Book Reviews

Michael D. Resnik (ed.), **Mathematical Objects and Mathematical Knowledge** (*The International Research Library of Philosophy*, vol. 13), Dartmouth Publishing Company, Aldershot, 1995, £110, pp. xxi + 647, ISBN 1-85521-638-8

The philosophy of mathematics is often thought to be a specialist and slightly esoteric area. And so in many ways it is. Yet it is a touchstone of very general importance. Mathematical knowledge has often been taken to be the most certain kind of knowledge that we have; yet its source appears to be in abstract objects as remote from us as can possibly be. Any account of metaphysics and epistemology that cannot resolve this tension satisfactorily is inadequate.

The first half of this century witnessed the most profound and exciting developments in the philosophy of mathematics since Ancient Greece-possible ever. The foundational movements of that period all appeared to have run out of steam by mid-century. But work has continued since then, if not in the same ground-breaking way, then at least in a spirit informed and transformed by the new developments.

Many of the seminal papers of the first half-century appeared in Benacerraf and Putnam's anthology *Philosophy of Mathematics* (Cambridge University Press, 1964; second — and thoroughly revised — edition, 1983). The current volume contains much important later work; and, like Benacerraf and Putnam, will provide an indispensable reference work for the philosophy of mathematics.

In all, 24 papers, which appeared originally between 1980 and 1991 as journal articles or as chapters of books, are reprinted here-literally. The book takes the unusual approach of simply reproducing each article from its source. This has some advantages: the original pagination and page numbers are preserved (with new page numbers added); and no doubt it made the book cheaper to produce. But it also has some drawbacks. References to work unpublished at the time of original publication have not been updated, cross references to other articles in the volume have not been made, and typographical errors appear to be reproduced *verbatim*. A name index is provided, but a subject index is not. Undoubtedly, the effort required to rectify these unhappinesses would have made the book even more welcome.

According to Resnik in his introduction, the volume contains "a fairly detailed and unified coverage of recent debates between mathematical realists and anti-realists" (p. xiii). This is not entirely true. There is nothing, for

Studia Logica **61**: 293–310, 1998.

example, on nominalist finitism (e.g., of van Bendegem); nothing on para-consistent Meinongianism (e.g., of Routley/Sylvan) and, most surprisingly, nothing at all on intuitionism. Resnik attempts to justify the omission of Dummett-inspired debates, saying: "I have not included anything on [Dummett's doctrines] here, since they have neither addressed the topic questions of this Volume nor have greatly influenced contemporary research on these questions" (p. xiv). This will strike British readers, in particular, as amazing.

In fact, what unifies all these articles is an agenda set largely by Quine (and, to a lesser extent, by Putnam), and, specifically, the assumption, not addressed at all in the volume, that only classical mathematics, or at least mathematics based on "classical logic", is to be taken seriously. It would be fairer, then, to say that the volume concerns the major papers on the realist/anti-realist problem by North American Philosophers. (Only one of the contributors, Wright, is not North American, and his paper stems from his time at Michigan.)

The papers in the collection fall, roughly and inexactly, into four major themes. An important Quinean argument for his mathematical realism is often called the "indispensability argument". Mathematics is applied in science and (holistically) confirmed to be true. Two papers in the volume by Field represent his well known reply to this argument. The appropriate mathematical apparatus is merely machinery for inferring empirical statements more expeditiously. Hence, the success of science in no way rebounds to the alethic credit of mathematics, which can be interpreted purely instrumentally. Papers by Malmet, Shapiro and Burgess examine aspects of Field's programme. A paper by Wright provides a critique of the programme, and defends Wright's account of Frege's logicism for arithmetic, based on second-order logic plus a "number of" operator. A paper by Boolos also defends Frege on similar lines.

One way to avoid the force of Field-like critiques of Quinean realism is to claim that mathematical statements can be verified directly by some kind of mathematical intuition (as did Gödel). Analyses of such intuition are provided in the volume by (Charles) Parsons and Tieszen. A more daring move is to argue that such statements can be confirmed by *ordinary* perception. This is a version of realism that has brought Maddy much recent attention. For her, certain sets just are physical objects, and so can be seen. Two papers in the volume represent Maddy's view. These are accompanied by a paper by Chihara comparing Maddy's view with Gödel's and providing critiques of her views along the way.

One way to attempt to avoid realism is by giving a structuralist account of mathematical objects. Mathematical objects are just *loci* in various structure, with no independent existence. Such a view was advocated by Putnam at one time — and expressions of a similar view are also to be found in Quine, at least according to Parsons (p. 543). Structuralist views of one kind or another are advocated in two papers by Resnik himself, Shapiro and Hellman (none of which, surprisingly enough, makes any use of, or even mentions, category theory, the mathematical theory of structure *par excellence*). A balanced critique of the project is provided in a second paper by Parsons.

Another Quinean theme is that second-order logic is disguised set theory. Hence any account of the foundations of mathematics that uses second-order logic is committed to realism (by other Quinean arguments). The fourth theme in the volume is this view of second-order logic. Two more of Boolos' papers argue for a line that has found favour with a number of writers, that second-order logic is just plural quantification over first order objects, and so not committed to the existence of sets. Critiques of this view are provided by Resnik and Shapiro in their second papers in the volume.

The other three papers in the volume are more miscellaneous. Chihara defends nominalism in mathematics, in his second paper, by interpreting an existential quantifier as saying "It is possible to construct a predicate such that...". Kitcher defends a Millian view that mathematical truths are just empirical generalisations. And Tait defends a common-sense realism of a Wittgensteinian kind.

There is hardly space here to comment on the substantive claims that any of these papers make, but let me say a few brief words about some of the Quinean problematic of the volume and, specifically, the indispensability argument, which, according to Field, is the only "serious argument" for the truth of mathematics (p. 314).

To discuss this, let us first distinguish between pure and applied mathematics. In pure mathematics, to put matters somewhat tendentiously, one establishes the facts about various mathematical structures; in applied mathematics one uses these to solve problems outside mathematics, whether in physics, economics, or elsewhere. Now, holism notwithstanding, when various empirical predictions, obtained using mathematics, are verified/falsified, it is not at all clear that these speak for/against the truth of the pure mathematical assertions applied. If they speak for anything relevant to mathematics, it would seem to be whether or not we have applied the right mathematical structure for the empirical system in question. If, for example, we applied theorems of Abelian group theory to some collection of physical operations, and subsequent predications were not borne out, we might infer that the operations in question had a non-commutative structure, but we would not consider Abelian group theory to have been refuted.

The facts of Group Theory, or any other part of pure mathematics that we apply, are established beforehand. How? By being proved, of course.

(Proof is the most distinctive phenomenological feature of pure mathematics, but one entirely ignored by *a posterior* accounts of mathematics.) And proof appears to provide an *internal* criterion for the truth of mathematical assertions. That is, after all, how we learn what *counts* as true in mathematics. Such a criterion may, of course, be questioned by sceptical considerations. But so can any criteria; this proves nothing. The nature of mathematical proof itself is an intricate question, and not one to be pursed here. We have at least seen not only that empirical considerations may not provide grounds for the truth of mathematics statements, but that there may be quite different grounds.

The arguments I have just rehearsed are, of course, contestable; but this is not the place to go into their ramifications. I hope only to have shown that there are considerations here that point beyond the Quinean problematic. To return to the book itself: the papers in the volume are all excellent and thought-provoking. Many are deservedly well known already. The book succeeds in capturing in once place much of importance in contemporary research in the philosophy of mathematics.

GRAHAM PRIEST
Department of Philosophy
University of Queensland

ANDRZEJ WIŚNIEWSKI, **The Posing of Questions: Logical Foundations of Erotetic Inferences**, Kluwer Academic Publishers, Dordrecht, 1995, $110 (US), pp. xiv + 247, ISBN 0-7923-3637-2

Loosely speaking, the author's aim is to analyze the raising of questions — that is, cases in which, e.g., a statement evokes a question, or one question implies another question, More generally. the aim is to show how erotetic logic (the logic of questions) can be conceived as the logic of arguments that have statements and/or questions as premises, and have questions as conclusions. (Example: From 'Who discovered X?' and 'The one who discovered Y is the one who discovered X' we may infer 'Who discovered Y?') The book makes important contributions to the theory of questions and their logic, and its main audience will probably be professionals and graduate students in the field, but some key parts of it can be read by beginners (at least, those who have had some introduction to logic), and it can serve as a good introduction to the field.

The author first specifies a class of formal languages and then defines his erotetic concepts for these languages. Loosely speaking, each of these