

DISCUSSION NOTE

Myers' paradox

Graham Priest^{1,2,3}

¹Department of Philosophy, CUNY Graduate Center, New York, NY

²Department of Philosophy, The University of Melbourne, Melbourne, Victoria, Australia

³Department of Philosophy, Ruhr University of Bochum, Bochum, Germany

Correspondence

Graham Priest, Department of Philosophy, CUNY Graduate Center, New York, NY, USA.

Email: priest.graham@gmail.com

Abstract

This note is an analysis of the paradox given by Meyers (2019). It is shown, assuming that the resources available in paraconsistent logic may be applied, how the conclusion of the paradox may be perfectly acceptable, but that the argument is, nonetheless, invalid. This provides a dialethic solution to the paradox.

KEYWORDS

dialetheism, intensional paradoxes, Meyers' paradox, paraconsistency, propositional identity, trivialism

1 | INTRODUCTION

In 'A paradox involving representational states and activities',¹ Blake Myers presents informally an interesting and novel paradox in the family of intensional paradoxes, the conclusion of which is that everything is possible—indeed, given certain assumptions about modality, that everything is true. Since such a conclusion is acceptable to nobody (except the trivialist—one who accepts everything), anyone concerned with paradoxes of this kind must explain why and how the argument fails.

Dialethic/paraconsistent solutions to the paradoxes of self-reference are, of course, well known.² Intensional paradoxes form a notable sub-class of these.³ The paradoxes of this kind produced to date have a natural paraconsistent/dialethic solution.⁴ In this paper, I will show that Myers' paradox does so too.

Specifically, I will present a formal version of the paradox (slightly cleaned up and simplified). I will use this to show how a dialetheist can accommodate the conclusion of Myers' argument without lapsing into trivialism. We shall also see, however, that dialetheism—indeed, even the weaker view that contradictions are true at *some* worlds—breaks the paradoxical argument.

The paradox deploys an intensional operator $S(x, p)$. Myers thinks of this as: x occurrently believes that p . But, as he notes, it could equally be: x says that p , x asserts that p , x writes that


p , and many other things. This does not affect the argument. In what follows, I will refer to it as *says that*.

Informally, the paradox goes like this. Let M be any sentence, and let G_M be the sentence ‘no one has said something which is true and entails M ’. First, it is not possible that $(G_M \wedge M)$ and someone says that $G_M \wedge M$. For if this were true, since $G_M \wedge M$ entails M , it would be false. Now consider a world where just one person says just one thing, namely, $G_M \wedge M$. It can’t be the case that $G_M \wedge M$. But G_M , since the only person who has said anything said $G_M \wedge M$, and this is not true. Hence $\neg M$.

2 | ARGUMENT STAGE 1

To formalise, let us break the argument up into its two stages. The informal argument uses the notion of entailment. For present purposes, the strict conditional, \rightarrow (where $A \rightarrow B$ is $\Box(A \supset B)$, \supset being the material conditional), is an object-language notion which is near enough the same thing.


So, let M be any sentence. Let G_M be the sentence:

- $\neg \exists x \exists p (p \wedge (p \rightarrow M) \wedge S(x, p))$ 

The first stage of the argument shows that for any a , $\neg \Diamond(G_M \wedge M \wedge S(a, G_M \wedge M))$. To show this, we suppose only that there is a class of possible worlds (for whatever notion of possibility is in question) and that these are closed under classical logic. Let w be a possible world, and write $w \Vdash A$ to mean that A holds at w . Suppose that:

- $w \Vdash G_M \wedge M \wedge S(a, G_M \wedge M)$

Since $(G_M \wedge M) \rightarrow M$ is a logical truth, we have:

- $w \Vdash G_M \wedge M \wedge ((G_M \wedge M) \rightarrow M) \wedge S(a, G_M \wedge M)$ 

Hence:

- $w \Vdash \exists x \exists p (p \wedge (p \rightarrow M) \wedge S(x, p))$

That is:

- $w \Vdash \neg G_M$

So:

- $w \Vdash \neg(G_M \wedge M \wedge S(a, G_M \wedge M))$

Thus we have:

- (0) $\neg \Diamond(G_M \wedge M \wedge S(a, G_M \wedge M))$

constant, $c, \delta(c) \in D$. For every monadic predicate, P , and world, w , $\delta_w(P) = \langle X, Y \rangle$, where $X \cup Y = D$. I will write X as $\delta_w^+(P)$ and Y as $\delta_w^-(P)$. $\delta_w^+(=) = \{\langle d, d \rangle : d \in D\}$.

To state the truth (\Vdash^+) and falsity (\Vdash^-) conditions for quantifiers, and to avoid the complexities of deploying the notion of satisfaction, we assume that the language has been augmented with a bunch of constants, k_d , for all $d \in D$, such that $\delta(k_d) = d$. The truth/falsity conditions are then as follows:

- $w \Vdash^+ Pa$ iff $\delta(a) \in \delta_w^+(P)$
- $w \Vdash^- Pa$ iff $\delta(a) \in \delta_w^-(P)$

- $w \Vdash^+ a = b$ iff $\langle \delta(a), \delta(b) \rangle \in \delta_w^+(=)$
- $w \Vdash^- a = b$ iff $\langle \delta(a), \delta(b) \rangle \in \delta_w^-(=)$

- $w \Vdash^+ \neg A$ iff $w \Vdash^- A$
- $w \Vdash^- \neg A$ iff $w \Vdash^+ A$

- $w \Vdash^+ A \vee B$ iff $w \Vdash^+ A$ or $w \Vdash^+ B$
- $w \Vdash^- A \vee B$ iff $w \Vdash^- A$ and $w \Vdash^- B$

- $w \Vdash^+ \exists x A$ iff for some $d \in D$, $w \Vdash^+ A_x(k_d)$
- $w \Vdash^- \exists x A$ iff for all $d \in D$, $w \Vdash^- A_x(k_d)$

- $w \Vdash^+ \diamond A$ iff for some w' such that wRw' , $w' \Vdash^+ A$
- $w \Vdash^- \diamond A$ iff for all w' such that wRw' , $w' \Vdash^- A$

$\Sigma \vDash A$ iff for every interpretation, I , if $@ \Vdash B$ for all $B \in \Sigma$, $@ \Vdash A$.

With this machinery available, we can now see the following.

Let τ be a world such that $\delta_\tau^+(P) = \delta_\tau^-(P) = D$, $\delta_\tau^+(=) = \delta_\tau^-(=) = D^2$. Then, as is easy to check with a simple induction, if A is any *non-modal* formula, $\tau \Vdash^+ A$. So if $@ R \tau$, $@ \Vdash^+ \diamond A$. Hence if every interpretation has such a world, $\vDash \diamond A$. Moreover, as a simple interpretation shows, $@$ is not the trivial world. For example, Pa may not be true there.

Next, if, in addition, $\tau R w$ iff $w = \tau$, then for any A , $\tau \Vdash^+ A$. So if every interpretation satisfies this extra condition, $\vDash \diamond A$, for *all* A . And as before, $@$ is not the trivial world.

Hence, a dialetheist can accept the conclusion of the argument without triviality.⁶ Indeed, the claim that everything is possible has been advocated by some paraconsistent logicians.⁷ The cost is that the accessibility relation is tightly constrained concerning τ . Other details concerning R are not determined, though. In particular, for worlds other than τ it can be universal. (That is, every world accesses every other world.) The trivial world has to be treated as a special case.

It would be natural enough to suppose that there are impossible worlds, and that τ is one of these. However, if \diamond is thought of as expressing logical possibility, the worlds accessible to $@$ under the accessibility relation should be restricted to the possible worlds. In that case, we would not have $\vDash \diamond A$. If, however, \diamond is thought of as expressing possibility in a much broader sense, possibility in *some* world, we would.

I note that accepting that everything is possible is no longer an option if the language is extended in certain ways. Thus, suppose that we add a “backwards looking” necessity operator, \square^{-1} , to the language, with the truth conditions:

- $w \Vdash^+ \Box^{-1}A$ iff for all w' such that $w'Rw$, $w' \Vdash^+ A$

(with appropriate falsity conditions). Then even with no constraints on R , if $@ \Vdash \Diamond \Box^{-1}A$, then $@ \Vdash^+ A$.⁹ Similarly, suppose that the language has an “actuality operator”, $\hat{@}$, with truth conditions:

- $w \Vdash^+ \hat{@}A$ iff $@ \Vdash^+ A$

(and appropriate falsity conditions). Then clearly, if $@ \Vdash^+ \Diamond \hat{@}A$, $@ \Vdash^+ A$.⁸

5 | ANALYSIS STAGE 2

Let us now move to a second stage of analysis. As we have just seen, with appropriate handling, a dialetheist can accept the conclusion of the argument; but they are by no means required to do so, as we shall see in this section. The argument of Stage 2 of the paradox does not work in a paraconsistent logic.

To see this, let us extend the language of the last section with propositional variables and quantifiers. We also add a propositional identity predicate, \doteq , and a binary relation $S(x, p)$.⁹

An interpretation for the language is a structure $\langle W, @, R, D, V, \delta \rangle$, where all is as before, except that $V = \{ \langle X, Y \rangle : X \cup Y = W \}$. If $v \in V$, I will write X as v^+ , and Y as v^- . One may think of members of V as propositions. (v^+ contains the worlds where the proposition is true; v^- contains the worlds where it is false.) $\delta_w^+(S), \delta_w^-(S) \subseteq D \times V$, where $\delta_w^+(S) \cup \delta_w^-(S) = D \times V$. $\delta_w^+(\doteq), \delta_w^-(\doteq) \subseteq V^2$, where $\delta_w^+(\doteq) \cup \delta_w^-(\doteq) = V^2$. $\delta_{@}^+(\doteq) = \{ \langle v, v \rangle : v \in V \}$.¹⁰

To specify the truth and falsity conditions of the language, we now also need to define the denotation of a formula, A (which is a proposition). Let us write this as $\delta(A)$. This and the truth/falsity conditions of formulas are defined by a joint recursion.¹¹ We assume (to handle to propositional quantifiers) that the language has been augmented with a new constant, k_v for each $v \in V$, such that $\delta(k_v) = v$.

For the denotation of formulas:

- $\delta(A) = \langle \{w : w \Vdash^+ A\}, \{w : w \Vdash^- A\} \rangle$

The truth/falsity conditions for the old language are as before. For the new vocabulary:

- $w \Vdash^+ k_v$ iff $w \in v^+$
- $w \Vdash^- k_v$ iff $w \in v^-$
- $w \Vdash^+ S(a, A)$ iff $\langle \delta(a), \delta(A) \rangle \in \delta_w^+(S)$
- $w \Vdash^- S(a, A)$ iff $\langle \delta(a), \delta(A) \rangle \in \delta_w^-(S)$
- $w \Vdash^+ A \doteq B$ iff $\langle \delta(A), \delta(B) \rangle \in \delta_w^+(\doteq)$
- $w \Vdash^- A \doteq B$ iff $\langle \delta(A), \delta(B) \rangle \in \delta_w^-(\doteq)$
- $w \Vdash^+ \exists p A$ iff for some $v \in V$, $w \Vdash^+ A_p(k_v)$
- $w \Vdash^- \exists p A$ iff for all $v \in V$, $w \Vdash^- A_p(k_v)$

The definition of validity remains unchanged.

Note that the propositional quantifiers behave as expected. Thus, one may prove by the appropriate joint recursion that:

- if $\delta(A) = \delta(B)$ then $\delta(C_p(A)) = \delta(C_p(B))$
- if $\delta(A) = \delta(B)$ then $@ \Vdash^\pm C_p(A)$ iff $w @ \Vdash^\pm C_p(B)$

Now suppose, for example, that $@ \Vdash^+ A_p(B)$. Let $v = \delta(B)$. Then $@ \Vdash^+ A_p(k_v)$. So $@ \Vdash^+ \exists pA$.¹²

Now, consider an interpretation and a world, w , where the following conditions are satisfied:

- $\delta(a) = d$
- $\delta_w^+(S) = \{\langle d, v \rangle\}$
- $w \in v^+ \cap v^-$
- M is some formula (e.g., Pa) which is just true at every world
- S , $=$, and $\dot{=}$ are classical predicates. (That is, their extensions and anti-extensions at all worlds are disjoint.)

Consulting Stage 1 of the argument, we can see that the deduction from G_M to $\neg G_M$ is valid, which shows that G_M is false at any world (since $G_M \vee \neg G_M$ holds there). Thus $w \Vdash^+ \neg(G_M \wedge M \wedge S(a, G_M \wedge M))$, which is (0). Verifying (1)–(3) requires a little preliminary work.

G_M is false at w . For G_M is: $\neg \exists x \exists p(p \wedge (p \rightarrow M) \wedge S(x, p))$. So $\neg G_M$ is:

- $\exists x \exists p(p \wedge (p \rightarrow M) \wedge S(x, p))$

When x is d and p is v , $w \Vdash p \wedge (p \rightarrow M) \wedge S(x, p)$. ($p \supset M$ is true at all worlds.) Hence, $w \Vdash \exists x \exists p(p \wedge (p \rightarrow M) \wedge S(x, p))$ as required.

But G_M is also true at w . G_M is: $\neg \exists x \exists p(p \wedge (p \rightarrow M) \wedge S(x, p))$, that is:

- $\forall x \forall p(\neg p \vee \neg(p \rightarrow M) \vee \neg S(x, p))$

If x is not d , then for any p , $w \Vdash \neg p \vee \neg(p \rightarrow M) \vee \neg S(x, p)$, because of the third disjunct; so $w \Vdash \forall p(\neg p \vee \neg(p \rightarrow M) \vee \neg S(x, p))$.

Suppose, then, that x is d .

If p is not v then $w \Vdash \neg p \vee \neg(p \rightarrow M) \vee \neg S(x, p)$, because of the third disjunct. If p is v then $w \Vdash \neg p \vee \neg(p \rightarrow M) \vee \neg S(x, p)$, because of the first disjunct. Hence $w \Vdash \forall p(\neg p \vee \neg(p \rightarrow M) \vee \neg S(x, p))$.

So $w \Vdash \forall x \forall p(\neg p \vee \neg(p \rightarrow M) \vee \neg S(x, p))$, as required.

Since $G_M \wedge M$ is both true and false at w , we may take v to be $\delta(G_M \wedge M)$. (1)–(3) now hold, as is easy to check.

Hence, all the premises of the argument hold at w . But the conclusion does not, since $w \Vdash^+ \neg M$. Where does the argument fail? It uses the disjunctive syllogism at the end to infer $\neg M$ from G and $\neg(G \wedge M)$. This is invalid. Indeed, both premises are true at w , and the conclusion is not.

The observant will have noted that the argument uses the syllogism earlier to infer (*), and also what amounts to the syllogism in two instances of the displayed deduction, in the form of *modus ponens*. Though these are not valid inferences, they are truth preserving, given that the relevant predicates are classical.

The very observant will also have noted that the premises are also true at w in our model if $\delta_w^+(S) = \{\langle d, v \rangle\} = \delta_w^-(S)$ and $v = \delta(G_M \wedge M)$. In this case, the argument breaks down because of the use of the disjunctive syllogism to infer (*). However, from the point of view of a solution to the paradox, supposing that d says something paradoxical is more plausible than supposing that d 's saying something is itself paradoxical.

6 | CONCLUSION

In this note, we have looked at Myers' paradox and how it may be handled if one is a dialetheist. Using a paraconsistent logic such as *LP* (or *FDE*) Myers' argument fails. Under certain circumstances, however, a dialetheist can simply accept the conclusion of Myers' argument. Everything is possible. Of course, if Myers' argument fails, it can provide no rationale for this conclusion or, therefore, for any dialetheism to which it might give rise. If such a rationale exists, it must concern the notion of possibility that is involved here, what properties it has, and what one is to make of a sense of possibility according to which everything is possible; and this is not the place to go into the matter.

What might be said about Myers' paradox by a logician of a more classical persuasion, I will leave for others to say. But at the core of the analysis of the failure of the paradoxical argument I have discussed, is a world where some person, a , says just one thing which is paradoxical. Perhaps a is a logician.¹³

ENDNOTES

¹ Myers (2019).

² See, for example, Priest (2006), Part 1, Bolander (2017).

³ Such paradoxes can be found in Russell (1903), Appendix B, Prior (1961), Kaplan (1995). Myers' paradox is, in fact, a modal variant of the kind of paradoxes considered by Prior.

⁴ See Priest (1991, 2018). Further discussion of the paradoxes can be found in Bacon and Uzquiano (2018).

⁵ See Priest (2008), chs. 7, 11a, 21. Modal semantics are often formulated without a "base world", @. However, it will be useful include such a world here to accommodate the non-classical behaviour of identity at some worlds, whilst allowing identity to maintain its standard properties.



⁶ But not without inconsistency. In modal *LP*, $\models \Box \neg (A \wedge \neg A)$, and hence $\models \neg \Diamond (A \wedge \neg A)$. Hence, if $\tau \Vdash^+ A \wedge \neg A$, $@ \Vdash^+ \Diamond (A \wedge \neg A) \wedge \neg \Diamond (A \wedge \neg A)$. I note that if we use modal *FDE* instead, where Excluded Middle does not hold, @ need not be an inconsistent world. Since modal *FDE* has no logical truths, anything of the form $\Diamond A$ can hold at it consistently. If we do this, however, the argument of Stage 1 of Myers' argument fails, since it uses Excluded Middle. (The argument of Stage 2 also breaks, since it breaks in the stronger *LP*, as we shall see.) I use *LP* in the analysis here not simply because I prefer it to *FDE* as the "one true logic", but to give Myers as much as possible whilst showing how his paradox may be accommodated.

⁷ For example, Mortensen (1989).

⁸ On these matters, see Humberstone (2011).

⁹ I note that when a propositional identity predicate is added to the language in the straightforward fashion, ambiguity may arise. Thus, consider $p \doteq Pa \wedge Pb$. This could mean either $p \doteq (Pa \wedge Pb)$ or $(p \doteq Pa) \wedge Pb$. To avoid this, one may formulate the syntax of the identity predicate as: if A and B are sentences, $(A \doteq B)$ is a sentence. As usual, outermost brackets can be dropped when they are not required for disambiguation.

¹⁰ These semantics for propositional quantifiers can be found outlined in Priest (2016), 13.2.1.

¹¹ Technically, this works as follows. Say that a formula, A , occurs in a subject position in contexts of the form $A \doteq B$, $B \doteq A$, and $S(a, A)$. Let the *length* of a formula be the greatest number of nested formulas in subject

position. Thus, if a formula has no occurrences of the form $A \doteq B$ or $S(a, A)$, its length is 0. And if A is of length n , and B is of length less than or equal to n , then formulas of the form $A \doteq B$, $B \doteq A$ and $S(a, A)$, are of length $n + 1$. Assuming that truth/falsity of formulas of length n are given, the denotations of formulas of length n are determined. And assuming that the denotation of formulas of length n are determined, the truth/falsity values of length of formulas of length $n + 1$ are determined.

¹² This raises the question of whether the existence of a world, τ , where everything is true and false, can be extended to this language. The answer, as things stand, is *no*. S and \doteq can be made to behave in the appropriate manner, but propositional quantification cannot. Thus, given the semantics, $\forall p p$ is just false at every world. A way around this is to deploy a variable-domain semantics for propositions. At @ the domain of propositions is as before (so propositional quantifiers still behave as required). But at τ we can take there to be but one proposition, v , which is both true and false there. k_v is then both true and false at τ ; and if $A_p(k_v)$ is both true and false there, so is $\exists p A$.

¹³ Many thanks go to Hartry Field and Blake Myers for comments on an earlier draft of the paper. Versions of the paper was given in 2019 to the Melbourne Logic Group and the conference Services to Logic: 50 Years of the Logicians' Liberation League, National Autonomous University of Mexico. Many thanks for comments go also to those present, and especially to Lloyd Humberstone and JC Beall. Finally, thanks go to some anonymous referees of this journal.

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