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Contradiction and the Instant of Change Revisited

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Abstract

Instantaneous changes may well be thought to give rise to contradiction. If one endorses an explosive logic, where contradictions entail everything, this is entirely unacceptable. However, if one deploys a paraconsistent logic, which keeps contradictions under control, one may give perfectly coherent and precise models of such changes. In *In Contradiction* the author showed how and he explored the philosophical implications of the model. Here, the author revisits the issue in the light of a recent critique by Greg Littmann.

Keywords

instant of change – contradiction – dialetheism – state of changing – Leibniz Continuity Condition

The problem of the instant of change is a thorny one, which exercised philosophers in both ancient and medieval Western philosophy.[1] One thing which

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1 For references and discussion, see N. Kretzmann, "Incipit/Desinit," in *Matter and Time, Space and Motion: Interrelations in the History of Philosophy and Science*, ed. P. Machamer and R. Turnbull (Columbus, OH, 1976), 101-136; S. Knuuttila and A. Inkeri Lehtinen, "Change and Contradiction: A Fourteenth Century Controversy," *Synthese* 40 (1979), 189-207; N. Kretzmann, "Continuity, Contrariety, Contradiction and Change," in *Infinity and Continuity in Ancient and Medieval Thought*, ed. N. Kretzmann (Ithaca, NY, 1982), 270-296; P.V. Spade,

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makes it so is the fact that instantaneous changes may well be thought to give rise to contradictions. That is, they may deliver dialetheias. Of course, if one endorses an explosive logic, where contradictions entail everything, this is entirely unacceptable. However, if one deploys a paraconsistent logic, which keeps contradictions under control, one may give perfectly coherent and precise models of such changes. In *In Contradiction*[2] I showed how and explored some of the philosophical issues surrounding such changes. Here, I revisit the issue in the light of a recent critique by Greg Littmann.[3]

First, some conceptual background. An instantaneous change is characterised by a triple $\langle I, t, \alpha \rangle$, where: I is an interval of time;[4] t is a time in the interior of I; and α is a statement where, for all $t' \in I$ such that $t'<t$, α is true at t', and for all $t' \in I$ such that $t' > t$, $\neg\alpha$ (its negation) is true at t'. We might illustrate it thus:

$$\begin{array}{cccc} & \alpha & ? & \neg\alpha \\ I & [--- & t & ---] \end{array}$$

The question mark we will come to in the next section.

One can ask many questions about instants of change—the most obvious being: are there such things? To which the answer is *yes*. A car accelerates smoothly from rest. Before a certain time, t, its velocity (or that of its centre of gravity) is zero; after that time it is non-zero. We have an instant of change. I is an interval running between the time of rest and the time of motion; t is the instant the car moves off; and α is: the velocity of the car is zero. Or: I throw a ball in the air. At a certain time, t, its velocity (or that of its centre of gravity) becomes zero. Before t it is positive; after t it is not positive. We have an instant of change. I is the set of times of the motion of the ball; t is the time of zero velocity; α is: the ball has positive velocity (upwards being positive).

"Quasi-Aristotelianism," in *Infinity and Continuity*, 297-307; S. Knuuttila, "Remarks on the Background of the Fourteenth Century Limit Decision Controversies," in *The Editing of Theological and Philosophical Texts from the Middle Ages*, ed. M. Asztalos (Stockholm, 1986), 245-266; N. Strobach, *The Moment of Change. A Systematic History in the Philosophy of Space and Time* (Dordrecht, 1998).

2 G. Priest, *In Contradiction: A Study of the Transconsistent* (2nd edition, Oxford, 2006). This is mainly in chapter 11, though there is related material in chapter 12. There is some additional discussion in 19.13 and 19.14 of the second edition.

3 G. Littmann, "Moments of Change," *Acta Analytica* 27 (2012), 29-44. Page and section references to his views are to this. Although, as will become clear, I disagree with Littman on many matters, his probing arguments have forced me to think about a number of issues more clearly than I had done before, for which I thank him.

4 In what follows, I take time to be continuous, and modelled by the real line.

1 What's for *t*?

The first non-trivial question is: what is the situation at t? There are two pieces in play, α and $\neg\alpha$, and each of these might either hold or fail. Hence there are four possible answers:

> A: α (and only α) holds at t
> B: $\neg\alpha$ (and only $\neg\alpha$) holds at t
> Γ: neither α nor $\neg\alpha$ holds at t
> Δ: both α and $\neg\alpha$ hold at t

I note that there is no *a priori* reason why all instants of change must fall into the same category. And *prima facie*, at least, there are certainly type A and type B changes. In the case of the car and the ball, their velocities are continuous. Hence, there is a last moment at which the car is at rest: we have a type A change. And there is no last moment at which the velocity of the ball is positive, but there is a first at which it is not: we have a type B change.

But are there type Γ and Δ changes? Of course, if one assumes the Principle of Excluded Middle (PEM), then type Γ changes are ruled out. And if one assumes the Principle of Non-Contradiction (PNC), then type Δ changes are ruled out. But both of these principles are contestable.[5] So it is ill-advised to close off those possibilities dogmatically. Indeed, we may even take what is at issue here to be whether there are counter-examples to those principles. To assume them would, in that case, simply beg the question.

In our car and ball cases, the fact that the velocity is continuous delivers asymmetry in the situation at t. If, in an instantaneous change, there is such an asymmetry, then we may naturally suppose that we are in case A or case B. There are cases, however, where there appears to be no asymmetry: t is symmetrically balanced between the prior and posterior states. Thus, for example, suppose that the ball (identified with its centre of gravity) is now thrown out of a window (to be identified with the vertical plane through its center of gravity) of a room. At the instant the ball goes through the window, is it in or out (not in) the room? It would seem to be just as much in as out; just as much out as in. Or again, let t be the instant of midnight. Is it the day before, or the day after (not the day before)? Symmetry strikes again. In such cases, the asymmetric possibilities, A and B, seem exactly the wrong way to go. We are therefore in one of the symmetric cases: Γ or Δ.

5 See, e.g., G. Priest, *Introduction to Non-Classical Logic* (Cambridge, 2008), ch. 7.

Of course, nothing so far tells us in which of these cases are our examples. Are there considerations that push one way or the other? Certainly in some cases. Take the instant of midnight example. Suppose that the day before is Monday 1st. Then if this were a type Γ change, it would be neither Monday 1st nor not Monday 1st at *t*. Symmetrically, it would be neither Tuesday 2nd nor not Tuesday 2nd at *t*. But it is clearly not some other day—e.g., Friday 5th. Hence *t* would seem to have no day or date. It would be a gap in time, as it were. But days are such that any time has to be in one of them; ditto dates. So this is not a type Γ change; it must be a type Δ one: *t* is both Monday 1st and not Monday 1st, both Tuesday 2nd and not Tuesday 2nd.

2 The Leibniz Continuity Condition

There is a more general argument to the effect that type Δ changes occur. This deploys a continuity condition endorsed by Leibniz and many other eighteenth-century mathematicians, and is a version of the venerable principle *natura non facit saltus* (nature makes no jumps). Roughly speaking, this says that anything that holds up to the limit of a sequence holds at the limit. The principle has to be treated with some care if it is not to produce manifestly unacceptable conclusions, but at least when it comes to states of affairs in time, we can formulate it as follows:

> Any physical state of affairs that holds at all times in an interval holds at any limit of those times.

Call this the *Leibniz Continuity Condition* (LCC). There is still slack in the formulation. Notably, we need to know exactly what a physical state of affairs is. For just one example: are there disjunctive states of affairs? Neither is the rationale for the principle entirely obvious, though it does comport with a certain intuition: for something to hold all the way to a limit, but fail only there, would appear to be capricious in a certain way.[6]

However, assuming this principle, we will have many type Δ changes. Consider an instant of change $\langle I, t, \alpha\rangle$, where α describes a state of affairs. Then

6 See, further Priest, *In Contradiction*, 11.4. Littmann criticizes as question-begging a reason I give there as to why we find changes that violate the LCC counter-intuitive (Littmann, "Moments of Change," 42). Whether or not this is the case, it does nothing to gainsay the fact that the principle has a certain pull, as is witnessed by the fact that continuity principles of this kind have appealed to so many thinkers.

α holds at all times up to t, and so by the LCC, at t; and $\neg\alpha$ holds at all times down to t, and so by the LCC, at t. So we have a type Δ change.

I note that the LCC will deliver a type Δ change even in some cases that we might have thought to be of types A or B. So consider the case of the car moving off from rest, which is naturally thought of as type A. By continuity (and the LCC) it has zero velocity at t; but by the LCC running in the other direction, it has non-zero velocity there. Type Δ.[7]

3 States of Changing

We now come to perhaps the thorniest question. Any instant of change obviously realises a state of change in some sense: before, one thing; after, another. But it need not be a state of *changing*. Or to put it another way, given our characterisation, there need be nothing intrinsic to an instant of change in this sense. Thus, suppose that we have an instantaneous change of type A; then there is no difference between the relevant state at t and the states at times before it. The time realises a state of change only in virtue of what comes later.

Now, are there states of changing, states that intrinsically indicate change? A natural, though unorthodox, thought is that there must be. If not, there would be no change, as such. There would simply be a series of prior and posterior states. This conflicts with the thought that real change requires some sort of state of fluxation.[8] What could such a state be like? The most obvious answer in the present context is that it is exactly the transition state in one of the type Γ or type Δ changes. For these realise a state intrinsic to the instant, different

7 Littmann ("Moments of Change," 42) argues against the LCC as follows. Let $\langle I, t, \alpha\rangle$ be an instant of change. *Ex hypothesi*, α does not hold after t. But now let t' be a time after t but 'arbitrarily close to it'. By the LCC, it follows that α holds at it, which it does not. I do not understand what Littman means by 'arbitrarily close' here. If t' is any point after t, there is an infinitude of points between the two. So t' cannot be a limit of the set of points where α holds, and the LCC cannot be applied. (In Priest, *In Contradiction*, 166, I formulate the LCC as 'anything going on arbitrarily close to a certain time is going on at that time too'. What this means is that if something holds at all points [in some interval] no matter how close they are to a time, it holds at that time as well. It is not a reference to some particular point.) Perhaps Littmann is taking the structure of the real line to be as given in non-standard analysis, where t' could be an infinitesimal distance from t. How to apply the LCC, with its talk of limits, is then an interesting question. But however one formulates it, the most one is going to get out of it is that there is no last non-standard number in the monad of t where α holds, which is quite compatible with t being the last standard point.

8 See the discussion in Priest, *In Contradiction*, 12.2 (172-175).

from the prior and posterior states. Of the two, it is the type Δ change that is the most natural candidate. For in a type Γ change, the transition state is one where neither α nor $\neg\alpha$ holds. Nothing in this requires $\neg\alpha$ to happen. Indeed, there is nothing in principle to stop the situation returning immediately to the prior state. Compare this with a type Δ change. In this, $\neg\alpha$ has already started. (So even if the system immediately reverted to being in just the state α, $\neg\alpha$ would still have obtained—if only for an instant.) So given an instant of change, $\langle I, t, \alpha\rangle$, let us take the state of changing to be the state at t, $\alpha \wedge \neg\alpha$.

But does every change require a state of changing? Absolutely not.[9] Suppose I die instantaneously at time t. If α is 'Priest is alive' then, for an appropriate interval, I the change is characterised by $\langle I, t, \alpha\rangle$, and the situation at t (delivered by the LCC) can be a state of changing: α and $\neg\alpha$ are both true. But let β be the sentence: Priest will be alive at some later time. This is true at all times before t (since time is continuous), but not true at any later time. So $\langle I, t, \beta\rangle$ is an instant of change; but we certainly cannot have a state of changing at t, β and $\neg\beta$. If β were true at t, I would be alive at some time after t, which I am not. In particular, then, β cannot define a state of affairs within the meaning of the LCC act.

4 A Higher Order Change

Against this background, I now want to consider two of the arguments put forward by Littman. Let $\langle I, t, \alpha\rangle$ be an instant of change, where t is a state of changing, so that α and $\neg\alpha$ hold at t. Consider the change from α holding to α failing to hold. This happens at t, but, says Littman, there is no state of changing: α simply goes from being true at t to not being true thereafter.[10]

Let T be the truth predicate, and $\langle . \rangle$ a name-forming device. Then we are asked to consider the change from $T\langle\alpha\rangle$ to $\neg T\langle\alpha\rangle$. (The latter, note, is not the same as $T\langle\neg\alpha\rangle$.) This happens at t, so $\langle I, t, T\langle\alpha\rangle\rangle$ is an instant of change. We might depict it thus:

$$\begin{array}{lccc} & T\langle\alpha\rangle & T\langle\alpha\rangle & \neg T\langle\alpha\rangle \\ & \alpha & \alpha \wedge \neg\alpha & \neg\alpha \\ I & [- - - & t & - - -] \end{array}$$

Note that since α holds at t, so does $T\langle\alpha\rangle$. So this looks like a type A change. But if we can apply the LCC to α, we can, presumably, apply it just as much to

9 This is pointed out in Priest, *In Contradiction*, 11.4.

10 Littmann, "Moments of Change," section 2 (32-34).

$T\langle\alpha\rangle$: these say much the same thing. And applying it right to left tells us that $\neg T\langle\alpha\rangle$ holds at t too. So there *is* an instant of change for the change Littman is concerned with: and it is just t itself.

One might argue that t cannot be the moment of changing, since $T\langle\alpha\rangle$ is still in place, so the change has not yet started. This does not follow. At t, $\neg T\langle\alpha\rangle$ is already in place, and that is just what a state of change is.

5 An Infinitude of Changes

Let $\langle I, t, \alpha\rangle$ be an instant of change, where t is a state of changing, so that α and $\neg\alpha$ hold at t. Before t the system is not in a state of change with respect to α; at time t it is. This itself is a change. So there must be a state of change between them. But then there must be another change between our original state and this state, and so on. If there is a minimum amount of time required for any change, then, it would follow, that change is impossible in a finite time, whilst it clearly is possible. And even if there is no minimum, the situation is still counter-intuitive, and we are committed to just too much change going on. So says Littmann.[11]

Consider the change from the pre-change state to the change state. I note, first, that this is not an instant of change as I have defined it, just because the second state is itself instantaneous, not holding through an interval. Nonetheless, the transition in question is certainly a change of some kind. Now, a state of changing is one where both the prior and the posterior states obtain. In this case, the prior state is characterised simply by α; the posterior state is characterised by $\alpha\wedge\neg\alpha$. So where both of these hold, we have $\alpha\wedge(\alpha\wedge\neg\alpha)$, which is just $\alpha\wedge\neg\alpha$. This is exactly our old state of changing. Odd though it may seem, then, there is a state of changing in this case, and it is exactly the posterior state! After all, to be changing into a state of change is already to be in a state of change. And if it be retorted that it is a contradiction for a time to be both one of a terminal state, and of transition into it, the reply is: just so—we already knew that t was a contradictory instant. The first step of Littman's regress therefore fails to get off the ground, and there is no infinitude of different states of changing, with whatever counter-intuitive consequences this might be thought to have. There is just the one.

11 Littmann, "Moments of Change," 31. There is a reprise of the point on p. 39, which does not, as far as I can see, add much to the matter. Since I take space and time to be continuous, we are in Littman's case (d), which just reiterates the claim that the infinity of changes is unacceptable.

One might object that the contradictory state could not be a state of changing for the following reason. Suppose that it is not just instantaneous, but lasts for some period of time, thus:

$$\begin{array}{cccc} & \alpha & \alpha \wedge\neg\alpha & \neg\alpha \\ I & [- \; - \; -] & [t \; - \; -] & [- \; - \; -] \end{array}$$

Then t cannot be an instant of changing because it continues afterwards. However, the point is question begging. A contradictory state is a state of changing by its nature. If it continues for a while, this does not imply that things are not changing: it just implies that the state teeters on the point of change for more than an instant.

Perhaps, also, one might object that our original instantaneous change was not a change from α to $\alpha\wedge\neg\alpha$; when spelled out correctly, it is a change from α holding and $\neg\alpha$ not holding, to both holding. The picture that we then have is:

$$\begin{array}{ccccc} & T\langle\alpha\rangle\wedge\neg F\langle\alpha\rangle & T\langle\alpha\rangle\wedge F\langle\alpha\rangle & \neg T\langle\alpha\rangle\wedge F\langle\alpha\rangle & \\ & \alpha & \alpha \wedge\neg\alpha & \neg\alpha & \\ I \; [& - \quad - \quad - & t & - \quad - \quad - &] \end{array}$$

Applying the LCC from both left and right to the top line then delivers the picture:

$$\begin{array}{ccccc} & T\langle\alpha\rangle\wedge\neg F\langle\alpha\rangle & \beta & \neg T\langle\alpha\rangle\wedge F\langle\alpha\rangle & \\ & \alpha & \alpha \wedge\neg\alpha & \neg\alpha & \\ I \; [& - \quad - \quad - & t & - \quad - \quad - &] \end{array}$$

where β is $T\langle\alpha\rangle\wedge\neg T\langle\alpha\rangle\wedge F\langle\alpha\rangle\wedge\neg F\langle\alpha\rangle$. The point is the same. The prior state of the top change is one conjunct of the middle state. So the transition state into the middle state is itself the middle state. One might note that since at t we have both $T\langle\alpha\rangle\wedge F\langle\alpha\rangle$ and $\neg T\langle\alpha\rangle\wedge\neg F\langle\alpha\rangle$, the change from α to $\neg\alpha$ is both a type Γ change and a type Δ change!

6 When Are There States of Changing?

Littman harnesses the arguments of the previous two sections in an attempt to show that there can be an instant of change without a state of changing. I have shown why I disagree with these arguments, but I already granted this conclusion in Section 3 anyway. And the existence of such things poses an issue which Littman puts as follows:

> Do all changes require a contradictory moment in which that which changes is in both the state changed from and the state changed to? … If the answer is "no" then contradictory moments are not needed for change *per se*, and the onus is thus on the proponent of the contradictory account to explain why such moments are ever needed for change.[12]

A good point. In this final section, let me address it.

First, I note that given an acceptance of dialetheism and a corresponding paraconsistent logic, one can accept that instants of change produce a contradiction, and this can be handled in a logically rigorous fashion.

Next, as we have seen, there are considerations that push towards there being such inconsistent states, considerations concerning symmetry, and particularly, the LCC.

Third, one does not have to accept such states as intrinsic states of change, that is, states of *changing*; but one can certainly do so. For the contradiction realised at an instant is not a relational fact, but intrinsic to the instant. And there are some notable advantages to doing so. If there are no states of *changing*, then all change is simply of the kind that occurs when one state of affairs is replaced by another, in the way that, in a film, lots of stills are shown so close together that something *appears* to change: the cinematic account of change. This is quite counter-intuitive: real changes would seem to involve some dynamic state of flux.[13]

Moreover, the existence of such dynamic states provides a ready solution to Zeno's paradox of the arrow. At each moment of its motion, the arrow is exactly where it is. The advance made on its journey is . . . nothing. But a sum of nothings (even infinitely many nothings) is still nothing. Yet the arrow does advance. So how is this possible? If there are contradictory instantaneous states of change, we have an answer. In particular, the arrow *can* make progress in an instant. It is where it is, certainly; but it is not there too: where it was and where it will be.[14]

12 Littmann, "Moments of Change," 31.

13 Priest, *In Contradiction*, 12.2. Littmann ("Moments of Change," 42) argues that to say that change is not like this simply begs the question against the cinematic account; but it is not question-begging to point out that the view is counter-intuitive. Littman also misses the main argument used in that section against the account: that concerning Zeno's paradox of the arrow.

14 Priest, *In Contradiction*, 12.2. For a general discussion of the virtues of an inconsistent account of motion, see G. Priest, "Motion," *Encyclopedia of Philosophy*, Vol. 6, ed. D. Borchert (2nd ed., New York, 2006), 409-411.

One might object that even if a point on the arrow can occupy more than one point in space at one time, still nothing during the instant changes: it occupies exactly those points that it is occupying. However, first, in the context, this is question-begging. On this account, to be in a state of changing precisely *is* to be in a contradictory state. More importantly, whether or not something is *changing* is irrelevant to the solution to the paradox. The relevant fact is that the point on the arrow occupies more than one point at a time, and so *does* make progress: passing through all those points.

But if not all changes occasion a state of changing, which ones do? The most general answer to this question is perhaps disappointing. One should accept that there are such inconsistent states when there is good evidence to this effect (and not otherwise). The evidence might come in the shape of the fact that this follows from other plausible things, such as the LCC, or helps us to solve problems, such as Zeno's Arrow Paradox.

A more principled answer must be more speculative, but a first shot might go something like this. The physical world may be thought of as composed of states of affairs, as in Wittgenstein's *Tractatus*.[15] Not every true sentence corresponds to a state of affairs, however. (Thus, for example, as noted, maybe a disjunction can be true, even though there are no disjunctive states of affairs.) It is instantaneous changes in those states of affairs that involve states of *changing*. (This may include states of affairs that themselves involve the truth predicate, as we saw above.) That, after all, is what the LCC delivers. Other changes of truth value must, in the end, supervene on these more fundamental changes (cf. the death example above), but do not, themselves, require a state of *changing* truth value, inconsistent or otherwise.

Conclusion

Change is, perhaps, the most conspicuous feature of our world. Like many obvious things, we are wont to take it for granted. But the job of philosophy—or one of its jobs—is to think through things we take for granted. Sometimes, we will come to the conclusion that something we took for granted is not, in fact, the case. Parmenides notwithstanding, this is unlikely to happen for change. But Parmenides saw in change violations of the Principle of Non-Contradiction. In that, he may well have been right.

15 See also D.M. Armstrong, *A World of States of Affairs* (Cambridge, 1997).