Review

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$A(a)$ is true in an interpretation, iff $A(x)$ is true relative to that assignment of denotations to variables which assigns x the denotation of a. But this fact is appealed to a couple of times in the Soundness proof on p. 108, and in Lemma 1 immediately after it; a similar fact is also required to verify the soundness of the substitutivity axioms for identity on p. 117.

Philosophically, the book is rather conservative. Howson doesn't have a great deal of sympathy with anything beyond simple first-order logic — second order logic, languages with truth predicates, modal logic, non-material conditionals, are all dismissed — indeed, the chapter on the last two of these is entitled 'Beyond the Fringe'. Students can add a little balance to this aspect of the book by augmenting it with Stephen Read's *Thinking about Logic*.

I also found the criticism of some of the more novel developments in logic a bit thin sometimes. For example, Howson suggests that we do not need special counter-factual conditionals: a material conditional can be taken as a counter-factual when it may be deduced from the laws of nature (p. 164). But this doesn't seem right. Since it is a law of nature that light travels at a constant speed, the following counterfactual would be true: if light were not to travel a constant speed, it would travel at a constant speed. Still, this sort of criticism is a somewhat harsh: the book does not pretend to be one of philosophical logic, and at least Howson raises philosophical issues and problems that many formal logic texts do not even mention.

There is a multitude of elementary logic books for the beginner. There are also many advanced logic books for the more sophisticated mathematical student. But books in the middle ground, where one wants to cover non-trivial mathematical results in classical logic, but at a level that does not entirely alienate non-mathematics students, are fairly hard to find. Despite the shortcomings of Howson's book that I have noted, I think that it fills this slot admirably.

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I. GRATTAN-GUINNESS and G. BORNET (eds.), **George Boole: Selected Manuscripts on Logic and its Philosophy**, Birkhäuser, Basel–Boston–Berlin, 1997, DM 58, pp. lxiv + 236, ISBN 3-7643-5456-9.

George Boole (1815-1864) wrote on the differential calculus and on logic. He was, perhaps, better known in his lifetime for his contributions to the former; but it is for his contributions to the latter that he is now mainly remembered. His work on "algebraicising" logic, first published in his short book, *The Mathematical Analysis of Logic* (1847), and expanded in *The Laws of Though* (1854), provided a pivotal point in the transition of logic from its traditional to its modern form.

Boole's work itself is historically fascinating. Though well aware of the limited nature of traditional logic, he is squarely entrenched in its problematic (for example, he had no notion of a quantifier, and held that 'All *A*s are *B*s' entails 'Some *A*s are *B*s'). And though he developed the algebraic techniques that now bear his name, he used methods that would drive any modern algebraist to distraction (for example, differentiating Boolean functions).

Throughout his working life, Boole also wrote a number of essays, notes and letters on logic, including chapters of a projected book on the philosophy of logic postdating the *Laws of Thought*. None of these was published in his lifetime. A few have appeared since, but the present book is an attempt to make a substantial part of them (about 40% the editors estimate) available for general consumption. The editors have done a thorough and commendable scholarly job. A 60 page introduction gives an account of Boole's life, the context of his work, the fate of his Nachlass, and provides an analysis of some of his philosophical ideas, notably his psychologism. This is rounded off with an account of the selection of the papers and the difficult job of dating them. There is also a set of helpful textual notes, a bibliography and indexes.

One thing the editors did not do, I think correctly, is punctuate. Boole's writing is clear (if a little wooden), but he shows a reluctance to use commas that would gladden the heart of the most traditional of lawyers. (This is not the case with his published books.) Here is an example (p. 86):

> And hence while we are able by the principle of substitution alone to combine the premises together through operations of Addition Subtraction and Composition we are compelled to have recourse to those canons of abstract thought which enable us to add to the premises whose truth is only assumed other propositions whose truth is not assumed but necessary in other words we must adopt the analytical and not the synthetical method before we can apply the same principle of substitution in connection with the operation of Abstraction.

Turning to the contents of the papers: there is, perhaps, little that will surprise those familiar with Boole's two books. But what the papers do do is bring to prominence things that are only implicit or gestured at there. One of these, predictably enough, is his psychologism. His views concerning logic are, in fact, strikingly similar to those concerning grammar expressed by Chomsky 100 years later. Both writers draw a distinction between competence and performance (p. 111); both take the appropriate laws of competence to be a universal feature of the operations of the mind (in Boole's case, the operations are those concerning classes of objects, as expressed in language); for both, these need not be known consciously.

One way in which Boole differs from Chomsky, though, is in thinking that the laws themselves, and not just individual data, may be known by introspection. Here, I think, Chomsky is to be preferred. Even in logic, the truth of a general rule cannot simply be read off from a particular case. For example, according to Boole (p. 75), in applying the term 'white flowers', the mind fixes on the class of flowers, and then selects out those that are white. It could equally well have fixed on the class of white things, and then selected out those that are flowers. This, Boole takes to be the obviously correct general analysis. But had he considered the term 'fake Rembrant', instead, he would have drawn rather different conclusions.

Another, perhaps more interesting, feature of the papers is the way that they make clear that Boole's driving inspiration was the analogy between arithmetic and logic. Specifically, he is guided by the idea that any algebraic manipulation that one can perform in arithmetic can also be performed in logic. Thus, for example, the distributive law, $x.(y + z) = xy + xz$, holds if we interpret '+' and '.' as the usual arithmetic functions or as set union and intersection, respectively. What distinguishes logic is an *additional* law, namely, idempotence: $x = x.x\,(= x^2)$.

But this idea leads Boole into all sorts of problems. In arithmetic, + and . have inverse operations; so they must have inverse operations governed by the same laws in logic (p. 113). What could these be? Consider the inverse of arithmetic addition, subtraction: $x + y = z$ iff $x = z - y$. What is the set-theoretic analogue? According to Boole, $z - y$ is the set obtained by removing from z the members in y. But this does not validate the inverse law. (Let $x = \{a, b\}$, $y = \{b, c\}$.) Boole solves the problem by insisting that set union makes sense only when the sets in question are disjoint; then the inverse law holds. He takes this to be an informal precondition of the applicability of the symbols. He even thinks that he can prove this precondition (p. 91f). By idempotence, $x+y = (x+y)^2 = x^2+xy+xy+y^2 = x+y+(xy+xy)$. Hence, $xy + xy = 0$, i.e., $xy = 0$. This is a curious proof. The penultimate step requires the subtraction of $x+y$ from both sides of the equation. But there is no reason to suppose that $x+y$ and $xy+xy$ are disjoint unless $xy = 0$, which is what we are supposed to be proving. Worse, since we cannot accept that $xy = 0$ in general, it follows that we cannot make arbitrary substitutions in logical laws. (We cannot infer $x + y = (x + y)^2$ from idempotence.) This flies in the face of Boole's stated aim of making the algebraic manipulations a purely formal matter (p. 98).

We are obviously a long way from modern Boolean algebra at this point, which *is* formal, and which simply gives up the inverse law (and so allows union to be non-exclusive). But worse is to come when we consider the inverse of multiplication. What are we to make of division? $x/y = z$ iff $x = zy$. But given x and y, there are, in general, many sets, z, such that $x = zy$. Hence division makes no sense at all. We see Boole struggling with this problem unsuccessfully at many places (e.g., pp. 58f, 75). But he is so convinced of the arithmetic/logic analogy that, rather than give up division, he goes on using it anyway. This requires him to use various *ad hoc* rules for manipulating certain quotients, e.g., setting to zero at the end of a computation, any term with coefficient 1/0. (One is struck, here, by the similarity with procedures in the old calculus, where, at the end of a computation, one sets to zero any term with infinitesimal coefficient). Luckily, Boole had the happy knack of choosing these rules so that everything works out right in the end. And this is much of the charm of his books for a modern reader, for whom logical division has — justifiably — disappeared from the scene.

Boole sometimes reflects on the fact that his algebraic computations require him to manipulate symbols that have no meaning. Indeed, he was pressed on just this by some of his contemporaries, notably, the algebraist Cayley. In his correspondence with Cayley (pp. 191–197) Boole compares his computations to those in arithmetic that go via imaginary numbers: provided that these all disappear by the end, it does not matter. One gets the impression that he is not entirely happy with this analogy, however. He knew perfectly well that one *could* makes sense of imaginary numbers. However, Boole's ideas at this point prefigure the views of another later figure: Hilbert. Hilbert, too, thought it legitimate to use reasoning in arithmetic that has no concrete meaning (infinitary reasoning), provided that this gives the right answers in the end.

Developments in the history of ideas often throw up transition figures; figures who see important parts of the way forward, but who cannot give up old views; figures who are driven by ideas and analogies which push them into new areas, but which ultimately have to be given up; figures whose awkward positions force them to articulate novel philosophical ideas. Boole is just such a figure as this. Modern logic has canonised him, and thus helped to ensure that the real Boole has been forgotten. But there is much that historians of ideas, and of logic in particular, may learn from Boole's work. Those who wish to learn it will find this book, which shows Boole's struggles behind the scenes, invaluable.

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