Meyer's Relevant Arithmetic: Introduction to the Special Issue

Thomas Macaulay Ferguson and Graham Priest

ILLC, University of Amsterdam and Arché Research Centre, University of St. Andrews

The Graduate Center, City University of New York

Abstract

We make some introductory remarks to set the stage for the present issue on Robert Meyer's program of relevant arithmetic.

1 Introduction

Over the decades of Bob Meyer's prodigious career as philosopher and logician, a topic to which he reliably—if intermittently—returned is *relevant arithmetic*. Fragmented across a series of abstracts, technical reports, and journal articles Meyer outlined a research program in nonclassical mathematics that rivals that of the intuitionists in its maturity, depth, and perspicacity.

The inaccessibility of the program's two foundational texts, however, has served as an artificial barrier to the circulation of Meyer's ideas. These 1976 monographs—*Arithmetic Formulated Relevantly* and *The Consistency of Arithmetic*—in which the program and core arguments saw their definitive expression remained unpublished. Meyer distributed copies of these works but the scope of this distribution was extremely limited, severely restricting the reach of Meyer's program. The project did not necessarily *languish* in the vacuum left by these monographs—Meyer continued to develop and promote the program in papers and talks—but the pronounced inaccessibility of its cornerstone documents no doubt hindered its *flourishing*. And, as the reader will soon discover, between the novelty of Meyer's insights into the relationship between logic and mathematics and the humor and wit with which he delivers those insights, this has inarguably been a loss.

This special issue of the *Australasian Journal of Logic* has been produced in an effort to correct this unfortunate state of affairs. Many hands have played a role in the production of this issue and among these contributors, it seems universally accepted that Meyer's project of relevant arithmetic still has a great deal to offer.

The centerpiece of this special issue the publication of corrected editions incorporating the most complete texts of *Arithmetic Formulated Relevantly* and *The Consistency of Arithmetic* available. While fragmented copies of the monographs have been in limited circulation, time and imperfect reproduction methods have significantly degraded their legibility, risking the loss of what remains. The present publication in a digital format will preserve their content for years to come, making the work available to anyone interested, irrespective of their geography, resources, or affiliation.

To place the monographs in context, we are presenting them alongside three additional texts on relevant arithmetic; these pieces complement the monographs either by providing historical context or filling technical lacunae (or at risk of being lost themselves). The presentation of Meyer's unpublished work is rounded out by a detailed bibliography of Meyer's published output on relevant arithmetic and a note from Chris Mortensen describing the genesis of the joint 1984 paper "Inconsistent Models for Relevant Arithmetics" [4].

Finally, we extended an invitation to contemporary researchers to appraise Meyer's program with a host of original, peer reviewed research papers that collectively illustrate the potential and continued relevance of Meyer's work. From philosophical and formal evaluations of Meyer's project to new technical results, the breadth of the papers included in this issue is testament to the continued relevance of Meyer's work to current trends in logic.

2 Meyer's Project

A *relevant arithmetic* is any formal theory in which number theory may be expressed that is based on a relevant logic. Meyer touched on a number of relevant arithmetics but his favored theory is $\mathsf{R}^\sharp$—the closure of the Peano axioms and induction schema under the relevant logic R.

2.1 Why $\mathsf{R}^\sharp$?

There is a range of possible entreaties one could make to motivate the study of $\mathsf{R}^\sharp$ and its properties. We can characterize two modes of these appeals on the basis of their *apparent scope*: the *conservative* and *radical*.

The conservative mode offers reasons to believe that *as a model of mathematical practice* $\mathsf{R}^\sharp$ is superior to classical PA. One way to demonstrate this superiority would be to show that classical methods demand a sacrifice in *expressivity* that collapses important distinctions in mathematical. The logical vocabulary of the relevant logic R is sufficiently rich to distinguish the *material conditional* ($\supset$) from the intensional notion of *entailment* ($\rightarrow$).

Meyer's description of the distinction between the two is:

> The material $\supset$ I take to be a *fact-laden* connective; it merely transmutes simpler statements of fact into a more complex statement of fact. As such, it neither rests on nor licenses any lawful connections between the facts so compounded. But good arguments, whether deductively general or inductively particular, rest on connections, not facts.
>
> The relevant $\rightarrow$, as I see it, is introduced *precisely* to take account of such connections. And this introduces, at the formal level, an opportunity to make distinctions which, though perhaps dimly present in the intuitive base, goes beyond it in sharpness and in formal clarity.

In the classical setting, the material conditional is the closest proxy for relation of entailment, leading to a conflation between the two. Applications of classical logic thus inherit this inability to distinguish between $\supset$ and $\rightarrow$.

Arguably, this limitation becomes especially apparent when formulating arithmetic with the limited resources of classical logic. A (possibly fuzzy) notion of *entailment*, after all, is a centerpiece of mathematics. Should it turn out that mathematical practice frequently invokes a notion of entailment *conceptually distinct* from $\supset$, then to formulate arithmetic in a classical setting is to rob from the mathematician the ability to express important distinctions.

An example that Meyer frequently deploys is an intuitive distinction between the sentences $0 = 2 \supset 0 = 1$ and $0 = 2 \rightarrow 0 = 1$. (This is an example that is touched upon by a number of the contributed papers in this issue.)

> [W]e should expect to find $A \supset B$ provable on some number-theoretic occasions when $A \to B$ is unprovable. And this happens. $0 = 2$, being arithmetically *false*, materially implies everything. But if we hypothetically overloaded arithmetic so as to identify 0 and 2, it's just gratuitous to suppose that we should thereby have identified, say, 0 and 1. Indeed, when we take the integers modulo 2, we *do* identify 0 and 2, without going on to identify 0 and 1.

The *conservative* appeal proceeds by providing reasons to *prefer* $\mathsf{R}^\sharp$ over PA. This mode of persuasion allows one to focus on $\mathsf{R}^\sharp$ itself, as success does not require that one shows PA to be *wrong*.

In contrast, the *radical* mode goes further. In *The Consistency of Arithmetic*, Meyer goes beyond the suggestion of $\mathsf{R}^\sharp$'s *superiority* and states that $\mathsf{R}^\sharp$ corrects a historical injustice triggered by incautious readings of Gödel's Second Theorem. In particular, Meyer is sympathetic to the *Hilbert program* of proving the correctness of arithmetic through finitary methods.

$\mathsf{R}^\sharp$ has several intriguing properties that can be established through finitary arguments. For one, it can be proven to be *Post consistent* by the construction of finite models. For two, it can be shown to be *arithmetically consistent* in the sense that no *false* arithmetical equations can be proven. Between these two properties, Meyer argues that $\mathsf{R}^\sharp$ can be shown to be *correct* in the sense proposed by Hilbert, thereby "overturning" Gödel and restoring Hilbert.

Of course, this position is risky—talk of "repealing" Gödel's Inconsistency Theorems runs dangerously close to the theses expounded by "Cantor cranks"—but Meyer bravely enters this minefield nevertheless. As the reader will see, however, Meyer comes prepared; the arguments outlined in *The Consistency of Arithmetic* are lucid and treat the subject of the correctness of arithmetic with incredible subtlety and care. Even in the typescripts' most pugnacious moments—and there are many pugnacious moments—Meyer refuses to cut corners.

2.2 The Friedman-Meyer Result

Given the laudatory tone of our description of Meyer's results on $\mathsf{R}^\sharp$, the reader may ask: Was the distribution of Meyer's typescripts *really* so limited as to deny the project the legacy it deserved? If finitary means suffice to show

$\mathsf{R}^\sharp$ to be arithmetically correct, why is this *a priori* vindication of Hilbert not more widely known? Surely, the publication of *e.g.* [2] and [4] made the results, if not the entire context, available to the logic community, after all.

As Meyer described in [1]:

> At first, all seemed promising. Ordinary theorems of classical Peano arithmetic PA... were readily provable in $\mathsf{R}^\sharp$. More or less trivially, PA can be exactly translated into $\mathsf{R}^\sharp$. But, unlike PA, there are simple and effective proofs of the consistency of $\mathsf{R}^\sharp$ in several interesting senses. Moreover, since intuitionist logic is also translatable into R, many devices of constructive proof theory are immediately available.[1, p. 824]

The turning point in the fortunes of $\mathsf{R}^\sharp$ was ultimately the inadmissibility of γ. In the background of the monographs, Meyer acknowledges the importance of showing that $\mathsf{R}^\sharp$ recovers all of number theory. One of the fundamental assumptions at the heart of Meyer's project is that mathematical practice is in many ways prior to logic; if the number theorist finds that there are theorems that fail in $\mathsf{R}^\sharp$, this would be a blow to the applicability of the theory. Throughout the monographs, Meyer knows that Ackermann's γ—*modus ponens* for the material conditional—would be necessary to ensure that the whole of classical number theory is recoverable in $\mathsf{R}^\sharp$.

But Meyer was unable to prove that γ is admissible in $\mathsf{R}^\sharp$. In [1], Harvey Friedman and Meyer published Friedman's result producing a recognizably true theorem of number theory that is not provable in $\mathsf{R}^\sharp$. Because $\mathsf{R}^\sharp$ is conservative over its negation-free subtheory $\mathsf{R}^{\sharp+}$, Friedman and Meyer can provide a necessary condition for the admissibility of γ, namely, that for an extension P^+ of $\mathsf{R}^{\sharp+}$, every *strictly positive* theorem of PA is provable in P^+. By showing that the ring of complex numbers $\mathbb{C}$ is a model of P^+, Friedman and Meyer show that the *Quadratic Residue Formula*:

$$\forall x \exists y \forall z \exists a \exists b (a(2x+1) + b(y - z^2) = +1)$$

is a PA theorem not provable in P^+.

It is not necessarily clear how damaging the Friedman-Meyer result *should* be to $\mathsf{R}^\sharp$, much less for the project of formulating arthmetic relevantly in general. Meyer left open avenues to explore. For one, Meyer continued to establish interesting results about $\mathsf{R}^\sharp$ (see *e.g.* [3]), demonstrating that worthwhile investigations remained in the wake of the Friedman-Meyer result. Additionally, Meyer showed that γ was admissible in $\mathsf{R}^{\sharp\sharp}$—$\mathsf{R}^{\sharp}$ enriched with the infinitary ω-rule—entailing that PA is included in $\mathsf{R}^{\sharp\sharp}$. Secondly, if $\mathsf{R}^{\sharp}$ lacks sufficient power to prove all number-theoretic truths—and $\mathsf{R}^{\sharp\sharp}$'s infinitary nature makes it overly powerful—there remains the possibility of a "sweet spot" $\mathsf{R}^{\sharp 1/2}$ in which γ is admissible yet is demonstrably Post consistent by finitary means.

> But the real question posed by our main result is the following: Given that the straightforward approach of simply grafting the first-order Peano postulates into R has failed, is there a relevant way of thinking about the natural numbers which will produce a more satisfactory result? Hence our question: Whither relevant arithmetic?[1, p. 825]

This issue may not provide a definitive answer to the question with which Meyer has left us. But by providing new resources for the study of $\mathsf{R}^{\sharp}$ and contemporary thoughts on the project, it is our hope that we have moved the needle in the direction of a satisfactory answer to the question.

3 This Issue

We'll break up the description of this issue's contents into three parts: The corrected monographs on relevant arithmetic, the reprinted papers by Meyer and Mortensen, and the pieces original to this issue.

3.1 Meyer's Two Monographs on Relevant Arithmetic

Of the many papers collected in this issue, one of the clear highlights is the appearance of Meyer's previously unpublished monographs on $\mathsf{R}^{\sharp}$: *Arithmetic Formulated Relevantly* and *The Consistency of Arithmetic*. After exhaustive efforts, we are confident that the copies included here represent as detailed and complete a picture of Meyer's typescripts as possible.

Although the two monographs are similar in scope—a philosophical and mathematical investigation into $\mathsf{R}^{\sharp}$—each is an independent, self-contained work. Although it is inevitable that independent surveys of a formal subject like $\mathsf{R}^{\sharp}$ should see some overlap, the relationship between the two is more complicated than the duplication of necessary formal definition.

We have remarked on two ways in which the study of $\mathsf{R}^\sharp$ can be motivated—one conservative, the other radical. This division is reflected by the distinct approaches taken by the two monographs. As illustration, consider Meyer's explicit goals for each work. In his own words, *Arithmetic Formulated Relevantly* has the following aim:

> The purpose of this paper is to formulate first-order Peano arithmetic within the resources of relevant logic, and to demonstrate certain properties of the system thus formulated.

The monograph serves as a *vade mecum*—an open invitation to the study of $\mathsf{R}^\sharp$—in which the reader is provided an overview of Meyer's primary technical results and provided with reasons to consider the system worth pursuing. While the work includes some of Meyer's hallmark poking fun of various idols of mathematics and logic, the tone never becomes vitriolic. Rather, the tone is good-humored and light, and the mode of argumentation is conservative in the sense that we have suggested.

In contrast, *The Consistency of Arithmetic* is far more radical in its aims and tone. Again, in Meyer's words:

> This paper offers an elementary proof that formal arithmetic is consistent... *repeal*[ing] Gödel's famous second theorem... Accordingly, this paper *reinstates* the formal program which is often taken to have been blasted away by Gödel's theorems—namely, the Hilbert program.

While *Arithmetic Formulated Relevantly* is content to engage in good-humored lampooning of what Meyer perceives to be the classical idols, *The Consistency of Arithmetic* sets out to *raze* them entirely.

In conjunction, then, the two monographs form a unified whole presenting a mature and accessible theory of arithmetic that is solid in its foundations and far-reaching in its goals.

We must make a remark about the completeness of the texts. Unfortunately, all copies of the two typescripts of which we are aware are *incomplete*; in all cases, the text breaks off at a "fracture point" that clearly occurs prior to typescript's intended conclusion. Consequently, at least some elements of the investigations promised in the respective introductions are victims of the lacuna. Moreover, Meyer employs *endnotes* rather than *footnotes* and denotes bibliographical references with integers, guaranteeing that the texts' marginalia and bibliographies are likewise counted as casualties. It seems clear that the *majority* of the works' text exists;[1] judging by the outlines that Meyer sketches in the typescripts' introductions, the extant sections of both typescripts appear to cover the lion's share of the topics set out for discussion. Despite this reassurance—the fact remains that the reproductions offered in this issue suffer from these gaps.

Cross-referencing against other copies of the typescripts shows a constancy in the locations of the two respective "fracture points," *i.e.*, all copies of *Arithmetic Formulated Relevantly* end at the same passage. This suggests that the copies of each of the two works share an ancestral typescript that itself was incomplete, making it likely that the present reproductions of Meyer's two works provide as complete a realization as possible.

While little besides the topic of Meyer's footnotes can be recovered from the the text, the identity of many of the works *cited* by Meyer can be inferred by textual clues. We have made an effort to reconstruct as much of the works' respective bibliographies as could be reasonably inferred from the text.

3.2 Selections from Meyer's Papers

Some efforts at preserving documents stem from an *archaeological* impulse, according to which saving obscure historical artifacts from degradation is a good in itself. The motivation driving the effort to prepare and publish Meyer's monographs, however, goes beyond this. Nearly half a century after their writing, the content of Meyer's two monographs remains as powerful and compelling as ever. Meyer's arguments and insights concerning $\mathsf{R}^\sharp$ continue to be relevant to the philosophies of logic and mathematics; they deserve not merely to be *preserved* but to be *debated* and *grappled with*. In short, our goal is to help make Meyer's work on $\mathsf{R}^\sharp$ *actionable*.

This goal requires that the typescripts are provided with sufficient context to ensure that researchers not familiar with the esoterica of relevant logic may still benefit. To this end, the monographs in this issue are coomplemented

[1]There is some uncertainty with respect to whether the works were indeed *completed*. No source of the extant texts has suggested having seen copies that were in fact complete, suggesting that Meyer never finished either work. On the other hand, gaps in the bibliographies suggest the existence of a more complete work. *E.g.*, the text of *The Consistency of Arithmetic* includes citations to items [1], [2], [3], [5], and [7]; were items [4] and [6] part of a *planned* bibliography, or were they cited in sections that are now lost?

by three reprinted papers by Meyer and Chris Mortensen:

- **Relevant Arithmetic**
– *Robert K. Meyer*

Meyer's extended abstract, published in the *Bulletin of the Section of Logic* in 1976, acts as a frontispiece to the entire issue. The work is essentially a précis of the two unpublished monographs, offering Meyer a venue to announce his formal results on $\mathsf{R}^\sharp$ alongside the philosophical conclusions he draws therefrom. Although the monographs themselves included no abstracts, the approachable sketch of the program given by Meyer in this work provides the needed orientation, preparing the reader for the monographs that follow.

- **Inconsistent Models for Relevant Arithmetics**
– *Robert K. Meyer and Chris Mortensen*

Meyer's unpublished typescripts (or, at least, the extant copies thereof) share an unfortunate—arguably, catastrophic—feature. The most important theses of the monographs rest on the Post consistency of $\mathsf{R}^\sharp$. That $\mathsf{R}^\sharp$ has this property is indispensable to the integrity of the work—indeed, this feature constitutes one typescript's *very title*. Yet although each advertises a proof of this property, neither work *includes that proof*. This 1984 paper is the first appearance of the proof. Not only does its inclusion complement Meyer's typescripts by making good on their promises, it has broader dividends for the issue as a whole. Meyer and Mortensen's general technique of establishing Post consistency by appeal to a finite, inconsistent model has become a standard tool in paraconsistent mathematics, one that is employed in several of the contributed research papers as well.

- **Alien Intruders in Relevant Arithmetic**
– *Robert K. Meyer and Chris Mortensen*

Meyer and Mortensen's 1987 ANU technical report is a detailed study of the rich and surprising model theory of nonstandard integers in $\mathsf{R}^\sharp$, culminating in the description of a *rational model* of $\mathsf{R}^\sharp$, *i.e.*, a model extending $\mathbb{Q}$ in which the model views each rational as a positive integer. In contrast to the foregoing pieces—each of which shore up some deficiency in the monographs' text—the inclusion of this piece is less clear-cut. Its appearance undoubtedly enhances the monographs in demonstrating the rich possibilities for the study of $\mathsf{R}^\sharp$. Other work on $\mathsf{R}^\sharp$ could have played a similar role. Instead, the work stands out for sharing with the monographs a nearly total *inac-*

cessibility remedied by its inclusion in this issue. The negligible scale of its publication—no library known to WorldCat holds a copy—guaranteed that the work has been as inaccessible as Meyer's unpublished monographs.

3.3 Original Contributions

When planning this project, we thought that looking forward to future directions for Meyer's project was as important as the backward-facing project of preserving the work. In this issue, Meyer's own work is complemented by a collection of research papers original to this issue.

First, Chris Mortensen shared a short piece that provides additional context for the genesis of his joint work with Meyer:

• **A Remark on Relevant Arithmetic**
– *Chris Mortensen*

This is a short piece in which Mortensen recounts facts about the process and deliberations between himself and Meyer that would lead to the 1984 "Inconsistent Models for Relevant Arithmetics."

Additionally, this issue includes a series of peer-reviewed research papers that collectively illustrate a number of directions in which future research on $\mathsf{R}^\sharp$ can be taken.

• **Episodes in Model-Theoretic Xenology: Rationals as Positive Integers in $\mathsf{R}^\sharp$**[2]
– *Thomas Macaulay Ferguson and Elisángela Ramírez-Cámara*

This paper applies the model-theoretic tool of *ultraproducts* to the rational models of $\mathsf{R}^\sharp$ described by Meyer and Mortensen in their 1987 technical report. Defining rational models as ultraproducts allows a view into their structure with a level of detail not available in the original formulation. It is shown how the approach serves to demystify several of the rational models' counterintuitive features—like a Lagrangian four-square representation of a negative integer—by showing how they naturally arise from the definition of an ultraproduct's elements.

[2]We note that Thomas Ferguson recused himself from any editorial responsibilities bearing on this contribution. With respect to this article—to include its refereeing and decision—Graham Priest served as the sole editor.

• The Formalization of Arithmetic in a Logic of Meaning Containment
– Ross T. Brady

Whereas Meyer uses a strong relevant logic to formulate arithmetic, Ross Brady has championed an alternative arithmetic $\mathsf{MC}^\sharp$ based on a weaker relevant logic than R. In this paper, Brady critiques several aspects of Meyer's approach to relevant arithmetic and provides an interesting contrast by outlining the strengths his own project. As a positive result, Brady strengthens his earlier results on encoding primitive recursion in $\mathsf{MC}^\sharp$, showing that *general* recursion can be consistently represented as well.

• On Consistency and Decidability in Some Paraconsistent Arithmetics
– Andrew Tedder

We have remarked that one of the enduring legacies of Meyer and Mortensen's 1984 paper is the utility of inconsistent, finite models in the study of paraconsistent arithmetic. Tedder's contribution is a perfect example of the versatility of the technique. Tedder considers several arithmetic theories yielded by evaluating two weak axiomatizations of arithmetic against several paraconsistent logics (including the system $\mathsf{RM3}$), establishing a number of interesting results—and proposing a number of alluring conjectures—considering their complexity and decidability.

• "A Smack of Irrelevance" in Inconsistent Mathematics?
– Luis Estrada-González and Manuel Eduardo Tapia-Navarro

The *relevance* of the subformulae of a sentence $j = k \to m = m$ to one another is not immediately recognizable in case m is distinct from j and k; however, all such sentences are provable in $\mathsf{R}^\sharp$. Estrada-González and Tapia-Navarro focus their contribution on this and analogous cases in which the relevance of a conditional is questionable. Several positions on this question are carefully evaluated in the contexts of arithmetic and set theory. The authors suggest that the standard notions of relevance in the literature are too coarse to adequately cover such cases and give a lucid account of the factors bearing on the adequacy of notion and the stakes involved, before proposing a new criterion—*q-relevance*—that improves upon the stock formalizations of relevance.

• On Not Saying What We Shouldn't Have To Say

– *Shay Logan and Graham Leach-Krouse*

This piece takes as a starting point Meyer's remarks on guaranteeing the distinctness of two distinct numbers' successors by means of an axiom, namely, that one shouldn't "have to say" that this holds. There are, after all, prearithmetical intuitions about *numerals* that already provide sufficient grounds for the thought represented by the axiom. Logan and Leach-Krouse respond by providing a formalization of intuitions about the identity between syntactic terms by primitive recursive methods and show that the formal system indeed captures important prearithmetical intuitions about the distinctness of successors. Among the many points of intersection with Meyer's project are lessons to be drawn about relevance between equations and frameworks for exploring the space of arithmetics between $\mathsf{R}^\sharp$ and $\mathsf{R}^{\sharp\sharp}$.

• **Relevant Arithmetic and Mathematical Pluralism**
– *Zach Weber*

Weber appraises Meyer's project of relevant arithmetic from twin perspectives: a *pluralist* perspective, in which $\mathsf{R}^\sharp$ is just a *different* arithmetic than PA, and a *monist* perspective, in which there exists a "true" account of arithmetic which may or may not coincide with $\mathsf{R}^\sharp$ (or $\mathsf{R}^{\sharp\sharp}$). Importantly, not only are Meyer's positive remarks considered, but also the import of the Friedman-Meyer result of the inadmissibility of γ. Of special interest over the course of this paper are reexaminations of the proofs of two critical results about γ: its inadmissibility in $\mathsf{R}^\sharp$ and its admissibility in $\mathsf{R}^{\sharp\sharp}$, with an eye to whether the arguments can work in a *relevant* metatheory.

4 Administrivia

We conclude by tending to the various items of administrivia that accompany such a issue. Happily, nearly all such matters are acknowledgments.

The core objective of this issue is widening the reach and availability of Meyer's work on $\mathsf{R}^\sharp$; success or failure of this goal is inextricably tied to the quality of the vehicle of its distribution. Releasing the issue with the *Australasian Journal of Logic* guarantees that anyone can access the work immediately and freely. The *AJL* eliminates virtually all barriers—financial, temporal, geographical, institutional—to this goal and we could not imagine a more appropriate setting for this issue. Ed Mares' suggestion that this collection take the form of a special issue of this journal was a pivotal event

in the preparation of this issue. We are grateful to him for providing this venue and for entrusting us with stewardship of an issue.

The survival of the two monographs across half a century was possible only for the stewardship of a number of individuals who allowed us access to their copies. We owe thanks not only to Mike Dunn and Chris Mortensen—whose copies made up the primary sources for the versions reproduced in this issue—but for the help of countless others who took the time to compare notes against their own copies.

Similarly, the three works reproduced in this issue provide critical context for *Arithmetic Formulated Relevantly* and *The Consistency of Arithmetic*. Their inclusion guarantees that this issue is coherent, self-contained, and accessible, providing immeasurable value to readers; these enhancements were won only through the assistance of a number parties. First, Chris Mortensen's permission to reproduce the two papers that he had coauthored with Meyer was critical—which is not to say that any other of the many ways in which this issue has benefited from his assistance were any *less* critical. We thank him for his willingness to share his work with us, doubly so for patiently answering queries that arose during typesetting.

Similar acknowledgments are due to the journals in which the previously published works originally appeared. Permission to include the abstract "Relevant Arithmetic" was kindly granted by Prof. Dr Hab. Andrzej Indrzejczak (who also tracked down the LaTeXfile used by the *Bulletin of the Section of Logic*); we thank him—as well as Łodz University Press—for these efforts.

Likewise, we are grateful to Prof. Richard Shore, the Association for Symbolic Logic, and the *Journal of Symbolic Logic* for granting permission to reprint "Inconsistent Models for Relevant Arithmetics" in this collection. The copyright for the work is retained by the Association for Symbolic Logic.

It is not hyperbole to suggest that reading Meyer's work is a treasure, a sentiment that applies no less to the monographs published in this issue. The quality of the *content*—in formal results, insights, and arguments—is matched by Meyer's *writing*, which is uniformly funny and engaging. As potential companions to Meyer's writing, the contributed research papers faced exceptionally high expectations.

We recognize the challenge for contributed papers' authors—to produce work of a high caliber carries costs payable in many currencies—emotional, mental, and temporal, among others. The reader will recognize that the authors delivered work that can proudly stand side by side with Meyer's work. The authors' willingness to pay this toll to support our efforts is

acknowledged and appreciated.

Likewise, high demands made of authors, of course, give rise to high demands of the researchers who agreed to referee the contributions. Many anonymous referees were willing to share their time and expertise reviewing these contributions; despite their anonymity, we are grateful to them for their efforts, a sentiment that has been communicated to us by the authors as well.

Graham Priest would like it noted that all the hard editorial work of turning Meyer's texts into polished LaTeXpieces—and in fact most of the hard editorial work on the issue—was done by Thomas Ferguson.

One final, unfortunate note is necessary. As the last touches were being put to this issue, we received the sad news of the death of Bob's long-time friend and co-worker in relevant logic, J. M. (Mike) Dunn. As we have indicated, there are myriad ways in which Dunn's help was critical in assembling the present collection, which likely would not have been started without Dunn's early encouragement.

References

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