Логические исследования
2021. Т. 27. № 1. С. 20–30
УДК 16+510.6

Logical Investigations
2021, Vol. 27, No. 1, pp. 20–30
DOI: 10.21146/2074-1472-2021-27-1-20-30

Jc Beall, Graham Priest

A Tale of Excluding the Middle

Jc Beall
University of Notre Dame,
Notre Dame, IN 46556, USA.
E-mail: jbeall@nd.edu

Graham Priest
CUNY Graduate Center,
365 5th Ave, New York, NY 10036, USA.
University of Melbourne,
Parkville, VIC 3010, Australia.
Ruhr University Bochum,
Universitätsstraße 150, 44801 Bochum, Germany.
E-mail: priest.graham@gmail.com

Abstract: The paper discusses a number of interconnected points concerning negation, truth, validity and the liar paradox. In particular, it discusses an argument for the dialetheic nature of the liar sentence which draws on Dummett's teleological account of truth. Though one way of formulating this fails, a different way succeeds. The paper then discusses the role of the Principle of Excluded Middle in the argument, and of the thought that truth in a model should be a model of truth.

Keywords: Negation, Principle of Excluded Middle, Liar Paradox, Teleological Account of Truth, Michael Dummett, Graham Priest, Truth, Validity, Truth in a Model

For citation: Beall Jc, Priest G. "A Tale of Excluding the Middle", *Logicheskie Issledovaniya / Logical Investigations*, 2021, Vol. 27, No. 1, pp. 20–30. DOI: 10.21146/2074-1472-2021-27-1-20-30

Prologue

Negation is one of the most important logical notions, and its properties are, and always have been, contentious. Many have held that satisfying the Principle of Excluded Middle is one of its properties. Many have denied this. Indeed, the Principle has been denied to provide an account of the open future, to allow for the identification of truth and provability in mathematics, to solve the liar paradox, and for a number of other reasons.

One could write a book (or two) on these matters. Our aim here is much less ambitious; it's to trace one strand of the story about the Principle. The liar paradox will be a central player in this, but so will truth and validity. The upshot of our story will be some important lessons about all these things.

Act I: In Which the Liar Paradox is Introduced

The Liar paradox is a familiar creature. For our purposes, we set it up as follows.

Angle brackets are a name-forming functor, and T is a truth predicate, satisfying the T-Schema rules of Capture and Release, respectively:

$$\frac{\alpha}{T\langle\alpha\rangle} \qquad \frac{T\langle\alpha\rangle}{\alpha}$$

A falsity predicate $F\langle\alpha\rangle$ may be defined as $T\langle\neg A\rangle$, so that the familiar dual F-Schema rules are enforced:

$$\frac{\neg\alpha}{F\langle\alpha\rangle} \qquad \frac{F\langle\alpha\rangle}{\neg\alpha}$$

The Principle of Excluded Middle (PEM), that every (declarative) sentence is either true or false, can be expressed by the schema $T\langle\alpha\rangle \vee F\langle\alpha\rangle$. Given Capture and Release, and standard rules concerning disjunction, the PEM can equivalently be expressed by the schema $\alpha \vee \neg\alpha$.[1]

The Liar sentence is a sentence λ equivalent to $F\langle\lambda\rangle$. The Liar paradox is now given by the following natural-deduction argument:

$$\dfrac{T\langle\lambda\rangle \vee F\langle\lambda\rangle \qquad \dfrac{\dfrac{\dfrac{\dfrac{\overline{T\langle\lambda\rangle}}{\lambda}}{F\langle\lambda\rangle}}{\neg\lambda} \quad \dfrac{\overline{T\langle\lambda\rangle}}{\lambda}}{\lambda\wedge\neg\lambda} \qquad \dfrac{\dfrac{\overline{F\langle\lambda\rangle}}{\neg\lambda} \quad \dfrac{\overline{F\langle\lambda\rangle}}{\lambda}}{\lambda\wedge\neg\lambda}}{\lambda\wedge\neg\lambda}$$

Clearly, one can break the argument if one rejects the PEM, as several logicians have argued should be done.[2] Those, like your narrators (JCB and GP), who think that the Liar serves as a strong witness to 'true contradictions' (gluts, dialethias) – true sentences of the form $\alpha\wedge\neg\alpha$ (dual of $\neg\alpha\vee\alpha$) – and who think that the Liar paradox (as *per* the above derivation) serves as a sound argument

[1]The two ways of expressing the PEM can, of course, come apart if the standard rules for disjunction fail, as they do, for example, on the familiar supervaluation account of van Fraassen [Fraassen, 1966], which validates the schema $\alpha\vee\neg\alpha$, though α may be neither true nor false in an interpretation.

[2]E.g., [Kripke, 1975; Field, 2008].

for just such a result, are naturally motivated to advance an argument for the PEM. And so the history unfolds.[3]

Act II: In Which the Teleological Account of Truth is Explained

In Chapter 4 of the first edition of *In Contradiction*,[4] GP, partly with a defense of the soundness of the Liar derivation in mind, advanced an argument for the PEM that rests on the teleological account of truth. This Act reviews the account; the next Act reviews the target argument for PEM.

The teleological account of truth is essentially Dummett's. Dummett notes that some notions are fully understandable only via an understanding of their point or *telos*. Thus, consider the notion of winning, as of a game. One could know what constitutes a winning position in a game of chess (or bridge, or cricket) and what constitutes a losing position, without understanding what it is to win. To understand this, one must know that the aim or point of playing a game, as such, is to achieve a winning position, not a losing one.

Similarly, says Dummett, one might know what constitutes a true statement and what constitutes a false statement without understanding what truth is. The T-Scheme for α states the condition under which α is true, and the F-Schema for α states the condition under which α is false. But if you don't understand what the point of calling something 'true' is, you don't understand truth.

What is the point (telos) of truth? The answer, according to Dummett, is that truth is the institutional (as opposed to personal) aim of assertion – the aim of asserting something.[5] As he puts it:[6]

> it is part of the concept of truth that we aim at making true assertions.

Or again:[7]

> the class of true sentences is the class the utterance a member of which a speaker of language is aiming at when he employs what is recognizably the assertoric use.

[3]One of us (viz., JCB) rejects the soundness of the Liar argument above without independent reason to accept the PEM, but JCB thinks that the methodological quest for completeness – sorting all sentences of the language of our theories into 'The True' or 'The False' – that drives systematic, truth-seeking theorists sufficiently pushes towards a glutty account of λ. We note this only to set it aside. For some discussion, see [Beall, 2017; Beall, 2018]. Both of us remain committed glut theorists, and equally interested in arguments for PEM.

[4][Priest, 1987]. In what follows, page references are to the *second* (viz., 2006) edition.

[5]And, one might add, other cognitive acts, such as believing.

[6][Dummett, 1959, p. 143].

[7][Dummett, 1973, p. 320].

Thus, as *In Contradiction* (§4.5) points out, if for whatever reason people played games in such a way as to realize what we currently take to be a losing position then winning would become losing, and vice versa. Similarly, if the institution of assertion morphed in such a way that it became the aim to assert the sentences we now take to be false ('the moon is a cube', '3+2 is not 5', etc.), people would understand these as expressing the opposite of what they are currently taken to express: the sense of a sentence would flip to what is currently expressed by its negation.

Now, one might think that the notion of truth (as applied to sentences, beliefs, propositions, etc.) is univocal, but one might not. Either way, at least with respect to the meaning of 'truth' that Dummett is talking about, his point is well taken. Without an understanding of the *point* of this notion, truth and falsity would be formally symmetrical notions, with nothing to break the symmetry. It is the way that they are applied in use that distinguishes them.

In what follows, it is truth in the given teleological sense with which we are chiefly concerned. Unless otherwise explicitly noted, when we speak of truth or falsity in what follows, we mean truth or falsity in this sense (if there is more than one).

Intermission: For the Sake of Transparency

Before our story continues, some points of clarification are useful. Call the following two-way rule the *Contraposed T-Schema Rule*:

$$\frac{\neg A}{\overline{\neg T\langle A\rangle}}$$

The T-Schema rules do not by themselves give the contraposed form. Nor is there anything in the teleological account of truth that delivers them; for the account says nothing about negation. On the other hand, just because of this, it is open to someone who endorses the teleological account of truth to endorse an account of negation that delivers the contraposed T-schema rule.

For the rest of this essay, we will call an account of truth *transparent* if for any A, A and $T\langle A\rangle$ are intersubstitutable *salva veritate* in all extensional contexts. If the only other contexts in play are those delivered by conjunction and disjunction (and those that can be defined from them and negation), and given that these behave in a standard fashion, transparency is equivalent to the T-schema rules plus the contraposed T-schema rules. Hence it is quite possible for someone to endorse an account of teleological truth that is transparent (the intersubstitutability holds), just as it's possible for someone to endorse an account of teleological truth that fails to be transparent (intersubstitutability fails).

JCB, in a variety of works, uses the word 'transparency' for an account of truth according to which there is neither more nor less to truth than is captured by (as he says) 'the transparency rules' – the given intersubstitutability rules in the above sense. Clearly, the transparency account of truth, so understood, is not compatible with the teleological account of truth. JCB has also argued that transparency in this sense – and so the relevant T-schema and F-schema rules – arise from the thought that (the target transparency notions of) truth and falsity are predicates that reflect (indeed, are just defined as 'abstractions from') the behavior of the logical connectives $\neg$ and $\dagger$ (the last of these being the monadic truth function whose output is identical with its input).[8]

Whether such an account of truth and the teleological account of truth are rivals or simply characterise different notions, is an issue on which we express no view here. As observed, both views deliver the T-Schema rules (viz., Capture and Release). For the teleological account of truth – our principal concern – it is exactly these rules that tell us which sentences *are* true. For JCB's account, it is the fact that truth 'supervenes' (in a sense that needn't detain us here) on $\dagger$. Moreover, arguably Capture and Release govern any notion of truth worth the name. (As Tarski noted, such is a condition of adequacy on any account of truth.) And, given this, all notions of truth are extensionally equivalent. For if T_1 and T_2 are any such notions, $T_1 \langle\alpha\rangle \dashv\vdash \alpha \dashv\vdash T_2 \langle\alpha\rangle$. Given the F-Schema rules, a similar equivalence holds for falsity.

Act III: In Which an Argument for the PEM is Seen to Crash

Now to the argument for the PEM mentioned at the beginning of Act II.

Deploying the teleological account of truth, Dummett notes (with a qualification meant to take certain complications off of the agenda) that if one utters something, one either achieves the point of doing so or one fails. Anything less than success is failure. As he puts it – in distinctly Dummettian terms:[9]

> A sentence, so long as it is not ambiguous or vague, divides all possible states of affairs into just *two* classes. For a given state of affairs, either the statement is used in such a way that a man who asserted it but envisaged the possibility of that state of affairs as a possibility, would be held to have spoken misleadingly, or the assertion of the statement would not be taken as expressing the speaker's exclusion of that possibility. If a state of affairs of the

[8] Further on these matters, see [Beall, 2009, Chapter 1], and especially [Beall, 2021].

[9] [Dummett, 1959, p. 149] f. Italics original.

> the first kind obtains, the statement is false. If all actual states of affairs are of the second kind, it is true. It is therefore *prima facie* senseless to say of any statement that in such-and-such a state of affairs it would be neither true nor false.

The game analogy is illuminating here. In many games, there are three possible outcomes: winning, losing, and drawing. But in some games, there are only two: winning and losing. In such games, not to win is, *ipso facto*, to lose. In asserting, there is nothing that corresponds to drawing. Anything less than success is, *ipso facto*, failure. *In Contradiction* (§4.7) used this point to argue for the PEM. The last sentence of the last quote by Dummett clearly sounds like a version of this.

In 'True Contradictions'[10] Terry Parsons criticised the argument from the teleological account of truth to the PEM. For Dummett's point, 'false', he said, has to be interpreted as *untrue*, not as *having a true negation,* as required by the PEM. And that seems right: nothing in Dummett's reflections says anything about how sentences containing negation work. And to appeal to the inference from $\neg T\langle\alpha\rangle$ to $F\langle\alpha\rangle$ would clearly beg the question.

The argument from teleology for the PEM thus fails to achieve its goal. But the play goes on: there is importantly more to the matter than this.

Act IV: In Which the Liar Paradox Reappears

In the second edition of *In Contradiction* (§19.6) GP accepted Parson's criticism, and whilst pointing out that the teleological account of truth still puts the onus of proof on one who wishes to draw a distinction within the category of untruths, he agreed with Parsons that Dummett's point shows only that (p. 267):

> anything that fails to live up to the aim of assertion is *ipso facto* not true.

But then there is a footnote (fn 13):

> Note that this conclusion, on its own, is not without its sting. It establishes, even if one denies the Law of Excluded Middle in general, that particular instances of the form $T\langle\alpha\rangle \vee \neg T\langle\alpha\rangle$ still hold. These are precisely the ones that give the "strengthened liar" its punch, as I noted in discussing ch. 1.

[10] [Parsons, 1990].

Matters are left there; but the import of the point can be brought out much more simply and directly, as follows.[11]

Instead of considering the Liar paradox in the form 'this sentence is false', consider it in the form 'this sentence is untrue'. Thus, let λ^* be a sentence of the form $\neg T\langle\lambda^*\rangle$. Given the preceding discussion, we have, as noted, $T\langle\lambda^*\rangle \vee \neg T\langle\lambda^*\rangle$, and a contradiction quickly follows. Merely consider the following argument in natural-deduction form:

$$\dfrac{T\langle\lambda^*\rangle \vee \neg T\langle\lambda^*\rangle \qquad \dfrac{\dfrac{\dfrac{\overline{T\langle\lambda^*\rangle}}{\lambda^*}}{\neg T\langle\lambda^*\rangle} \quad \overline{T\langle\lambda^*\rangle}}{T\langle\lambda^*\rangle \wedge \neg T\langle\lambda^*\rangle} \qquad \dfrac{\dfrac{\dfrac{\overline{\neg T\langle\lambda^*\rangle}}{\lambda^*}}{T\langle\lambda^*\rangle} \quad \overline{\neg T\langle\lambda^*\rangle}}{T\langle\lambda^*\rangle \wedge \neg T\langle\lambda^*\rangle}}{T\langle\lambda^*\rangle \wedge \neg T\langle\lambda^*\rangle}$$

Hence, *without relying on PEM the teleological notion of truth delivers a direct argument for true contradictions*, an argument for the truth and – assuming double negation – falsity of $\neg T\langle\lambda^*\rangle$, that is, λ^*, and so for the truth of the contradiction $\lambda^* \wedge \neg\lambda^*$.

We end this Act by noting a corollary concerning transparency. The scheme $T\langle\alpha\rangle \vee \neg T\langle\alpha\rangle$ is obviously a restricted case of PEM; let us call it *Restricted Excluded Middle* (REM). If truth is transparent[12] then $T\langle\neg\alpha\rangle$ is equivalent to $\neg\alpha$, which is equivalent to $\neg T\langle\alpha\rangle$. That is, truth commutes with negation. In this case, REM clearly delivers PEM. Hence the argument in Act I for the contradictory nature of λ goes through as well as that for λ^*.

Act V: In Which Validity Comes Onstage

What we have discussed so far concerns the truth of the REM and PEM, not their logical validity, which is a quite different matter. Indeed, the preceding discussion does *not* require these to be logically true (i.e., logically valid schemata, each instance of which is logically valid), simply true; and it is perfectly coherent to take them to be true without being logically so.

To show this, we need some machinery to deliver an account of validity; we shall take this to be model-theoretic machinery. To handle the inconsistency of Liars, a paraconsistent validity relation is required. Some paraconsistent logics, such as *LP*, defended by GP in *In Contradiction*, validate PEM, and so cannot be used to show this. But, *FDE*, preferred by JCB,[13] does not validate PEM.

[11] Perhaps GP overlooked the point; perhaps he forgot it. It was noted by JCB in a talk given at a conference at the National Autonomous University of Mexico in 2019, where GP was in the audience.

[12] As [Beall, 2009] and [Field, 2008] hold, though [Priest, 1987, §4.9] does not.

[13] [Beall, 2017; Beall, 2018].

So we can use this. The semantics of FDE can be set up in many equivalent ways. We use the four-valued semantics, where the values are t (true only), f (false only), b (both), and n (neither). t and b are the designated values. LP has the same semantics, except that it does not use the value n.[14]

Let the language be a first-order one that contains a monadic truth predicate, T, and for every expression e in the language, a suitable name, $\langle e \rangle$. The extension of any monadic predicate P (including T) is the set of objects in the domain which the interpretation of P maps to t or b; the anti-extension is the set of objects it maps to f or b. If in every interpretation every sentence is in either the extension or anti-extension of T then REM is validated (that is, it is true on every interpretation over which the given validity relation is defined) but, crucially, PEM is not thereby validated, since the validity of REM is compatible with some formulas (not involving T) having value n.

Such interpretations may not validate the T-Schema rules, but they can be made to do so if, in addition, every sentence, α, has the value t or b iff $T\langle \alpha \rangle$ does. But PEM is still not validated. Again, suppose that P is some (other monadic) predicate and 'a' is the name of some object which is in neither the extension nor anti-extension of P. Then Pa has the value n, and neither Pa nor $\neg Pa$ is in the extension of T, and so (by assumption) is in its anti-extension.

Matters are more complicated if the truth predicate is transparent in every interpretation. For then, α and $T\langle \alpha \rangle$ have the same value in every interpretation. So if the REM holds in an interpretation then, for any α, $T\langle \alpha \rangle$ or $\neg T\langle \alpha \rangle$ holds. But then $T\langle \alpha \rangle$ has the value t, b, or f, and so, given transparency, α does too. Hence $\alpha \vee \neg \alpha$ is true in all interpretations.[15]

However, with a small change, one can accommodate the situation in which T behaves (let us say) 'transparently at a distinguished interpretation' (but not at all interpretations), REM is true at the given interpretation, but PEM is not valid (true at all interpretations). Fix on some interpretation, @. One may think of @ as an interpretation which verifies some true theory – maybe the theory of T itself – or is, in some other way, such that everything that holds in it is actually true (*simpliciter*) – whatever one takes that to mean. The way that T behaves in this interpretation may well not be the way in which it

[14]See [Priest, 2008, chs. 8 and 22]. For natural-deduction systems for for FDE, LP, and related systems, see [Priest, 2019]. For a wider discussion of FDE see [Omori and Wansing, 2017; Omori and Wansing, 2019].

[15]This result is implicit in the discussion of an FDE-based 'compromise' in [Beall, 2009, p. 104], which shows that if a PEM-demanding negation-like device ('exhaustive device' in said work) is added to a language with its own transparent truth predicate then the only models of the resulting theory are LP models: one kicks out any sentence that can be gappy, that is, has value n. In a language with its own transparent truth predicate, imposing REM is, then, tantamount to imposing a PEM-satisfying negation.

behaves at other interpretations. After all, interpretations represent situations. These may be actual, they may be possible, or they may be impossible. What validity gives is a way of preserving truth-in-an-interpretation in all of these.

In particular, then, @ may 'make true' REM and the transparency of truth (at @), and so likewise the schema $\alpha \vee \neg\alpha$. But there is no reason why these should hold in other interpretations too. In other words, they can hold (at @), but not be valid – not hold at all interpretations. And let us stress, again, that the argument that λ^* is a true contradiction requires only that the REM (or PEM) be *true*, not that it be valid.

Act VI: In Which an Old Adage is Dissected

Of course, whether the PEM *is* logically valid is another matter. FDE is *prima facie* a more attractive account of validity than LP, simply because of its symmetry. It also accommodates naturally the apparent duality between the liar and the truth-teller, 'this sentence is true'. The liar looks like a case of overdetermination (*both*); the latter looks like a case of underdetermination (*neither*).[16] So it is fair to say that if someone prefers LP to FDE as an account of logical validity, the onus is on them to make the case.

This is not the place to go into this matter, since it raises the whole question of the methodology determining the rational choice of logical theory.[17] But since truth has been very much a major concern of what has gone before, let us conclude this story by discussing just one aspect of the matter, which concerns truth. It involves the natural thought expressed in the old adage:

- truth-in-a-model ought to be a model of truth

that is, truth-in-an-interpretation should behave in the same way as does truth *simpliciter* (whatever, exactly, one takes this to be). Exactly how to understand this thought might be debated but one may naturally take it to have the following consequence. If PEM is true (i.e., all instances are true), so that for any α, either $T\langle\alpha\rangle$ or $T\langle\neg\alpha\rangle$ holds, then either α or $\neg\alpha$ should be true in an interpretation. But if either α or $\neg\alpha$ holds in every interpretation, then nothing has the value n in any interpretation, and we have the semantics of LP (not FDE). So on this understanding of the adage, the truth of the PEM entails its validity.

The adage may well be resisted, however. As observed, when we reason, we reason about all sorts of situations, actual, possible, and maybe impossible. And there is no reason why truth at such situations must behave like truth *simpliciter*. Thus, suppose the actual situation to be as classical as one likes.

[16] But on this, see *In Contradiction*, p. 66, and [Mortensen and Priest, 1981].

[17] For some discussion, see [Priest, 2014], and similarly [Beall, 2019].

We may yet want to reason about situations that are gappy or glutty. One may think of these as merely possible or impossible situations. And, one needs a canon of inference which preserves what holds at *every one* of these interpretations. Truth in some interpretations will not, then, mirror truth *simpliciter*.

Certain understandings of the adage may, in fact, be problematic even if one takes LP to deliver the correct notion of logical validity. For if both $T\langle\alpha\rangle$ and $\neg T\langle\alpha\rangle$ may hold then, it might be thought to follow, there should be interpretations where α can both hold and not hold; that is, where α may both have the value t or b, and not. Obviously this cannot be the case in a consistent semantics. The possibility may be accommodated by moving to a semantics itself formulated in a paraconsistent logic. However, that raises many issues of its own, and this is not the theatre for that tale.[18]

Epilogue

This brings our story to an end. Of course, we know that there are likely to be many other players in the wings who will want to jump on stage. And there may well be other acts to be written. Nonetheless, we think our tale an illuminating one, and trust that the reader has enjoyed it.

References

Beall, 2009 – Beall, J. *Spandrels of Truth*, Oxford: Oxford University Press, 2009.

Beall, 2017 – Beall, J. "There is no Logical Negation: True, False, Both and Neither", *Australasian Journal of Logic*, 2017, Vol. 14, `https://ojs.victoria.ac.nz/ajl/article/view/4025/3576`.

Beall, 2018 – Beall, J. "The Simple Argument for Subclassical Logic", *Philosophical Issues*, 2018, Vol. 28, pp. 30–54; reprinted in *The Philosopher's Annual*, 2018, `http://www.pgrim.org/philosophersannual/past.html`.

Beall, 2019 – Beall, J. "On Williamson's New Quinean Argument Against Nonclassical Logic", *Australasian Journal of Logic*, 2019, Vol. 16, `https://ojs.victoria.ac.nz/ajl/article/view/5915`.

Beall, 2021 – Beall, J. "Transparent Truth as a Logical Property", to appear in *The Nature of Truth: Classical and Contemporary Perspectives*, ed. by M. Lynch, J. Wyatt, J. Kim, and N. Kellen, Cambridge, MA: MIT Press, 2021.

Dummett, 1959 – Dummett, M. "Truth", *Proceedings of the Aristotelian Society*, 1959, Vol. 59, pp. 141–162; reprinted as ch. 1 of *Truth and Other Enigmas*, London: Duckworth, 1978.

Dummett, 1973 – Dummett, M. *Frege: Philosophy of Language*, London: Duckworth, 1973.

Field, 2008 – Field, H. *Saving Truth from Paradox*, Oxford: Oxford University Press, 2008.

[18] A discussion can be found in [Priest, 2020].

Fraassen, 1966 – Fraassen, van, B.C. "Singular Terms, Truth-Value Gaps, and Free Logic", *Journal of Philosophy*, 1966, Vol. 63, pp. 481–495.

Kripke, 1975 – Kripke, S. "Outline of a Theory of Truth", *Journal of Philosophy*, 1975, Vol. 72, pp. 690–716.

Mortensen and Priest, 1981 – Mortensen, C., and Priest, G. "The Truth Teller Paradox", *Logique et Analyse*, 1981, Vol. 95–96, pp. 381–388.

Omori and Wansing, 2017 – Omori, H., and Wansing, H. "40 Years of *FDE*: An Introductory Overview", *Studia Logica*, 2017, Vol. 105, pp. 1021–1049.

Omori and Wansing, 2019 – *New Essays on Belnap-Dunn Logic*, ed. by H. Omori and H. Wansing, New York: Springer, 2019.

Parsons, 1990 – Parsons, C. "True Contradictions", *Canadian Journal of Philosophy*, 1990, Vol. 20, pp. 35–53.

Priest, 1987 – Priest, G. *In Contradiction*, Dordrecht: Martinus Nijhoff, 1987; 2nd edn, Oxford: Oxford University Press, 2006.

Priest, 2008 – Priest, G. *Introduction to Non-Classical Logic: from If to Is*, Cambridge: Cambridge University Press, 2008.

Priest, 2014 – Priest, G. "Revising Logic", *The Metaphysics of Logic*, ed. by P. Rush, Cambridge: Cambridge University Press 2014, ch. 12.

Priest, 2019 – Priest, G. "Natural Deduction for Systems in the *FDE* Family", pp. 279–292 of Omori and Wansing (2019).

Priest, 2020 – Priest, G. "Metatheory and Dialetheism", *Logical Investigations*, 2020, Vol. 26, pp. 48–59.