Thus, according to Ockham's analysis, the essential exclusion between horses and bears — an exclusion between essences that holds eternally and is necessary in the way that Aristotelian laws of nature are necessary — on application to the current state of the world, says, of every currently existing horse (no matter how it is described), that it is not a possible-bear. And this is necessary, just in the sense that it is an *application* of an essential exclusion which is necessary.

This is a valuable thing for Patterson to have done. It provides a framework for a plausible hypothesis telling why Aristotle would have wanted to construct the modal syllogistic in the way he did. The hypothesis is that Aristotle was trying to formalise the logic of statements in which the eternal verities of metaphysical theory are applied to the fluctuating contingencies of the everyday world. Two problems remain. First, if some of Aristotle's results hold only for strong modals, and some only for weak ones, then the system is not a unified one. Second, if strong modals make ineliminable use of metaphysical concepts then the system lacks logical purity. Whether these are problems with Aristotle's system, or with Patterson's interpretation, is a question that can only be answered by comparing his interpretation with others on offer.

There are several misprints, including the following:
p. 2: "Prantle" should be "Prantl";
p. 32: "nA a ppB" should be "A a ppB"; "ppA a C" should be "A a C";
p. 86. "Two-footed *N o* Animal" should be "Two-footed *N o* Moving";
p. 186. "Barbara and Celarent *pp*, *N/p*" should be "... *pp*, *N/pp*".

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J. HINTIKKA (ed.), **From Dedekind to Gödel: essays on the development of the foundations of mathematics**, Kluwer Academic Publishers, Dordrecht, 1995, US $194, pp. ix + 459, ISBN 079-233-484-1

This volume contains 16 papers, most of which were given at a conference at Boston University in 1992. As its name (not to be confused with the van Heijenoort collection *From Frege to Gödel*) says, they cover aspects of the development of studies in the foundations of mathematics between about the years 1850 to 1930. Much of the work of this period is, of course, well worked over by scholars, and the ideas of Cantor, Frege, Russell, Wittgenstein and

Gödel are standard fare in undergraduate courses. But as Hintikka suggests in his introduction to the volume, we often do not know historical periods as well as we think we do, especially when these have become part of folklore, as this one has. The essays in the present volume, delve more closely into some of the less well known aspects of work at that time, and hence paint a fuller picture of it; and one, it might be said, that sometimes conflicts with more popular views of what, exactly, happened. The essays are a somewhat mixed bag, in all of content, style, level of historical scholarship, formal technical content and, I thought, quality. It is unlikely that all the essays in the volume will interest everyone for whom the area is important; on the other hand, there should be something here to interest every such person.

One may divide the essays in the volume, loosely, between those that deal with particular topics and those that deal with particular people. The topics papers include one by Judson Webb on the development of the notion of a counter-model from the problems of non-Euclidean geometry, and one by Jaakko Hintikka, who argues that the emergence in the 19th century of the notion of an *arbitrary* function resulted in what we would now call the standard interpretation of second order logic (or more generally, the power-set operation), more restrictive notions corresponding to nonstandard interpretations. The other three papers in this category are of a more technical nature. Philip Erlich traces the development of the modern theory of Archimedean ordered rings and fields, with particular reference to the work of Hahn. Akihiro Kanamori, traces the development of descriptive set theory after Cantor. And Jan von Plato describes Borel's work on probability, and its relation to constructivism.

The papers on particular people include one by Harold Edwards, who describes and defends an interpretation of Kronecker's philosophy of mathematics; another is by William Boos who attempts to forge a connection between Skolem's set-theoretic relativism and Weyl's constructivism; another is a paper by Jan Woleński on Tarski, which discusses the tension between Tarski's official philosophical nominalism, and his heavy use of model-theoretic methods. There are two papers on Frege in the collection. The first is by Claire Hill, who tells the story of the destruction of Frege's correspondence, and speculates on what might have been in it. The second is by Richard Heck, who points out that the results of Frege's Basic Law V in the *Grundgesetze*, could equally well have been accomplished by "Hume's Principle" (that if X and Y can be put into 1-1 correspondence, the number of Xs is equal to the number of Ys). But Basic Law V gives rise to paradox and Hume's Principle does not. Heck discusses why Frege did not simply take Hume's Principle as an axiom instead of Basic Law V. There are also two papers on Wittgenstein in the collection. In the first of these, Mathieu Marion suggests that Wittgenstein's account of identity in the *Tractatus* is incompatible with the notion of an arbitrary function (the standard account, see the paper by Hintikka, above). Ramsey, he argues, accepted Wittgenstein's account of identity but wanted to endorse arbitrary functions, and so came to grief. The second paper is on the later Wittgenstein. In this, Juliet Floyd tries to make sense of Wittgenstein's gnomic utterances concerning Gödel's incompleteness theorems, by appealing to his somewhat more intelligible discussions of the impossibility of trisecting an angle.

Frege and Wittgenstein were, of course, big players in the game. One of the things that comes over in the volume is the important influence of people who are not usually thought of as major players. One of these is Husserl. In her paper already mentioned, Claire Hill discusses Husserl's correspondence with Frege, and notes that Husserl had already critiqued Frege's Basic Law V, 10 years before Russell's discovery that it lead to paradox. In her second paper in the collection, she discusses the connection between Husserl's notion of intensionality and Hilbert's distinction between ideal and non-ideal reasoning. In yet another paper, one that 1 found particularly illuminating, Dagfinn Føllesdal documents the clear influence of Husserl on Gödel's philosophy of mathematics.

Another surprising influence on the development of the subject also becomes evident in the collection of papers. The influence of Kant on Brouwer (on whom, incidentally, there is very little in the volume) is well known. But the non-constructivists of the period are usually depicted as resoundingly reacting against German Idealism. Think of Frege on psychologism, or Russell's much trumpeted rejection of his philosophical education. (In section 433 of the *Principles of Mathematics*, Russell describes the effect of his arguments on Kant's philosophy of mathematics as, like Samson, pulling down the pillars of the edifice.) Yet the influence of German Idealism on affairs is clearly demonstrated by the volume. Føllesdal, for example, talks of the influence of Kant on Husserl (and hence Gödel). But the influence comes out most clearly in the two papers of the volume that 1 have not yet mentioned, and which 1, personally, found the most interesting. The first is by David McCarty, and is on Dedekind. McCarty locates the site of a couple of aporias in the foundational writings of Dedekind. One of these is the famous proof of the infinitude of things in Section 66 of *Was sind und was sollen die Zahlen*. He then argues persuasively that the aporias are resolved once one understands that Dedekind is working against a backdrop of Kant's Transcendental Dialectic, and is assuming a number of its central ideas.

The second paper is by Gregory Moore, and is on the origins of Russell's paradox, as revealed, in part, in Russell's correspondence with Couturat. Until nearly the end of the century, Russell was a paid-up Hegelean, who accepted the view that many of our mathematical concepts are contradictory; and in particular, that the notion of infinity is inconsistent due to the antinomy of the number of numbers. At this stage, he rejected Cantor's work altogether. By 1900, he had come to accept Cantor's work, but the antinomy clearly generalised to the paradox of the greatest cardinal (and ordinal) number. At this stage Russell thought that the paradox could be solved by finding a mistake in some of Cantor's proofs. Even after simplifying this paradox to the one that now bears his name, Russell came to think of it as a serious difficulty only after digesting Frege's reaction to it. But by this time Russell had, in effect, reverted to his original view, that the infinite really is embroiled in contradiction, only now he held that such contradictions require solution — a reasonable summary of Kant's own view of the matter!

The history of ideas is never a straight and naked line. Unlike the distilled histories that often get told, the truth is full of dead ends, hidden heroes, turns and twists — sometimes ironical ones. It is books like this, that look more closely at its details, which do the invaluable service of reminding us of this fact.

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LASZLO PÓLOS and MICHAEL MASUCH, editors, **Applied Logic: How, What and Why, Logical Approaches to Natural Language**, Dordrecht, Boston, London, Kluwer Academic Publishers, Synthese Library, Studies in Epistemology, Logic, Methodology, and Philosophy of Science, 1995, pp. viii + 392, Dfl. 175; US $115; UK £74, ISBN 0-7923-3432-9.

The idea of applied logic is an abstraction. There has been fruitful interaction between the theory and application of symbolic logic throughout its history. Many innovations of syntax and semantics have been designed to meet the formal modeling demands of particular kinds of discourse or natural phenomena. An enhanced repertoire of logical methods encourages as it facilitates more ambitious formalization projects. What, then, is applied logic, as opposed to pure or just plain good old fashioned logic?

The editors and contributors to this interesting new volume of essays have something specific in mind by the concept of applied logic. These logicians