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BOOLEAN NEGATION AND ALL THAT

INTRODUCTION

The point of this paper is to reply to a certain charge that may be raised against a dialetheic solution to the semantic paradoxes. A major argument against consistent accounts of the paradoxes is they all make perfectly intelligible notions ineffible. The charge is simply that the dialetheist is no better off. Section 1 gives the background to the issue. Section 2 explains the charge. Sections 3–5 answer it, Section 5 being the heart of the matter. Sections 6–8 tidy up a few loose ends, generalise the issue and draw some final conclusions.

1. BACKGROUND TO THE CHARGE: INCONSISTENCY AND INCOMPLETENESS

First, then, the context of the issue. I advocate a dialetheic solution to semantic paradoxes such as the liar. The contradictions at the bottom line of such paradoxical arguments are *true*. Naturally, I also hold that the correct logic is a paraconsistent one, where the inference $\alpha, \neg\alpha \vdash \beta$ fails.[1]

One of the most damning objections to all other approaches to the semantic paradoxes turns on the fact that they end up making certain notions ineffable in the language for which semantics is being provided. Thus, on the orthodox Tarskian view, truth itself is ineffable. On Kripke's view it is non-truth (or maybe exclusion negation) that suffers the same fate. In Gupta and Herzberger's approach it is stability (or stable truth), and so on.[2] These "solutions" therefore purchase consistency at the price of expressive incompleteness. When one sees this pattern repeating itself one is forced to wonder whether it is more than coincidence. Indeed it is: there are deep reasons why consistent solutions to the semantic paradoxes all end up in the same logical boat. I will not pursue the issue here;[3] of

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more immediate importance is the question of why this is objectionable. To see why, just ask what is to be made of these ineffable notions by consistent theorists. An heroic stance is to argue that there are no such notions, that their intelligibility is illusory. Few people take this line; nor can they: for the notions concerned are usually an integral part of the semantics, and hence the solution, being offered. Indeed, normally one cannot even state the solution without using the notion in question. (Thus, the central claim of Kripe's account is that paradoxical sentences are non-true and non-false; the Gupta/Herzberger solution is that they are unstable.) The heroic view would therefore be self-refuting. An apparently more moderate approach is to relegate the notion in question to a different language, the meta-language, where the discouse of the theorist is therefore located. This manoeuvre, though, fares no better. The whole point of *solutions* to the liar paradox (as opposed to reformist suggestions as to how to change our language) is to show that our semantic discourse (about truth etc.) is, appearances notwithstanding, consistent. An attempt to show this which produces more such discourse, not in its own scope, therefore fails.[4]

The whole beauty of a dialetheic/paraconsistent solution to the paradoxes is that the tired old distinction between object language and meta-language is finally put to rest. This is not to say that we cannot distinguish between discourse and discourse about discourse – of course we can. It is to say that the claim that the meta-language is different from, in fact, expressively stronger than, the object language, is finally given its come-uppance. There is but one language, which is able, amongst other things, to spell out is own semantics. In particular, the *T*-scheme, $T\boldsymbol{\alpha} \leftrightarrow \alpha$, for *every* closed formula, α (with canonical name $\boldsymbol{\alpha}$), of the language, can be endorsed quite happily. Inconsistent the theory may be; expressively incomplete it is not.

2. THE CHARGE: THE INEXPRESSIBILITY OF BOOLEAN NEGATION

Bearing all this in mind, it was suprising to see this approach to the semantic paradoxes classified in a recent paper by Rich Thomason under the heading: *Hold certain seemingly intelligible notions ineffable*[5] (though he admits that the classification is not entirely happy). If this classification is justified, then obviously a large part of the dialetheic case against consistent theorists is under-cut.

The next question is 'Which notion is it that is supposedly inexpressible for the dialetheist?'. Thomason points to the failure of the disjunctive syllogism (DS), $\alpha \wedge (\neg\alpha \vee \beta) \rightarrow \beta$, and observes, quite correctly, that to reject a principle is equally to reject the existence of notions satisfying that principle. Which notion is in question here? Obviously one of: conjunction, negation, disjunction and implication. We can discount implication since the principle could equally relevantly be put in deductive form: $\alpha \wedge (\neg\alpha \vee \beta) \vdash \beta$. We can also discount disjunction since Thomason might equally have pointed to the failure of *ex falso quodlibet* (EFQ), $\alpha \wedge \neg\alpha \vdash \beta$. Finally, we can discount conjunction since the point applies equally to the inference $\alpha, \neg\alpha \vdash \beta$. Thus we are left with negation. The criticism is therefore that the dialetheist cannot express a notion of negation satisfying certain conditions.

At this point the argument engages with another issue: Boolean negation. The negation of relevant logics (which are *ipso fact* paraconsistent) does not, of course, satisfy DS or EFQ either. However, Meyer and others[6] have investigated logics which have two negations. One, so called De Morgan negation, does not satisfy DS or EFQ. The other, Boolean negation, satisfies all the proof-theoretic rules of classical negation. Both negations are given semantics, proof-theories etc. And a dialetheist approach to the semantic paradoxes can work only if Boolean negation is not allowed into the language. If it is allowed then, using the *T*-scheme and self-reference in the usual way, we can produce a sentence equivalent to its own Boolean negation, and hence deduce a Boolean contradiction, whence everything follows by Boolean EFQ. This seems to make Thomason's point precisely.

I shall explain why I take this argument to be incorrect, or at least, to seriously beg the question. Let us start by considering how one characterises Boolean negation. There are two ways one may do this, semantically and proof-theoretically. Consider the proof theoretic characterisation first.

3. BOOLEAN NEGATION BY PROOF THEORY

This way of characterising Boolean negation is to specify that it be governed by a set of rules which generate all (and only) those rules of inference that are valid according to the classical theory of negation. And this way of characterising Boolean negation certainly makes it inexpressible in a dialetheic solution to the semantic paradoxes – or at least makes it expressible only on pain of triviality.

How worried should a dialetheist be by this? Not at all. If one is free to introduce a connective by stipulating that it satisfy a certain collection of rules, then there is any number of ways of inducing triviality. For example, we might just introduce a new zero place connective, $*$, satisfying the rule $\alpha \leftrightarrow * \vdash \beta$. Then triviality follows from the instance of the T-scheme $T* \leftrightarrow *$.

The point, of course, is that stating that a connective be such as to satisfy a certain set of rules is no way to guarantee it a sense, that is, to insure its intelligibility. The point is a familiar one thanks to Prior and Tonk.[7] A dialetheist may just deny the intelligibility of Boolean negation as specified proof-theoretically. An independent argument for its intelligibility therefore needs to be given. As I noted above, the dialetheist can argue *ad hominem* against consistent theorists that the notions they declare to be ineffable are intelligible, because they themselves use these very notions to explain their views. A similar *ad hominem* point is not available against the dialetheist: a use of Boolean negation is by no means necessary in explaining dialetheic semantics. (*A* notion of negation may be, but this certainly need not be Boolean negation. This will become clearer below to those for whom it is not obvious.) Thus, a direct argument for the intelligibility of Boolean Negation is required.

4. CONSERVATIVE EXTENSION

The obvious arguments are provided by the standard replies to Prior. There are two of these, both of the form that some constraint must be applied to a set of rules if it is to determine sense. The first (Belnap's[8]), and I think less satisfactory, is that the addition of the new connective must produce a conservative extension. Does Boolean negation pass this test? Obviously not. Semantically closed theories with suitable underlying paraconsistent logics are known to be non-trivial. The addition of Boolean negation produces triviality.[9]

The notion of being a conservative extension is relative to a pre-existing notion of deducibility, and in particular, to the logical apparatus already present (as Belnap was at pains to note). It might, therefore, be replied that though Boolean negation produces a non-conservative extension if added to a theory with a dialetheic truth predicate (i.e., one satisfying the *T*-scheme), it does not if the theory has no such predicate. Its is therefore the addition of a dialetheic truth predicate which produces triviality; and it is this, therefore, which lacks sense.

Now, it is not always true that the addition of Boolean negation to a logic without a truth predicate produces a conservative extension. For example, the addition of Boolean negation to positive intuitionist logic is well known to be non-conservative (giving rise to Peirce's law, $((\alpha \rightarrow \beta) \rightarrow \alpha) \rightarrow \alpha$). However, it is not intuitionist logic that is at issue here (since Curry paradoxes rule this out for dialetheic purposes anyway, as we shall see). And for many relevant logics at least, adding Boolean negation does produce a conservative extension. Unfortunately, it is equally true that adding a truth predicate to many such logics also produces a conservative extension – even when machinery of self-reference is avialable.[10] Thus, it is the *joint* addition of Boolean negation and the truth predicate that produces a non-conservative extension. Where, then, should blame be laid? The conservative-extension test is silent on this issue, and we witness its limitations.

5. SOUNDNESS: BOOLEAN NEGATION BY SEMANTICS

The second, and I think more satisfactory reply to Prior (Stevenson's[11]), is that the rules of inference in question must answer to a satisfactory semantic account of the connective, in the sense that the rules are demonstrably sound according to the semantics.[12] This brings us to the second way of charactersing Boolean negation: in semantic terms.

What is an adequate semantic characterisation of Boolean negation to which the rules of Boolean negation answer? This raises the prior question of what an adequate semantics should be like anyway. The question is an important one. For if one is an intuitionist, or constructivist of some other stripe, no semantics can be adequate unless it gives semantic conditions which we can effectively recognise as obtaining when they do. If this view is correct then Boolean negation is an immediate casualty. For, as intuitionists have stressed, classical negation does not have such semantics.[13]

However, the majority of people who have discussed Boolean negation in the present context are not intuitionists. Neither do I want to argue that a semantics must be constructive to be adequate. (Indeed, I think the view is not correct, but that is another matter.) So I will set problems of effectivity aside. This only partly solves the problem of what a semantic account for Boolean negations should look like, however; for even the defenders of Boolean negation have given different kinds of semantic account (four-valued ternary relational, two-valued ternary relational, algebraic etc.). Fortunately, then, the differences between them are not relevant for what I wish to say. We may therefore take one semantics as an example. Similar points can be made for the others.

I will work with the 3-valued semantics of LP.[14] This is not only simple, but using it will bring home the point that most of the complexity of the semantics of relevant logics is irrelevant to the present issue. Truth values are just non-empty subsets of $\{0, 1\}$. Let us say that α is true (under an evaluation) just if 1 is in its truth value; it is false (under an evaluation) if 0 is in its truth value. The truth conditions of conjunction and disjunction are:

$\alpha \wedge \beta$ is true iff α is true and β is true
$\alpha \wedge \beta$ is false iff α is false or β is false

The truth conditions for disjunction are:

$\alpha \vee \beta$ is true iff α is true or β is true
$\alpha \vee \beta$ is false iff α is false and β is false

Notice that these truth conditions are already sufficient to show the equivalence of DS and EFQ, regardless of the truth conditions for negation. For $\alpha \wedge (\neg\alpha \vee \beta)$ entails $(\alpha \wedge \neg\alpha) \vee \beta$; and this entails β if $\alpha \wedge \neg\alpha$ does. Conversely, if DS holds then so does EFQ, by the usual Lewis argument. All these inferences involve only conjunction and disjunction essentially and are validated by the above semantics. Thus, to endorse either of DS or EFQ is to endorse both.

Let us turn to the truth conditions for negation. I will use $\neg$ for Boolean negation, now thought of as characterised semantically as follows:

$\neg\alpha$ is true iff α is not true
$\neg\alpha$ is false iff α is not false

For "De Morgan" negation I will use $\sim$. Its truth conditions are:

$\sim\alpha$ is true iff α is false
$\sim\alpha$ is false iff α is true

Note that the truth conditions for Boolean negation (unlike those for De Morgan negation) actually use the notion of negation, and hence are ambiguous, depending on whether this is itself De Morgan or Boolean negation (assuming for the present that the negation of English might be either). We do not need to resolve this ambiguity now. All we need to note is that a similar ambiguity will be inherited by any consequence drawn from these conditions.

Now, are the rules of Boolean negation, and crucially, EFQ, demonstrably sound with respect to these semantics? The argument for the soundness of EFQ goes essentially as follows:

(A) α and $\neg\alpha$ cannot both be true under an evaluation (from the truth conditions for Boolean negation). Thus the inference $\alpha, \neg\alpha \vdash \beta$ is semantically valid.

What we are to make of (A) depends on how, exactly, we take semantic consequence to be defined. The natural (and correct) definition of semantic consequence is:

$\alpha_1, \ldots, \alpha_n \models \beta$ iff for all evaluations, v, if $\alpha_1, \ldots, \alpha_n$ are true under v then so is β.

In this case argument (A) is an instance of EFQ. Is it valid? This may depend on what 'not' is taken to mean. If it is a De Morgan negation, then obviously not. If the argument is to work it must therefore be a Boolean negation. But now to insist that this inference is valid is just to insist on the correctness of the inference EFQ for Boolean negation, and this is exactly what we were supposed to be proving. The argument therefore begs the question viciously.

The situation is somewhat different if we take another possible (though incorrect) definition of semantic consequence:

> $\alpha_1, \ldots, \alpha_n \vDash \beta$ iff there is no evaluation, v, such that $\alpha_1, \ldots, \alpha_n$ are true under v but β is not.

In this case, argument (A) is a contraposed version of conjunction elimination, an inference that no one in the debate has (yet) called into question. But now the correctness of this definition itself comes into question, and its adequacy can be defended only by a similar question-begging. For we require of a semantically valid inference that it have inferential force. That is, we require that if $\alpha \vDash \beta$ then we can detach the consequent β from the premise α. But if the definition of semantic validity is as suggested, then to infer that β is true (under an evaluation) from the fact that α is true and $\alpha \vDash \beta$, is just (a variant of) DS: $\gamma, \neg(\gamma \wedge \neg\delta) \vdash \delta$, and so of EFQ itself. Thus, this argument for the semantic validity of EFQ in an adequate sense again begs the question.

It is sometimes said, by friends and foes of relevant logics alike, that the meta-language in which the semantics of relevant logics is given is classical. And if things are conceived of in this way then of course it begs no question to endorse EFQ or DS in the meta-language. But whether or not relevant logicians have conceived of their meta-language in this way, this is obviously not the way it should be conceived of in the present context. For, as I explained in Section 1, the whole *point* of the dialetheic solution to the semantic paradoxes is to get rid of the distinction between object language and meta-language. The logic for which semantics are being given must therefore be the logic in which the semantics are given. The arguments for the soundness of Boolean EFQ therefore constitute a serious *petitio*, and so fail.

6. BOOLEAN NEGATION NEGATED

I have now, in effect, answered the charge against the dialetheist that their solution to the semantic paradoxes works only by making some intelligible notion ineffable. If Boolean negation is characterised proof-theoretically, it is certainly inexpressible (on pain of triviality). However, in this case it cannot be shown to have determinate sense. Alternatively, if it is characterised in semantic terms it is certainly not ineffable: its truth conditions were given in the previous section. But now the supposed catastrophic consequences of its expressibility (in particular, the correctness of EFQ and DS) do not follow, or at least, can be shown to follow only by begging the question.

It may come as a shock to some that the rules of classical negation characterise a connective that lacks sense. (We have been talking gibberish since Boole!) But one should not forget that whether or not something makes sense is *theory dependent*. And the dialetheist lines up with the intuitionist in this battle: the classical theorist has got it wrong. Of course, there is nothing to prevent someone saying: I will, by fiat, use the sign $\neg$ according to the rules EFQ, DS etc. But this proves nothing. Someone can equally say: I am going to act according to the rule that people do not fall when they walk out of 10th storey windows. All may, indeed, go well, until a semantic paradox, or a 10th story window is confronted. Both theorists must then prevaricate or take the consequences.

There remains to be answered the question of which set of truth conditions capture our ordinary notion of negation (rather than the negation of the received theory). As we have seen, as far as the issues we have discussed go, it makes little difference. For just this reason, I doubt that it is possible to find decisive considerations one way or the other. But for my money, the correct conditions are the "De Morgan" ones. The fundamental symmetry between truth and falsity these enshrine constitutes the primordial intuition about negation. The fact that De Morgan negation does not satisfy inferences such as the DS, which we do appear to invoke, certainly constitutes a *prima facie* case against this claim; but this can be answered by giving an account of the legitimacy of these invocations which does not appeal to the deductive validity of the inference.[15] I suspect that one of the appeals of the view that the Boolean truth conditions are the correct conditions of negation arises from the mistaken thought that these deliver the validity of the classical rules of proof for negation. As we have seen, they do not (at least on their own); and once one sees this, this appeal is markedly diminished.

Having answered this question, the question left hanging in the previous section concerning what negation is used in giving the truth conditions of Boolean negation is also anwered. For this was just an ordinary English, that is "De Morgan", negation. It follows that the argument we discussed in the previous section for the validity of Boolean EFQ (or DS) is not just question begging, but downright fallacious.

Though it is not strictly relevant, this is a good place to say a word about the history of relevant logic.[16] Modal logic was born of a dissatisfaction with classical logic and in particular with material implication. Lewis took himself to be offering a *rival* to classical logic. History has changed this perception of the situation. Modal logic is not a rival to classical logic, but an *extension.* A modal logic can be seen as classical logic extended by an intensional functor or two. History has a strange way of repeating itself. Relevant logic, too, was born out of a dissatisfaction with material implication, and was proposed as a rival to classical logic (and modal logic to the extent that strict implication is supposed to give an account of implication). There are now certain voices, even those of erstwhile relevance logicians, according to whom it, too, should be seen as a mere extension of classical logic.[17] Classical logic is the logic of conjunction, disjunction and negation – Boolean of course. Relevant logic merely adds to this two funny functors, De Morgan negation and a relevant $\rightarrow$. Whatever relevant logicians as such think about this proposal, it should be clear that from the perspective of dialetheism, this view of the situation is not only wrong, but highly misguided. De Morgan negation is not some funny add-on functor: It is the correct theoretical account of the negation we commonly use and love. Boolean negation – to the extent that the term 'negation' is justified at all – is either unintelligible, or else does not have the properties of classical negation. *It* is therefore the additional funny functor if anything is. Dialetheic logic, unlike modal logic, does, therefore, provide a genuine rival theory to that provided by classical logic.

7. CURRY IMPLICATIONS

To return to the subject: Perhaps the most fundamental lesson to be learned from the previous sections, and one that is already well known to most perceptive logic teachers, is that to show the validity of certain rules of inference truth-conditions alone are not enough. Inference must be made from those conditions: and these inferences *may* be just those whose validity one is trying to demonstrate.[18]

A way of illustrating the same point is by considering another way dialetheic semantics might be thought to be expressively inadequate. It is not only EFQ that may cause triviality in a semantically closed theory. As was demonstrated by Curry, amongst others, and is now well known, the rule of inference Absorption (ABS), $\alpha \rightarrow (\alpha \rightarrow \beta) \vdash \alpha \rightarrow \beta$, does so too in the presence of *modus ponens* (MP).[19] Let us call any connective that satisfies ABS and MP a *Curry implication.* Then a dialetheist must hold that implication is not Curry. In fact, it is not difficult to give an account of implication according to which ABS is obviously invalid. For example if we take $\rightarrow$ to be a strict implication then the failure of reflexivity of the accessibility relation (in general, though it might hold on a restricted class of worlds) is sufficient to invalidate ABS, as even a straight classical logician will agree. (The philosophical rationale for this failure is, of course, another matter, which I shall not discuss here.[20])

But it might again be suggested that a dialetheist cannot allow *any* connective which is a Curry implication, and that this does result in an intelligible notion being made ineffable. Making the case for this fails for exactly the same reasons that the corresponding case for Boolean negation failed, however. If such an implication connective is characterised proof-theoretically then we may simply deny its intelligibility: it clearly fails the conservative extension test. Establishing that it passes the other test (that of answering to a satisfactory semantics) is the same task as demonstrating that an appropriate semantic characterisation for the connective shows it to validate ABS and MP. In the case of an implication there is much greater scope for giving different semantics than in the case of Boolean negation (due, in part, to the variety of restrictions one might put on accessibility relations). Fortunately, again however, exactly the same situation arises whatever semantics are given. I will therefore illustrate the situation in the simplest case, where implication is "material", that is, does not involve a "world-shift". (Throwing in a binary relation, a ternary relation or wot not, produces exactly the same dilemma.) To give truth conditions for a material connective, $\rightarrow$, one might use clearly extensional connectives such as disjunction and negation, as in:

$\alpha \rightarrow \beta$ is true iff α is not true or β is true.

or else one might use a conditional itself:

$\alpha \rightarrow \beta$ is true iff if α is true then β is true.

Both of these notions are quite intelligible, and effable. (I have just effed them.) But the first definition reduces MP to DS; whence it is not semantically valid. The argument to catastrophe is therefore broken.

What happens in the second case depends, of course, on what properties the conditional is taken to have. As may easily be checked, $\rightarrow$ inherits its properties from *if*. In particular, assuming that this *if* satisfies MP then so does $\rightarrow$. And the only way that $\rightarrow$ can be shown to satisfy ABS is to assume ABS for *if*. But this is to assume, in effect, that there is an intelligible Curry implication, which is just what was to be shown. The argument is therefore viciously circular, as was that for Boolean negation.

8. CONCLUSION

We have seen that proofs of soundness of (Boolean) DS, EFQ and of ABS – and hence the legitimation of these inferences – can be achieved only be appealing to the very form of reasoning in question. But this by no means implies that we have to fall back on classical reasoning willy-nilly. Many logical theories can provide the relevant boot-strapping. Decision between them has, therefore, to be made on other grounds. The grounds include the many criteria familiar from the philosophy of science: theoretical integrity (e.g., paucity of *ad hoc* hypotheses), adequacy to the data (explaining the data of inference – all inferences, not just those chosen from consistent domains!) and so on. This paper has not attempted to address these issues in general. All it demonstrates is that the charge that a dialetheist solution to the semantic paradoxes can be maintained only by making some intelligible notion ineffable cannot be made to stick. The dialetheist has a coherent position, endorsing the *T*-scheme, but rejecting DS, EFQ (even Boolean DS and EFQ) and ABS. And any argument to the effect that the relevant notions are both ineffable and intelligible begs the question. The case against consistent "solutions" to the semantic paradoxes therefore remains intact.

NOTES

[1] On these matters see Priest (1979, 1984a, 1984b) and esp. (1987b), part I. The following paper is a much fuller discussion of some comments in (1984b). See esp. fn 24 and the text thereto.

[2] See Priest (1987a), esp. sec 4, and Priest (1987b), ch 1, esp. sec. 7.

[3] It is pursued in Priest (1984a), sec 5, and the references in fn 2.

[4] This is discussed at greater length in the references cited in fn 2.

[5] Thomason (1986), pp. 226ff.

[6] See, e.g, Meyer and Routley (1973, 1974).

[7] Prior (1960).

[8] Belnap (1962).

[9] The point is quickly made for naive set theories. A number of logics are known to be non-trivial (though some of them are inconsistent) when augmented by the abstraction scheme of naive set theory.

$$(1)\qquad \forall x(x \in \{y; \varphi\} \leftrightarrow \varphi(y/x))$$

where / denotes substitution, and x is free for y in φ. (See, e.g., White, 1979; Brady, 1983, 1988.) Suppose we add Boolean negation, $\neg$, however. We quickly obtain $\varrho \leftrightarrow \neg\varrho$, where ϱ is $\{y; \neg y \in y\} \in \{y; \neg y \in y\}$, and hence triviality.

The point extends simply from set theory to semantics. Let $\boldsymbol{\alpha}$ be $\{x; \alpha\}$, where x is the least variable, in some standard enumeration, not occurring in α; and let Tx be $\phi \in x$. Then $T\boldsymbol{\alpha} \leftrightarrow \phi \in \{x; \alpha\} \leftrightarrow \alpha$ (by (1)). Hence, any such logic can non-trivially model the T-scheme; but the addition of Boolean negation occasions collapse. Indeed, under this mapping ϱ is just the liar sentence since $\varrho \leftrightarrow \neg T\boldsymbol{\varrho}$, as may easily be checked.

[10] For example, if $\nvdash_{LP}\alpha$ then there is a classical interpretation in which α is false (Priest (1979), III.13). But given any classical interpretation, it is possible to extend this to an LP model of the T-scheme in which all formulas without T retain their classical truth values (Dowden, 1984). A similar result for certain logics that contain a relevent $\rightarrow$ can be extracted from Brady (1988). I indicate the proof. Let X be any relevent logic of the kind that Brady considers; and let $\beta \notin X$. It is well known that for most relevant logics, X can be extended to a normal theory, X^+, such that $\beta \notin X^+$. (See Dunn, 1986, 2.4, 5; Routley *et al.*, 1982, 5.6.) Using X^+ and Brady's fixed-point constructions, we can define a model of the logic and the T-scheme in which members of X^+ are designated but β is not. (See the proof of Theorem 0, Priest, 1989.) The result follows.

[11] Stevenson (1961).

[12] '. . . we must show that, given a statement of the syntactic properties of a connective, the soundness of certain rules of inference can be demonstrated. . . we can state the syntactic [*sic*] properties of, say, a truth-functional binary sentence connective, 'o', by stating in the meta-language, the way in which the truth-value of the well-formed formula 'poq' is a function of (all possible combinations of) the truth values of the components '*p*' and '*q*'. We can then deduce from these statements, in a very rigorous

way, a meta-theorem of the calculus (again stated in the meta-language) to the effect that such-and-such permissive rules are sound.' *Op. cit.*, p. 126.

[13] On these points see, e.g., Dummett (1978a).

[14] See, e.g., the appendix of Priest (1980), or Priest (1987b), ch 5.

[15] As in Priest (1987b), ch 8 or Priest (1988).

[16] The parallel between the historical development of modal logic and that of relevant logic is discussed further in Priest (1986).

[17] See Meyer (1985), though the seeds of the idea can be found in Meyer (1974).

[18] For two essays on this theme, see Dummett (1978b) and Haack (1976).

[19] See, e.g., Meyer, Dunn, and Routley (1979), Priest (1980), sec. 8, Priest (1987b), ch. 6.

[20] The issue is taken up in Priest (1987b), ch, 6.

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