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MULTIPLE DENOTATION, AMBIGUITY, AND THE STRANGE CASE OF THE MISSING AMOEBA

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The paper falls into two parts. Taking off from some insights in the semantics of paraconsistent logic, the first part sets up the formal semantic machinery for a language in which predicates and the terms to which they are applied may have multiple denotations. The second part discusses some aspects of the philosophical import of the semantics, including their relationship with the notion of ambiguity, conceptual fission, and their application to solve a puzzle of Prior concerning the identity of objects that split into two.

1. *Formal Considerations*

1.1. *Classical Semantics*

In formulating the semantics of the classical propositional calculus, it is standard to take an interpretation, v, to be a function that assigns to each sentence exactly one of the values 1 (*true*) or 0 (*false*). v may be thought of as defined on propositional parameters in the first instance; it is then extended to one defined on all formulas by recursive clauses. Let $\oplus$, $\otimes$, and $\ominus$ be the usual truth functions for disjunction, conjunction and negation, respectively. (Thus, $1 \oplus 1=1$, $\ominus 1=0$, etc.) Then:

$$v(\neg\alpha) = \ominus v(\alpha)$$
$$v(\alpha \wedge \beta) = v(\alpha) \otimes v(\beta)$$
$$v(\alpha \vee \beta) = v(\alpha) \oplus v(\beta)$$

Validity is defined in terms of truth-preservation. That is, $\Sigma \models \alpha$ iff for every interpretation, v, if $v(\sigma)=1$ for all $\sigma \in \Sigma$ then $v(\alpha)=1$.

To extend the semantics to a first order language, we take an interpretation to be a pair $\langle D, d \rangle$ where D is a non-empty domain, and d is a function such that for each constant, c, $d(c) \in D$, and for each n-place predicate, $P, d(P) \subseteq D^n$. (To keep things simple, we assume that there are no function symbols.) Truth values are now assigned relative to an evaluation of the free variables, s (that is, a map from the variables into D). For atomic formulas:

$$v_s(Pt_1 \ldots t_n) = 1 \text{ iff } \langle f(t_1), \ldots, f(t_n) \rangle \in d(P)$$
$$v_s(Pt_1 \ldots t_n) = 0 \text{ iff } \langle f(t_1), \ldots, f(t_n) \rangle \notin d(P)$$

where $f(t_i)$ is $d(t_i)$ if t_i is a constant, and $s(t_i)$ if t_i is a variable. Truth values are assigned to other sentences by the same recursive clauses for the connectives as in the propositional case (relativised to s). For the quantifiers, if we represent the natural generalisations of $\oplus$ and $\otimes$ by the same signs (so that $\oplus X{=}1$ iff $1 \in X$, etc.), then the truth conditions of the quantifiers are as follows:

$$v_s(\forall v\alpha) = \otimes\left\{v_{s(v/a)}(\alpha); a \in D\right\}$$
$$v_s(\exists v\alpha) = \oplus\left\{v_{s(v/a)}(\alpha); a \in D\right\}$$

where $s(v/a)$ is that evaluation of the variables that is the same as s, except that its value at v is a. Validity is defined in terms of truth preservation with respect to all interpretations and evaluations of the free variables.

1.2. *LP Semantics*

Over recent years, some logicians have suggested that certain sentences, notably, paradoxical ones, may have more than one truth value. Classical semantics are easily modified to accommodate this possibility. Instead of taking an evaluation to be a function, we take it to be a relation, R, between formulas and truth values, such that each formula relates to at least one truth value. In the propositional case, the relation may be taken as defined on atomic formulas in the first instance. It can then be extended to all formulas according to the following simple idea: the truth values of a compound are exactly those that can be obtained by combining the truth values of its components in all possible ways. More precisely:

$$R(\neg\alpha, z) \text{ iff } \exists x(R(a, x) \& z = \ominus x)$$
$$R(\alpha \wedge \beta, z) \text{ iff } \exists x \exists y(R(a, x) \& R(\beta, y) \& z = x \otimes y)$$
$$R(\alpha \vee \beta, z) \text{ iff } \exists x \exists y(R(a, x) \& R(\beta, y) \& z = x \oplus y)$$

Validity is defined in terms of truth preservation:

$$\Sigma \models \alpha \text{ iff for all } \not{r}, \text{ iff } R(\beta, 1) \text{ for all } \beta \in \Sigma \text{ then } R(\alpha, 1)$$

For the first-order case, we now take an interpretation to be a pair, $\langle D, d \rangle$, where D is a non-empty domain of objects, d assigns each constant a member of the domain, and each n-place predicate, P, a pair $\langle E_P, A_P \rangle$ (the extension and anti-extension of P, respectively), such that $E_P \cup A_P = D^n$. The relation between formulas and truth values is now relative to an evaluation of the free variables, s. In the atomic case, this is defined by the clauses:

$$R_s(Pt_1\ldots t_n, 1) \text{ iff } \langle f(t_1), \ldots, f(t_n)\rangle \in E_P$$
$$R_s(Pt_1\ldots t_n, 0) \text{ iff } \langle f(t_1), \ldots, f(t_n)\rangle \in A_P$$

Truth values (relative to s) are then extended to all sentences. The recursive clauses for the connectives are the same as in the propositional case (relativised to s). For the quantifiers, if F is a map from D into $\{0,1\}$ such that $R_{s(v/a)}(\alpha, F(a))$ for all $a \in D$, let us say that F *tracks* α. Then:

$$R_s(\forall v\alpha, z) \text{ iff } \exists F(F \text{ tracks } \alpha \,\&\, z = \otimes\{F(a); a \in D\})$$
$$R_s(\exists v\alpha, z) \text{ iff } \exists F(F \text{ tracks } \alpha \,\&\, z = \oplus\{F(a); a \in D\})$$

Validity is defined in terms of truth preservation in all interpretations and evaluations of the variables, as usual.

It is easy to check that the recursive truth conditions may be put in an equivalent, but slightly more familiar, form as follows:

$$R_s(\neg\alpha, 1) \text{ iff } R_s(\alpha, 0)$$
$$R_s(\neg\alpha, 0) \text{ iff } R_s(\alpha, 1)$$

$$R_s(\alpha \wedge \beta, 1) \text{ iff } R_s(\alpha, 1) \text{ and } R_s(\beta, 1)$$
$$R_s(\alpha \wedge \beta, 0) \text{ iff } R_s(\alpha, 0) \text{ or } R_s(\beta, 0)$$

$$R_s(\alpha \vee \beta, 1) \text{ iff } R_s(\alpha, 1) \text{ or } R_s(\beta, 1)$$
$$R_s(\alpha \vee \beta, 0) \text{ iff } R_s(\alpha, 0) \text{ and } R_s(\beta, 0)$$

$$R_s(\forall \upsilon\alpha, 1) \text{ iff for every } a \in D,\ R_{s(\upsilon/a)}(\alpha, 1)$$
$$R_s(\forall \upsilon\alpha, 0) \text{ iff for some } a \in D,\ R_{s(\upsilon/a)}(\alpha, 0)$$

$$R_s(\exists \upsilon\alpha, 1) \text{ iff for some } a \in D,\ R_{s(\upsilon/a)}(\alpha, 1)$$
$$R_s(\exists \upsilon\alpha, 0) \text{ iff for every } a \in D,\ R_{s(\upsilon/a)}(\alpha, 0)$$

The above semantics are one version of the semantics for the paraconsistent logic *LP*.[1] If the interpretation of a predicate, *P*, is such that $E_P \cap A_P = \phi$, I will call it *classical*. As is easy to see, if all the predicates of an interpretation are classical, then the interpretation is essentially a functional interpretation of classical logic. Hence, *LP* is a sub-logic of classical logic. It is, however, a proper sub-logic, since it is paraconsistent. I note, without proof, that the logical truths of *LP* are exactly those of classical logic.[2]

To extend the machinery to identity is not difficult. We simply require identity to be a logical (semi-)constant: $E_=$ is always $\{\langle x, x\rangle; x \in D\}$. ($A_=$ can be anything, except, of course, that $E_= \cup A_= = D^2$.) The logical truths in the language with identity are, again, exactly the same as those of classical logic.[3] I note also that the semantics verify the standard substitutivity principle for identity: $a = b, \alpha(a) \models \alpha(b)$.

1.3. *Multiple Denotation*

There is a certain asymmetry in the preceding semantics.[4] The semantics of classical logic take both evaluation and denotation to be functions; so the appropriate value of a sentence, constant or predicate is unique. *LP* allows for multiple truth values, but retains the functionality of denotation. Yet the thought that constants, say, may have multiple denotations is at least as natural as the claim that sentences may have multiple truth values. Hence a natural thought is to treat denotation in the same way that *LP* treats truth values. Let us take constants first.

For these, we take denotation, *d*, to be a relation between constants and members of the domain, *D*, such that each constant relates to at least one object.[5] For uniformity of notation, we now also assume that evaluations of the variables are relations between variables and the domain, but we maintain the necessity for each variable to relate to exactly one member of D. By analogy with the case of truth values, the appropriate truth conditions for atomic sentences are now:

[1] They are formulated this way in Priest (1984). Note that if we allow the possibility that sentences may relate to no truth value as well, but leave everything else the same, we get a logic with truth value gaps satisfying the Fregean principle: gap-in/gap-out. In particular, then, the alternative truth conditions just given are no longer equivalent. If we preserve these truth conditions instead, we obtain First Degree Entailment.

[2] See Priest (1987), ch. 5.

[3] Ibid.

[4] This was pointed out to me by someone a long time ago. I forget now who it was.

[5] If we relax the restriction and allow a constant to relate to no member of *D*, we generate a free logic.

$R_s(Pt_1...t_n, 1)$ iff for $1 \leq i \leq n$, there are x_i such that $f(t_i, x_i)$ and $\langle x_1, ...x_n \rangle \in E_P$

$R_s(Pt_1...t_n, 0)$ iff for $1 \leq i \leq n$,, there are x_i such that $f(t_i, x_i)$ and $\langle x_1, ...x_n \rangle \in A_P$

where *f* is either *d* or *s*, as appropriate). That is, an atomic sentence is true (false) iff there are *some* denotations of its terms that make it so. The rest is as before.

Clearly, an atomic sentence, say *Pc*, may now have multiple truth values if *c* has multiple denotations, even if *P* is classical. Multiple denotation therefore requires a logic of multiple truth values, even if all predicates are classical. Multiple denotations do not automatically give multiple truth values, however; if *P*, is classical, and all the denotations of *c* agree with respect to *P*, then *Pc* has a single truth value.

The logic of these semantics is a sub-logic of *LP*. (Any *LP* interpretation can be seen as a special case of these semantics, one where denotation relates each constant to exactly one object.) In fact, it is a proper sub-logic. For example, the rule of Existential Generalisation fails in them (though not in LP). Consider an interpretation in which 'a' denotes two distinct things, but in which the identity predicate is classical. Then $a \neq a$ is true (and false), but $\exists x x \neq x$ is not true.[6]

The logic of identity for multiple denotation is also weaker than that of *LP* . Though the semantics verify the identity law, *t=t*, they do not verify the law of substitutivity: $a = b, \alpha(a) \models \alpha(b)$. An easy way to see this is to consider one notable case of substitutivity, transitivity:

$$a = b, b = c \models a = c$$

Suppose that *x*, and *y* are distinct members of the domain of an interpretation, and *d* is such that $d(a, x)$, $d(b, x)$, $d(b, y)$, and $d(c, y)$. Then the premises are true (and false as well) in the interpretation, but the conclusion is not true.

Although substitutivity fails in this form, it does hold in the form:

[6] If we allow evaluations of the variables to relate variables to more than one thing, Existential Generalisation holds. But now the logic is not a sub-logic of *LP*. For example, these semantics (though not *LP*) validate $\models \forall x \forall y\, x = y \vee \exists x\, x \neq x$. (The first disjunct holds in any interpretation with only one member; the second in any with more.)

$$\models (a = b \wedge \alpha(a)) \supset \alpha(b)$$

where $\beta \supset \gamma$ is $\neg\beta \vee \gamma$. If a and b have unique denotations, then $\alpha(a)$ and $\alpha(b)$ have the same truth values, and the material conditional is true. If either of a and b has multiple denotations, $a=b$ is false, making the whole conditional true. More generally, the logically valid sentences of the language with identity coincide with those of *LP* (and so of classical logic), though I shall not prove this here.[7]

This deals with multiple denotations of constants. Multiple denotations for predicates can be dealt with very simply. We now take d to be also a relation between predicates and extension/anti-extension pairs, such that each predicate relates to at least one such pair. The natural truth/falsity conditions for atomic formulas are now:

$R_s(Pt_1 \ldots t_n, 1)$ iff for some $\langle \text{E, A} \rangle$, $\text{d}(\text{P}, \langle \text{E, A} \rangle)$, and for $1 \le i \le n$, there are $x_i \in D$, such that $f(t_i, x_i)$, and $\langle x_1, \ldots, x_n \rangle \in E$

$R_s(Pt_1 \ldots t_n, 0)$ iff for some $\langle \text{E, A} \rangle$, $\text{d}(\text{P}, \langle \text{E, A} \rangle)$, and for $1 \le i \le n$, there are $x_i \in D$, such that $f(t_i, x_i)$, and $\langle x_1, \ldots, x_n \rangle \in A$

This produces no further novelties. Indeed, as the truth conditions make clear, any interpretation where a predicate, P, has multiple denotations, $\langle E_i, A_i \rangle$, $i \in I$, is equivalent to one where it has a single extension and anti-extension: $\bigcup\{E_i; i \in I\}$ and $\bigcup\{A_i; i \in I\}$, respectively —even if the predicate is identity. Thus, multiple denotations of predicates occasions no change in the logic.

Note, though, that if we insist that every denotation of a predicate is classical, then interpretations where predicates have multiple denotations will not have single-denotation equivalents. (If each predicate in the family I is classical, this will not, in general, be the case for their unions.)

Summary: the preceding semantics provide for constants and predicates of the language to have multiple denotations. Their logic is weaker than *LP*, though the propositional parts coincide. A notable failure of the logic is the substitutivity of identicals.

[7] Any *LP* countermodel can be seen as a relational countermodel. Conversely, take any relational countermodel, and consider the *LP* interpretation formed by selecting *one* of the denotations for its *LP* denotation. This obviously does not increase the truth values of atomic sentences. A simple induction shows that this is, in fact, true for all sentences. Hence the interpretation is an *LP* countermodel.

2. *Philosophical Considerations*

2.1. *Predicates with Multiple Denotation*

So much for the formal details. In the second part of this essay I want to discuss some of the philosophical issues that a logic of multiple denotation raises.[8] In particular, I want to address the question of whether a semantics of the kind given is ever appropriate for a natural language. The answer, I take it, is 'yes', and I will give some examples where this is plausibly the case.

Let us start with predicates with multiple denotations. The first thing that might occur to one here, is where a predicate has different denotations in virtue of different senses. Thus, 'is a bat' has one extension composed of animals and one composed of implements for hitting balls. Such examples are not very interesting, however. When someone uses such a word, they normally have in mind one particular sense, and this serves to fix a unique denotation.

More interesting is when a predicate has a unique sense, but still multiple referents. Are there such things? Well, if we start, in the first place, by assuming that predicates need not be classical, then as we saw in 1.3, nothing can force us into this conclusion. If, however, we are to take predicates to be classical, there may be such grounds. One case where this may plausibly happen concerns multi-criterial predicates.[9] There is more than one criterion (sufficient condition) for applying a predicate such as 'is x^0 absolute'; e.g., measurement by a correctly functioning alcohol thermometer, electrochemical thermometer, or by the frequency of black-body radiation emitted. And if the world behaves in an unkind fashion, these may well determine inconsistent results. The fact that a predicate has multiple criteria of application, does not imply that it has different senses. (For example, the fact that we have different criteria for applying numerical size to people in a small room and molecules in a volume of gas, does not imply that these quantifications have different senses.) But if, as seems plausible, we take the criteria each to determine different classical extensions of the predicate, we have a case of multiple denotation. Doubtless, if it is ever discovered that the criteria determine different answers, the old predicate is likely to be discarded in favour of new ones whose senses (and denotations) correspond to the different criteria. However, for the old predicate, the multiple denotation semantics are correct.

[8] As we have seen, the semantics can be modified to give one of denotation failure as well. The philosophical issues here —at least in the case of constants— are well known. What it is for a predicate to suffer denotation failure (as opposed to having an empty extension and anti-extension) is not at all clear.

[9] See Priest and Routley (1989), 2.2.1.

Cases of a related kind are given by Field (1973), who argues that there actually have been cases where a scientific notion comes to be seen as really a confusion of two distinct notions. His main example is that of mass. He argues that though there is nothing that answers to the Newtonian notion of mass, the Newtonian use of the word may reasonable be taken (with hindsight) as applying to two distinct notions, rest mass (which is frame independent) and inertial mass (which is not)—both of which, it is reasonable to assume, are classical. It would be wrong to think that the Newtonian predicate 'x is the mass of y' had more than one sense. It was just that its sense failed to pick out a unique referent. As it happened, there were two equally good candidates, and it picked out both indiscriminately.[10]

Field gives an account of the semantics of such language different from the one given here; essentially, it is a supervaluational one. He defines a notion of truth/falsity with respect to each referential disambiguation. Truth (falsity) *simpliciter* is then truth (falsity) on every disambiguation. Thus, multiple denotation occasions truth value gaps, not gluts. I have no knockdown arguments against Field's proposal, but a multiple denotation semantics strikes me as more plausible. The simple reason is that Field's account patently mishandles the situation. The truth values of sentences with multiple denotations are, in a very clear sense, *over-determined*, not under-determined.[11] Compare this with a case of denotation failure, which is, plausibly, a case of under-determination. It is true that a *unique* truth value is under-determined; but that is quite a different matter. In many ways, the situation is similar to that concerning the sentences 'this sentence is true' and 'this sentence is false'. Intuitively, the truth value of the former is under-determined; that of the latter is over-determined.[12]

[10] There are other cases of conceptual fissions in science, e.g., that concerning the notion of angular size in the Seventeenth Century. See Priest (1987), p. 250f.

[11] As Field himself seems to note, *op. cit.*, p. 476.

[12] Field's semantics naturally give rise to a supervaluational account of validity in terms of the preservation of truth in some (or all) disambiguations, which gives classical logic. Lewis (1982) objects to this notion of validity in the present context, on the ground that since we cannot know that a sentence is ambiguous, we cannot know that the premises are true in a particular (or every) disambiguation. Hence, we would never be in a position to apply an inference. I do not think that this objection works. We might not know that we are in such a position, but we can certainly *assume* it: specifically, we can assume that our language is univocal, and thus that we are entitled to classical logic. The assumption may also be warranted by the fact that it engenders correct predictions. It may also turn out to be false, and may have to be revised. But in both these ways it is no different from any other scientific hypothesis.

It might replied that the semantics advocated here will make certain sentences true that are not true on any single disambiguation. For example, if there are two disambiguations, and α is true on only one of them, and β on only the other, then $\alpha \wedge \beta$ will be true on neither disambiguation, whilst it will be true on the present semantics. The point is correct, but not a telling one. Even when ambiguity is generated by ambiguous senses, we are quite happy to accept a sentence as true when different tokens of the same word have to be disambiguated differently. Consider: When playing his shot, the cricketer missed the ball, but hit the bat that flew past him with his bat.

2.2. *Singular Terms with Multiple Denotations*

Let us now turn to singular terms, which we can divide up into proper names and descriptions. The case of descriptions is very similar to that concerning predicates. In particular, there would seem to be cases where a description with a unique sense may pick out multiple referents, as in, e.g., 'the author of *Isiah*'—it has turned out, it would seem, that *Isiah* was written by more than one person. There is another obvious option here, though. This is to apply a theory such as Russell's theory of definite descriptions. If the sense of a description fails to single out a unique denotation, we may take (broad scope) sentences containing it to be false—or at least truth-valueless. Again, I have no knock-down arguments against such an approach, but there are reasons which suggest that the multiple-denotation approach is better. It seems natural to suppose, for example, that the claim that the author of *Isiah* was Jewish is (simply) true, not false (or neither true nor false).

Turning to proper names, the situation might appear to be somewhat different. Proper names having multiple bearers are commonplace. Just think of the English name 'John Smith', for example. This hardly seems very interesting, however. As with predicates with different senses, when we talk about some John Smith or other, we usually have a particular person in mind, which fixes a unique referent. Cannot proper names with multiple referents always be disambiguated in such a way? Plausibly, no.

Suppose that we have an amoeba. Call it a. At some time, t, it divides down the middle to form two new amoebas, b and c. After t, where is a? If it exists at all, it must be either b or c: it has not transmigrated elsewhere. But by symmetry, if it is b it is also c and vice versa. Hence either it has gone out of existence, or it is both b and c. Now it is difficult to suppose that it has gone out of existence. It has not, after all, died. So it is both b and c. Moreover, if b had died immediately on fission, it would seem clear that a was c. It is difficult to see how extrinsic features can affect the identity of an entity. Hence, even if b still exists, a must be c; and by symmetry, b. Hence, again, a is both b and c.

It is plausible to diagnose here a case of multiple denotation. After t, 'a' denotes b and c, even though these are distinct. Hence, $a=b$ and $a=c$. (For the same reason, $a=a$ is also false—as well as true—after t.) Notice that we cannot apply substitutivity of identity to infer that $b=c$ because, as we saw in 1.3, this principle of inference breaks down in the case of multiple denotations.

The example can be varied in many ways. We can consider a case of fusion rather than fission. We can consider things other than amoebas that fis or fuse, for example, people who undergo split brain transfers. The particular example I have employed here comes from Prior ((1968), p. 83), who is clearly sympathetic to the idea that it is a genuine counter-example to the law of the substitutivity of identicals, though he has no coherent semantic account of how this is possible. He also observes that there must, presumably, be some close relative of the law that does hold, or why we are attracted to it in the first place would be a mystery. According to the present account, the close relative that holds is the law in the form of a material conditional, and detachment is permissible provided that we do not have a case of multiple denotation.

There are certainly other possible solutions to Prior's problem (though they all face well-known objections), and I have not attempted to argue here that the logic of multiple denotations is the best solution. However, it is certainly a simple and elegant solution.

2.3. *Truth and Ambiguity*

I want to conclude with some comments on the notion of ambiguity. A sentence, some of whose parts have multiple referents, is clearly ambiguous in a certain sense. In this sense, the logic of Part 1 can be seen as a logic of ambiguity. Why should one need such a thing? It is part of logical folklore that discourse should be disambiguated *before* any reasoning is performed. The standard justification for this is that failure to do so may allow invalid arguments, and so false conclusions. This is no longer so if one uses a logic which accommodates the ambiguity. The folklore is correct though. Once disambiguated, the discourse permits the use of a stronger logic, *LP* (or even classical logic if all predicates are classical). The stronger logic allows access to consequences that would not otherwise be accessible. Hence, disambiguation is sound methodology. But if there are situations where disambiguation is impossible, then the advice to disambiguate is vacuous, and a logic that accommodates the ambiguity is mandatory. In the situations that we looked at in the previous sections, disambiguation is impossible. In the first sort of case, we are in a position to disambiguate only with hindsight, i.e., maybe never. In the second, the ambiguity arises only after a certain time, but we wish to reason diachronically. Disambiguation is therefore impossible.

In his 'Logic for Equivocators' (1982), David Lewis discusses the logic *LP* and recommends its use if one has to reason employing ambiguous sentences. (He is concerned only with propositional logic, and hence multiple referents are not on the agenda. Ambiguity may arise for all sorts of reasons, not just multiple referents.) He gives the usual advice that disambiguation is preferable when possible, and thinks that only pessimists would suppose that it is not—though he concedes that pessimists might just be right. As we have seen, in some cases, Lewis' pessimism is justified; though those of us who are persuaded of the correctness of paraconsistent logic for other reasons, will hardly suppose this to be a case of things turning out for the worse. Quite the contrary.

A more substantial disagreement with Lewis is the following. Lewis assumes—dogmatically—that all multiple truth values are due to ambiguity. Now of course, if, *à la* Frege, we take the referents of sentences to be truth values, then sentences with multiple truth values are ambiguous in the same sense that sentences whose parts have multiple denotations are. But this seems little more than a fact of terminology. Of more importance is the question of whether there are sentences with multiple truth values that do not express multiple propositions in any interesting sense.

I take the answer to this to be that there are. In (1987) I gave arguments that many different kinds of sentences have multiple truth values: paradoxes of self-reference ('the Russell set is a member of itself'), certain sentences about states of change ('the car is in motion'—at the instant it changes from rest to motion), certain sentences concerning obligations and other norms ('Jan may vote'—when she is enfranchised under one law and disenfranchised under another). I shall not repeat the arguments here. The important point is that in none of these cases is it even vaguely plausible to suppose that the sentences in question express more than one proposition.

In only one of these cases (that I am aware of) has it even been suggested that the multiple truth values correspond to different propositions that the sentence expresses. This is the case of the liar paradox. Let *a* be the sentence '*a* expresses a proposition that is not true'. *a* hardly appears to be ambiguous: the subject and predicates have unique senses, and the grammatical construction is unambiguous.[13] But Slater has argued that this is, in

[13] The predicate 'is true' is, in fact, ambiguous. Its sense, when applied to statements (claims, beliefs, propositions, etc.), is different from that which it has when applied to friends, dice, etc. But there is no doubt which sense is in question here.

fact, the case.[14] His argument, in effect, is simply that if the sentence were not ambiguous a contradiction would arise. This argument is hardly persuasive: one can take the liar argument to be a *reductio* of *any* assumption that one can contrive to pack into it. Thus, to give this kind of solution any bite, one needs to give some independent account of what these two meanings are—something I have not seen attempted.

Moreover, appealing to ambiguity does not really solve the problem. For we may reformulate it, not in terms of sentences, but in terms of propositions themselves. Merely consider the proposition: this proposition is not true. The usual argument establishes a contradiction. Of course, one may now say that there is no such proposition. Another *ad hoc* move, and one that does not really help either, since we may consider instead the proposition:

This proposition is not true, or there is no such proposition.

We cannot now say that there is no such proposition. If this were the case, the sentence displayed would be true, and *a fortiori* it would express a proposition. Hence there is such a proposition, and its second disjunct is false. It is therefore logically equivalent to its first disjunct. The usual reasoning then gives the expected contradiction. Of course, there are other moves that can be made at this point, but they are all familiar and problematic.[15] More importantly in the present context, the initial claim that *a* was ambiguous has become irrelevant to a solution. There is nothing, then, in the situation, that speaks for this counter-intuitive claim.

2.4 *Conclusion*

I have argued that multiple truth values cannot always be attributed to ambiguity in any interesting sense. I have also argued that multiple truth values can arise due to a form of ambiguity: multiple denotation of predicates and constants. As I indicated, the examples used to show this are not mandatory, but they seem to me to be very plausible ones. What makes them interesting and important is that, unlike the example concerning the

[14] E.g., (1994), ch. 1. Burge (1979) and others have argued that the predicate 'is true' is indexical, in the way that 'is happening now' is. I have argued against this elsewhere (Priest (1987), ch. 2). But even if the claim were right, the liar sentence would not give us a case of multiple senses. 'The Vietnam war is happening now' is not ambiguous: it has different truth values in different contexts.

[15] See, e.g., Priest (1993), section 5.

name 'John Smith', disambiguation is not a possibility. In these, and any similar cases, the semantics of multiple denotation recommends itself.[16]

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