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WHAT NOT? A DEFENCE OF DIALETHEIC THEORY OF NEGATION

1 INTRODUCTION

The primary concern of logic is inference; and in particular, the question of what constitutes a valid inference. In investigating this issue, a certain class of notions has always appeared to be of crucial importance. We now call them logical constants, though they have been called by different names at different times. (For example, they were called *syncategoremata* by medieval logicians.) Much of logic has therefore been devoted to an analysis of these notions. Historically, the most contentious have been the quantifiers and the conditional. Consensus concerning the former has been achieved this century, due to the work of Frege and others. The debate concerning the latter shows no similar sign of convergence.

Amongst the logical constants, negation is, perhaps, the most crucial, dealing as it does with a certain polarity of thought, without which there could, some have thought, be no thought—or inference—at all. Historically, its behaviour may not have been terribly contentious. At least until this century. During this, our understanding of logical structures has become sharper and more profound by an order of magnitude that is historically unheard of; and this has allowed logicians to reflect on, and question, many traditional assumptions about the behaviour of negation. Two movements, in particular, stand out in this context: intuitionism and paraconsistency; the former can be seen as challenging the law of excluded middle; the latter as challenging the law of non-contradiction.

For these reasons, the nature of negation is a contemporary question that is both important and difficult. In this essay, I want to address it and suggest a dialetheic answer.[1]

2 NEGATION OR NEGATIONS?

How, then, does negation behave?[2] There is a short way with this question. There is no such thing as negation; there are lots of different negations: Boolean negation,

[1] Sainsbury [24, p. 142], discerns a challenge for dialetheism: to provide an account of what understanding negation involves. I hope that this essay goes a reasonable way towards meeting that challenge.

[2] I will concern myself only with propositional negation, though this fits into a much broader family of negative constructions. See Sylvan, [28].

D.M. Gabbay and H. Wansing (eds.), What is Negation?, 101–120.

intuitionist negation, De Morgan negation. Each of these behaves according to a set of rules (proof-theoretic or semantic); each is perfectly legitimate; and we are free to use whichever notion we wish, as long as we are clear about what we are doing. If this is right, there is nothing left to say about the question, except what justifies us in categorising a connective as in the negation family. And I doubt that there is anything very illuminating to be said about that. Virtually *every* negation-like property fails on some account of a connective that is recognisably negation-like: the law of excluded middle, the law of non-contradiction, double negation, De Morgan's laws, contraposition, and so on. All we are left with is a family-resemblance whose fluid boundaries are largely historically determined.

I do not think that the answer is right, however. It makes a nonsense of too many important debates in the foundations of logic. Doubtless, philosophical debates do rest on confusion sometimes, but questions concerning the role of negation in discourses on infinity, self-reference, time, existence, etc., are not to be set aside so lightly.

At the root of this kind of answer is a simple confusion between a theory and what it is a theory of.[3] We have many well worked-out theories of negation, each with its own proof-theory, model-theory and so on. And if you call the theoretical object constituted by each theory a negation, then, so be it: there are many negations. But this does not mean that one can deploy each of these theoretical objects at will and come out with the correct answer. The theoretical object has to fit the real object; and how this behaves is not a matter of choice.

A comparison with geometry may be helpful here. There are, in a sense, many geometries. Each has its own well defined structure; and, as an abstract mathematical structure, is worthy of investigation. But if we think of each geometry, not as an abstract mathematical structure, but, suitably interpreted, as a theory about the spatial (or spatio-temporal) structure of the cosmos, we are not free to choose at will. The theory must answer to the facts—or, if one is not a realist, at least cohere in the most satisfactory way with the rest of our theorising.

There is always an extreme conventionalist line to be run here. One might say, as Poincaré [12] did, that we are free to choose our geometry at will, e.g. on the grounds of simplicity, and then fix everything else around it. Similarly, we might insist that we are free to employ a certain notion of negation and make everything else fit. But such a line is not only philosophically contentious, but foolhardy, at least in advance of a good deal of further investigation. The tail may end up wagging a dog of a considerable size. For example, as Prior [20] pointed out a long time ago, we can determine to use a connective $*$ (tonk) according to the rules of

[3]See Priest [14, Ch. 14]. The confusion is manifested by, e.g. Quine [21, p. 81] when he complains that someone who denies *ex contradictione quodlibet* just doesn't know what they are talking about, since changing the laws is changing the subject. A similar confusion is apparent in those who argue that someone who suggests adopting a non-classical logic wants to revise logic, i.e. correct reasoning. Such a person need only be suggesting a revision of a theory of logic, *not* logic itself. One cannot simply *assume* that classical logic gets it right. That's exactly what is at issue here.

inference $\alpha \vdash \alpha * \beta$ and $\alpha * \beta \vdash \beta$. But the cost of this is accepting that if anything is true, everything is!

3 CONTRADICTORIES

We see, then, that a simple voluntarism with respect to negation is unsatisfactory. If it is to be applied, an account of negation must be considered not just as an abstract structure, but as a theory *of* something, just as a geometry is a theory of physical space. And this will put substantial constraints on what an acceptable account is.

The next question is what, exactly, an account of negation is a theory of. It is natural to suggest that negation is a theory of the way that the English particle 'not', and similar particles in other natural languages, behaves. This, however, is incorrect. For a start, 'not' has functions in English which do not concern negation. For example, it may be used to reject connotations of what is said, though not its truth, as in, for example, 'I am not his wife: he is my husband'.[4]

More importantly, negation may not be expressed by simply inserting 'not'. For example, the negation of 'Socrates was mortal' may be 'Socrates was not mortal'; but, as Aristotle pointed out (*De Interepretatione*, ch. 7), the negation of 'Some man is mortal' is not 'Some man is not mortal', but 'No man is mortal'.

These examples show that we have a grasp of negation that is independent of the way that 'not' functions, and can use this to determine when 'notting' negates. But what is it, then, of which we have a grasp? We see that there appears to be a relationship of a certain kind between pairs such as 'Socrates is mortal' and 'Socrates in not mortal'; and 'Some man is mortal' and 'No man is mortal'. The traditional way of expressing the relationship is that the pairs are *contradictories*, and so we may say that the relationship is that of contradiction. Theories of negation are theories about this relation.

As usual in theorisation, we may reach a state where we have to reassess the situation. For example, it may turn out that there are several distinct relationships here, which need to be distinguished. But at least this is the data to which theorisation must (and historically did) answer, at least initially.

Having got this far, the next obvious question is what the relationship of contradiction is a relationship between: sentences, propositions, some other kind of entity? There are profound issues here; but, as far as I can see, they do not affect the question of negation substantially. For any issue that arises given one reasonable answer to this question, an equivalent one arises for the others. So I shall simply call the sorts of thing in question, non-commitally, statements, and leave it at that.

[4] See, e.g. Horn [9, pp. 370 ff].

4 THE LAWS OF EXCLUDED MIDDLE AND NON-CONTRADICTION

So if α is any statement, let $\neg\alpha$ represent its contradictory. (Contradictories, unlike contraries and sub-contraries are unique—at least up to logical equivalence.) What relationships hold between these? Traditional logic and common sense are both very clear about the most important one: we must have at least one of the pair, but not both.[5] It is precisely this which distinguish contradictories from their near cousins, contraries and sub-contraries. If we have two contraries, e.g., 'Socrates was black' and 'Socrates was white', it is necessarily false that Socrates was black $\wedge$ Socrates was white; but it is not necessarily false that Socrates was black $\vee$ Socrates was white. Dually, if we have two subcontraries, e.g., 'Socrates was under 2m. tall' and 'Socrates was over 1m. tall', it is necessarily false that Socrates was under 2m. tall $\vee$ Socrates was over 1m. tall, but not necessarily true that Socrates was under 2m. tall $\wedge$ Socrates was over 1m. tall.

This fact about contradictories obviously gives immediately two of the traditional laws of negation, the law of excluded middle (LEM), $\alpha \vee \neg\alpha$, and the law of non-contradiction (LNC), $\neg(\alpha \wedge \neg\alpha)$.[6] (Note that the LNC, unlike the LEM, is not only a principle about contradictories, but is itself a negative thesis. This is important, and we will return to it later.) Now, maybe the traditional claim about contradictories—and consequently these two laws—is wrong; but it would certainly seem to be the default position. The onus of proof is therefore on those who would dispute it.

Disputation comes from at least two directions. The first is that of some (though not all) paraconsistent logicians. The argument here is that some contradictories are both true, i.e., for some βs we have $\beta \wedge \neg\beta$. We do not, therefore, have $\neg(\beta \wedge \neg\beta)$. We will look more closely at the first part of this argument later. For the moment, just note that if it is correct, it undercuts the second part of the argument (at least without some further considerations). For if some contradictions are true, we may well have both $\beta \wedge \neg\beta$ and $\neg(\beta \wedge \neg\beta)$. Hence, the fact that some contradictions are true does not, of itself, refute the LNC (at least in the form in question here).

The second direction from which one might dispute the traditional characterisation is that of some logicians who suppose there to be sentences that are neither true nor false, notably intuitionist logicians. The argument here is that if α is neither true nor false, so is $\neg\alpha$. Hence, assuming that disjunction behaves normally, $\alpha \vee \neg\alpha$ is not true.[7] The claim that certain statements are neither true nor false is clearly a substantial one. The claim that disjunction behaves normally is also challengeable. (If we give a supervaluationist account, $\alpha \vee \neg\alpha$ may be true even

[5]Classically, these facts actually characterise contradictories up to logical equivalence. This, however, is moot. If β satisfies the conditions $\Box\neg(\alpha \wedge \beta)$ and $\Box(a \vee \beta)$, and γ is any necessary truth, then so does $\beta \wedge \gamma$; but β does not entail $\beta \wedge \gamma$ unless one identifes entailment with strict implication.

[6]I express the laws in the form of schemas. I will use lower case Greek letters schematically throughout this essay.

[7]If conjunction behaves normally, the LNC may also fail for truth-valueless sentences.

though each disjunct fails to be so.) However, we need discuss neither of these issues here. For from the present perspective there is an obvious objection. If $\neg$ behaves as suggested, it is not a contradictory-forming operator at all, but merely a contrary-forming one. This would seem particularly clear if we consider the intuitionist account of negation. According to this, $\neg\alpha$ is true (= assertable) just if there is a proof that there is no proof that α. This is obviously a *contrary* of α.[8]

A genuine contradictory-forming operator will be one that when applied to a sentence, α, covers *all* the cases in which α is not true. Thus, it is an operator, $\neg$, such that $\neg\alpha$ is true iff α is either false or neither true nor false. (In English, such an operator might be: it is not the case that.) For this notion, which is the real contradictory-forming operator, the LEM holds.

Those who believe in simple truth-value gaps would seem to have little reply to this objection. The intuitionist does have a reply to hand, however. They can argue that a contradictory-forming operator, as traditionally conceived, literally makes no sense.[9] The argument is a familiar one from the writings, notably, of Dummett.[10] *In nuce*, it is as follows. If a notion is meaningful there must be something that it is to grasp its meaning. Whatever that is, this must be manifestable in behaviour (or, the argument sometimes continues, the notion would not be learnable). But there is no suitable behaviour for manifesting a grasp of a connective satisfying the conditions of a classical contradictory-forming operator. In particular, we cannot identify the behaviour as that of being prepared to assert $\neg\alpha$ when (and only when) α fails to be true. For this state of affairs may well obtain when there is no principled way for us to be able to recognise that it does.

There are subtle issues (and a substantial body of literature) here. And to deal with them satisfactorily would require taking up a disproportionate part of this essay. But let me at least say something about the matter. For a start, I do not see why the grasp of a notion must be manifestable. There is no reason why, in general, certain notions should not be hard-wired in us. If, for example, there is a Fodor-style language of thought,[11] it is quite natural to suppose that single-bit toggling is a primitive operation. One might even tell an evolutionary story as to how this came about: it is the simplest and most efficient mechanism for implementing the polarity of thought. In particular, then, a contradictory operator does not have to be learned; its use is merely triggered in us by certain linguistic contexts, in much the

[8] Most perspicuously, consider the embedding of intuitionist logic into $S4$ where the modal operator $\Box$ is considered as a provability operator. Then $\neg\alpha$ is translated into $\Box\neg\alpha^{+}$ (where α^{+} is the translation of α). In other words, $\neg\alpha$ is intuitionistically true iff the negation of α is *provable*.

[9] They might even suggest that $\neg T\langle\alpha\rangle \rightarrow T\langle\neg\alpha\rangle$ is perfectly acceptable provided the negation in the antecedent is understood as intuitionist negation. But this is highly problematic, for it leaves them no way of expressing their view concerning instances of the Law of Excluded Middle that fail: if α is undecided, one can no longer say that $\alpha \vee \neg\alpha$ is not true, let alone false, since $\neg T\langle\alpha \vee \neg\alpha\rangle$ now entails $T\langle\neg\alpha \wedge \neg\neg\alpha\rangle$.

[10] See, e.g. Dummett [4], esp. pp. 224–5 of the reprint. A somewhat different argument is explained and dispatched in Read [22, pp. 220–230].

[11] See Fodor [6].

same way that the categories of universal grammar are, according to Chomsky.[12]

But even granting that the grasp of a notion must be manifestable, I do not see why it must be manifestable by anything as strong as the argument requires (which is, I agree, impossible). In particular, it can be manifested by being prepared to assert $\neg\alpha$ when in a position to recognise that α fails to be true, and refusing to assert it when in a position to recognise that α is true.[13] It could well be suggested that such a manifestation would not be adequate. There will be many cases where we are not in a position to recognise either. People could therefore manifest the same behaviour whilst disagreeing about how to handle new cases when these become recognisable, and so meaning different things. This is true. But if the people not only behave as suggested, but also manifest a disposition to agree on new cases, this is sufficient to show (if not, perhaps, conclusively, then at least beyond reasonable doubt) that they are operating with the notion in the same way. In just this way, the fact that we are all prepared to apply, or refrain from applying, the word 'green' to hitherto unseen objects when they come to light, shows that we all mean the same thing by the word. This is essentially what following an appropriate rule comes to, in Wittgensteinian terms.[14]

There is much more to be said here. But if the onus of proof is on an intuitionist, as it would seem to be in the case of a contradictory-forming operator, I know of no argument against the LEM that I find persuasive. (That one can tell a coherent epistemological/metaphysical intuitionist story is not at issue.)

Before we leave the LEM it is worth noting that the fact that for every pair of contradictories one must be true (period), does not entail that for every situation one of each pair must be true *of it*. If one thinks of a situation as *part* of the world, then it may well be argued that neither of a pair of contradictories need be true of it. Thus, consider the situation concerning my bike. It may be the case that neither 'Gent is in Belgium' nor 'Gent is not in Belgium' is true of this situation. See Restall [23]. The question of whether or not one needs to consider partial situations, as so conceived, is important in discussions of the semantics of conditionals. But since conditionality is not the issue here, I will discuss the matter no further.

5 TRUTH AND FALSITY

So far, we have met two of the classical laws of negation, LEM and LNC. A third, the law of double negation (LDN) is simply derivable. The relationship of being contradictories is symmetric. That is, if β is the contradictory of α, then α is the contradictory of β. In particular, α is the contradictory of $\neg\alpha$. Hence, $\neg\neg\alpha$ just is α.

[12] See, e.g., Chomsky [3].

[13] A different suggestion, though not one I would make, is that an understanding can be manifested by using classical logic. This raises quite different issues.

[14] See *Philosophical Investigations*, Part I, esp. sections 201-40.

We are now in a position to look at another important feature of negation: its truth conditions. To do this we will need a definition of falsity. Let us define 'α is false' to mean that $\neg\alpha$ is true. This is not the only plausible definition; one might also define it to mean that α is not true. It may turn out that these two definitions are equivalent, of course. However, to assume so here would be to beg too many important questions. And the present definition is one that all parties can agree upon, classical, intuitionist and paraconsistent.

The definition of falsity assures us that $\neg\alpha$ is true iff α is false. Dually, $\neg\alpha$ is false iff $\neg\neg\alpha$ is true (by the definition of falsity) iff α is true, by LDN. Hence, the traditional understanding of the relationship between truth and falsity falls out of the understanding of negation as contradiction, and the definition of falsity.

Two more of the classical laws of negation, the Laws of De Morgan (LDM), can now also be dealt with. These involve conjunction and disjunction essentially; and so we need to make some assumption about how they behave. Since this is not an essay on conjunction/disjunction, this does not seem the place to discuss the matter at great length. For present purposes, let us suppose that they behave as tradition says they do: a conjunction is true iff both conjuncts are true, and false iff at least one conjunct is false. The conditions for disjunction are the obvious dual ones.

One of De Morgan's Laws is the equivalence of $\neg(\alpha \wedge \beta)$ and $\neg\alpha \vee \neg\beta$. This can now be demonstrated thus: $\neg(\alpha \wedge \beta)$ is true iff $\alpha \wedge \beta$ is false iff α is false or β is false iff $\neg\alpha$ is true or $\neg\beta$ is true iff $\neg\alpha \vee \neg\beta$ is true. Dually, $\neg(\alpha \wedge \beta)$ is false iff $\alpha \wedge \beta$ is true (LDN) iff α and β are true iff $\neg\alpha$ and $\neg\beta$ are false (LDN) iff $\neg\alpha \vee \neg\beta$ is false. The other of De Morgan's Laws is an equivalence between $\neg(\alpha \vee \beta)$ and $\neg\alpha \wedge \neg\beta$, and can be verified by a similar argument.

The connection between negation and the conditional is more difficult to deal with, but this is because the conditional is itself more contentious. Indeed, the claim that there are different kinds of conditional (entailments, causal conditionals, indicative conditionals, subjunctive conditionals) is well known; some of these distinctions are well motivated; and negation may well interact with different conditionals in different ways. A minimal condition for a conditional of any kind would seem to be that it preserve truth in an appropriate way from antecedent to consequent. From this, it follows that *modus ponens*, $\alpha, \alpha \rightarrow \beta \vdash \beta$, is valid. The most important question concerning a conditional in the present context is whether it preserves falsity in the reverse direction. For some conditionals, at least, this would seem to fail, as, e.g., Stalnaker and Lewis have argued.[15] And if $\alpha \rightarrow \beta$ fails to preserve falsity backwards, $\neg\beta \rightarrow \neg\alpha$ will fail to preserve truth forwards, and so will not be true. The law of contraposition (LC), $\alpha \rightarrow \beta \vdash \neg\beta \rightarrow \neg\alpha$ is not, therefore, to be expected to hold for an arbitrary conditional. Of course, there may well be conditionals which do preserve falsity in the appropriate way; in fact one can always *define* one, $\Rightarrow$, in a simple fashion: $\alpha \Rightarrow \beta$ is just $(\alpha \rightarrow \beta) \wedge (\neg\beta \rightarrow \neg\alpha)$. For such conditionals contraposition will hold.

[15] See, e.g., Stalnaker [27], Lewis [10]. See also Priest [14, 6.5].

In this section, I have talked of truth. I have said nothing about truth-in-an-interpretation, as required, for example, for a model-theoretic account of validity. It is important to distinguish these two notions, for they are often confused. The first is a property (or at least a monadic predicate); the second is a (set-theoretic) relation. It is natural enough to suppose that truth is at least coextensive with truth-in-g, where g is some one privileged interpretation (set). And this may provide a constraint on the notion of truth-in-an-interpretation. But it, even together with an account of truth, is hardly sufficient to determine a theory of truth-in-an-interpretation. It does not even determine, for example, how to conceptualise an interpretation. So how are an account of truth-in-an-interpretation, appropriate for the connectives we have been discussing, and a corresponding model-theoretic notion of validity, to be formulated? The details of this are a bit more technical than the rest of this essay, and I will defer them to an appendix: the rest of the material does not presuppose them.

6 TRUTH AND CONTRADICTION

Starting with a conception of negation as a contradictory-forming operator, we have now validated five standard laws of negation (LEM, LNC, LDN and the two LDM), and a sixth (LC) in certain contexts. We have hardly settled all the central issues concerning negation, however.

It is common to distinguish between the LEM and the Principle of Bivalence: every statement is either true or false.[16] Though these are natural mates, either can hold without the other, given the right account of other things. Similarly, we need to distinguish between the LNC and what I will call, for want of a better term, the Principle of Consistency: no statement is both true and false. Again, though these are natural mates, it is quite possible to have one without the other. In particular, as I have already observed, the fact that every instance of $\neg(\alpha \wedge \neg\alpha)$ is true does not, on its own, prevent some instances of α and $\neg\alpha$ from being true. So what is one to say about the Principle of Consistency? This is the next issue that needs to be addressed.

The traditional view endorses this Principle. But the traditional view has been called into question by some paraconsistent logicians, who assert that some contradictions are true, dialetheists. The case for this is a long one, and, like the intuitionist case against classical negation, is too long to take up in detail here; but let me say a little.[17]

Many examples of dialetheias have been suggested, but the most impressive ones are those generated by the paradoxes of self-reference. Here we have a set of arguments that appear to be sound, and yet which end in contradiction. *Prima facie*, then, they establish that some contradictions are true. Some of these arguments

[16] See Haack [8, p. 66f].

[17] The case is made in Priest [14].

are two and a half thousand years old. Yet despite intensive attempts to say what is wrong with them in a number of logical epochs, including our own, there are no adequate solutions. It is illuminating to compare these paradoxes with ones of equal antiquity: Zeno's. Zeno's paradoxes have also been the subject of intensive study over the years, and for these there *is* a well recognised and stable solution.[18] (Philosophers may still argue over some of the details, but then philosophers will argue over anything.) The fact that there is no similar thing in the case of the paradoxes of self-reference at least suggests that in their case, trying to solve them is simply barking up the wrong tree: we should just accept them at face value, as showing that certain contradictions are true.

Because a major part of what is at issue in this essay is the semantics of negation, the semantic paradoxes are particularly pertinent. Every consistent solution to these is generally acknowledged as wrong (except by the few who propound it). Moreover, there are general reasons why, it would seem, no consistent solution will be forthcoming. The reason is the following dilemma (actually, trilemma).[19]

The paradoxes arise, in the first place, as arguments couched in natural language. One who would solve the paradoxes must show that the semantic concepts involved are not, despite appearances, inconsistent. And it is necessary to show this for all the concepts in the semantic family, for they are all deeply implicated in contradiction. Attempts to do this, given the resources of modern logic, all show how, given any language, L, in some class of languages, to construct a theory, T_L, of the semantic notions for L, according to which they behave consistently.

The first horn of the dilemma is posed by asking the question of whether the theory T_L is expressible in L. If the answer to this is 'yes' it always seems possible to use the resources of the theory to construct new semantic contradictions, often called extended or strengthened paradoxes. Nor is this an accident. For since T_L is expressible, and since, according to T_L, things are consistent, we should be able to prove the consistency of T_L in T_L. And provided T_L is strong enough in other ways (for example, if it contains the resources of arithmetic, as it must if L is to be a candidate for English), then we know that T_L is liable to be inconsistent by Gödel's second incompleteness theorem.

It would seem, then, that the answer to the original question must be 'no'. In that case we ask a second question: is English, or at least, the relevant part of it, E, one of the languages in the family being considered? If the answer to this is 'yes' then it follows that T_E is not expressible in English, which is self-refuting, since the theorist explained how to construct each T_L in English (assuming the theorist to speak English—and if they do not, just change the language in question). If, on the other hand, the answer is 'no' then the original problem of showing that the semantic concepts of English are consistent has not been solved.

Hence, all attempts to solve the paradoxes must swing uncomfortably between

[18] See, e.g. Sainsbury [24, Ch. 1].

[19] For versions of this argument, see Priest [14, 1.7] and [16, Section 1].

inconsistency, incompleteness and inexpressibility, a pattern that is clear from the literature.[20]

Let us take it, then, that truth and falsity overlap (at least to see where this takes us): for some αs we have both α and $\neg\alpha$. (This does not imply that the LNC fails, as we have already seen. If $\neg$ is a contradiction-forming operator, it should hold.) We can now deal with another law of negation: *ex contradictione quodlibet* (ECQ): $\alpha, \neg\alpha \vdash \beta$. Unlike the other laws we have already met, this one has always been contentious historically. And its presentation to a class of students before they have been taught a logic course, is sure to draw pretty universal dissent. Given the present discussion, it can clearly be seen to fail. For we can simply take an α which is both true and false, and a β that is not true. This instance of the inference is not truth-preserving, and hence the inference is not valid (truth-preservation being at least a necessary condition for validity). For good measure, the equally contentious inference of Antecedent Falsity (AF), $\neg\alpha \vdash \alpha \rightarrow \beta$, must also be invalid, for exactly the same reason (*modus ponens* holding); as, again, and more contentiously, must be the disjunctive syllogism (DS): $\alpha, \neg\alpha \vee \beta \vdash \beta$.

7 BOOLEAN NEGATION

In the case of intuitionism, where truth and falsity are not exhaustive, I argued that intuitionist negation is not a contradictory-forming operator, and that we can define a genuine such operator by the condition: $\neg\alpha$ is true iff α *fails* to be true. It is therefore natural to suppose that a similar objection can be made here. Dialetheic negation is merely a sub-contrary forming operator. The same clause still defines the genuine contrary-forming operator.[21]

The case that dialetheic negation is not really a contradictory-forming operator is harder to make out, precisely because the negation validates so much of the classical account, and especially the LNC, LEM and the flip-flop between truth and falsity. It can rest solely on the fact that the truth of $\neg\alpha$ does not rule out that of α. The crucial question, then, is whether the alternative negation fares any better.

Suppose we define an operator, $\sim$, such that $\sim \alpha$ is true iff α is not true, and, let us say, false otherwise. As can easily be checked, given what we know about negation, the operator $\sim$ toggles between truth and untruth, and satisfies the LEM and LNC; it might also appear that for no α can we have both α and $\sim \alpha$. In the literature on relevant logics $\sim$ is called 'Boolean negation'.[22]

The behaviour of $\sim$ requires more careful examination, however. In particular, why should one suppose that we can never have both α and $\sim \alpha$? The natural argument is simply that if α and $\sim \alpha$ are true then α is both true and not true, and

[20] This is documented in Priest [18]. See especially the last chapter.
[21] This is claimed, for example, by Slater [25].
[22] The following material is covered in more detail in Priest [16].

this cannot arise. But we cannot argue this way without seriously begging the question. If, as the dialetheist claims, some statements and their negations are both true, maybe α can be both true and not true. Indeed, if a dialetheic solution to the semantic paradoxes is correct, and α is 'α is not true' then α *is* both true and not true. The Boolean properties of 'Boolean negation', may therefore be an illusion.[23]

To rub this in, a dialetheist may even endorse the principle that if α is false, α is not true, (the Exclusion Principle). If we use T for a truth predicate (and F for a falsity predicate) and angle brackets as a name-forming device, this can be expressed as: $T\langle\neg\alpha\rangle \rightarrow \neg T\langle\alpha\rangle$. If a person does not accept truth value gaps, they also accept the converse of the Exclusion Principle. Hence we have $T\langle\neg\alpha\rangle \leftrightarrow \neg T\langle\alpha\rangle \leftrightarrow T\langle\sim\alpha\rangle$. We also have that $F\langle\neg\alpha\rangle \leftrightarrow T\langle\neg\neg\alpha\rangle \leftrightarrow T\langle\alpha\rangle \leftrightarrow F\langle\sim\alpha\rangle$. Hence negation and Boolean negation collapse into each other: there is no difference; so it cannot be argued that one is the correct negation whilst the other is not.[24]

Even though for some αs, such as the 'Boolean liar', $\alpha\wedge\sim\alpha$ may be true, it still follows from the truth conditions of $\sim$ that for every α, $\alpha\wedge\sim\alpha$ is not true. This is, of course, a contradiction. And some have felt this to be an objection.[25] However, it is not. If certain contradictions are true, then we should not seek to avoid them, but to embrace them. And the contradiction that is both true and not true in this case is one generated by semantic concepts, negation and self-reference. This is exactly what one would expect if a dialetheic account of the semantic paradoxes is correct.

Another wrinkle on this argument is as follows. Since $\alpha\wedge\sim\alpha$ is never true, then, according to the definition of validity as necessary truth-preservation, the principle of inference $\alpha\wedge\sim\alpha\vdash\beta$ is valid. Any contradiction of the form $\alpha\wedge\sim\alpha$ (such as a Boolean liar) will therefore induce triviality (everything follows), which is unacceptable. This argument is also fallacious, however. For to establish the conclusion we need to argue: since it is impossible for $\alpha\wedge\sim\alpha$ to be true then, necessarily, if $\alpha\wedge\sim\alpha$ is true then β is true. This is just a modalised version of AF, and just as fallacious.[26]

I have sometimes heard it argued that AF is acceptable in the present context, since the context is metatheoretic, and metatheory is (must be?) classical. This is a short-sighted argument. Any intuitionist or dialetheist takes themself to be giving an account of the correct behaviour of certain logical particles. Is it to be sup-

[23] Slater [25, p. 453], seems to think that the fact that contradictories cannot both be true, *by definition*, settles the matter. This is essentially just Quine's argument to the effect that changing the logic is changing the subject (see fn. 2). But in any case, even if it is definitionally true that contradictories cannot be simultaneously true, there is no *a priori* reason why definitional truths may not also be paradoxical.

[24] As a matter of fact, I have suggested elsewhere (Priest [14, 4.2]) that there are good methodological reasons for rejecting the Exclusion Principle, since it spreads contradictions apparently unwarrantedly: given the Principle, any contradiction, $\alpha\wedge\neg\alpha$, gives rise to a contradiction concerting truth, $T\langle\alpha\rangle\wedge\neg T\langle\alpha\rangle$.

[25] See, e.g. Parsons [11].

[26] One might vary the argument by employing, not the modal definition of validity, but the model-theortic one: an inference is valid if it is truth-in-an-interpretation preserving. The essential point is the same, however. See Priest [16].

posed that their account of this behaviour is to be given in a way that they take to be incorrect? Clearly not. The same logic must be used in both 'object theory' and 'metatheory'. Indeed, even this distinction is bogus for someone who espouses a dialetheic solution to the semantic paradoxes. The idea that the metatheory must be a distinct, more powerful, theory, is a response to the first horn of the dilemma we looked at in the last section. It has nothing to recommend it once we give up trying to solve the semantic paradoxes. The distinction between a theory (say about numbers) and its metatheory makes perfectly good sense to a dialetheist. But there is no reason to insist that the metatheory must be stronger than, and therefore different from, the theory. Indeed, if the original theory deals with, say, numbers and truth, then the metatheory may be a *subtheory* of the theory.

It is always possible for someone to reply to all of this by agreeing that it is impossible to prove that $\sim$ behaves classically, but saying, none the less, that they intend to employ a connective, $\sim$, and let it be governed by the rules of classical negation, such as $\alpha\wedge \sim \alpha \vdash \beta$. But then it is always possible for someone to say that they will employ a connective $*$ and let it be governed by the rules of *tonk*. The obvious reply in both cases, is that they are simply using a connective that has no well-defined sense, as is shown by the fact that triviality ensues.

8 ARROW *FALSUM*

Negation, then, does not satisfy ECQ, even Boolean negation. But how can this be? There must be some sense of negation that satisfies ECQ. For example, let $\perp$ (*falsum*) be a logical constant such that it is a logical truth that $\perp \rightarrow \alpha$, for every α. For example, if we have a truth predicate, T, satisfying the T-schema ($T\langle\alpha\rangle \leftrightarrow \alpha$, for some conditional operator, $\rightarrow$, and every α), $\perp$ can be defined as $\forall x Tx$. We can then define $-\alpha$ simply as $\alpha \rightarrow \perp$ to obtain the appropriate ECQ. For we have $\alpha, -\alpha \vdash \perp$ and $\perp \vdash \beta$.[27] ($-\alpha$ is, of course, equivalent to $\neg\alpha$ in both classical and intuitionist logic.)

The point is well made. There is such a logical constant, and such a notion defined by employing it. But it is not negation. Its properties depend, of course, on the notion of conditionality employed. In particular, the LEM and LNC reduce to $\alpha \vee (\alpha \rightarrow \perp)$ and $(\alpha \wedge (\alpha \rightarrow \perp)) \rightarrow \perp$, respectively. There is no reason to accept the first of these. The only ground could be that the falsity of α suffices for the truth of $\alpha \rightarrow \perp$, i.e., AF; and we have already seen that this is to be rejected. The latter may appear more plausible, but in fact fails in a number of accounts of the conditional, namely those that reject contraction (absorption): $\gamma \rightarrow (\gamma \rightarrow \delta) \vdash \gamma \rightarrow \delta$. The other laws also depend on properties of the conditional, often of a dubious nature.[28]

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[27] See Priest [14, 8.5].

[28] If $\alpha \supset \beta$ is defined, as usual, as $\neg\alpha \vee \beta$ then it is not difficult to see that $\neg\alpha$ is equivalent to $\alpha \supset \perp$. But $\supset$ is not a conditional operator: *modus ponens* for it fails. (This is just the DS.)

Crucially in the present context, $-$ is not to be identified with $\sim$. Given the situation, there is no plausible inference from $\neg T\langle\alpha\rangle$ to $T\langle\alpha \to \bot\rangle$. (Though the converse is more plausible: if $T\langle\alpha \to \bot\rangle$ then, assuming that T distributes over the conditional, we have $T\langle\alpha\rangle \to T\langle\bot\rangle$; and by the T-schema $T\langle\alpha\rangle \to \bot$. If contraposition holds we have $\neg\bot \to \neg T\langle\alpha\rangle$. But $\neg\bot \to \neg\bot$ and $\bot \to \neg\bot$, so $\neg\bot$ by the LEM; and $\neg T\langle\alpha\rangle$, by *modus ponens*.)

One might wonder whether a dialetheic solution to the semantic paradoxes can be sustained once the connective $-$ is at our disposal, due to the reappearance of triviality-inducing extended paradoxes. It can. For example, the form of the Liar using $-$ is just a sentence, β, such that $T\langle\beta\rangle \leftrightarrow (T\langle\beta\rangle \to \bot)$. If we could help ourselves to the principle of contraction then we could infer $T\langle\beta\rangle \to \bot$, and so $T\langle\beta\rangle$, and so $\bot$. This is just, in fact, a Curry paradox. But contraction fails in numerous accounts of the conditional, and there are reasons to suppose that the conditional of the T-schema does not satisfies it.[29]

9 DENIAL

So far I have discussed two aspects of negation: its semantics and its inferential relations. There is a third aspect that must be discussed in any adequate treatment of the subject: its pragmatics. To this I now turn. The pragmatic issues concerning negation all relate to denial.[30]

Assertion is a linguistic act. It is normally performed by uttering an indicative sentence with a certain illocutory force. Typically, its aim is to indicate to a hearer that the utterer accepts, that is, believes, the sentence asserted (or, at least, to get hearers to believe that the utterer believes it). Denial is a linguist act with a different illocutory force. Typically, its aim is to indicate to a hearer that the utterer rejects, that is, refuses to believe, something or other (or at least, to get the hearer to believe this).[31] Note that denial indicates something much stronger than the mere absence of belief: we do not deny something when we are undecided about it.

This much all can agree on. Henceforth it is more contentious. Although assertion and denial are distinct linguistic acts, Frege argued[32] (and many now accept[33]) that to assert the negation of a sentence is always to deny it. (It is certainly not the only way: one can say 'no', shake one's head, or even stomp off in a rage.) The

[29] For arguments against contraction, see Priest [14, Ch. 6] and [16, Section 7].

[30] The material in this section draws on Priest [17].

[31] I am not claiming that 'deny' is always used in this way in the vernacular: 'negation' and 'denial' are often, in fact, used interchangably. However, it is important to distinguish clearly between propositional content and linguistic act. For this reason I will stick to using these two words as explained. There is, in fact, a tradition in philosophy, going back to Aristotle himself, for using 'denial' and 'assertion' for the propositional contents of negated and unnegated sentences (respectively). In the case of 'assertion', this invited a confusion that bedeviled logic until Frege cleared it up. (See, e.g. Frege [7].) I suspect that the confusion in the case of 'denial' is still taking its toll.

[32] See Frege [7].

[33] See, e.g. Smiley [26].

Fregean move is not terribly tempting if negation is what I have taken it to be. If I assert a sentence expressing the contradictory of α, I certainly do not, in general expect a hearer to believe that I reject α. The sentence may well be a complex one; and the fact that it expresses the contradictory of α may not be obvious—or even known by me.

If, by asserting $\neg\alpha$, one means something like asserting a sentence expressing α with a 'not' inserted at an appropriate place (which is what, I take it, Frege had in mind), the Fregean move is tempting, but still incorrect. This is because one can assert $\neg\alpha$, even in this sense, without denying it. Consider a dialetheist (like me) who asserts both 'The Liar sentence is true' and 'The Liar sentence is not true', for example. In asserting the latter I most certainly do not intend you to come to believe that I reject the former: I don't.

Even setting dialetheism aside, there are reasons for supposing that asserting a negation (in the Fregean sense) is not a denial. In explaining their views, people often assert contradictions, unwittingly. In this way, they discover—or someone else points out to them—that their views are inconsistent. In virtue of this, they may wish to revise their views. But in asserting $\neg\alpha$ in this context, they are not expressing a refusal to accept α, i.e., denying it. It is precisely the fact that they accept *both* α and $\neg\alpha$ that tends to promote belief revision. It may even be rational sometimes—as a number of classical logicians have suggested—to hang on to both beliefs and continue to assert them: consider, for example, the Paradox of the Preface.[34] If to assert $\neg\alpha$ is to deny α, mooting this possibility would not even make sense.

Denying is not, therefore the same thing as asserting a negation—even if one interprets negation to mean sticking in a 'not'. Yet acts of denial *can* be performed by asserting negations. If, for example, I am in a discussion with someone who claims that the truth is consistent, it is natural for me to mark my rejection of the view by uttering 'it is not', thereby denying it.

This raises the question of when the uttering of a negated sentence is to be interpreted as an assertion, and when as a denial. There is, in general, no neat answer to this question. One has to asses the intentions of the utterer. The information provided by tone of voice, context, etc. will provide relevant clues here. Nor is there anything novel in this kind of situation. The utterance of the sentence 'would you close the door' can constitute linguistic acts with quite different illocutory forces (e.g., commanding, requesting). One often needs to be a very fluent speaker of a language (including having a knowledge of the social practices and relations in which the language is embedded), and have detailed knowledge of the context, to be able to determine which linguistic act is, in fact, being performed.

In most contexts, an assertion of $-\alpha$ would constitute an act of denial. This is because, in uttering this, one thereby commits oneself to $\alpha \rightarrow \perp$ (for a suitable conditional, $\rightarrow$). But no one in their right mind would accept $\perp$. (Why this is, is

[34] See Priest [14, 7.4].

a substantial and interesting question, but not one we need to go in to here.) And since one may be understood as refusing to accept $\perp$, one may be understood as refusing to accept α. The qualifier 'in most contexts' is there because it is, I suppose, possible that one might meet someone who is not in their right mind and who believes that everything is true. In the mouth of such a person $-\alpha$ would not constitute a denial: nothing would.

10 *REDUCTIO AD ABSURDUM* AND EXPRESSIBILITY

While we are in this area, a couple more points are worth noting. The first concerns the principle of *reductio ad absurdum* (RAA).[35] The purpose of a *reductio* argument is often to establish something of the form $\neg\alpha$ by deducing a contradiction from α. Dialetheism need not affect this enterprise. If the deduction establishes that $\alpha \rightarrow (\beta \wedge \neg\beta)$, and the $\rightarrow$ in question contraposes, then we have $(\beta \vee \neg\beta) \rightarrow \neg\alpha$ by contraposition LDM and LDN. $\neg\alpha$ follows by the LEM.

In a polemical context, the point of a *reductio* argument is not normally to establish something, but to try to force an opponent to give up a view. In a dialetheic context, establishing its negation is not *logically* sufficient for this. However, whilst a contradiction may be logically possible, it does not at all follow that it may be rational to believe it. That I will turn into a fried egg tomorrow is logically possible, but a belief in this is ground for certifiable insanity. (*A fortiori* that I both will and will not turn into a fried egg, since this entails it.) And an argument against an opponent who holds α to be true *is* rationally effective if it can be demonstrated that α entails something that ought, rationally, to be rejected, β. For it then follows that they ought to reject α. β may be a contradiction, or it may be the claim that I will turn in to a fried egg. Not all contradictions may work. For example, that the Liar sentence is both true and not true may be (in fact, is) perfectly rationally acceptable. This raises the question of when and why something *is* rationally acceptable. But to explore this issue would take us too far afield here.

The second issue is this. It is sometimes urged as an objection to dialetheism that dialetheists cannot express their own views.[36] Notably, they cannot express α in such a way as to rule out $\neg\alpha$. Several points are pertinent here. First, it is not clear that dialetheists need to be able to express α in such a way as to rule out $\neg\alpha$ (however one interprets this notion). Their situation is not like that in which many who try to give consistent accounts of the semantic paradoxes find themselves: the very notions that are rendered ineffable in their theory are required to explain it. (See the dilemma argument of Section 6.) It is sometimes argued that if a statement cannot rule out its negation, it cannot rule out anything, and that no statement can have a meaning unless it rules out *something*. This last claim is just false. The

[35] The following material is covered at greater length in Priest [14, 7.5] and Priest [15].

[36] See, e.g. Parsons [11], Batens [2]. The following material draws on Priest [19].

statement 'everything is true', entails everything, including its own negation, and so rules nothing out; yet it is quite meaningful.[37]

Next, it is not clear that non-dialetheists can do any better as far as ruling out goes. If dialetheism is correct then, like it or not, no one can rule out $\neg\alpha$ by asserting α. Maybe they would like to; but that does not mean they succeed. Maybe they intend to; but intentions are not guaranteed fulfillment. Indeed, it may be logically impossible to fulfill them, as, for example, when I intend to square the circle. Even if dialetheism is false and Boolean negation makes sense, asserting $\sim \alpha$ does not rule out α, at least in one sense: someone who asserts $\sim \alpha$ may still assert α. The cost is that they are certifiably insane, since they are committed to everything being true. But there are no *logical* guarantees against insanity. And the use of Boolean negation is not necessary to rule things out in this sense anyway; the use of $-$, which makes perfectly good dialetheic sense, will do just as well.

The third, and conclusive, point is that anyone (dialetheist or otherwise) *can* express themself in such a way as to rule things out. They cannot rule out α by asserting $\neg\alpha$—or anything else. But they can simply deny α (which, as we have seen, is not equivalent to asserting anything).

In the discussion of the last three sections we have seen the necessity of distinguishing between negation, denial, and arrow *falsum*. As we have seen, all these things make perfectly good sense from a dialetheic perspective—they are just not the same thing. The orthodox modern account of negation fuses them together. But this fusion is a confusion: they need to be kept distinct. It is common enough in the development of science for us to come to realise that we had run together different concepts (e.g., rest mass and inertial mass). This is another case. And in this case, the antipathy towards dialetheism is, in part, I think, a product of this confusion.

11 CONCLUSION

We have now (I hope) looked at all the main aspects of negation. The discussion has hardly been comprehensive. At many points we have had to leave discussions which would have taken us too far afield in a simple essay on negation, e.g., into areas concerning meaning, paradox, conditionals, rationality. It is a mark of the conceptual centrality of negation that it is integrally related to these other important and contentious concepts. For these reasons, definitive accounts of negation are hardly to be expected. But this certainly does not mean that all accounts are equally good; and I hope that this essay shows at least that a dialetheic account of negation ends up with a not, rather than in a knot.[38]

[37] See, further, Priest [14, 7.2].

[38] Earlier drafts of parts of this essay were read at the University of Edinburgh, King's College, London, and the University of Queensland. I am grateful to colleagues in these places for a number of helpful comments. I am also grateful to Diderik Batens, Nick Denyer and Greg Restall for written comments on an earlier draft.

12 APPENDIX: A LITTLE MODEL-THEORY

In this appendix I will give some details of a formal model-theory that is appropriate for the account of negation I have advocated. Let us start by considering a language that contains only the connectives $\wedge$, $\vee$, and particularly, $\neg$.

We can take an interpretation to be a relation, R, between sentences and two objects: t, f. αRt $[\alpha Rf]$ is read as: α is true [false] in the interpretation R. Given this conception, it is natural to generalise the truth/falsity conditions of conjunction, disjunction and negation, by requiring that every interpretation, R, satisfy the following conditions:

$\alpha \wedge \beta Rt$ iff αRt and βRt
$\alpha \wedge \beta Rf$ iff αRf or βRf

$\alpha \vee \beta Rt$ iff αRt or βRt
$\alpha \vee \beta Rf$ iff αRf and βRf

$\neg\alpha Rt$ iff αRf
$\neg\alpha Rf$ iff αRt

As usual, these clauses suffice to determine R for all sentences, once it is determined for atomic sentences. I will call any evaluation, R, of this kind a *simple* evaluation. Validity is then defined in the usual way, viz.: an inference is valid iff, for all simple evaluations, R, if all the premises are true in R, the conclusion is true in R.

The semantics, as described so far are those of First Degree Entailment,[39] and do not validate the LEM. To do this, as I have argued should be the case, it is natural to require that for all α, $\alpha R1$ or $\alpha R0$. (If this constraint is satisfied for all atomic sentences, the truth/falsity conditions suffice to ensure that it is satisfied for all sentences.) If we were to impose the dual condition that for no α, $\alpha R1$ and $\alpha R0$, we would, of course, have the classical propositional calculus. But since some statements are both true and false, it is natural for us to allow sentences to be both true and false in an interpretation. Hence we do not make this requirement.

These semantics give us the system LP of Priest [14, Ch. 5].[40] As is easy to check, the semantics validate the LEM, LNC, LDM and LDN. In fact, they validate all the tautologies of the classical propositional calculus.[41] It is also easy to

[39] See, e.g. Anderson, Belnap and Dunn [1, 50.3]. For a general discussion of this style of semantics, see Dunn [5].

[40] Except that the semantics there employ an evaluation function instead of a relation. These approaches are equivalent clasically, but the latter leads to triviality in the context of a dialetheic approach to the semantic paradoxes, whilst the former appears not to. See Smiley [26] and Priest [17].

[41] See Priest, [14, 5.2].

check that they invalidate ECQ and DS. Hence the semantics deliver an account of validity having just the right properties—at least for a language with just those three connectives.

Now consider a propositional language with the additional connectives $\sim$ (Boolean negation), $\rightarrow$ (an intensional conditional operator), and (for good measure) $\perp$. For this richer language the simple notion of interpretation is inadequate, or at least, highly problematic. The inadequacy of extensional semantics for intensional notions is well known, and hardly needs discussing. The problem with Boolean negation is more novel. Suppose we employ the simple semantics just described, and give the recursive conditions for $\sim$ in the obvious way:

$\sim \alpha Rt$ iff it is not the case that αRt
$\sim \alpha Rf$ iff αRt

The semantics quickly give us the validity of $\alpha\vee \sim \alpha$, $\neg(\alpha\wedge \sim \alpha)$ and $\sim (\alpha\wedge \sim \alpha)$. But now consider Boolean ECQ: $p\wedge \sim p \vdash q$. They do not deliver the validity of this—as long as we remember that AF is not a valid principle of reasoning—for reasons that we have already seen in Section 7. A counter-model for this ECQ would, however, require us to come up with an interpretation, R, in which, for some propositional parameter, p, pRt and it is not the case that pRt. In other words, a counter-model would itself be an inconsistent entity. Clearly, one cannot rule out the possibility of this from a dialetheic perspective. But, equally, it is not clear what the status of such an interpretation should be.

An adequate semantics for the extended language can be obtained by changing the notion of interpretation and the corresponding notion of truth in an interpretation as follows.[42] A propositional structure is a pair, $\langle \mathbf{L}, T\rangle$. $\mathbf{L}$ is itself a structure, $\langle L, \wedge, \vee, \neg, \rightarrow, \sim\rangle$. Intuitively, L is thought of as a set of propositions, or Fregean senses. The other components are operators on L of obvious arity. I use the same sign for the operator and the logical connective for which it is to be the interpretation. (Disambiguation will be provided by the style of variable it is used with.) $\langle L, \wedge, \vee, \neg\rangle$ is a complete De Morgan lattice, i.e., a distributive lattice, where $\neg$ is an involution of period two ($\neg\neg a = a$ and $a \leq b \Rightarrow \neg b \leq \neg a$). It is natural enough to suppose that propositions have the structure of such a lattice, with the lattice ordering capturing the idea of containment of sense, that is, entailment.

T is a subset of L. Intuitively, it is thought of as the set of true propositions. This makes it natural for it to satisfy the following conditions. (1) T is a filter on the lattice. (2) If the binary relation $R \subseteq L \times \{t, f\}$ is defined by:

aRt iff $a \in T$
aRf iff $\neg a \in T$

[42] The following draws on, but also modifies, Priest [13].

then R is a simple evaluation. (This could be broken down into simpler components, but I leave it like this here to emphasise the connection between these semantics and the extensional ones.) (3) $a \rightarrow b \in T$ iff $a \leq b$. (4) $\sim a \in T$ iff $a \notin T$. These last two conditions are exactly the ones one would expect for an entailment operator and Boolean negation. Finally, since the lattice is complete, it has a minimal element. I will write this as $\perp$ (typographic identity again indicating semantic function.)

Now, an interpretation for the language is a pair, $\langle P, \nu \rangle$, where P is a propositional structure and ν is a map from the language into P, satisfying the natural homomorphism. We may read $\nu(\alpha) \in T$ as: α is true in the interpretation. A sentence, α, is a logical truth iff it is true in every interpretation. An inference with set of premises Σ and conclusion α is valid iff in every interpretation $\nu[\Sigma] \leq \nu(\alpha)$, where $\nu[\Sigma]$ is the meet of $\{\nu(\beta); \beta \in \Sigma\}$. Thus, a valid inference is one where the senses of the premises contain that of the conclusion.

It is not difficult to show that simple evaluations and truth filters are essentially inter-translatable. (See Priest [13, 10.3, 10.4].) Hence, these semantics subsume the simple semantics. It is therefore not surprising that these semantics validate LEM, LNC, LDM and LDN, and that they invalidate ECQ and DS. It is a simple exercise to show that they also validate: $\perp \rightarrow \alpha$, *modus ponens*, LC (reasonable for an entailment operator), $\alpha \vee \sim \alpha$ and $\sim (\alpha \wedge \sim \alpha)$. They also invalidate AF and Boolean ECQ. I leave the former as an exercise. To see the latter, just consider the propositional structure where L is the lattice of integers (positive and negative), $\neg a$ is $-a$, T comprises the non-negative integers, and $\sim a$ is -3 if $a \in T$, and $+3$ if $a \notin T$. ($\rightarrow$ is irrelevant.) Map p to 6. Then the lattice value of $p \wedge \sim p$ is -3. Now map q to -6 to give a counter-example to $p \wedge \sim p \models q$.

The semantics therefore validate or invalidate exactly the required inferences.

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