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IV
Classical Logic *aufgehoben*

G. Priest

> The history of quasi-empirical theories is a history of daring speculation and dramatic refutations. But new theories and spectacular refutations . . . do not happen every day in the life of quasi-empirical theories, whether scientific or mathematical. There are occasional long *stagnating periods* when a single theory dominates the scene without having rivals or acknowledged refutations. Such periods make many forget about the criticizability of basic assumptions. Theories, which looked counterintuitive or even perverted when first proposed, assume authority.
>
> Lakatos, 1967, p. 41.

> Almost no one, perhaps no one at all, needs to be told that the vitality of science depends upon the continuation of tradition-shattering innovations. But the apparently contrary dependence of research upon a deep commitment to established tools and beliefs receives the very minimum of attention.
>
> Kuhn, 1963, pp. 370–371.

This paper[1] has two basic aims, and I will start by explaining what they are.

1) Many logicians' conception of the history of logic has not, as yet, outgrown a positivism which takes the currently accepted theory on some subject as the final truth on the matter and any historical theory to be either a crude forerunner of the present system or else a wild blunder. In recent years we have seen the philosophy of science grow out of a similar sterile phase. It is now widely accepted that the nature of science is to be appreciated only via the dynamics of its growth. Past scientific theories may be valuable—indeed indispensable—to the growth of science even though wrong, and current theories may be wrong even though accepted. The dynamics of theory growth in logic has met with little attention. Fortunately however, some of the recent views of the dynamics of science have wide application to the dynamics of theory growth in general. Kuhn's notions of paradigm and revolution (see Kuhn 1962) and Lakatos' notion of research programs which progress and degenerate (see Lakatos 1970) are particularly useful. I shall assume the reader to have a basic familiarity

with these views and although I would subscribe wholeheartedly to neither of them I shall use them without hesitation where I feel their application can provide genuine insights into the history of logic. Thus my first aim is to show how the understanding of theory-dynamics emerging from the history of science can be fruitfully applied to the history of logic.

2) The upshot of this discussion will be that we are now in a revolutionary situation in logic. An old paradigm which was a great advance in its day, has now become a rigid, entrenched dogma, unable to cope with, or hold out any hope of solving its basic anomalies. It is liable to be replaced by a better one, and in the last part of my paper I will say what that is. This brings me to the second aim of my paper, namely, to make the revolution self-conscious: by bringing the situation out into the open and allowing logicians and philosophers to see the dynamics of the process underway, I hope to speed the revolution.

1. Nineteenth century logic

So much for the twin aims. Let me now turn to the first. Most of what I have to say concerns "classical logic"—by which I mean the logical theory of the two-valued propositional and predicate calculus, whose foundations were laid by Frege and Russell, which developed through Hilbert, Carnap and Tarski (to name but a few) and which now finds its staunchest defenders in Quine and Geach. However, I do not want to give the impression that logic started with Frege, so we must trace the story back a little before him.

The dominant paradigm in nineteenth century logic was a form of Aristotelian logic.[2] The theory of the syllogism underpinned nineteenth century logic. However, eventually the internal difficulties of the theory led to its overthrow. Its restrictive doctrine of the subject/predicate form, and the poverty of its forms of inference, could not account for a great deal of deductive reasoning. In particular the theory collided with the nineteenth century drive for rigour in mathematics. Once the logic used by mathematicians was put under the microscope, it became clear that Aristotelian logical theory could in no way do it justice. Hence it gave way to classical logic.

The victory of classical logic was by no means automatic, the fight being carried on in philosophy journals for many years (with rear-guard actions being fought by Aristotelians well into the twentieth century). However, the problem-solving ability of classical logic was greater than that of nineteenth century logic and this gave it the edge. In particular, classical logic could solve problems that had troubled Aristotelians. To take an example

(that of De Morgan), classical logic could cope with, and show the validity of inferences such as

All horses are animals
———————————————
All heads of horses are heads of animals

which Aristotelian logic could not.

As another example, in rejecting the subject/predicate form doctrine and introducing the notion of a quantifier, Frege produced a solution to perhaps the most irritating semantic problem of Aristotelian logic, namely, to give an account of the denotations of phrases such as 'a man', 'no woman', etc. This conundrum had led logicians into all kinds of tangles and even plagued Russell as late as 1900 when in the *Principles of Mathematics* he had not yet managed to free himself entirely from the grips of the Aristotelian problematic.[3]

The classical revolution will be a rich hunting ground for historians and methodologists of logic, but since my interest is more in the current state of our discipline I will pass it over quickly.

2. Classical logic: the progressive phase

Before continuing the success story of classical logic, I owe it to the reader to show that a logical theory can be seen as a Kuhnian paradigm or a Lakatosian research program. This I will now do using classical logic as an example.

Let us start with Kuhn. A paradigm (or disciplinary matrix) is a collection of elements shared by a community of scientists. Of these the most important (especially for a non-experimental science) are a) an explanatory theory, and b) model examples of applications of the theory to solve problems. Scientists working in the paradigm use the theory to solve problems, modelling their solutions on the standard examples, and at the same time articulate (develop) the theory and model solutions. The theory of classical logic is the well-known propositional and predicate calculus and can be found in modern dress in any modern text book. In its unarticulated form it can be found in Frege's *Begriffsschrift* and the early parts of Whitehead's and Russell's *Principia Mathematica.* The theory was used by Frege and Russell to solve certain problems, viz. to give a theoretical account of inference as it is found in mathematics, and their model solutions to these problems are found in Frege's *Grundgesetze* and the latter part of Whitehead's and Russell's *Principia.* Of course as the classical theory was articulated it found applications in new areas and therefore acquired new model solutions, but these two books were paradigms in another of Kuhn's senses of the word—classic works which provided reference points for a generation of research.

Let us now turn to Lakatos. The central item of a research program is the "hard core" of theoretical notions which constitute the defining characteristics of the program. At any time the "refutable variant" of the program is a theory based on these central notions. As the program meets problems the "refutable variants" are changed, but only auxiliary hypotheses, not the hard core, can be changed. The overall development of the program is guided by a "positive heuristic" which tells the research workers which problems to tackle, which to ignore, etc. The hard core of classical logic is the theory of the propositional and predicate calculus, the Kuhnian paradigm. The "refutable variant" of the research program (at a certain time) is the total extant theory of classical logic, its semantics, proof theory and other meta-logical aspects. The positive heuristic of the program was (note the past tense; I shall return to that later) precisely to articulate the logical theory implicit in the hard core.

We can now pick up the historical story again. We have seen that classical logic had greater explanatory power than its nineteenth century rival and this is part of the explanation of why it superseded it. However, another important factor is that the classical program was incredibly *progressive.*[4] That is, under the guide of its positive heuristic it successfully solved its problems, gave rise to many new notions and results and was the fruitful basis of many new sub-programs. At the same time its rival was doing nothing—except defend itself against the attacks of classical logicians—i.e. it was degenerating.

To give a decent account of this phase of the classical program would be a task in itself, and I shall have to be content with a brief summary. (The people indicated are major though not necessarily the only figures in the areas indicated.)

The progressive phase of classical logic:[5]

1879: Isolation of a fragment of language adequate to express a substantial amount of reasoning, its formalization and axiomatization. (Frege)
1910: The proof that with certain assumptions, this fragment could account for all reasoning used in contemporary mathematics. (Russell, Whitehead)
1915–20: Foundations of model theory laid. (Skolem, *et al.*)
1918–20: Truth tabular semantics for propositional logic. Propositional logic proved consistent, complete, decidable. (Post, *et al.*)
1915–30: Foundations of proof theory laid. (Hilbert, *et al.*)
1929: Predicate calculus proved consistent. (Hilbert)
1930: Completeness proof for predicate logic. (Gödel)
1936: Characterization of algorithms. Foundations of recursion theory laid. (Turing *et al.*)
1936: Predicate logic proved undecidable. (Church)

1930–35: Discovery of formal semantics for classical logic. (Tarski)
1930–40: Discovery of natural deduction. (Gentzen)
1940–50: Formulation of important metatheoretic notions, e.g. maximal consistency, saturation. (Henkin)
1950–60: Reduction of modal logic to classical logic. (Carnap, Kripke, *et al.*)[6]

These are by no means the only novel events in logical theory in the eighty years in question but they are certainly among the most important, and they show just how progressive and fruitful classical logic was during this time.

So much for the development of the logical theory itself. But the story would not be complete without reference to the metaphysical and foundational programs to which classical logic gave rise. We can list some of the major ones thus:[7]

1880–1910: Logicism. The attempt to show that pure mathematics is logic. (Frege, Russell, Whitehead)
1918–20: Logical atomism. The attempt to read off the ontological structure of the world from (classical) logic. (Russell, Wittgenstein)
1920–30: Constructionalism. The attempt to "build" the external world by logical constructions. (Russell, Carnap)
1920–30: Formalism. The attempt to distinguish between finitary and "ideal" mathematical reasoning and prove the consistency of mathematics in finitary terms. (Hilbert)

These offshoots of classical logic between 1880 and 1960 serve to underline how fruitful and stimulating of novel ideas classical logic was. During this period logic was in its most progressive phase certainly since the 13th century and arguably since Aristotle.

3. Classical logic: the degenerating phase

So much for the progressive phase of the classical program/paradigm. Unfortunately the success story of classical logic begins to tail off around 1940 and ends around 1960. The main thrust of this section is that classical logic has now hit a stagnant period (see the quotation from Lakatos at the start of the paper), where anomalies provide the objective conditions for revolution.

Let us look at it first in Lakatosian terms, and let us start with the fate of the offshoots of classical logic. Each of the four sub-programs I mentioned in the last section has degenerated and faded out. For each program this of course took time, but with hindsight we can see that the following incidents were crucial in killing off the programs.[8]

1) Logicism. The discovery of the paradoxes and the failure to handle these in a suitable way. Gödel's 1931 discovery of the unaxiomatizability of classical mathematics.
2) Atomism. The dismantling of the *Tractatus* by Wittgenstein himself.
3) Constructionalism. The failure of Carnap's *Aufbau*.
4) Formalism. Gödel's 1931 discovery of the unaxiomatizability of classical mathematics and the non-provability of consistency by finitary means.

The rise and fall of each of these programs would make interesting material for any historian of metaphysics but it suffices for our purposes to note that all these programs are now defunct (at least in their classical form. See 4.3).

Of course, metaphysical and foundational studies have not ceased since 1960 but even though many of them assume the correctness of classical logic it is only as an auxiliary hypothesis and not as part of the hard core. Moreover, those to which classical logic is essential, such as Quine's systematic philosophy (where classical logic provides canonical forms—see Quine, 1960) are in serious trouble. (See Routley, 1980.)

Let us now turn to the fate of classical logic itself since the early 1960's. Since then, classical logic has ceased to be an important source of insights into logic. It is a seam that has been worked out. To some, especially those with philosophical interests who have worked in mathematical logic, this situation, once noted, may be painfully obvious. To others it may sound a ridiculous claim. So let us now examine the situation more closely.

What are the most significant results, methods, problem-solutions etc. that have occurred in the area of orthodox logical studies in the last 20 years? It might be difficult to obtain a consensus concerning some suggested answers to this question such as 'relevant logic'. However, the following would occur on most people's lists: forcing and the Cohen independence results; non-standard analysis; the solution to Hilbert's 10th problem; the application of model theory to algebra; the semantics of intuitionist logic. Now ask yourself what relevance these results and methods have to *logic* as the theory of valid inference. The first four belong to the domains of set theory, analysis, axiomatic arithmetic and algebra respectively. Their relevance to logic *as such* is at best tangential. The fifth is certainly relevant to logic but not to classical logic! Intuitionism has long been a rival research program/paradigm to classical logic.[9]

What this shows is that the *Zeitgeist* of orthodox "logic" has shifted significantly in the last twenty years. Part of it has gone to inhabit the nether regions where the rivals to classical logic (such as intuitionism and relevant logic) are fomented. This minor shift is itself significant. For as Kuhn has

emphasized the proliferation and development of rivals to the paradigm is an index of crisis. However, the major part of the *Zeitgeist* which inhabited classical logic, has come to reside in mathematical logic, the domain inhabited by resplendent models, hyper-hyper-simple vector spaces and ever larger countable ordinals. And mathematical logic has about as much to do with logic as material implication has to do with implication. This shift is underlined vividly by the changes in the *Journal of Symbolic Logic*, the house journal of classical logic. Its pages have become increasingly closed and esoteric and its estrangement from the interests of real logicians is virtually complete.[10]

What does all this show? It shows that research in classical logic, or what it has now become, mathematical "logic" has ceased to be important for logic. And this in turn shows that the positive heuristic of the program which guided logicians to problems of logical theory and their solutions, has dried up. (Hence the use of the past tense in describing the heuristic of classical logic in section 2, above.) The driving force which produced so much new insight into the nature of logic has gone. Classical logical theory stagnates. At this point in theory dynamics another factor—the existence of anomalies—becomes crucial.

As both Kuhn and Lakatos emphasize, all paradigms or research programs face anomalies all the time, i.e. there are facts, results, problems, etc. which do not fit into the theory but go against it. Classical logic is no exception. The anomalies of classical logic are manifold: intensional contexts, discourse about non-existent objects, etc. but the two most important as far as I am concerned at the moment are a) the paradoxes of implication, b) the logical paradoxes.

The paradoxes of implication (e.g. that anything is a [classical] logical consequence of a logical falsity) were regarded as problematic from early this century. Indeed in its early years the paradigm was much criticized on these grounds, and a number of attempts were made to modify the paradigm in order to avoid them. None of them was successful. With the entrenchment of the classical paradigm these anomalies were pushed into the background and ignored. There are several *ad hoc* accounts of the paradoxes. However, the received position with respect to the paradoxes now is the C. I. Lewis[11] one, that however counter-intuitive the paradoxes of implication appear, they are in fact true. However, this quiescent acceptance of an anomaly—without even the excuse of an *ad hoc* hypothesis to cover its nakedness, will not do. It is possible to accept the Lewis position only by forgetting that logic is a normative subject. It is supposed to lay down the canons of good reasoning.[12] But any one of us who found students actually employing these paradoxical principles would rebuke them quite sharply.[13] Neither are these local anomalies of limited importance. They spill over into many other fields. For example, united with the reasonable view that bi-entailment is sufficient for propositional identity they issue in the result that all logical truths have the same sense, i.e. that there is only one necessary proposition. They also render an inconsistent but interesting theory such as the early infinitesimal calculus totally trivial and pointless.[14]

The second example of an anomaly, the logical (set theoretic and semantic) paradoxes, is much more difficult to dismiss (partly because of the orthodox view concerning the paradoxes of implication) and has been a pain in the side of logicians ever since Russell's famous letter to Frege.[15] For eighty years now logicians have been looking for a solution to the paradoxes. Yet no solution has been found. Of course we know of *ad hoc* ways of defusing the paradoxes but these are not solutions (see Priest, 1979, §1).[16] In fact this historical affair is a good example of the way *ad hoc* hypotheses can reasonably be used to protect a theory during its progressive phase. However, it is important that these are only *ad hoc* ways (albeit, ones of increasing sophistication) of avoiding the problem presented by the logical paradoxes to classical logic.

So classical logic has been faced with anomalies all the time. However, anomalies are more important at some times than at others. In particular, if there are fundamental problems (such as the above two) which have resisted solution by the brightest adherents of the discipline, the discipline will enter a period of crisis, that is, a period when the discipline is ripe for revolution. (Let me say that by "crisis" I intend to describe the objective state of the discipline, not the psychological states of individual logicians.) This is precisely the state logic is in now. For despite the attention of so many of the outstanding classical logicians this century there are no satisfactory classical solutions to these problems.

Let me put the general Kuhnian point in a slightly different Lakatosian way: when a program is progressing we may reasonably suppose that anomalies will get sorted out when their times come. However, once the program stagnates and the positive heuristic—the thing which gives the program the power to digest anomalies—wanes, we know that time is unlikely to come. Anomalies then, especially old ones, become quite crucial and telling facts against a stagnant program.

I have now made the main point of this section, that the time is objectively ripe for logical revolution. But no discussion of the current situation in logic would be complete without a few words about dogmatism. As Kuhn has emphasized, dogma plays an important role in any normal scientific tradition. The acritical attitude that scientists have to the fundamental tenets of their theory makes it possible for them to get on working with them without worrying about them. This is acceptable when a theory is progressing, but when, as in the case of classical logic, the theory ceases to progress, dogmatism acts as a reactionary force protecting the old paradigm.

That dogmatism about logical theory exists amongst logicians and philosophers is, I think, undeniable. A subjective measure of this is the reaction one normally meets when attacking the basic tenets of classical logic.

Logicians, not now used to having these attacked, except perhaps by intuitionists, are soon reduced to unsupported professions of faith.[17] A much more objective measure of the dogmatism of classical logic is the way it is taught. It is not taught as a problematic theory concerning logic. It is taught as a received doctrine. Very few of us would think it good to teach a philosophical theory without discussing its weaknesses and alternatives, but such is absent from the usual way that logic is taught. I think any of us who has learnt classical logic will sympathize with this observation and I am sure it will strike a chord with most logic teachers. (I am as guilty as anyone of teaching classical logic as a received doctrine.) The dogmatism in teaching logic is reflected in the institution of the logic text book. Text books abound in logic (though not in philosophy). They all cover much the same ground, the only difference between them being pedagogic. And most expound classical logical theory, quite dogmatically, as a received doctrine. As Kuhn says when discussing text books, their overt function is to teach the theory. Their covert function is to present it as a *fait accompli*, to stifle criticism and discourage objections to its fundamental principles. A glance through any selection of logic texts is enough to substantiate this claim.

So dogmatism exists. When a theory is progressing it has a positive function: to defend the basic principles of the subject while people get on with exploiting the good part of the theory. Once a theory starts to degenerate, however, dogmatism has no function other than to preserve the entrenched and inadequate theory. This is precisely how dogmatism functions now with respect to classical logic. It is the main (irrational) barrier to revolution.

Of course, to many adherents of the old paradigm, heavily indoctrinated with orthodoxy, the situation will appear somewhat different. Whilst not denying that mathematical logic has not produced much of interest to logic[18] for the last twenty years, they will interpret the situation differently. Classical logic is the final and ultimate truth about logic. Of course there can be no new insights to be obtained: the truth is already known. Of course there are no important problems left to be solved: all important problems have been settled. This view that the ultimate truth has been found embodies a blindness that ill behoves anyone with a passing knowledge of the history of their discipline. (But then wiping out the history of the discipline is an essential part of the dogmatic indoctrination of the orthodox paradigm). However, it really will not do. It can be maintained only by *ignoring* the anomalies that classical logic faces or down-grading them to insignificant problems. This in turn can be done only by *assuming* that classical logic is right and refusing to countenance a serious alternative.

This situation, which undoubtedly exists in some logical circles has been well described by Feyerabend, 1963, pp. 30-31. He is actually describing the current state in quantum physics but I have taken the liberty of changing his target. (However, I have indicated by square brackets where I have done this.)

> Assume that [logicians] have adopted, either consciously or unconsciously, the idea of the uniqueness of [classical logic] and that they therefore elaborate the orthodox point of view and refuse to consider alternatives. In the beginning such a procedure may be quite harmless. After all, a man can do only so many things at a time and it is better when he pursues a theory in which he is interested rather than a theory he finds boring. Now assume that the pursuit of the theory he chose has led to successes and that the theory has explained in a satisfactory manner circumstances that had been unintelligible for quite some time. This gives ... support to an idea which to start with seemed to possess only this advantage: It was interesting and intriguing. The concentration upon the theory will now be reinforced, the attitude towards alternatives will become less tolerant. Now if it is true ... that many facts become available [or at least that their significance becomes apparent] only with the help of such alternatives, then the refusal to consider them *will result in the elimination of potentially refuting facts*. More especially, it will eliminate facts whose discovery would show the complete and irreparable inadequacy of the theory. Such facts having been made inaccessible, the theory will appear to be free from blemish and it will seem that all the evidence points with merciless definiteness in the [direction of the theory]. This will further reinforce the belief in the uniqueness of the current theory and in the complete futility of any account that proceeds in a different manner. Being now very firmly convinced that there is only one good [logic], the [logicians] will try to explain even adverse facts in its terms, and they will not mind when such explanations are sometimes a little clumsy. By now the success of the theory has become public news. Popular [logic] books (and this includes a good many books on [philosophy]) will spread the basic postulates of the theory; applications will be made in distant fields. More than ever the theory will appear to possess tremendous ... support. The chances for the consideration of alternatives are now very slight indeed. The final success of the fundamental assumptions of [classical logic] will seem to be assured.

Those who take classical logic to be essentially correct but having a few "minor" difficulties to sort out are like those late nineteenth century physicists (such as Kelvin) who held that physics was all sewn up; that contemporary physical theory was the essential truth about the universe and that there were only a few small problems (e.g. about black body radiation) to be sorted out! Many nineteenth century logicians would also, no doubt, have maintained that logic was all sewn up. Kant of course maintained that both logic and dynamics were sewn up and was cruelly shown up by history. Let us hope that those, at least, who know enough history to be aware of Kant's mistakes, do not repeat them.

4. The shape of things to come

So much for history. Now for a little futurology. We have seen that by both Kuhn's and Lakatos' standards, logical theory is in a state of crisis, i.e. a time when the conditions are objectively ripe for revolution. Now if during

a crisis a new theory emerges which a) solves some of the anomalies of the old paradigm, b) preserves a substantial amount of the problem-solving ability of the old paradigm, and c) poses new open problems with promise of solution (i.e. promises to be the basis of a new and fruitful research program), a revolution is liable to occur.[19] But such a theory is already emerging: paraconsistent logic.[20]

The basic idea of paraconsistency is as simple as it is radical. The fundamental classical postulate that truth and falsehood are mutually exclusive is rejected and replaced by the idea that there may be sentences of a language such that both they and their negations are true. Let us call this the strong paraconsistency principle.[21] This is the central part of the hard core of the paraconsistent research program. That it is radical can hardly be denied: it runs against virtually the whole tradition of Western logic with perhaps the exception of Hegel and the dialecticians. However, it satisfies the above conditions. Before I show this however, let me point out that the actual case for paraconsistency does not rest on its satisfying these conditions. There are independent arguments for the strong paraconsistent principle based on the nature of proof and the expressive power of language.[22] However, these are not my main concern at the moment, which is to show that paraconsistency satisfies the theoretical conditions for a successful revolutionary theory. Let us see how.

4.1. *The solution of classical anomalies*

How paraconsistency solves (or perhaps better, dissolves) the problem of logical paradoxes is fairly obvious. The logical paradoxes are precisely what they appear to be: sound arguments showing that something is both true and false! Seen like this the paradoxical sentences cease to appear as anomalies and fit neatly into the conceptual framework. It is clear that such a position requires the rejection of certain principles of classical logic. However, this is no loss; for they are precisely ones that were anomalous anyway! The paradoxes of implication serve to spread contradiction like an infectious disease. A decent logic (like a decent doctor) prevents the spread. The paradoxical postulate that $A \& \sim A \models B$ (and others like it) has to go. However, most importantly the paraconsistent principle shows how and why it goes. This is simply seen. We suppose that some atomic sentences are true (and true only), some false (and false only) and some paradoxical (i.e. both). The truth (and falsity) conditions for negation and conjunction are given in the obvious way:

$\sim A$ is true iff A is false
$\sim A$ is false iff A is true

A & B is true iff A is true and B is true
A & B is false iff A is false or B is false

Clearly compound sentences may also be both true and false. Now take any sentence A which is both true and false. Then A & ~A is true (and false). But if B is an arbitrary sentence that is false (only), then clearly the inference A & ~A/B is not truth preserving, let alone valid. Thus paraconsistency solves this anomaly of classical logic too. The solution of other paradoxes of implication requires a more general framework incorporating the paraconsistency principle. However, we need not go into that.[23] We have shown all that is necessary: that the paraconsistent principle solves anomalies that have brought the old paradigm to crisis.

4.2. *Preservation of problem-solving ability*

For a new theory to be successful it must also preserve a substantial part (though not necessarily all) of the problem-solving ability of the old paradigm. But this the paraconsistent principle does. Let me explain.

Classical propositional logic can be formulated in a language with the two connectives ~ and & and the quantifier ∀. Semantics of paraconsistent logic show that any sentence of this language is paraconsistently valid if and only if it is a two-valued logical truth (see Priest, 1979, §3.13). Thus any axiom of classical logic is paraconsistently valid. The situation with classical rules is a bit more complex. The only rule that classical logic needs is disjunctive syllogism:

$$\frac{A \quad \sim(A \,\&\, \sim B)}{B}$$

but this is paraconsistently invalid. (Any paradoxical A makes the premisses true.) However, things go wrong only if A is paradoxical. Hence this inference is usable provided we are not in the realm of paradox.[24] Now many theories for which we need an underlying logic don't contain any paradoxical sentences. And we may therefore go ahead and use full classical reasoning.

Let me put it this way: provided a theory is consistent then we can use classical logic. But of course classical logicians are interested only in consistent theories anyway. Others are pointless. Thus the full strength of classical logic, wherever it can be used non-trivially, is taken over by paraconsistency. Its problem-solving power is preserved as required. Where its *extra* strength comes from is precisely in being able to function as the underlying logic of inconsistent theories. Disjunctive syllogism cannot be used in such circumstances and the theory does not collapse into triviality.

Thus we see that paraconsistent logic is related to classical logic in much the same way that, say, special relativity is related to Newtonian mechanics. Just as Newtonian mechanics is generally incorrect but usable for low velocities (i.e. agrees with special relativity for low velocities)[25], so classical logic is incorrect generally but usable for consistent situations (i.e. it agrees with paraconsistency in consistent situations).[26] Hence the problem-solving power of paraconsistent logic captures that of classical logic in just the same way that special relativity captures that of Newtonian mechanics, as required.

4.3. *Providing the basis for a new and fruitful research program*

So the paraconsistent principle both preserves and improves upon the problem-solving ability of classical logic. However, a further condition is required for a successful revolutionary theory. It must provide the basis for a new period of normal science. It must provide new and interesting problems and the possibility of their solutions. It must provide the promise of new ideas, results, and insights. This paraconsistency does and I must finally show how. There is so much material here that I must, of necessity, be sketchy.[27]

The first and most basic line of research is the articulation of paraconsistent logical theory. The foundations have been laid but there still remain important problems in paraconsistent logical theory, especially in the areas of normal form theorems in proof theory, algebraization, and perhaps most of all in the semantics and model theory of paraconsistency.[28]

The next line of research is rather an ironical one. For paraconsistency opens up the possibility of progressive research in those very programs in the foundations of mathematics which degenerated in classical hands. For a start, logicism.[29] This was killed by the logical paradoxes and Gödel's first incompleteness theorem. Paraconsistency solves the first and avoids the second. For we can replace Frege's classical logic with paraconsistent logic whilst retaining his naive set theory. How much of the Frege/Russell reduction of arithmetic to logic is possible in these circumstances is an open problem. Paraconsistent naive set theory also avoids Gödel's first incompleteness theorem; for this applies only to *consistent* theories. This means that the question of whether the set of true arithmetic sentences is axiomatizable is open. I have conjectured that it is. (See Priest, 1979, §4.13). Solving this conjecture promises a number of fruitful investigations.

A second foundational program, Hilbert's formalism, is also reopened by paraconsistency.[30] The death blow to this was dealt by Gödel's two incompleteness theorems. We have seen how, paraconsistently, the first is avoided and, for similar reasons, so is the second. It is also encouraging to have a positive result in this area. (So that progress is occurring; it is not just promise.) Meyer, 1976, has shown that a theory that he calls R#, which is essentially Peano's postulates for arithmetic with an underlying paraconsistent logic, has the following properties. a) All recursive functions are representable in R#. b) R# has a half page absolute consistency proof which is certainly finitary and formalizable in R#.

The next line of research is perhaps the most obvious one: to investigate interesting but inconsistent theories. Of these, two cry out for investigation: naive set theory and naive semantics. To take the first, just how much set theory can be done without making local consistency assumptions? Is classical ZF a (consistent) sub-theory of naive set theory? What new things can be proved in naive set theory? There is already a proof of the axiom of choice in a paraconsistent set theory (see Routley, 1977, §8). Can the continuum hypothesis be decided by naive set theory?

Let us turn to semantics. Any decent theory which contains its own semantics is inconsistent. This has been thought to rule out semantically closed theories, but obviously doesn't. What interesting properties do semantically closed theories have? For the first time it seems a theoretical possibility to give a semantics of English in English. This obviously has significant (positive) implications for the Davidsonian program. I should not leave the area of inconsistent theories without mentioning the subject of non-triviality proofs. An inconsistent theory is useless if you can prove everything. Every inconsistent theory therefore poses a non-triviality problem: are there things which can not be proved and, if so, what? The only inconsistent theory whose non-triviality has been much investigated is naive set theory. Brady has proved that this is non-trivial,[31] that not all contradictions are provable. However, it is not yet known how far the inconsistency spreads. Can we, for example, prove that any small (e.g. finite) sets have inconsistent properties? This whole area provides plenty of scope for the development and application of new mathematical technology.

The fourth line of research opened up is the use of paraconsistent logic to handle what we might call inconsistent data. For example, a fusion of paraconsistency and probability theory leads to non-trivial assessments of probability relative to inconsistent data—a practical problem for many. Another example is the use of deontic logic based on an underlying paraconsistent logic. This will provide the theoretical framework for handling inconsistent obligations. Inconsistent obligations are an (unfortunate) fact of life, but cannot even be countenanced in classical deontic logic without total collapse. A final example is the possible use of paraconsistency as a logical base for quantum theory. Quantum logic blocks causal anomalies by jettisoning distributivity. Suppose we instead keep distributivity, accept the anomalies as objectively true contradictions, but use a paraconsistent logic to prevent contradictions spreading. What consequence does this line of work have?[32]

I must not leave the research programs that paraconsistency opens without mentioning the area of the philosophical implications of paraconsistency. This is, perhaps, the most important area of all. The philosophical implications of the existence of contradictions which are true, could be profound. Two are worth especial mention. First, all standard accounts of rationality presuppose that it is irrational to accept a contradiction (at least in the form $A \& \sim A$). However, if some contradictions are true then it may well be rational to believe them. A very different account of rationality will have therefore to be produced. Secondly, many writers in the tradition of analytic philosophy (such as Popper) have dismissed the area of dialectics on the grounds that if dialecticians mean what they literally say about contradictions, it is absurd. If paraconsistency is correct then obviously this is not so. In fact, paraconsistency will be very important in making dialectics much more rigorous.[33]

Clearly I have had to give a brief outline of the paraconsistency program. However, I think I have said enough to show the great scope there is for future research. Thus paraconsistency shows the third *desideratum* for a revolutionary theory, promise of a fruitful period of research: new ideas, new theories, new applications and above all new insights into the nature of logic to be gained. In comparison with this, the future promise of the classical program is exceedingly bleak.

5. Conclusion

I have now, I think, fulfilled the two aims of the paper. I have shown that theoretical notions concerning the theory-dynamics of science can fruitfully be applied to the history of logic. In doing this we have gained insights into the progress of the classical paradigm and seen in particular that the time is ripe for revolution. Progressive and fruitful as it was in its time, it has been stagnant for the last twenty years. It should be overtaken by the paraconsistent program: a new theory with a strong positive heuristic which solves anomalies which plagued the classical theory, and holds the promise of exciting research in logic and its application for a good time to come.[34]

Notes

[1] The paper was given under the title "Classical Logic: a degenerating research program in crisis" at a meeting of the Australasian Association of Philosophy in

Melbourne, 1979. This largely explains its style, polemical nature and brevity on many issues. I gave quite a lot of consideration to rewriting it in a more extended and literary style but eventually rejected the idea, contenting myself merely with bringing reports on current work up to date, and conceding the footnotes to erudition. Most of the claims made in the paper concerning paraconsistent logic are spelt out in more detail elsewhere in this volume. As for the remarks on classical logic: like most potted histories they are over-simple. However, the few simple strokes of a Japanese painting may well contain insights not to be found in, say, a Brueghel packed with detail.

[2] Of course people such as Boole, De Morgan, etc. were trying to produce somewhat different accounts before Frege. However, this is exactly a Kuhnian sign of a paradigm beginning to break down, and a transitional stage emerging.

[3] See for example Russell's discussion of quantifiers in Ch. 5 of Russell, 1903.

[4] For the notions of progress and degeneration in general (i.e. not specifically in empirical science) see Lakatos, 1968, p. 128.

[5] The history is so well known that it hardly needs references. These, however, can be found in any standard reference book. See, for example, the relevant discussions and bibliography in Fraenkel, Bar Hillel and Levy, 1973.

[6] This requires some comment. Modal logic was produced in response to certain anomalies of classical logic, the paradoxes of implication (see next section). Unfortunately it never really succeeded in solving the problems, because of the paradoxes of strict implication. However, it remained an object of investigation. What the reduction did was to show how the semantics of modal logics could be understood in terms of certain sets of classical models (plus a binary relation). This absorbed modal logic into the classical paradigm, thus defusing its challenge to classical logic once and for all.

[7] Again, details are extremely well known. Standard reference books that can be consulted are Fraenkel, Bar Hillel and Levy, 1973 and Passmore, 1957.

[8] See note 7.

[9] This is sufficient to show that Kuhn's claim that rival paradigms can not co-exist during a period of normal science, is false—it is perfectly possible for there to be a dominant one and a minor one. This obviously raises the question of why intuitionism hasn't succeeded classical logic since 1960. To a certain extent it has been taken more seriously recently. However, it seems to me unlikely that it will ever supersede classical logic, simply because it fails to preserve a good deal of classical reasoning. See 4.2 below.

[10] It is not by accident that by the early 1970's it was felt necessary by some to produce the *Journal of Philosophical Logic*, a journal of formal but not mathematical "logic".

[11] See Lewis and Langford, 1932, pp. 248–251.

[12] For a further discussion, see Priest, 1979a.

[13] See Anderson and Belnap, 1975, pp. 17–18.

[14] On this see Priest and Routley, 1989, section 2.1.

[15] In case it is not clear that these are anomalies for *classical logic*, let me point out that they are anomalous precisely when viewed through the classical problematic that nothing can be both true and false. Their anomalous nature is brought out most clearly against the background of paraconsistent logical theory—an interesting example of how a rival theory can turn a mere anomaly into a refuting fact.

[16] For further discussion see Priest and Routley, 1989b.

[17] An unusually candid example of this is David Lewis, 1982:

> The reason we should reject the proposal [that things can be both true and false] is simple: nothing is and nothing could be, literally both true and false. This we know for certain and *a priori*... This may seem dogmatic. And it is: I am affirming the very thesis that... [is being]... called into question and—contrary to the rules of debate—I decline to defend it.

Lewis goes on to say that the position he rejects is uncriticizable. It is not. See Priest and Routley, 1989b, 3.1.2.

[18] Assuming of course they can remember what *logic* is and have not simply identified logic with mathematical "logic".

[19] Of course, it can not be said that it will occur. There may be a "progressive problem-shift" in the heuristic of classical logic. Or change may be prevented for external reasons such as war (cf. the fate of the Polish School of logicians in the 1930's).

[20] Perhaps, more correctly, paraconsistent/relevant logic. For, though strictly speaking, these are separate enterprises, they go naturally together. Relevance is unstable without paraconsistency (see Priest, 1989, section 7) and paraconsistency can not be adequately formulated irrelevantly (see Priest and Routley, 1989a).

[21] We now call this dialethism. A weaker version of paraconsistency is that there are interesting and important inconsistent but non-trivial theories. Again, in the last instance the weak form of paraconsistency is unstable without dialethism.

[22] Interested readers can consult Priest, 1979 and 1984, and Priest and Routley, 1989b.

[23] Details can be found in Priest, 1980, and Priest and Routley, 1989a.

[24] Though the exact understanding of this claim is a sensitive business. See Priest, 1989.

[25] In practical terms anyway. Our theoretical understanding of what is going on is fundamentally changed.

[26] As note 25.

[27] A number of the topics are taken further in Routley, 1977, and in this volume.

[28] See Part Two of this volume.

[29] See Priest and Routley, 1989b, 3.3.2.

[30] See Priest and Routley, 1989b, 3.3.3.

[31] See Brady, 1989.

[32] Further details on all these issues can be found in Priest and Routley, 1989c, 3.4.

[33] For a further discussion of these areas, see Priest and Routley, 1989b, 3.1, 3.2.

[34] I am grateful to Ivor Grattan-Guinness and Richard Routley for comments on an earlier draft of this paper.

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