

# Torn by Reason: Łukasiewicz on the Principle of Contradiction

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It is surprising how strongly certain opinions can persist within the sciences that are not only incorrectly formulated and without justification, but which are plainly *false*—most likely, as I believe, because what has been declared in the past is repeated uncritically again and again.

*Jan Łukasiewicz*<sup>1</sup>

## 1 Introduction: a Seminal Book

In 1910, the young Jan Łukasiewicz published his ground-breaking *On the Principle of Contradiction in Aristotle*.<sup>2</sup> About two and a half millennia earlier, Aristotle had launched an attack on those who would violate the Principle of Non-Contradiction (PNC, or the Principle of Contradiction, as Łukasiewicz calls it). That attack established the principle as orthodox in Western Philosophy—in a way that perhaps no other philosophical claim has ever been so entrenched. The only Western philosopher who balked seriously was Hegel (and perhaps some of his intellectual descendants). Łukasiewicz subjected the Aristotelian arguments to a detailed, penetrating attack, and

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<sup>1</sup>Heine (2013), p. 125.

<sup>2</sup>Łukasiewicz (1910a). He was 21 at the time.

put the question of the cogency of the PNC on the table for 20th Century philosophy.

He was well aware of what he was doing. He says in the introduction to the book:<sup>3</sup>

There are two moments in the history of philosophy in which disputes over the principle of contradiction excited the minds of an age—one is connected with Aristotle's name—the other with Hegel. Aristotle formulated the principle of contradiction as the highest law of thought and being. He pursued everyone who would not recognize the principle with stubborn polemics in which, at times, anger and annoyance find a voice: Antisthenes and his school, Eristics from Megara, followers of Heraclitus, students of Protagoras. Aristotle won the fight. And whether it was the persuasive force of his arguments or the correctness of the position he defended—for centuries no one dared to contradict this highest of laws. Only Hegel allowed the convictions that had been buried by Aristotle to come back to life and instructed us to believe that reality is simultaneously rational and contradictory.

With considerable prescience, Łukasiewicz foresaw a third moment in the historical debate, one which would make use of the newly emerging symbolic logic:<sup>4</sup>

If I am not mistaken, the *third* moment in the history of the principle is approaching now, a moment that will remedy old shortcomings... [A] time has come in which logicians are beginning to review ... [formal logical principles] and to dedicate themselves to those investigations that had not been considered by Hegel.

Indeed, he saw himself and his book as a harbinger of this moment. Before the third moment can be pursued properly, however:<sup>5</sup>

one has to first return to Aristotle himself; some unresolved problems (related to the principle), which nowadays have been forgotten, need to be brought to mind and new investigations should

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<sup>3</sup>Heine (2013), p. 81. Quotations and page references in what follows are to this translation. All italics are original.

<sup>4</sup>Introduction, pp. 84-5.

<sup>5</sup>Introduction, pp. 86-7.

then connect to them. I want to convince the reader that this principle is not as unshakable as one might expect with its general acceptance. I want to show that it presents a thesis which demands proof, and that despite the Stagirite's words ... this proof can be found.

## 2 The Book's Two Halves

In the same year in which he published the book, Łukasiewicz published a paper in German, '*Über den Satz von Widerspruch bei Aristoteles*',<sup>6</sup> summarising the first half of his book. It is in this half that Aristotle is clinically dissected. The paper was—somewhat belatedly—translated into English after some 60 years; and its contents are now well appreciated by English-speaking philosophers. The book itself has not been translated into English until recently.<sup>7</sup> The present paper is an analysis of the contents of its second half.

In this part of the book we find Łukasiewicz giving his own views of the PNC, including the proof referred to in the last quote of the previous section. Łukasiewicz's categorical statement of the existence of a proof might lead one to expect him to have a similar categorical attitude to the PNC itself; but this is not what we find. Łukasiewicz, as we shall see, is badly torn. It is true that he comes down on the side of the PNC eventually, but the journey is a tortured one, and the conclusion disappointingly lame. The material, however, shows us an acute mind wrestling with a principle it would really like to believe, in *despite* of the considerations it marshals. It also presents a fascinating window on a period in the history of logic, a century yore, when the new symbolic logic, the logical paradoxes, the thought of Meinong, and the thought of Hegel, delivered a heady and stimulating cocktail.

## 3 The Demolition of Aristotle

The critique of Aristotle in the first half of the book is clinical and devastating. It is also, as I have said, well known. Before we turn to the second half of the book, however, I want to note four of its aspects for future reference.

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<sup>6</sup>Łukasiewicz (1910b)

<sup>7</sup>Heine (2013).

1. Aristotle's arguments for the PNC in the *Metaphysics* comprise one long argument, and then a series of about half a dozen brief arguments. The long argument is tangled, torturous, and how best to understand it is not at all clear. According to Łukasiewicz, it works, at best, only if applied to claims of the form  $Pa$ , where  $P$  is essentially predicated of  $a$ .<sup>8</sup>

Even if ... [the argument] were correct, it would prove the principle of contradiction only for a narrow range of objects: it would merely concern the essence of things, but not accidental properties.

Moreover, Łukasiewicz argues:<sup>9</sup>

Everything appears to be in favour of the view that Aristotle limited the significance of the principle of contradiction to substantial being.

Łukasiewicz' claim is certainly contentious.<sup>10</sup> I cite these passages to show that he was well aware that one might hold that the principle has only limited validity. Indeed, in discussing the difference between the PNC and the principle of double negation, he himself appears to state that the PNC does not have universal validity. Discussing squaring the circle (a mathematical impossibility), he says:<sup>11</sup>

There are instances in which the principle of double negation is true and the principle of contradiction is not applicable or, put simply, where it is false... Whoever has studied geometry will without doubt understand what this is: "A square constructed with the aid of compass and ruler, whose surface area is identical to the surface area of a circle that has radius 1"... Such a square—let's designate it simply by  $Q$ —is a contradictory object...  $Q$  has  $S$  [sides that are expressible by an algebraic number] and simultaneously *does not have*  $S$ . It is precisely because of this that  $Q$  is a contradictory object...

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<sup>8</sup>Chapter XI, p. 153.

<sup>9</sup>Chapter XIV, p. 178.

<sup>10</sup>Indeed, I do not agree with it. See Priest (1998), esp. 1.5-1.10.

<sup>11</sup>Chapter X, pp. 145-6.

Contradictory objects will loom large in our discussion of the second half of the book. So let us leave further discussion of this example till then.

2. Aristotle's battery of small arguments are easier to understand than the long argument. And concerning these, Łukasiewicz makes two points worth noting (Chapter XII).

First, several of these arguments have a conclusion to the effect that it is not the case that *all* contradictions are true. Łukasiewicz notes, correctly, that this conclusion is beside the point: what needs to be established is that it is not the case that *some* contradictions are true. In these arguments, then, Aristotle has made an illicit slide from *some* to *all*.

Secondly, several of these arguments deduce supposedly unacceptable consequences of violating the PNC, and then apply *modus tollens*. Such arguments fail, says Łukasiewicz, since *modus tollens* presupposes the PNC. Suppose that  $A$  entails  $B$  and that  $\neg B$ . Then without the PNC one cannot infer  $\neg A$ , for we might simply have  $B$  and  $\neg B$ .

Łukasiewicz's point is moot. If entailment is defined (as usual) in terms of the preservation of truth forward, it is correct. However, if it is defined as the preservation of truth forward *plus* the preservation of falsity backward, *modus tollens* is perfectly acceptable. However, I note the point only to show that Łukasiewicz holds that standard inferences concerning negation may fail without the PNC.

3. Łukasiewicz notes and defends Aristotle's claim in the *Analytics* that syllogistic validity does not presuppose the PNC (Chapter XV).<sup>12</sup> Indeed, in Chapter XVI, we find a remarkable thought experiment aimed to show that most reasoning does not require the PNC. Łukasiewicz asks us to consider a society where people take every negated sentence to be true (and so *some* things are both true and false). Such people would not care about negation at all, but could still reason by induction and by syllogism. Thus, a doctor can recognise the symptoms of diphtheria, remember that a certain drug has been successful at curing such symptoms, and so prescribe it. None of this concerns negation. He concludes (p. 191):

The example shows that beings that do not recognize the principle of contradiction, that ascertain matters of experimental fact, are able to reason inductively and deductively and are able to act effectively on the basis of such conclusions. If, however, these

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<sup>12</sup>*An. Post.* 77a10-23. Aristotle's claim bears a small—and opaque—qualification. Łukasiewicz argues that this does not seriously limit the categorical claim.

thought processes are possible in *one* case, then they must be possible in *all* cases. If, by the way, the intellectual organization of our fictional beings would not be different from the human one, then they would be in a position to develop the same sciences as the ones developed by man. From that society, a second Galileo would emerge who would calculate the paths of balls rolling along tilted chutes and who would postulate the laws of free falling objects based on the foundation of these facts; there would be a second Newton, who would synthesize the discoveries of Galileo, Kepler and Huygens into one unified account by determining the highest principle of mechanics. There would be a second Lavoisier...

and so on. Indeed, we would have scientific business as normal.

The conclusion is somewhat surprising. It would seem obvious to suppose that sometimes scientific testing involves the refutation of hypotheses. This requires *modus tollens*, and Łukasiewicz' himself has already said that this does not work without the PNC. What is happening here? I take it that the model of science that Łukasiewicz is working with is an "inductivist" one.<sup>13</sup> Science starts by making observations. Thus, given that the observed *As* are  $a_1, \dots, a_n$ , and that  $Ba_1, \dots, Ba_n$  are observed, we may infer that all *As* are *Bs*. We can then infer further things by deduction. Thus, if we have also established that all *Bs* are *Cs*, we can infer that all *As* are *Cs*. Few would now subscribe to this view of science. But it must be remembered that when Łukasiewicz was writing, Popper's *Logic of Scientific Discovery* was still 24 years into the future. However, again, I note Łukasiewicz' view, not for its correctness, but for its relevance to matters to emerge.

4. Łukasiewicz' closes the chapter, and so the first part of his book, with a summary worth noting. Amongst its points are the following:<sup>14</sup>

b) The principle of contradiction is not a *final* law but it demands proof.

c) Aristotle has not provided proof because his arguments were insufficient. Thus, as long as no one else delivers a proof, the principle of contradiction remains an *unjustified* principle in which we have blind faith.

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<sup>13</sup>See Chalmers (2014), esp. ch. 1.

<sup>14</sup>Chapter XVI, p. 192.

f) There are cases in which the principle is certainly *false*, namely with respect to contradictory objects.

## 4 Łukasiewicz' Proof

To the second part of the book, then, which contains Łukasiewicz' positive considerations on the PNC.<sup>15</sup> This begins in Chapter XVII, as follows (p. 194):

I have completed the primarily critical part of the investigations. The more the results turned out to be negative, the stronger the need grew to add a positive part. Despite all this, nobody seriously doubts the principle of contradiction.

His ambivalence concerning the PNC is already clear to the not-so-discerning eye. How could Łukasiewicz fail to doubt the principle, given that he concludes the previous chapter by saying that there are cases where it is 'certainly false'!

At any rate, this chapter contains Łukasiewicz' proof promised in the Introduction to the book. Right at the start of the book, he defines an object as any *something*:<sup>16</sup>

*By object I mean any something whatever that is "something" and not "nothing"...*

In the present chapter, we then find:<sup>17</sup>

There is only *one* way ... [to prove the PNC]: it has to be assumed that contradictory objects are no objects at all, that they are not something but *nothing*. Anything, then, that is an object and therefore is something and not nothing, does not contain contradictory properties... And here we have the proof of the principle of contradiction, *the only strict and formal proof* that, in my opinion, does exist.

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<sup>15</sup>In the first half of the book Łukasiewicz finds three versions of the PNC in Aristotle. Logical:  $A$  and  $\neg A$  cannot be true together. Metaphysical: an object cannot both have and not have a property. Psychological: no one can believe  $A$  and  $\neg A$ . The third, he argues, is just factually false. The first two are, however, equivalent, and provide the subject of the following discussion.

<sup>16</sup>Chapter I, p. 89.

<sup>17</sup>Chapter XVII, p. 201.

I'm sure that the reader will not be impressed. Neither was Łukasiewicz. He says (p. 201) 'I doubt that this proof will deceive anyone', and then goes on to explain why (p. 202):

According to the first definition we will call an "object" everything that is something and not nothing, i.e., things persons, phenomena, events, relations, the entire external world and everything that takes place within ourselves. Also all scientific concepts and theories are objects. According to the second definition, we will call everything an "object" which does not contain a contradiction. The question arises: *are objects in the first sense also objects in the second sense?...* This is the real problem and we have been looking for its solution from the beginning.

And it is the one to which Łukasiewicz turns in the subsequent chapters.

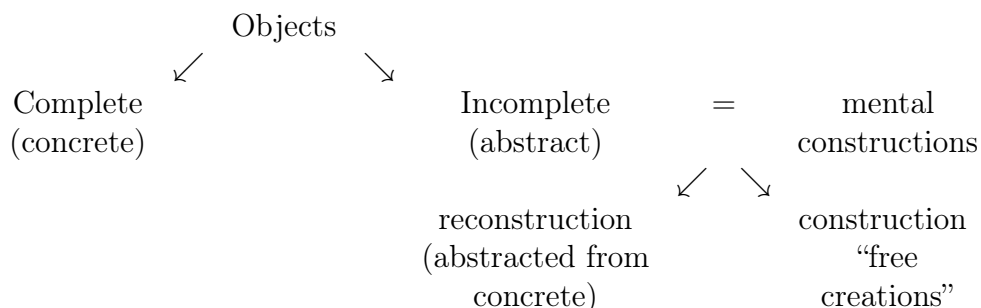
## 5 Impossible Objects

He begins his discussion in Chapter XVIII with a taxonomy of objects. First, there is a division between *complete* objects—i.e., objects,  $x$ , such that for any property,  $P$ , either  $Px$  or  $\neg Px$ —and *incomplete* objects. (So far, no commitment to whether one can have, for either kind, an object such that  $Px$  and  $\neg Px$ .) Concrete objects are complete. Abstract objects are incomplete. Abstract/incomplete objects do not exist in reality, but are 'merely products of the human mind' (p. 205).

Incomplete objects themselves are of two kinds. *Reconstruction objects* are those obtained by abstraction from concrete objects. Thus, *the tree as such*, is obtained from trees by abstracting away all properties which some trees have and some trees lack, leaving the rest. Thus, the tree as such has a wooden trunk and bears leaves. It is neither deciduous nor not deciduous; it is neither ever-green nor not evergreen. Construction objects, on the other hand, are the 'objects of *a priori* concepts, which are primarily the concern of mathematics and logic' (p. 205), such as numbers and geometric figures. These are free creations of the human mind (p. 205). Hence we have the



following taxonomy:



Obviously one may take issue with Łukasiewicz’ taxonomy. Thus, for example, platonists about abstract objects will standardly hold that numbers are neither incomplete nor free mental creations. However, this is not relevant here. We are simply tracking Łukasiewicz’ thought at this point.

The consistency of concrete objects is taken up in the next chapter, which also comments on reconstruction objects. The rest of the present chapter is taken up with the issue of the consistency of construction objects. The consistency of such objects, it seems, is easily established. Since such objects are free creations of the human mind:<sup>18</sup>

we have unlimited freedom in their construction... it depends only on *us* whether these objects turn out contradictory or not contradictory: but because we do believe in the principle of contradiction, we construct them in such a way that they would not be contradictory. Accordingly, we may assert at least about construction objects with certainty that none of them can simultaneously contain and not contain a property.

This is rather suprising. What one might have expected him to say is that since they are entirely free creations, of course we can construct contradictory objects if we wish. But in any case, Łukasiewicz backtracks immediately (p. 206):

even in the domain of such objects contradictions nevertheless do occur. It is enough to mention “the greatest prime number” or the “square constructed with ruler and compass that has the same area as a circle of radius 1”.

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<sup>18</sup>Chapter XVIII, pp. 205-6.

The influence of Meinong on Łukasiewicz' is obvious.<sup>19</sup> He is endorsing the Meinongian principle now often called the *Characterisation Principle* (CP): the thing which is so and so, is indeed so and so. Thus, an object characterised by an inconsistent condition is indeed inconsistent. The CP is a vexed principle, and no one can endorse it without triviality.<sup>20</sup> But such vexations are not on Łukasiewicz' horizon; and given that, one might think, it settles the matter once and for all.

But Łukasiewicz' takes away with one hand what he has given with the other (p. 206):

To this one can reply that these contradictory objects, which obviously are *not* objects, have found their place among other constructions only erroneously and by accident, and because our imperfect intellect is unable to grasp the entire manifold of properties and relations in a single moment and cannot in all cases detect a contradiction right away. But we have immediately removed such objects from science as soon as it became apparent that the objects mentioned were contradictory, and nowadays we already know that the squaring of a circle is impossible and that the greatest prime does not exist.

The remark is doubly puzzling. In the case of the two examples given, their inconsistency is not, indeed, immediately apparent. But what of *the round square* and its like? If one endorses the CP, then this is round and square: its inconsistency is patent. Perhaps, then, it was never constructed mentally? But it seems that it *must* have been constructed if we can think of *it*. Worse, even if it be the case that examples of the kind given are removed when we discover that they are contradictory, it remains the case that the *were* constructed in the first place, and were contradictory. Something done by accident, such as an insult, has still been done.

Łukasiewicz' seems oblivious to this, but raises another problem of his own making (p. 206):

But the doubt remains: If we are unable to recognize contradictions right away, then how do we know that constructions, which are held to be not contradictory, do not contain a contradiction?

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<sup>19</sup>Łukasiewicz attended lectures by Meinong in Graz in 1908-1909. (See Simons (1989), p. 257.)

<sup>20</sup>See Priest (2005), esp. the Preface.

Perhaps we have not discovered it until now. This doubt can be expressed in the form of a *charge of principle*: *Where is the guarantee that non-contradictory objects exist at all?*

He points out, quite correctly, that things constructed can have properties that go beyond those explicit in their construction. Thus, that it is contradictory is no part of the explicit characterisation of the greatest prime number. So how does one know of *any* construction that it is consistent? He continues (p. 207):

And once again, someone might object at this point that if every construction were to contain a contradiction, we then would, more frequently than up to now, encounter contradictory objects. In the meantime, one should consider the examples just mentioned merely *exceptions*. They are simply the leftovers in the workshop of science, impurities on the surface of grey molten iron.

Łukasiewicz's remarks are exceptionally puzzling. He moots a doubt to the effect that all construction objects are contradictory. This is irrelevant in the context: it is a slide from *some* to *all*, of a kind for which Łukasiewicz himself has rightly castigated Aristotle. More: even if the worry is real, it fails to address the question on the table: are *some* objects contradictory? Indeed, the passage seems to concede that they are. If there are exceptions, then there are *exceptions*.

## 6 Paradox

Łukasiewicz fails to note any of this, but takes issue, instead, with the suggestion that the constructions are *mere* intellectual flotsam (p. 207):

However, that this nevertheless is not the case, and that the purity of the metal itself is strongly under suspicion, that is testified by the newest investigations of the foundations of mathematics.

Contradictory objects are not metallic froth: they are at the very core of mathematics, in the form of the theory of sets.

Łukasiewicz shrewdly observes (p. 209) that solving the traditional paradoxes of the infinite by enforcing the condition that two sets are the same size iff they can be put in one to one correspondence, has merely succeeded

in shifting paradoxes from the infinite to the absolute infinite—in the form of paradoxes such as Russell’s and Burali-Forti.<sup>21</sup>

Łukasiewicz says that he *suspects* that a solution to Russell’s paradox, compatible with the Principle of Excluded Middle, can be found (p. 212),<sup>22</sup> but he clearly has nothing to offer here. He then continues:

I want to return to the problem to which this chapter is dedicated. We did ask whether construction objects, that is, the *a priori* concepts of mathematics and logic, are objects in the second sense of the word, which is to say, whether they contain contradictory properties or not. The presented examples show that we cannot answer the question unequivocally. In fact, we do encounter strange contradictions in these objects and can never know with certainty whether apparently contradiction-free objects really are such.

His conclusion to the chapter, then, is one of agnosticism (p. 213):

*In fact, we do encounter strange contradictions in [construction] objects and never can know with certainty whether apparently contradiction-free objects really are such.*

But it is hard to avoid the conclusion that Łukasiewicz is loth to accept the force of his own arguments. Twice in the chapter he has had counter-examples to the PNC at his finger tips, and twice he has backed away from them. In the first case, the examples concerned contradictory Meinongian objects. These counter-examples were rejected by what can only be described as disappointingly sloppy thinking from a mind as acute as Łukasiewicz’. In the second case, these concerned the paradoxical objects of set theory. The best he can do concerning these is to express the hope that there is something wrong with the arguments concerned. But in a context where the PNC is at issue, and so cannot be assumed, this would appear to betray another failure of rationality. The wise person, as Hume noted,<sup>23</sup> apportions their beliefs according to the evidence. The evidence available to Łukasiewicz in this case—less than conclusive though it may be—is that these objects really are contradictory.

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<sup>21</sup>See Priest (1995), p. 126 of 2nd ed.

<sup>22</sup>He notes that the argument for the Russell paradox uses the Principle of Excluded Middle: the Russell set is either a member of itself or it is not. Since construction objects are incomplete, perhaps he had doubts about this.

<sup>23</sup>*Enquiry Concerning Human Understanding*, Part X, Sect. 1.

## 7 Motion

The next chapter, XIX, turns from the consistency of abstract objects to that of concrete objects. Łukasiewicz starts by saying (p. 214) that though reconstruction objects are mental constructions, because they are abstractions from concrete reality, they can be contradictory only if this is so. Could it be? Łukasiewicz says (p. 214-5):

There does not seem to be anything easier than the answer to the question... If there is anything that cannot be doubted, then it is *this* fact: real existing phenomena, things and their qualities, do not contain contradictory properties. If I am now sitting at my desk and write, then it cannot be true at the same time that I am not sitting and writing... [There is then a series of such homely examples.] In fact, such and similar considerations taken from daily life are the strongest arguments for the principle of contradiction.

The strongest argument, then, is one by induction. But of course, even if such an induction is valid, it establishes the principle only for medium-sized dry goods; and Łukasiewicz is well aware, as we have seen, that the PNC might have only limited validity. Indeed, he indicates this almost immediately—in a grudging sort of way (p. 215):

Things are completely different, if someone is not satisfied with the merely superficial considerations of the appearances and engages in a more subtle analysis. Whoever does this, moves away from “healthy common sense” and has only to blame himself if he gets caught in contradictions.

Łukasiewicz’ lead-off batter in such less superficial considerations is Zeno of Elea, whose arguments appeared to establish the contradictory nature of motion. It was not only some of the presocratics who were persuaded. So was Hegel, whom Łukasiewicz quotes as saying:<sup>24</sup>

[One] has to concede to the old dialecticians the contradictions that they demonstrate in motion, but from this it does not follow

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<sup>24</sup>P. 216. Łukasiewicz’ text then refers back to Ch. V, where he has quoted a passage from Hegel’s *Logic* expressing the point even more explicitly.

that motion does not exist, but rather than motion is the being-present of contradiction itself..

He then adds—somewhat in tension with his discussion of paradoxical objects in the previous chapter:

Thus, it appears that if it is possible to cast doubt on the principle of contradiction at all, then it will be in the area of concrete objects, that is, in the area of facts of experience.

What, then, is one to say of apparent contradictions concerning motion? Says Łukasiewicz, they may be resolved by drawing an appropriate distinction: time. ‘Concrete objects may contain contradictory properties, but not “simultaneously”, that is, not at one and the same time’ (p. 216). Perhaps. But Łukasiewicz is troubled by Zeno’s arrow paradox (pp. 217-8):

Let us imagine a transverse section cut across the entire world of phenomena, performed at some arbitrary point of time. In this transverse section, on its immobilized surface, there would be no change and no time. The arrow would have to freeze motionless at some location. But how do we know that it would have to be at only *one* location? As long as it was moving, it continuously changed its location in space and, consequently, it was present at many locations even within each smallest of time moments. Why, then, could it not also be at least at *two* different places in the not extended time-point of the transverse section?

Łukasiewicz goes on to answer his own question (p. 218):

There can be no answers to these questions. One cannot gain anything here with *a priori* considerations because one would have to already rely on the principle of contradiction, which is what we are trying to ground. Experience, too, is silent on the matter, since a not extended time-moment is not an object of experience.

But he misses the obvious here. Zeno does not just raise the *possibility* that the arrow is instantaneously in a contradictory state. He gives an argument for it: if it made no advance at each instant of its journey, it could make

no advance at all.<sup>25</sup> And just as Łukasiewicz would have to find an error in the argument for Russell's paradox, he would have to find an error in Zeno's argument. It does not seem to occur to him that he needs to do this. At any rate, Łukasiewicz draws the same agnostic conclusion he draws in the previous chapter: one just cannot know whether reality is inconsistent.

But Łukasiewicz, torn, seems to feel that he cannot leave it at that. He then says—contradicting what he said earlier (after his quotation from Hegel) about the realm of the concrete providing the most likely counter-examples to the PNC (p. 219):

But despite all this, the case of the principle of contradiction is stronger in the domain of real objects than in the sphere of mental constructions. There we encountered *factual* contradictions, whose solution is not at all easy, and here, on the other hand, the existence of the contradiction is merely *possible*.

Moreover, he says, if at some time we seemed forced to conclude that an object were in two places at the same time, we could just conclude that being in two places at the same time is not a contradiction. After all, if  $p_1$  and  $p_2$  are distinct places, ' $x$  is at  $p_1$  and  $x$  is at  $p_2$ ' is not a literal contradiction. For that, we need the extra premise that if  $x$  is at  $p_1$ , it is not at  $p_2$ .<sup>26</sup> He concludes that we might treat the principle of non-contradiction in the same way that we treat the principle of causation: everything has a cause. If we ever come across an apparent counter-example to this, we can always assume that there is a cause; we have just not found it yet. The two principles, though, are evidently not the same. Were we to find ourselves in the situation of having to suppose that an object is in two places at the same time, we do not have to suppose that there is something going on which we have not yet discovered: we just have to take it, according to Łukasiewicz, that being at two different places at the same time is not a contradiction.

In any case, this supposed resolution of the supposed contradiction is somewhat lame. Of course, an *extended* object can be in two places at the same time: my left and right hands are at different points in space. But the very notion of a point is one which can have no such extension. Its being in

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<sup>25</sup>See Priest (1987), 12.2.

<sup>26</sup>Łukasiewicz makes the point only concerning the *experience* of an object being in two places at the same time. But he might equally have applied the thought to instantaneous situations, thus attempting a solution to the Arrow Paradox.

more than one place is ruled out by very definition. And any point on the arrow—which moves in tandem with the rest of the arrow—is such a point.<sup>27</sup>

The chapter closes with the following paragraph (p. 221):

The final result of the last two chapters has in principle been negative: it is impossible to show in an indubitable manner that contradictory objects exist. Long centuries of scientific work separate us from the moment during the origins of philosophy when Aristotle sought to prove the existence of at least one contradiction-free being. Today we are older, and thus more modest.

The statement is telling. We cannot prove the existence of a contradictory object. Perhaps so. But that was not what was at issue. What was at issue is whether we could prove that there is *not*. Like Aristotle sliding from *some* to *all*, Łukasiewicz, sensing that he cannot prove what he has sought, slides to a different claim, without remarking on the fact.

## 8 A “Practical-Ethical” Principle

In the last substantive chapter of the discussion, Łukasiewicz addresses two questions concerning the PNC:<sup>28</sup>

- (a) Why is it that we believe in a principle whose truth cannot be demonstrated?
- (b) Why do we attribute a value to it that exceeds even the value attributed to statements that are true with certainty?

The answer to (a) is simply the magisterial authority of Aristotle (p. 224):

It is a mark of the genius of the Aristotelian spirit, that it was able to convince all humanity of two things: First, that the principle of contradiction is true *even though there is no proof for it*; and further that the principle of contradiction *does not require a proof at all*. Has there ever been anything comparable in the history of science?

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<sup>27</sup>One might, I suppose, suggest that the length which is the arrow, is not made up of points. So much the worse for contemporary science. And supposing spatial distance to be quantized drives us into the arms of another of Zeno’s paradoxes anyway: the Stadium.

<sup>28</sup>Chapter XX, p. 230.



That seems about right. The problematic nature of Aristotle's arguments is patent to anyone who considers them with an open mind; and there has been no substantial defence of the PNC since. What was there, then, but Aristotle's name?

The answer to (b) is more substantial, and it is here that Łukasiewicz finally lays his own cards about the PNC on the table. He says (p. 226):

... this is the place to present the final and probably most important idea of this treatise. It seems to me that so far nobody has brought this thought into clear awareness, even though Aristotle was perhaps closest to it: *the value of the principle of contradiction is not of a logical but of a practical-ethical nature: this practical-ethical value, however, is so great that the lack of logical value does not count in comparison.*

Łukasiewicz illustrates what he means. Suppose that I am accused of a crime. I can show that I was not there. I have an alibi, a witness who will vouch that I was somewhere else, and so on. But another witness comes forward who swears that I was there, and that he saw me commit the crime. Given the principle of non-contradiction, both witnesses cannot be correct, and the judge has to decide who is the more reliable witness, where the balance of evidence lies, etc. But without the principle, the judge may just decide that I was there, even if I wasn't, and find me guilty.<sup>29</sup> The example (p. 228):

show[s] what the practical and ethical significance of the principle of contradiction consists in. The principle is the only weapon against mistakes and lies. If contradictory statements were to be reconcilable with each other, if affirmation were not to nullify denial, but if the one were to meaningfully co-exist next to the other, then we would have no means at our disposal to discredit falsity and unmask lies.

This is a somewhat amazing argument for Łukasiewicz to use. As we have seen, both he and (according to him) Aristotle have noted that the PNC might hold only in a limited realm. The fact that we may apply the principle in this case says nothing, therefore, about whether it applies to mental

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<sup>29</sup>What my evidence must do is cause the judge to *reject* the claim that I was at the scene of the murder. However, the distinction between asserting a negated sentence and denial—the linguistic correlate of rejection—is not on Łukasiewicz' radar. (For more on the distinction, see Priest (1987), 7.3, and Priest (2006), 6.2.)

constructions and instantaneous states, for example. Łukasiewicz even comments on the fact a page later, where he says (p. 229) that there is no danger to the empirical sciences ‘if some contradiction in the *a priori* sciences, for example, Russell’s contradiction cannot be resolved’.

Indeed, it is not even true that one needs the *impossibility* of contradiction in this case. The *improbability* will do. As Hume noted in his discussion of miracles,<sup>30</sup> we may well justifiably refuse to accept that something has happened if it is most unlikely—especially if there is a better explanation of the situation (in this case, that someone has lied). After all, even if everything is entirely consistent, given the laws of quantum mechanics, there is some probability that my atoms might have collectively disappeared and rematerialised at the scene of the crime long enough for me to pull the trigger. Pity the poor barrister for the prosecution who tried to use this as an argument!

Łukasiewicz then generalises his point to a more grandiose scenario: the development of science in Ancient Greece. At that time the new empirical sciences were coming into being, but many of the sophists denied the PNC. Had Aristotle not defended it, the nascent science would have been still-born. Commenting on Philip of Macedonia’s defeat of the Thebans and Athenians, he explains, in the following colourful passage, that Aristotle must have (p. 230-2):

felt heart break over the battle of Chaeronea. In this politically most difficult moment, which made practical life so distasteful to Aristotle and encouraged him to treasure theoretical life above all, the sophists introduced the elements of intellectual and moral laxness. This constituted a hundred times greater defeat than the the might of Macedonia. It not only destroyed the foundations of social being but also the foundations of the individual; it destroyed the principles of the understanding. Aristotle saw the future of his homeland in the cultural work that was to remain the only free area of activity for the Greeks for centuries to come. And he took on an inexhaustible part of this labour by establishing the considerable foundations of a *scientific* culture and, by combining his research and that of others, created a series of new and systematic branches of knowledge. The sophists, in

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<sup>30</sup> *An Enquiry Concerning Human Understanding*, Section X.

particular, had proven themselves as the enemies of such goal-oriented, creative, and systematic labour... The sophistries and paradoxes of these clever speakers were known all over Greece. Perhaps there wasn't anyone who took these strangely distorted thoughts too seriously, but they nevertheless ridiculed science in the public eye and instigated chaos in the minds of men. These sophists denied the principle of contradiction. Even though their charges were vacuous, the positive proof of the principle was not possible and ... Aristotle was himself aware of the weakness of his arguments. Thus, there was nothing else left than to declare the principle of contradiction a *dogma*, and to set authoritarian limits to any destructive works. Only in that way was the Stagirite able to forge armours against sophistries and errors and the clear the path for positive work.

Whether or not this is true, it is a bizarre statement for Łukasiewicz to make, precisely because, as we have already noted, he has argued a few chapters before that scientific development would continue in exactly the same way, even without the PNC. Aristotle did not need the dogma!

## 9 Conclusion: *Tu Quoque*

Let us draw the threads of our discussion together. Łukasiewicz clearly accepts the PNC; indeed, he declares that no one doubts it, and asserts confidently at the start of the inquiry that it can be proved. But as the investigation proceeds, he concedes that the proof he gives is of little value. Objects, by definition, may be consistent, but the real question is then whether there are some things that are not objects. He discusses contradictory Meinongian objects, paradoxes of set theory, and the views of Zeno and Hegel on motion. His official conclusion is that it is impossible to show these situations do not generate contradiction. But in many places he seems forced to concede that there are arguments for contradiction that he does not know how to answer—sometimes he does not even try. Indeed, at one point earlier in the investigation, he actually lets slip the admission that the PNC is indeed false. Finally, he resorts to a pragmatic justification of the PNC which is not only rather feeble, but fails given his own prior considerations.

In the first half of the book, commenting on Aristotle's arguments for the

PNC, he says:<sup>31</sup>

It seemed as if the Stagirite, filled with trust in his own strength and certain of victory, entered head-on into a fight. Like arrows from a quiver, he takes out one proof after the other; but he does not notice that the arrows have no effect. He has exhausted the arguments and none of them has demonstrated the principle that was so dear to him. So he defends with what is left of his strength and faith the last position: that of only one contradictory free being and only one contradiction free truth.

Or perhaps it has been different? Perhaps his pride and self-assuredness were just pretended? Sometimes one wants to assume that Aristotle, sensing the practical and ethical importance of the principle of contradiction with his acute and deep understanding, *deliberately* formulated it as an untouchable dogma in order to replace the lack of factual arguments with his *sic volo sic iubeo*. But in the depth of his soul, he himself was not sure about this matter. He hid himself behind this thought; the debate, however, made him grow passionate. And in this way, he let slip a moan of despair against his will... It will be shown soon that he had in fact enough reasons to doubt the universal significance of the principle of contradiction, but apparently did not have enough courage to admit this outright.

Though Łukasiewicz is speaking of Aristotle, he might just as well have been speaking of himself.<sup>32</sup>

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<sup>31</sup>Ch. XIII, pp. 170-1.

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