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To Be and not to Be: Dialectical Tense Logic

Abstract. The paper concerns time, change and contradiction, and is in three parts. The first is an analysis of the problem of the instant of change. It is argued that some changes are such that at the instant of change the system is in both the prior and the posterior state. In particular there are some changes from p being true to $\neg p$ being true where a contradiction is realized. The second part of the paper specifies a formal logic which accommodates this possibility. It is a tense logic based on an underlying paraconsistent propositional logic, the logic of paradox. (See the author's article of the same name *Journal of Philosophical Logic* 8 (1979).) Soundness and completeness are established, the latter by the canonical model construction, and extensions of the basic system briefly considered. The final part of the paper discusses Leibniz's principle of continuity: "Whatever holds up to the limit holds at the limit". It argues that in the context of physical changes this is a very plausible principle. When it is built into the logic of the previous part, it allows a rigorous proof that change entails contradictions. Finally the relation of this to remarks on dialectics by Hegel and Engels is briefly discussed.

1. Introduction

This paper concerns dialectical tense logic. In the first part (§ 2) I discuss considerations which make it plausible to suppose that true contradictions are realized in some changes. The next part (§ 3) specifies and investigates the properties of a basic tense logic which accommodates this idea. In the final part (§ 4) I discuss a principle which implies the existence of dialectical contradictions, and some of its ramifications.

2. The instant of change

2.1. This part is devoted to the problem of the instant of change. The problem is this. Let us suppose that before a time t_0 a system, S, is in a state S_0. After $t_0 S$ is in a state S_1. Accordingly, at t_0 it changed from S_0 to S_1. What state was it in at t_0? There appear to be three possibilities:

α) S is in exactly one of S_0, S_1.

β) S is in neither S_0 nor S_1.

γ) S is in both S_0 and S_1.

I shall call a change of the first kind a 'type-α change' and so on. Of course we should not suppose that all changes are necessarily of the same kind. What concerns me first is whether there are some type-γ changes. Accordingly I will examine type-α and type-β changes to see whether all changes could fall into these categories.

2.2. Let us start with type-α changes. A standard assumption is that all changes are of this kind. Let us examine this assumption. Changes are often represented by mathematical functions. Of course there is no guarantee that an arbitrary mathematical function represents a physically possible change, but let us assume for the moment that the following function v of one variable t, does:

$$\begin{aligned} v &= 0 \quad \text{for} \quad t \leqslant 0 \\ v &= t \quad \text{for} \quad t \geqslant 0. \end{aligned}$$

We may suppose that λ represents the velocity of a car moving off from rest. At time $t = 0$ the car changes from S_0, $v = 0$, to S_1 $v > 0$. At $t = 0$ the car is in state S_0. Hence this is unproblematically a type-α change. But consider now the acceleration of the car $\frac{dv}{dt}$. At $t = 0$ the car changes from S_0', $\frac{dv}{dt} = 0$ to S_1', $\frac{dv}{dt} = 1$. At time $t = 0$ $\frac{dv}{dt}$ is undefined. Thus this change is a type-β change.

Now let us address the question of whether all changes are type-α changes. What the previous example illustrates is that a change must be very "smooth" or it will give rise to type-β changes. More precisely, if any change is represented by a function which has a singularity where some derivative is undefined, it is a type-β change, for at the singularity the derivative is in neither the state it was in before nor the state it was in afterwards. On the other hand, if all changes are represented by functions all of whose derivatives are everywhere continuous, all changes are type-α changes. Thus, the answer to our question hinges on that of whether all physical changes are "smooth" enough.

A negative answer is provided by Charles Hamblin [4]. According to him, if an object moves off from rest "if no derivative ever changes discontinuously, nothing ever changes". (A point discontinuity of course gives an undefined singularity in the derivative.) Unfortunately this claim is incorrect, as is shown by the following example (for which I am grateful to Phil Schultz):

$$x = \begin{cases} 0 & \text{for} \quad t \leqslant 0 \\ e^{-1/t} & \text{for} \quad t > 0. \end{cases}$$

For every $n \geqslant 0$, $\frac{d^n x}{dt^n}$ is continuous everywhere, including $t = 0$.

I think that, in fact, there must be sufficiently "unsmooth" changes in nature to produce type-β changes. I have no knock-down argument for this, but I shall marshall what evidence I can find.

First, notice that the above function is unusual, in the sense that virtually any other common mathematical function substituted for $e^{-1/t}$ would produce discontinuities. This does not prove that physical changes described by the above function are unusual, but it is suggestive, especially since there is doubt that the function describes a physically possible change. For at $t = 0$ *nothing* is changing. Yet at any instant after $t = 0$, however close to zero, the object moves.

Secondly, and more importantly, there seem to be many changes which produce discontinuities of the appropriate kind. At the instant I hear a song or realize a problem solution, there are phenomenological changes which appear to be discontinuous. There are discontinuous quantum events such as the change of an atomic particle from one quantum state to another, and we needn't plumb the depths of modern physics to find discontinuous changes. They occur just as much in classical physics. The instant an electromagnetic wavefront propagated from an electromagnet hits an electron there is a discontinuity in the electron's acceleration.

It could be said that all these changes are only *prima facie* discontinuous — that despite appearances the changes really are continuous, though the changes occur so fast that we do not notice them and do not need to consider them for theoretical purposes. There is, perhaps, no refuting this. However, there is no evidence for this; quite the contrary, since it flies in the face of well-confirmed scientific theories. Thus it seems more than reasonable to suppose that there are some discontinuous changes and hence some type-β changes.

2.3. Could it be then that all changes are type-β changes, or at least either type-α changes a type-β changes? The answer to this is 'no' and is quickly seen. Simply consider a state changing from S_0 to S_1 at t_0 and suppose this to be a type-β change (i.e., at t_0 the system is in neither S_0 nor S_1). Consider the state of not being in the state S_0. Call this $\overline{S_0}$. Similarly for S_1. For $t \geqslant t_0$ the state is not in S_0; hence it is in state $\overline{S_0}$. Similarly, for $t \leqslant t_0$ the object is in state $\overline{S_1}$. Hence at t_0 the state is in both S_0 and S_1. This is therefore a type-γ change. In exactly the same way a type-γ change between S_0 and S_1 gives rise to a type-β change between $\overline{S_1}$ and $\overline{S_0}$. Thus type-β changes and type-α change always occur together. So if there are type-β changes, there must be type-γ changes. Hence we seem driven to the conclusion that there are type-γ changes.

2.4. I now want to concentrate on one particular sort of change. Let us consider a change from p being true to p being false (i.e. $\neg p$ being true.). We have seen that in general we must suppose that there are changes of all three types. I wish to argue that in the particular kind of change we are now considering we must still suppose there to be changes of types

β and γ. Of course — if classical logic is right — since all sentences are either true or false but not both, only type-α changes are possible here. However, classical logic is built on the unargued assumption that truth and falsity are exclusive and exhaustive, and it is exactly this point which is now at issue. The principles of classical logic cannot therefore be invoked without begging the question. I will try to show that type-β/γ changes occur, even in this context, by three examples. Here is the first.

As I write I come to the end of a word and my pen leaves the paper. The instant it leaves the paper is it on or not on (i.e. off) the paper? There seems no reason to say either. Certainly the matter is not settled by any physical theory of which I am aware. Neither, of course, is it a matter to be settled by observation. As if we could see whether it was on or not on at the instant of change! However, an objector could say that the pen is either on or off but not both, although it is in principle impossible for us to find out which. However, let us try a phenomenological example. For days I have been puzzling over a problem. Suddenly the solution strikes me. We are all familiar with this kind of situation (though for my part I wish it occurred to me more frequently!). Now, at the instant the solution strikes me, do I, or do I not know the answer. There appears to be no good reason for saying one rather than the other. All I know is that before, I could not say what the answer was, whilst after I could. The situation is symmetrical. Nor will it really do to say that at that instant I determinately either did or did not know the answer: It is just that we do not know and can not tell which. In the previous case it was possible to suppose that there was a determinate physical situation at the instant of change which obtained independently of our epistemological and perceptual abilities. However, there is no independent physical state in the present example. My epistemological state is all there is. It makes little sense to suppose that at the instant of change I either did or did not know the answer, though it is impossible to know which.

One more example will suffice. I am in a room. As I walk through the door, am I in the room or not in (out of) it? To emphasize that this is not a problem of vagueness, I will identify my position with that of my centre of gravity and the door with the vertical plane passing through the centre of gravity of the door frame. As I leave the room there must be an instant at which the point lies on the plane. Am I in or out of the room? Clearly there is no reason for saying one rather than the other. This can not therefore be a type-α change. One could, I suppose, *stipulate* that I was in rather than out. However, this is not a solution. Rather, it underlines the problem. I am free to stipulate either way, and this is because there is no determinable answer, i.e. it is neither in rather than out, nor out rather than in.

2.5. We have seen that some changes from p being true to $\neg p$ being true (or vice versa) are not type-α changes. Are they type-β changes or type-γ changes? I wish to argue that they are at least type-γ changes. To this end let us assume that they are type-β changes. We will see that they are type-γ changes too. If the change from p being true to $\neg p$ being true is a type-β change, then, at the instant of change both p and $\neg p$ fail and are false. Since p is false $\neg p$ is true, and since $\neg p$ is false, $\neg\neg p$ is true. Hence both $\neg p$ and $\neg\neg p$, and presumably therefore p, are true and we have a type-γ change. Another way to look at the matter is this. The following is valid classically, intuitionistically, relevantly and virtually every other way

$$\neg(p \vee \neg p) \leftrightarrow \neg p \wedge \neg\neg p.$$

The lefthand side appears to correctly describe the situation at the instant of a type-β change from p to $\neg p$. The righthand side describes the situation at the instant of a type-γ change from $\neg p$ to $\neg\neg p$. Yet they hold or fail together. This train of thought, surprising at first, is in fact little more than a corollary of something we have already noted. In **2.3** we saw that type-β changes and type-γ changes are dual and hold or fail together. In the case we are considering the duality is, in fact, an identity: to be neither true nor false, is to be both true and false!

This reasoning, though I think correct, is not mandatory. One can avoid it if one is prepared to reject the claim that if a sentence is not true, its negation is true. For then, even though one admits that at the instant of change, neither p nor $\neg p$ is true, one does not have to admit that both $\neg p$ and $\neg\neg p$ are true. Similarly if neither p nor $\neg p$ is true then $p \vee \neg p$ is not true either. However, one cannot move from this to the claim that its negation is true and hence to the truth of $p \wedge \neg p$. This line can be maintained, however, only by a rejection of the T-scheme. For the crucial negation principle, i.e.

$$\neg Tp \rightarrow T\neg p$$

(with the obvious notation) follows from

$$T\neg p \leftrightarrow \neg p$$
$$\neg Tp \leftrightarrow \neg p$$

and the first of these is an instance of the T-scheme, whilst the second is the contraposition of one.

Now I think that a rejection of the T-scheme is wrong, and hence this way of avoiding the existence of type-γ changes incorrect. However, backing this up would require a detailed defence of the T-scheme, requiring a long detour. Hence I will summarise this section by saying that there are very plausible arguments pushing us towards the conclusion that there are type-γ changes from p to $\neg p$; i.e., in some changes contradictions are realized.

2.6. There is one more thing that needs to be discussed briefly. The whole discussion so far has been predicated on the assumption that time has instants. Obviously the problem of the instant of change (and the conclusion I have drawn from it) disappears if this is denied. This had led some people to suggest that time is composed of intervals, rather than instants (see [4]), and systems of tense logic [6] have been constructed on this assumption. I wish to make just a couple of remarks about this kind of approach to the problem.

First, a good part of our science is based on the assumption that physical continua have a structure represented by the real line and therefore that we can speak of instants. In particular, any science which uses the differential and integral calculus presupposes this. Therefore the rejection of this assumption would cause the demise of a good part of science. This more than justifies solutions to the problem of the instant of change which retain the assumption that time can be represented by the real line. Of course it is possible that some or much of the mathematics used in science can be reconstructed on the interval theory of the continuum. However, the extent to which this can be done is not at all clear.

Secondly, and perhaps more importantly, the interval thesis may well solve the problem of the instant of change. However, it does so only by producing a curious account of change. For suppose that during a certain time a state S changes from S_0 to S_1. Then there must be two abutting intervals a and b such that a wholly precedes b, S_0 is true throughout a and S_1 is true throughout b. Now given that there is no instant dividing a and b we can not ask whether S is in S_0 or S_1 at it. However, because there is no such instant, there is no time at which the situation *is changing*: a is before the change, b is after it. Thus, in a sense, there is no change in the world at all, just a series of different states patched together! The universe would appear to be more like a series of photographic stills shown consecutively, than something in a genuine state of flux or change. And this certainly runs counter to our intuitions concerning the way things are. Again this makes it highly desirable to investigate alternatives.

3. Dialectical tense logic

3.1. I have argued for the conclusion that change may involve the realization of a true contradiction. It might be thought impossible to build this insight into a system of formal logic and this, in fact, might be considered as an argument against it. However, it is impossible only if we work with classical or a similar logic which *presupposes* that nothing is both true and false. Obviously a dialectical logic needs to be based on a paraconsistent propositional logic. This section will show that it is possible to have a formal dialectical logic based on a paraconsistent logic. I will use as a base the system *LP* of my paper [10] as reformulated in the appendix to [11]. This is not the only system of paraconsistent logic that has been mooted, but it is certainly the simplest.

The semantics concern a language L whose formulas, F, are those obtained from a set of propositional parameters P by means of the connectives $\wedge$, $\vee$, $\neg$. The "truth values" are the set, V, $\{\{0\}, \{1\}, \{0,1\}\}$ (false, true, and both). A fourth truth value $\varnothing$ (neither) is possible. However, as argued in **2.5** I will take *neither true nor false* to be the same as *both true and false*. Hence this fourth value is unnecessary. Given any evaluation, v, of the propositional parameters ($v: P \to V$) this is extended to an evaluation (which we will also write as v) of all formulas by the following conditions:

1(a)	$1 \in v(\neg A)$	iff	$0 \in v(A)$
(b)	$0 \in v(\neg A)$	iff	$1 \in v(A)$
2(a)	$1 \in v(A \wedge B)$	iff	$1 \in v(A)$ and $1 \in v(B)$
(b)	$0 \in v(A \wedge B)$	iff	$0 \in v(A)$ or $0 \in v(B)$
3(a)	$1 \in v(A \vee B)$	iff	$1 \in v(A)$ or $1 \in v(B)$
(b)	$0 \in v(A \vee B)$	iff	$0 \in v(A)$ and $0 \in v(B)$.

These are of course exactly the classical truth conditions, except that in the classical case, **(b)** of each pair is redundant. It is easily checked that $v: F \to V$.

3.2. To obtain a tense logic, the language L is extended in the usual way to a language L' by the addition of two new one-place operators F, P. We will call the new set of formulas F'. An *interpretation* for L' is a pair $\langle <, v \rangle$ where $<$ is a relation with domain X and v is a function with domain X such that for all $x \in X$, v_x: $P \to V$. I will allow $<$ to be totally arbitrary. Hence the system of tense logic I specify, which I will call DTL, will be the paraconsistent equivalent of Lemmon's minimal tense logic K_t (For this and other details of the classical case referred to, see Rescher and Urquhart [12] Chs 6–8.) I will discuss briefly extensions of this system in **3.5**.

Given an interpretation, v_x can be extended to an evaluation (which we will also write as v_x) of all formulas by the following conditions.

The condition for $\wedge$, $\vee$, $\neg$ are as for *LP*.

4(a)	$1 \in v_x(PA)$	iff	$\exists y < x$	$1 \in v_y(A)$
(b)	$0 \in v_x(PA)$	iff	$\forall y < x$	$0 \in v_y(A)$
5(a)	$1 \in v_x(FA)$	iff	$\exists y > x$	$1 \in v_y(A)$
(b)	$0 \in v_x(FA)$	iff	$\forall y > x$	$0 \in v_y(A)$.

Again these are just the truth conditions of standard tense logic except that in the classical case, **(b)** of each pair is redundant. Thus PA is true if at some past time A was true and PA is false, if A has always been false. It is easy to check that for all $x \in X$ v_x: $F' \to V$. As usual GA can be defined as $\neg F \neg A$ and HA can be defined as $\neg P \neg A$. It is then simple to check that

6(a)	$1 \in v_x(HA)$	iff	$\forall y < x$	$1 \in v_y(A)$
(b)	$0 \in v_x(HA)$	iff	$\exists y < x$	$0 \in v_y(A)$
7(a)	$1 \in v_x(GA)$	iff	$\forall y > x$	$1 \in v_y(A)$
(b)	$0 \in v_x(GA)$	iff	$\exists y > x$	$0 \in v_y(A)$

Finally, we can define semantic consequence in the standard way:

$\Sigma \models A$ iff for all interpretations I and all x in the domain of the first member of I, either $1 \in v_x(A)$ or for some $B \in \Sigma$ $1 \notin v_x(B)$.

3.3. I will now give a proof theory for the above semantics. In contrast to the semantics, the proof theory could hardly be said to be simple or natural. It could, I am sure, be simplified. However, the version given facilitates the completeness proof of the next section. I will give the proof theory in the form of a natural deduction system in the style of Prawitz [9]. I will use his terminology and notation with which I shall assume familiarity. In addition, a two-way rule $\dfrac{A}{\overline{B}}$ is equivalent to the two rules $\dfrac{A}{B}$ and $\dfrac{B}{A}$. The rules are as follows:

$$(1)\quad \frac{A \wedge B}{A \quad (B)} \qquad\qquad (2)\quad \frac{A \quad B}{A \vee B}$$

$$(3)\quad \frac{\neg(A \wedge B)}{\overline{\neg A \vee \neg B}} \qquad\qquad (4)\quad \frac{A \quad (B)}{A \vee B}$$

$$(5)\quad \frac{\begin{matrix}[A] & [B] & \\ \vdots & \vdots & \\ C & C & A \vee B\end{matrix}}{C} \qquad\qquad (6)\quad \frac{\neg(A \vee B)}{\overline{\neg A \wedge \neg B}}$$

$$(7)\quad \frac{A}{\neg\neg A} \qquad\qquad (8)\quad \frac{}{A \vee \neg A.}$$

The above rules provide a sound and complete rule system for LP as the completeness proof of the next section, in effect, shows. To obtain the rules for DTL we add:

$$(9)\quad \frac{A}{HFA} \qquad\qquad (10)\quad \frac{G\neg A}{\overline{\neg FA}} \qquad\qquad (11)\quad \frac{FHA}{A}$$

$$(12)\quad \frac{GA \wedge FB}{F(A \wedge B)} \qquad\qquad (13)\quad \frac{G(HB \vee C)}{B \vee GC}$$

(14) $$\dfrac{\begin{matrix}\overline{A}\\ \vdots\\ B\end{matrix}\quad FA}{FB}$$ where A is the only undischarged assumption on which B depends.

(15) $$\dfrac{\begin{matrix}\Pi\\ B\end{matrix}\quad GA_1 \ldots GA_n}{GB}$$ Where $A_1, \ldots, A_n$ are all the undischarged assumptions of Π. They are discharged by an application of this rule.

(16) (Mirror-image rule). Any rule obtained from rules (9)–(15) by systematically replacing 'F' by 'P' and 'P' by 'F' is a rule.

Proof-theoretic consequence is defined as usual, viz.:

$\Sigma \vdash A$ iff there is a proof tree whose bottom formula is A and all of whose undischarged assumptions are in Σ.

A few facts about proof-theoretic consequence are useful. Since their proofs are mostly trivial, I will just state them.

i) If $\Delta \vdash A$ and $\Sigma \cup \{A\} \vdash B$, $\Delta \cup \Sigma \vdash B$
ii) If $\Delta \cup \{A\} \vdash C_1$ and $\Sigma \cup \{B\} \vdash C_2$, $\Delta \cup \Sigma \cup \{A \vee B\} \vdash C_1 \vee C_2$
iii) If $A \vdash B$ and $B \vdash C$, $A \vdash C$.
iv) $HA \vee HB \vdash H(A \vee B)$
v) If $A \vdash B$, $FA \vdash FB$
vi) $GA \wedge FB \vdash F(A \wedge B)$
vii) $GA \wedge GB \vdash G(A \wedge B)$
viii) If $A \vdash B$, $GA \vdash GB$
ix) $G(HA \vee B) \vdash A \vee GB$
x) $G\neg A \vdash \neg FA$ and $\neg FA \vdash G\neg A$.

I will use these facts in what follows without further mention.

THEOREM 1. *If* $\Sigma \vdash A$, $\Sigma \vDash A$.

PROOF. This is proved in the usual way, by a recursion over the formation of proof trees. All that needs to be checked is that the rules preserve truth in the appropriate sense. This is straightforward and left to the reader.

3.4. This section will present a completeness proof for DTL. The proof proceeds via a couple of definitions and lemmas and uses the canonical model construction:

DEFINITIONS.

(i) A set of formulas of L' is *deductively closed* iff, if $\Delta \vdash A$, then $A \in \Delta$. (Evidently the converse condition is always true.)

(ii) A set of formulas of L' is *prime* iff if $\Delta \vdash A \vee B$ then $\Delta \vdash A$ or $\Delta \vdash B$.

LEMMA 1. *If Δ is prime and deductively closed.*

i) $A \wedge B \in \Delta$ *iff* $A \in \Delta$ *and* $B \in \Delta$
ii) $A \vee B \in \Delta$ *iff* $A \in \Delta$ *or* $B \in \Delta$
iii) $\neg(A \wedge B) \in \Delta$ *iff* $\neg A \in \Delta$ *or* $\neg B \in \Delta$
iv) $\neg(A \vee B) \in \Delta$ *iff* $\neg A \in \Delta$ *and* $\neg B \in \Delta$
v) $A \in \Delta$ *iff* $\neg\neg A \in \Delta$
vi) $A \in \Delta$ *or* $\neg A \in \Delta$.

PROOF. For i): if $A \wedge B \in \Delta$, $\Delta \vdash A$ (by rule 1). So $A \in \Delta$. Similarly $B \in \Delta$. Conversely if $\{A, B\} \subseteq \Delta$, $\Delta \vdash A \wedge B$ (by rule 2). Hence $A \wedge B \in \Delta$. For ii) if $A \vee B \in \Delta$, $A \in \Delta$ or $B \in \Delta$ by primeness. Conversely if $A \in \Delta$ (or $B \in \Delta$) $\Delta \vdash A \vee B$ (by rule (4)). Hence, $A \vee B \in \Delta$. iii)-vi) are proved similarly by invoking rules 3 and 6–8.

To state the next lemma a piece of notation will be useful. If Σ is a set of sentences, $\Sigma_\vee$ will be the closure of Σ under disjunction.

LEMMA 2. *Let Σ, Π be two sets of formulas, such that for no $A \in \Pi_\vee$, $\Sigma \vdash A$. Then there is a set Δ such that*

i) $\Delta \supseteq \Sigma$
ii) *for no* $A \in \Pi_\vee$, $\Delta \vdash A$
iii) Δ *is deductively closed*
iv) Δ *is prime*.

PROOF. Let $A_1, A_2 \ldots$ be an enumeration of the formulas of L'. Define a sequence of sets of formulas $\Delta_0, \Delta_1 \ldots$ thus:

$$\Delta_0 = \Sigma.$$

If there is a $B \in \Pi_\vee$ such that $\Delta_n \cup \{A_n\} \vdash B$.

$$\Delta_{n+1} = \Delta_n.$$

Otherwise $\Delta_{n+1} = \Delta_n \cup \{A_n\}$

$$\Delta = \bigcup_{n<\omega} \Delta_n.$$

i) and ii) clearly hold since $\vdash$ is compact. To show iii) suppose $\Delta \vdash A$ but $A \notin \Delta$. Then for some n and $B \in \Pi_\vee$, $\Delta_n \cup \{A\} \vdash B$. Hence $\Delta \vdash B$, which is impossible. To show iv) suppose that $\Delta \vdash A \vee B$ but $A \notin \Delta$ and $B \notin \Delta$. Then for some n, m and $C_1, C_2 \in \Pi_\vee$, $\Delta_n \cup \{A\} \vdash C_1$, $\Delta_m \cup \{B\} \vdash C_2$. Hence if $k = \max(m, n)$ $\Delta_k \cup \{A \vee B\} \vdash C_1 \vee C_2$. Thus $\Delta \vdash C_1 \vee C_2$ which is impossible since $C_1 \vee C_2 \in \Pi_\vee$.

LEMMA 3. *Let Γ be any prime deductively closed set such that $FA \in \Gamma$. Then there is a prime-deductively closed set Δ such that*

i) $A \in \Delta$
ii) *if* $GB \in \Gamma$, $B \in \Delta$
iii) *if* $HD \in \Delta$, $D \in \Gamma$ (*i.e., if* $D \notin \Gamma$, $HD \notin \Delta$)

PROOF. Let $\Sigma = \{A\} \cup \{B \mid GB \in \Gamma\}$, $\Pi = \{HD \mid D \notin \Gamma\}$. I claim that for no $C \in \Pi_\vee$, $\Sigma \vdash C$. Applying Lemma 2 now gives the result. To prove the claim, suppose that for some $C \in \Pi_\vee$, $\Sigma \vdash C$. Then for some $B_1 \ldots B_n \in \Sigma - \{A\}$ and some $D_1 \ldots D_m, \notin \Gamma$,

$$\{B_1 \ldots B_n, A\} \vdash HD_1 \vee \ldots \vee HD_m.$$

Hence

$$\begin{aligned} B_1 \wedge \ldots \wedge B_n \wedge A &\vdash H(D_1 \vee \ldots \vee D_m) \\ F(B_1 \wedge \ldots \wedge B_n \wedge A) &\vdash FH(D_1 \vee \ldots \vee D_m) \\ F(B_1 \wedge \ldots \wedge B_n \wedge A) &\vdash D_1 \vee \ldots \vee D_m \\ G(B_1 \wedge \ldots \wedge B_n) \wedge FA &\vdash D_1 \vee \ldots \vee D_m \\ GB_1 \wedge \ldots \wedge GB_n \wedge FA &\vdash D_1 \vee \ldots \vee D_m. \end{aligned}$$

But $GB_1 \wedge \ldots \wedge GB_n \wedge FA \in \Gamma$. Thus $D_1 \vee \ldots \vee D_m \in \Gamma$. Hence $D_1 \in \Gamma$ or ... or $D_m \in \Gamma$, which is impossible.

LEMMA 4. *Let Γ be any prime deductively closed set such that $\neg FA \notin \Gamma$. Then there is a prime-deductively closed set Δ such that*

i) $\neg A \notin \Delta$

ii) *If* $GD \in \Gamma$, $D \in \Delta$

iii) *If* $HD \in \Delta$, $D \in \Gamma$ (*i.e., if* $D \notin \Gamma$, $HD \notin \Delta$).

PROOF. Let $\Sigma = \{B \mid GB \in \Gamma\}$, $\Pi = \{\neg A\} \cup \{HD \mid D \notin \Gamma\}$. I claim that for no $C \in \Pi_\vee$, $\Sigma \vdash C$. Applying Lemma 2 now gives the result. To prove the claim suppose that for some $C \in \Pi_\vee$, $\Sigma \vdash C$. Then for some $B_1 \ldots B_n \in \Sigma$ and some $D_1 \ldots D_m \notin \Gamma$

$$\{B_1 \ldots B_m\} \vdash HD_1 \vee \ldots \vee HD_m \vee \neg A.$$

Hence

$$\begin{aligned} B_1 \wedge \ldots \wedge B_m &\vdash HD_1 \vee \ldots \vee HD_m \vee \neg A \\ B_1 \wedge \ldots \wedge B_m &\vdash H(D_1 \vee \ldots \vee D_m) \vee \neg A. \\ G(B_1 \wedge \ldots \wedge B_m) &\vdash G\big(H(D_1 \vee \ldots \vee D_m) \vee \neg A\big). \\ GB_1 \wedge \ldots \wedge GB_n &\vdash G\big(H(D_1 \vee \ldots \vee D_m) \vee \neg A\big). \\ GB_1 \wedge \ldots \wedge GB_n &\vdash D_1 \vee \ldots \vee D_m \vee G\neg A. \\ GB_1 \wedge \ldots \wedge GB_n &\vdash D_1 \vee \ldots \vee D_m \vee \neg FA. \end{aligned}$$

Since $GB_1 \wedge \ldots \wedge GB_n \in \Gamma$,

$$D_1 \vee \ldots \vee D_m \vee \neg FA \in \Gamma.$$

Hence D_1 or ... or D_m or $\neg FA \in \Gamma$, which is impossible.

THEOREM 2. *If* $\Sigma \models A$, $\Sigma \not\models A$.

PROOF. I will prove the contrapositive. Suppose $\Sigma \not\models A$. Define the interpretation $\langle <, v \rangle$ as follows. As the domain of $<$, take the set of prime-deductively closed sets of sentences, and let $\Delta_1 < \Delta_2$ iff

i) for all A if $GA \in \Delta_1$, $A \in \Delta_2$

ii) for all A if $HA \in \Delta_2$, $A \in \Delta_1$.

(In the classical case, where maximal consistent sets are the elements of the domain of $<$, these two conditions are equivalent. This is not so (as far as I can see) in the present case.)

v is defined thus:

$$(*)\quad \left.\begin{array}{lll} 1 \in v_\Delta(p) & \text{iff} & p \in \Delta \\ 0 \in v_\Delta(p) & \text{iff} & \neg p \in \Delta \end{array}\right\} \text{ for all } p \in P.$$

v is an evaluation by Lemma 1 part vi). I claim that (*) holds for all formulas of L'.

The result then follows, or by applying Lemma 2 with $\{A\}$ as Π we can find a prime-deductively closed Δ such that $\Delta \supseteq \Sigma$ and $A \in \Delta$. By the above condition $1 \notin v_\Delta(A)$, but $1 \in v_\Delta(B)$ for all $B \in \Sigma$. Hence $\Sigma \not\models A$. The claim is proved by induction over the formation of formulas. (*) provides the basis. The cases for $\wedge$, $\vee$, $\neg$ are simple applications of parts i)-v) of Lemma 1. The case for F is as follows, and that for P is similar.

$$\begin{array}{llll}
1 \in v_\Delta(FA) & \Rightarrow & \exists \Pi > \Delta\ 1 \in v_\Pi(A) & \\
 & \Rightarrow & \exists \Pi > \Delta\ A \in \Pi & \text{(Induction hypothesis)} \\
 & \Rightarrow & \exists \Pi > \Delta\ HFA \in \Pi & (\Pi \text{ is deductively closed}) \\
 & \Rightarrow & FA \in \Delta & \\
FA \in \Delta & \Rightarrow & \exists \Pi > \Delta\ A \in \Pi & \text{(Lemma 3)} \\
 & \Rightarrow & \exists \Pi > \Delta\ 1 \in v_\Pi(A) & \text{(Induction hypothesis)} \\
 & \Rightarrow & 1 \in v_\Delta(FA) & \\
0 \in v_\Delta(FA) & \Rightarrow & \forall \Pi > \Delta\ 0 \in v_\Pi(A) & \\
 & \Rightarrow & \forall \Pi > \Delta\ \neg A \in \Pi & \text{(Induction hypothesis)} \\
 & \Rightarrow & \neg FA \in \Delta & \text{(Lemma 4)} \\
\neg FA \in \Delta & \Rightarrow & G\neg A \in \Delta & (\Delta \text{ is deductively closed}) \\
 & \Rightarrow & \forall \Pi > \Delta\ \neg A \in \Pi & \\
 & \Rightarrow & \forall \Pi > \Delta\ 0 \in v_\Pi(A) & \text{(Induction hypothesis)} \\
 & \Rightarrow & 0 \in v_\Delta(FA). &
\end{array}$$

Hence the result holds for all formulas and the theorem is proved.

3.5. The basic dialectical tense logic can be extended, as in the classical case, by putting conditions on $<$ (on the semantic side) and adding extra rules of proof (on the proof-theoretic side). For example, the proof theory obtained by adding the following rules is sound and complete with respect to the semantics obtained by imposing the corresponding condition on $<$.

(1) $\dfrac{FFA}{FA}$ $\quad \dfrac{PPA}{PA}$ $\quad$ If $x < y$ and $y < z$ then $x < z$

(2) $\dfrac{GA}{FA}$ $\quad$ $\forall x \exists y\ x < y$

(3) $\dfrac{GA}{A}$ $\quad \dfrac{HA}{A}$ $\quad$ $\forall x\ x < x$.

This can be shown by a simple extension of the proof of the previous section. All this is as in the classical case (except that in (1) and (3) both the rules appear to be necessary whilst classically they are equivalent.) In the classical case the following rules and conditions pair off in a similar way.

(4) $\dfrac{FA}{FFA}$ $\dfrac{GA}{GGA}$ if $x < y$ then $\exists z\ x < z < y$.

(5) $\dfrac{PFA}{A \vee PA \vee FA}$ if $x < y$ and $x < z$ then $y < z$ or $z < y$ or $y = z$.

In the paraconsistent case soundness is easily shown. However, completeness is at the present time, an open question.

More complex extensions of the basic tense logic are often made. For example, if '$\Box A$' is '$A \wedge HA \wedge GA$' then classically the rule

$$\frac{\Box(GA \to PGA)}{GA \to HA}$$

corresponds to the second order condition on $<$:

Let X, Y be such that $\forall x \in X\ \forall y \in Y,\ x < y$; then X has a $<$-last member or Y has a $<$-first member.

This and similar rules can not even be formulated in the language of DTL since L' has no implication operator. ("Modus ponens" for "material implication" fails.) However LP can be extended with a satisfactory implication operator [11], and the result of adding this kind of rule can then be investigated. I will leave the details of this to another paper.

4. Leibniz' principle of continuity

4.1. In the previous two sections we have seen that there are plausible reasons for supposing that there are type-γ changes, even between p being true and p being false. Moreover, we have seen how a formal tense logic can accommodate this insight. In this section I want to examine a principle which definitely entails the existence of type-γ changes. The principle was stated by Leibniz. Hence I will call it 'Leibniz' continuity principle (LCP)'. His most explicit statement of it is this:

"When the difference between two cases can be diminished below any given quantity, in the data or what is posited, it must be similarly diminished below any given quantity in what is sought or in that which results. Or to speak more familiarly: when two cases (or that which is given) continually approach one another and eventually merge, their consequences or results (or what is sought) must do so too." (Leibniz [8]).

Now, with 300 years of mathematical hindsight, it is easy to think that Leibniz is just saying that if (s_n) and (t_n) are two mathematical sequences such that $\lim_{n\to\infty} s_n - t_n = 0$, then $\lim_{n\to\infty} s_n = \lim_{n\to\infty} t_n$.

No doubt he would take his principle to imply this. However, the scope of the principle is, in fact, much wider than this. The principle is intended to apply to all limiting processes, including geometrical and physical ones. This is quite clear from the applications of the principle that Leibniz gives. For example, he points out [8] that since a parabola can be approached "as close as we please" by taking an ellipse and sending one focus off to infinity, then "any geometric theorem established for an arbitrary ellipse, can be applied to a parabola". As another example, Leibniz reasons as follows. According to Descartes' second rule for impacts, if two bodies travelling with the same speed collide head on, then the less massive will have its velocity reversed whilst the more massive will maintain its velocity. This is supposed to be true however small the difference between the masses of the particles. Now by the principle, the same effect must hold in the limit when the differences between the masses is zero. This, says Leibniz, contradicts Descartes' first rule, according to which a collision between equal masses produces a symmetrical effect.

It is clear from these examples that a, perhaps better, way of stating the LCP is this: given any limiting process (whether arithmetic, geometric, physical or whatever) whatever holds up to the limit, holds at the limit. In fact this is precisely how the principle was taken after Leibniz. For example in 1786, Lhuiler wrote "if a variable quantity at all stages enjoys a certain property, its limit will enjoy the same property" (Boyer [1], p. 256).

The continuity principle must be treated with some care. For using it carelessly one could prove, e.g. that every real number is rational (since every real is the limit of a sequence of rationals), that the limit of every sequence of continuous functions is continuous, etc. However, it is quite clear that Leibniz must have held that there are some bounds on the application of the principle. For example: every ellipse is a bounded and closed figure. However, it does not follow that a parabola is a bounded and closed figure, even though "every geometric theorem established for an arbitrary ellipse can be applied to a parabola", and this must have been obvious to Leibniz. What exactly Leibniz took these bounds to be, I do not know. However, we need not try to decide the issue at the moment. In **4.4** I will give a precise and unproblematical formulation of the LCP in the context of tense logic. Let us turn instead to the rationale for the LCP.

4.2. I want to fix, in particular, on the principle as applied to physical limits. In this form, it says that any physical state of affairs which holds arbitrarily close to a given time holds at that time. The principle is a very plausible one with a good deal of intuitive appeal. I am not absolutely certain why. It is not for the reason that Leibniz gave (viz. that a change that violates the principle is incompatible with the wisdom and perfection of God the designer.) Rather I think the reason is this. A change that violates this principle would have to take place in no time. We are prepared to buy changes over very short times, but a change that occurs in no time at all is difficult to swallow. For exactly the same reason theories of action at a distance have always been thought philosophically puzzling. For they require an event, viz. the transmission of an effort to occur in zero time. Surely if something happens, it must take *some* time, even if just an instant. The idea that something can happen in *no* time appears close to self contradictory.

A similar way of putting the point is that if this situation arose, there would be no time at which the state was *changing*. Hence the situation would be more like a succession of photographic stills than a genuine dynamical flux. (We have discussed this in another context in § 2.5.) Thus the LCP seems well grounded intuitively.

4.3. What has this to do with the instant of change? Simply this: the LCP implies that there are type-γ changes. Suppose that prior to t_0 system S is in state S_0, whilst after it, it is in state S_1. Since S_0 occurs arbitrarily close to t_0, it occurs at t_0. Similarly S_1 occurs at t_0. Thus both S_0 and S_1 are realized at t_0, i.e. this is a type-γ change. Of course if S_0 is p's being true and S_1 is $\neg p$'s being true, t_0 realizes a contradiction. Thus LCP entails that contradictions are realized at the point of change. I will give a more formal proof of this in **4.6.**

4.4. Leibniz' principle is easily built into the semantics of tense logics. We need only suppose that the domain of the order in an interpretation comes with a topology T. We then demand that for every x in the domain of the ordering and every propositional parameter p:

(**) If X is any set such that $\forall x \in X\ 1 \in v_x(p)$ $[0 \in v_x(p)]$ and y is any limit (accumulation) point of x with respect to the topology T, $1 \in v_y(p)$ $[0 \in v_y(p)]$.

(Equivalently: if p is true (false) at every point in X, it is true (false) at every point in its closure.)

It may seem arbitrary to impose this condition on propositional parameters only, and not on all formulas. However, it is easily proved that provided the condition holds for propositional parameters it holds for all formulas in which a tense operator does not occur. The proof of this is by induction over the formation of formulas. (**) provides the basis. Here are the cases for $\neg$ and $\wedge$. The case for $\vee$ is similar.

i) Suppose $1[0] \in v_x(\neg A)$ for all $x \in X$. Then $0[1] \in v_x(A)$ from all $x \in X$. By induction hypothesis $0[1] \in v_y(A)$ i.e. $1[0] \in v_y(\neg A)$.

ii) Suppose $1 \in v_x(A \wedge B)$ for all $x \in X$. Then $1 \in v_x(A)$ and $1 \in v_x(B)$ for all $x \in X$. By induction hypothesis $1 \in v_y(A)$ and $1 \in v_y(B)$. Thus $1 \in v_y(A \wedge B)$.

On the other hand, suppose that $0 \in v_x(A \wedge B)$ for all $x \in X$. Then $0 \in v_x(A)$ or $0 \in v_x(B)$ for all $x \in X$. Let $X_A = \{x \in X \mid 0 \in v_x(A)\}$ and $X_B = \{x \in X \mid 0 \in v_x(B)\}$. Then $X_A \cup X_B = X$, and since y is an accumulation point of X it is an accumulation point of either X_A or X_B. Hence either $0 \in v_y(A)$ or $0 \in v_y(B)$ by induction hypothesis. In either case $0 \in v_y(A \wedge B)$.

However, the condition does not hold for formulae containing tense operators. Neither should one expect it to. For if it did, nothing would ever finish! To be precise, suppose that time has the structure of the real line and let A be any statement which can not hold at an isolated point of time (i.e., if $1 \in v_x(A)$ there is a nondegenerate interval X such that $x \in X$ and for all $y \in X$, $1 \in v_y(A)$); then if A were ever to hold it would hold at a later time too. To see this suppose A holds at x. Let X be a maximal interval containing x throughout which A holds. By the LCP X must contain all its limit points. Hence X is closed. If x is not the right hand end point of X, we are home. So suppose x is the righthand end point. Consider any interior point in the interval, y. Then there is a z such that $y < z < x$, and since $1 \in v_z(A)$, $1 \in v_y(FA)$. Applying the LCP to FA it follows that $1 \in v_x(FA)$. Hence A must hold at some point to the right of x.

Assuming that one can not live for but an instant, this would be a proof of immortality, or at least of reincarnation! Thus the LCP applies only to present tensed sentences.

4.5. An obvious question is what effect the LCP has on the rules which hold in a tense logic. If the topology of an interpretation is discrete, then there are no limit points and hence the LCP will be satisfied vacuously. Hence the LCP will begin to bite only when we consider continuous or at least dense time. For the reasons given in **3.5**, an investigation of the proof-theoretic effects of the LCP goes beyond the bounds of this paper. However, some of its effects can be glimpsed from the following considerations. Provided we restrict ourselves to interpretation in which time has the structure of the real line, then the principle

$$\frac{L(A \vee B) \wedge L(A \rightarrow HA) \wedge L\big(B \rightarrow P(B \wedge GB)\big) \wedge A \wedge FB}{(A \wedge B) \vee F(A \wedge B)}$$

(where A and B contain no tense operators) holds in all structures in which the LCP holds. To see this, suppose the LCP holds and let the premise of the rule be true at some point, x, i.e. Let $\boldsymbol{A} = \{y \mid 1 \in v_y(A)\}$ and $\boldsymbol{B} = \{y \mid 1 \in v_y(B)\}$. Because of the final two conjuncts $\boldsymbol{A}$ and $\boldsymbol{B}$ are nonempty and because of the first conjunct $\boldsymbol{A}$ and $\boldsymbol{B}$ are exhaustive. Since $L(A \rightarrow HA)$ holds then $\boldsymbol{A}$ is a left semi-infinite interval and since $L\big(B \rightarrow \rightarrow P(B \wedge GB)\big)$, $\boldsymbol{B}$ is a right semi-infinite interval whose left hand end is open. But $\boldsymbol{A} \cap \boldsymbol{B} \neq \varnothing$. For suppose not, then $\boldsymbol{A}$ must be closed at its right hand end. Let y be its end point. Then y is a limit point of $\boldsymbol{B}$ and hence by the LCP $1 \in v_y(B)$. Thus $1 \in v_y(A \wedge B)$. Finally, since A and B overlap there must be a point $z \geqslant x$ in the overlap. Hence

$$1 \in v_x\big(F(A \wedge B) \vee (A \wedge B)\big).$$

Moreover, the above principle may fail if the LCP fails. As a counter-example, let B hold in and only in any right semi-infinite interval whose left hand end is open, and A hold in and only in its non-empty complement. At any point of the complement, the premise of the rule is true and its conclusion is not.

I will leave a full investigation of the proof-theoretic effects of the LCP to another paper.

4.6. We can now show quite precisely that the LCP implies that change involves contradictions. To be precise, take any interpretation in which time is linear and continuous and the topology is the order topology. Then if there are a formula A and points x_1, x_2 such that $v_{x_1}(A) \neq v_{x_2}(A)$, there is a point y such that a contradiction is true at y.

PROOF. We may suppose A to be a formula without tense operators. Let $X_1 = \{x \mid 1 \in v_x(A)\}$ and $X_2 = \{x \mid 0 \in v_x(A)\}$. By the above conditions, X_1 and X_2 are non-empty and $X_1 \cup X_2$ is exhaustive. But since the order is linear and continuous, the order topology is connected (see Kelley [7], p. 58). Hence $X_1 \cap X_2^C \neq \varnothing$ or $X_1^C \cap X_2 \neq \varnothing$ (where X^C is the closure of X). But by the LCP $X_1^C = X_1$ and $X_2^C = X_2$. Hence $X_1 \cap X_2 \neq \varnothing$ as required.

This little theorem vindicates dialecticians such as Hegel, who asserted that change was impossible without (true) contradictions:

> ... Contradiction is the root of all movement and life, and it is only in so far as it contains a contradiction that anything moves or has impulse or activity.
>
> [5] Book 2, section 1, chapter II part C observation 3.

The LCP can also be used to vindicate another cryptic pronouncement of Hegel. He said (*loc. cit.*)

> We must grant the old dialecticians the contradictions they prove in motion... Something moves not because it is here at one point of time and there at another, but because at one and the same point of time it is here and not here...

To see this, suppose that an object is in continuous motion from, say, left to right. Take some instant of time t_0. At t_0 the object must be at some point. Call this a. So at t_0 'The object is at a' is true. But for any point prior to t_0 'The object is not at a' is true. And since t_0 is a limit of all these points, this is true at t_0 too.' I am not sure that this is a legitimate application of the LCP, but it is certainly an interesting one.

4.7. So much for the LCP, its plausibility and some of its applications. I want to discuss one final issue concerning change and contradiction and I will do this via one final application of the LCP. In [15], Thomason defined a "super-task" as an infinite number of tasks and argues that it is impossible to complete super-tasks. His argument for this concerns a system which can be in the states *on* or *off* (i.e. not on). There is one switch. If pressed when the system is on, the system goes off and vice versa. We suppose that at $t = 0$ the system is off. The switch is then pressed at times $1-(\frac{1}{2})^n$ for all $n \geqslant 1$. If it were possible to complete a super-task, Thomason argues, it would be possible to complete this one. In fact it would be completed by $t = 1$. But it is impossible, for at $t = 1$ the state must be either on or off. But it can not be on, since it has never been on without having been turned off, and similarly it can not be off.

Now let us apply the LCP to the situation. In any neighbourhood of the point $t = 1$ there are times when the system is on. Hence at $t = 1$ the system is on. (Of course the original specifications of the problem leave the situation at $t = 1$ undefined. Thus, in effect, the LCP acts as a sort of principle of transfinite induction.) Similarly at $t = 1$ the system is off. Thus Thomason's conclusion that at $t = 1$ the light is neither on nor off, is correct. For it is both on and off, and this is the same thing, as we saw in **2.5.** However, it does not follow, and Thomason thinks it does, that the situation at $t = 1$ is not physically realisable. $t = 1$ realizes a contradictory situation.

This will leave some people cold. How can a contradiction be realised physically. What would it be like for a light to be both on and off? This description seems to paralyse the imagination. However, the mental block is removed once we realise that a lights being both on and off is a situation with which we are very familiar. Contradictions occur at the nodal points of a type-γ change between on and off. Thus, assuming the LCP, we literally witness a true contradiction whenever we turn the light on or off! It is just how things are at the point of change.

There is an important lesson to be learnt here. An objection mooted against paraconsistency (perhaps *the* most commonly mooted objection) goes something like this: 'I just can't see (= understand) what it would be like for a contradiction to be true. What would it be like for something to be a cup and not a cup, for someone to be in the room and not in the room?' The answer should now be obvious. A cup is both a cup and not a cup the instant it fractures into smithereens. Someone is both in the room and not in the room the instant he leaves. Contradictions are the nodal points of type-γ changes and, as such, are perfectly familiar. One finds it difficult to grasp what a true contradiction is like only because one fixes on a frozen, static (metaphysical) state of the world and forgets its dynamical aspects. Engels censures Dühring for just this

> True, so long as we consider things as at rest and lifeless, each one by itself, alongside and after each other, we do not run up against any contradictions in them. We find certain qualities which are partly common to, partly different from, and even contradictory to each other, but which in the last-mentioned case are distributed among different objects and therefore contain no contradiction within. Inside the limits of this sphere of observation we can get along on the basis of the usual, metaphysical mode of thought. But the position is quite different as soon as we consider things in their motion, their change, their life, their reciprocal influence on one another. Then we immediately become involved in contradictions. Motion itself is a contradiction: even simple mechanical change of position can only come about through a body being at one and the same moment of time both in one place and in another place, being in one and the same place and also not in it. And the continuous origination and simultaneous solution of this contradiction is precisely what motion is.
>
> [3], p. 139.

Engels' comment is one of which modern logicians interested in formalizing dialectics would do well to take note. A number of formal dialectical logics have appeard in recent years (Thomason [14], Routley and Meyer [13], Da Costa and Wolf [2]). However, none of these concerns time or change, which is at the very heart of dialectics.

Finally it is worth noting the following. I have argued that there are type-γ changes between p being true and $\neg p$ being true. During these changes $p \wedge \neg p$ is true. In fact it is plausible to suppose that for a contradiction to be true is for the situation to be in a state of change, to be *changing*. Let us call such a state, a *flux state*. This raises the question of whether there is an infinite regress here. What about the change from p being true to the flux state? Might this not be some new state? The answer is 'no'. For if this is a type-γ change, it is characterized by both p and $p \wedge \neg p$ holding at it. But $p \wedge (p \wedge \neg p)$ is of course equivalent to $p \wedge \neg p$. Thus to be in a state of changing to a flux state is to be in the flux state itself, and no regress arises.

5. Conclusion

Let me conclude by summarizing the main points of the paper. I started by showing that there are good reasons for supposing there to be type-γ changes, i.e. changes at which, at the instant of change, a system is in both antecedent and posterior states. In particular, if the change is from p being true to p being false a contradiction is realized at the instant of change. I then specified a basic system of tense logic DTL, with an underlying paraconsistent logic, which accommodates this observation. Finally, I discussed Leibniz' continuity principle and its dialectical consequences.

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