.*

It is worth nothing that, according to Buddhism, reality is also multi-faceted in a certain sense. As we have already seen, it is standard in Buddhist philosophy that reality has both a conventional aspect and an ultimate aspect. However, it is generally agreed that ultimate reality, as the name suggests, is what is really there; and conventional reality is, in some sense, less real. Hence this is quite different from the Jain view that the different aspects of reality are equally real. Moreover, there was no attempt to aggregate these two realities into a compound picture as, we are about to see, the Jain *saptabhaṇgī* does.

3.2 … and the *Saptabhaṇgī*

Given this background, we can now understand the Jain *saptabhaṇgī* (sevenfold division). A sentence may have one of seven truth values; or, as it may be put, there are seven predicates that may describe its semantic status. The matter is explained by the 12th century theorist, Vādideva Sūri, in *Pramāṅa-naya-tattvālokālaṁkāra*, ch. 4, vv. 15-21 (Battacharya (1967)):

> The seven predicate theory consists in the use of seven claims about sentences, each preceded by 'arguably' or 'conditionally' (*syāt*) [all] concerning a single object and its particular properties, composed of assertions and denials, either simultaneously or successively, and without contradiction. They are as follows:
>
> (1) Arguably, it (i.e., some object) exists (*syād esty eva*). The first predicate pertains to an assertion.
>
> (2) Arguably, it does not exist (*syād nāsty eva*). The second predicate pertains to a denial.
>
> (3) Arguably, it exists; arguably it does not exist (*syād esty eva syād nāsty eva*). The third predicate pertains to successive assertion and denial.
>
> (4) Arguably, it is non-assertable (*syād avaktavyam eva*). The fourth predicate pertains to a simultaneous assertion and denial.
>
> (5) Arguably, it exists; arguably it is non-assertable (*syād esty eva syād avaktavyam eva*). The fifth predicate pertains to an assertion and a simultaneous assertion and denial.
>
> (6) Arguably, it does not exist; arguably it is non-assertable (*syād nāsty eva syād avaktavyam eva*). The sixth predicate pertains to a denial and a simultaneous assertion and denial.
>
> (7) Arguably, it exists; arguably it doesn't exist; arguably it is non-assertable (*syād esty eva syād nāsty eva syād avaktavyam eva*). The seventh predicate pertains to a successive assertion and denial and a simultaneous assertion and denial.

A perusal of the seven possibilities indicates that there are three basic ones, (1), (2), and (4), and that the others are compounded from these. (1) says that the statement in question (that something exists) holds from a certain perspective. (2) says that from a certain perspective, it does not. (4)

says that from a certain perspective, it has another status, non-assertable. Exactly what this is, is less than clear. We will return to the matter in a moment. Let us call these three values t, f, and i, respectively.

In understanding the other possibilities we hit a *prima facie* problem. Take (3). This says that from some perspective the sentence is t, and from some perspective it is f. That's intelligible enough, but unfortunately, it would seem to entail both (1) and (2). If it's true from some perspective and false from some perspective, it's certainly true from some perspective.

The solution is straightforward, however. We have to understand (1) as saying not just that the sentence is true from some perspective, but as denying the other two basic possibilities: it is t from some perspective, and there are no perspectives from which it is f or i. (3) is now to the effect that there is a perspective from which the sentence is t, a perspective from which it is f, and no perspective from which it is i. In fact, all the seven cases now fall into place. Thus understood, each of the three basic possibilities may hold of fail—except that they cannot *all* fail, since there must be at last one perspective. Hence there are $2^3 - 1 = 7$ values.

But what are we to make of the value i? A natural possibility is that i means *both true and false*. That is essentially how Vādideva Sūri glosses case (4) in the quotation above. Unfortunately, he also glosses i as *unassertable*. So the status of i is more like *neither true nor false*. Which is the most plausible interpretation of i in Jain logic, all things considered, is a moot point. Stcherbatsky (1962: 415), Bharucha and Kamat (1984), and Sarkar (1992) argue that i is most plausibly interpreted as *both true and false*. Ganeri (2002: sect. 1 and 2001: 5.6) favours *neither true nor false*.

We may leave scholars to debate the matter. In what follows, we will take both possibilities into account

3.3 K_3 and LP

We may now turn to matters of contemporary formal logic. Concentrate, first, on the basic logic with three values, t, i, f. If i means *both true and false*, we can think of this as the value b of the FDE semantics. And we then get an appropriate 3-valued logic simply by ignoring the value n in the 4-valued semantics. This gives a logic known as LP. Alternatively, if i as means *neither true nor false*, we may think of it as the value n. We then obtain an appropriate 3-valued logic simply by ignoring the value b (both as a value and as a designated value) in the 4-valued semantics. This is a logic known as K_3, usually referred to as *strong Kleene*. (See Priest (2008: ch. 7).) We might therefore picture the two logics thus. *LP* is on the left; K_3 is on the right:

$$\begin{array}{ccccc} t & & & & t \\ \uparrow & & & & \uparrow \\ b & = & i & = & n \\ \uparrow & & & & \uparrow \\ f & & & & f \end{array}$$

Notice that as far as the two lattices go, there is now nothing to distinguish between b and n. (The lattices are isomorphic.) The difference between the two logics lies only in the fact that b is designated, and n is not.

Proof theoretic characterisations of these two logics can be obtained using the rules:

$$\frac{.}{B \vee \neg B} \qquad\qquad \frac{A \wedge \neg A}{B}$$

(The first rule means that $B \vee \neg B$ can always be added as a line in a deduction. If it is an assumption it is immediately discharged.) Adding the first to the rules of *FDE* gives the logic *LP*, which validates the PEM. Adding the second to the rules of *FDE* gives the logic K_3, which validates the PNC. This is exactly as one would expect, since the first uses the value b, but rules out the value n; and the second uses the value n, but rules out the value b. Adding both rules to those for *FDE*—ruling out both b and n—delivers a system of rules for classical logic. (See Priest (2019).)

3.4 Plurivalent Logic

Now, as we have seen, the *saptabhaṇgī* allows statements to take any combination of our three basic values (except the combination with none of them). Formally, this can be handled with a construction called *plurivalient logic*. (See Priest (2014a).) In a plurivalent logic, sentences may have one or more of the available values. In the present case, they may have any number of the values t, i, and f—except none of them. (Technically speaking, then, an assignment of values is not a *function*, but a one-many *relation*—or equivalently, a function whose values are non-empty subsets of $\{t, i, f\}$.)

An interpretation assigns some values to propositional parameters, and the values of compound formulas are then determined recursively by computing all possible combinations. (That is, the values are computed point-wise, as mathematicians say.)

Thus, suppose that A has the values t and f, and B has the values t and i. We compute the result of combining these under the rules for LP or K_3. (As noted, it does not matter whether one thinks of i as b or n.) We might draw the result in the following table (the values of A are in the left hand column; the values of B are in the top row):

$\wedge$	t	i
t	t	i
f	f	f

As the matrix values show, the possible values for $A \wedge B$ are t, i, and f. (f occurs twice, but that is irrelevant.) So these are the values assigned to $A \wedge B$. (If we draw up a table of this kind, there will be from one to three columns and from one to three rows, depending on how many values each formula has.)

We do the same thing for disjunction. Thus, suppose that A has the values t and f, and B has the values t and i. The result of combing these is shown in the following table:

$\vee$	t	i
t	t	t
f	t	i

The possible values for $A \vee B$ are t, and i. So these are what $A \vee B$ is assigned.

Tho compute the values of $\neg A$, we simply negate all the values of A. Thus, suppose that A has the values t and i. The result of negating these is shown by the following table:

A	t	i
$\neg A$	f	i

Hence, $\neg A$ has the values f and i.

The definition of validity is now given in a natural way. Say that a formula is designated under the new regime if at least one of its values is designated under the old. That is: one of its values is t or, if i is b, i. Then an inference is valid in the plurivalent logic if whenever all the premises are designated in this sense, so is the conclusion.

The consequence relation for plurivalent LP is the same as that of LP itself. Hence it is characterised by the same set of rules. The consequence relation for plurivalent K_3 is, in fact, the same as FDE, and so is characterised by the rules for this. (See Priest (2014a.))

Notice, then, that even though K_3 validates Explosion, plurivalent K_3 does not. To see this, suppose that A has the values t and f, and that B has just the value f. Then $\neg A$ has the values f and t (that is, the same as A; the order does not matter). And $A \wedge \neg A$ has the values t and f. So $A \wedge \neg A$ is designated, but B is not.

I note that one could equally apply the plurivalent construction to the four values of FDE or the five values of $FDEe$. (Though I know of no Indian texts which suggest or countenance this.) In the first case, we obtain a $2^4 - 1$, that is, 15-valued logic. In the second case, we obtain a $2^5 - 1$, that is, 31-valued logic! Plurivalent FDE is characterised by the same set of rules as those of FDE itself; and Plurivalent $FDEe$ is characterised by the same set of rules as those of $FDEe$. (See Priest (2014a).)

4 Conclusion

We have now looked at the Buddhist *catuṣkoṭi*, the Jain *saptabhaṇgī,* how these work, and what underlies them. We have also seen how the ideas can built into some contemporary non-classical logics. This makes it clear that the *catuṣkoṭi* rejects the PNC and the PEM. If i is n the *saptabhaṇgī* rejects the PEM and the PNC; if i is b it rejects only the PNC.

Of course, using the techniques of contemporary logic to interrogate Ancient Indian texts is anachronistic. But the anachronism is not a pernicious one. Contemporary logicians, in fact, do exactly the same to Ancient and Medieval Western texts. Thus, if one browses the issues of the journal *History and Philosophy of Logic*, one will find many examples of this. Here are a few more. Versions of Ontological Argument for the existence of God have been given by a number of philosophers, including Anselm, Descartes, and Leibniz. These arguments are often analysed with the tools of modern logic. (See many of the papers in Oppy (2018).) Another: there have been many attempts to analyse Hegel's dialectics using the techniques of modern logic. (See many of he papers in Marconi (1979).) Finally, one can find analyses of views of, amongst others, Berkeley, Kant, and Hegel, which employ the tools of contemporary logic in Priest (1995).

Moreover, it is clearly sensible to investigate something using the tools one has at one's disposal, even if they were not available at the time when the thing to which the tools are applied was proposed/discovered. In biology is silly not to use a microscope if one has one available. In logic, it is silly not to use the tools of modern mathematics if they are available.

At any rate, what we have seen concerning the PEM and PNC is this. These principles have been high orthodoxy in Western logic/philosophy. However, the principles were being challenged by Indian thinkers at the same time as (or just before) Aristotle was fixing them into orthodoxy in the West. Contemporary Western logicians have now cast doubt on both the PEM and the PNC. In particular, they have constructed systems of logic in which they fail. Moreover, some of these systems—constructed in ignorance of the relevant parts of Indian thought—provide just what is needed for a rigorous development of these profound and Ancient Indian ideas.

5 Appendix: Technical and Historical Details Concerning Some Paraconsistent and Paracomplete Logics

A logic where Explosion fails is called *paraconsistent*. A logic where Implosion fails is often now called *paracomplete*. Classical logic is neither paraconsistent nor paracomplete. Not all paraconsistent and paracomplete logics are many-valued logics. In this appendix I will discuss a few points of technical and historical interest concerning some that are.

5.1 FDE

The logic FDE is the core of a family of logics called *relevant logics*. It is both paraconsistent and paracomplete. It was invented/discovered by the US logicians A. R. Anderson and N. D. Belnap in (1962). The main concern of relevant logic is that if A entails B, A should be relevant to B. If a logic satisfies Explosion or Implosion, this is obviously not the case. The 4-valued semantics was invented/discovered a little later, by J. M. Dunn. (For discussion and references, see Anderson and Belnap (1975), ch. 3.)

One way of setting up the semantics of FDE is as follows. The language contains a set of propositional parameters, P, and the connectives, $\wedge$, $\vee$, $\neg$. An interpretation is a binary relation $\rho \subseteq P \times \{0, 1\}$. Given an interpretation, truth and falsity are assigned independently to all formulas as follows. $\Vdash^+ A$ means that A is true; $\Vdash^- A$ means that A is false. If $p \in P$:

- $\Vdash^+ p$ iff $p\rho 1$
- $\Vdash^- p$ iff $p\rho 0$

Then:

- $\Vdash^+ \neg A$ iff $\Vdash^- A$
- $\Vdash^- \neg A$ iff $\Vdash^+ A$
- $\Vdash^+ A \wedge B$ iff $\Vdash^+ A$ and $\Vdash^+ B$
- $\Vdash^- A \wedge B$ iff $\Vdash^- A$ or $\Vdash^- B$
- $\Vdash^+ A \vee B$ iff $\Vdash^+ A$ or $\Vdash^+ B$
- $\Vdash^- A \vee B$ iff $\Vdash^- A$ and $\Vdash^- B$

If Σ is a set of formulas, then $\Sigma \models A$ iff for all ρ : if $\Vdash^+ B$, for all $B \in \Sigma$, then $\Vdash^+ A$.

We may define a conditional, $A \supset B$, as usual in classical logic. As is easy to check, both $\models A \supset A$ and $A, A \supset B \models B$ fail—the first, because A may have the value n; the second, because A may have the value b. Full relevant logics add a new conditional, $\rightarrow$, to the language, and give it an appropriate (and more complex) semantics. These inferences hold for $\rightarrow$.

As is clear, the relational FDE truth/falsity conditions are exactly those of classical logic (though, in the case of classical logic, the falsity conditions are redundant). The definition of validity is also exactly the same as that of classical logic. If an interpretation is a total function (that is, it relates every p to exactly one member of $\{0, 1\}$), then it is an interpretation of classical logic. Hence, FDE expands the possibilities (interpretations) countenanced by classical logic.

Given a relational FDE interpretation, there are obviously four possibilities for a formula, A:

- $\Vdash^+ A$ and $\not\Vdash^+ A$
- $\not\Vdash^+ A$ and $\Vdash^- A$

- $\Vdash^+ A$ and $\Vdash^- A$
- $\nVdash^+ A$ and $\nVdash^- A$

If we write these four possibilities as t, f, b, and n, then the relational truth/falsity conditions deliver the Diamond Lattice and its operators, as may easily be checked. And relational validity is equivalent to preserving the values t and b. Hence, the relational semantics and the 4-valued semantics are equivalent. (See Priest (2008), 8.4.)

The system $FDEe$ was introduced in Priest (2018), and, unlike the other logics mentioned in this essay, *was* motivated by Buddhist considerations.

5.2 K_3 and B_3

If in the relational semantics one requires that for no p, $p\rho 1$ and $p\rho 0$ then, as is easy to check, this is so for all formulas. The semantics is then one for the logic K_3. K_3 is paracomplete, but not paraconsistent. As indicated, the 3-valued version of the semantics is obtained by taking the right-hand side of the Diamond Lattice of 2.3.

K_3 was invented/discovered by the US mathematician S. C. Kleene in (1938). (See also his book (1952: §64).) Kleene was concerned with partial recursive functions. The value of such a function may not be defined. Hence, if f is such a function the equation $f(i) = j$ may be neither true nor false. Hence, Kleene calls the value n 'undefined'.

If we replace the value n by the value e of 2.5, the resulting logic is often called 'weak Kleene logic', but it is better called Bochvar Logic (B_3), since it was invented by the Russian logician D. A. Bochvar in a paper in Russian in 1938. (An English translation appears as Bochvar and Bergmann (1981).) Like K_3, B_3 is paracomplete, but not paraconsistent. Bockhvar interprets the value e as *nonsense*. (So, for the connectives: nonsense in, nonsense out.) As the title of the paper indicates, he takes sentences involved in paradoxes of self-reference, such as that involved in Russell's paradox, $\{x : x \notin x\} \in \{x : x \notin x\}$, to be nonsensical.

A rule system that is sound and complete with respect to B_3 can be obtained from that of K_3 by removing $\vee$-introduction, and adding:

$$\frac{A^\dagger}{A \vee \neg A}$$

(Recall that $A^\dagger$ is any formula containing all the propositional parameters of A.) See Priest (2019).

5.3 LP and H_3

If in the relational semantics one requires that, for every p, $p\rho 1$ or $p\rho 0$, then, as is easy to check, this holds for all formulas. The semantics is then one for the logic LP. LP is paraconsistent, but not paracomplete. As indicated, the 3-valued version of the semantics is obtained by taking the left-hand side of the Diamond Lattice of 2.3.

The logic LP was invented/discovered by G. Priest (1979). Like Bochvar, he thought of the value b as applying to paradoxical sentences. (He calls the value *paradoxical*.) But unlike Bochvar, he read it as *both true and false*—and so as a species of truth.

To round out the picture: B_3 may equally be obtained from LP by replacing the value b with e—since e is is not designated. However, if we then take e to be designated we obtain a logic usually now often called 'Paraconsistent Weak Kleene', though it would be better called Halldén logic (H_3), since it was invented/discovered by the Swedish logician Sören Halldén in (1949). Like LP, H_3 is paraconsistent, but not paracomplete. As the title of Halldén's work indicates, he interprets the middle value as *nonsensical*, like Bochvar. Why he takes the value to be designated is somewhat opaque, however.

A sound and complete system of rules for H_3 can be obtained by taking the rules for LP, deleting the rule for $\wedge$-elimination, and replacing it with:

$$\frac{A \quad \neg A}{A^\dagger} \qquad \frac{A \wedge B}{A \vee B^\dagger} \qquad \frac{A \wedge B}{A^\dagger \vee B}$$

See Priest (2019).

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6 Definitions of Key Terms

- Principle of Excluded Middle: a logical principle according to which statements are either true or false.
- Principle of Non-Contradiction: a logical principle according to which statements are not both true and false.
- *Catuṣkoṭi*: a logical/metaphysical principle deployed by Buddhist philosophers, according to which statements may be true, false, both, or neither.
- *Saptabhaṇgī*: a logical/metaphysical principle deployed by Jain philosophers, according to which statements may be true, false, or "non-assertible", or any combination of the three.
- *Anekānta-Vāda*: a Jain principle according to which reality is multi-faceted.
- Many-valued logic: a logic in which statements may take one of more than two values.
- Plurivalent logic: a logic in which statements may take more than one of the available values (at the same time).
- FDE: A 4-valued logic in which the values are: *true*, *false*, *both*, and *neither*.
- LP: a 3-valued logic in which the values are: *true*, *false*, and *both*.
- K_3: a 3-valued logic in which the values are: *true*, *false*, and *neither*.
- Explosion: the inference $A \wedge \neg A \vdash B$.
- Implosion: the inference $A \vdash B \vee \neg B$.
- Paraconsistent logic: a logic in Explosion is not valid.
- Paracomplete logic: a logic in which Implosion is not valid.

7 Summary Points

- The Principle of Excluded Middle (PEM) and the Principle of Non-Contradiction (PNC) are highly orthodox in Western philosophy.
- However, they have been rejected by some important Indian philosophical traditions.
- Buddhist philosophers deploy a logical/metaphysical principle called the *catuṣkoṭi*, according to which statements may be true, false, both, or neither.
- Jain philosophers deploy a logical/metaphysical principle called the *saptabhaṇgī,* according to which statements can be true, false, or "non-assertible"—sometimes interpreted as both truth and false; sometimes interpreted as neither true nor false—or any combination of the three.
- Contemporary logicians have investigated systems of logic in which both the PEM and the PNC fail.
- One of these is the system FDE, a system of many-valued logic, which is based on the four possibilities of the *catuṣkoṭi*.
- The technique of plurivalent logic allows statements to have more than one value (at the same time). This can be used to construct systems which encode the ideas of the *saptabhaṇgī*.
- The invention of these logics had nothing to do with Indian philosophy. However, putting the Indian ideas together with the modern logical constructions can benefit both.
- The formal logic shows how the Indian ideas can be put on a rigorous mathematical basis. Conversely, the Indian ideas can show that the logical systems are no mere formalisms, but can be seen as encoding profound metaphysical views of the world.