[7] Battigalli, P., Bonano, G., 'Synchronic information, knowledge and common knowledge in extensive games', *Research in Economics* 53, 1999, 77–99.

[8] Blackwell, D., Dubins, L. E. , 'On existence and non-existence of proper, regular, conditional distributions,' *Annals of Stats.* 3, 741–752.

[9] Brandenburger, A., 'On the existence of a 'complete' belief model', Harvard Business School, Working Paper, 03/20/98.

[10] Gärdenfors, P., *Knowledge in Flux*, Bradford Book, MIT Press, Cambridge, Mass., 1988.

[11] Hintikka, S., *Knowledge and Belief*, Cambridge University Press, Cambridge, UK, 1962.

[12] Kyburg, Henry E., Jr., *Probability and the Logic of Rational Belief*, Wesleyan University Press, Middletown, 1961.

[13] Lismont, L., Mongin, P., 'A non-minimal but very weak axiomatization of common belief', *Artificial Intelligence* 70, 1994, 363–374.

[14] Rényi, A., *Foundations of Probability*, Holden Day, 1970.

[15] Seidenfeld, T., 'Remarks on the need to use extremely non-proper conditional probability distributions for Lebesgue measure,' typescript CMU, May 1999.

[16] Stalnaker, R., 'Belief revision in games: forward and backwards induction,' forthcoming *Mathematical Social Sciences.*

[17] van Fraassen, B., 'Fine-grained opinion, probability, and the logic of full belief', *Journal of Philosophical Logic* 24 (1955), 349–377.

Horacio Arlo-Costa
Department of Philosophy
Carnegie Mellon University

N. C. A. da Costa, **Logiques Classiques et Non Classiques: Essai Sur les Fondements de la Logique**, Masson, Paris, 1997, FF 240, pp. 275, ISBN 2-225-85257-2.

1. *General introduction*

This book is a revised edition of da Costa's *Ensaio Sobre os Fundamentos de Lógica* (Huicitec, São Paulo, 1980), translated from the Portuguese by his erstwhile student and collaborator, J.-Y. Béziau. Béziau also contributes a foreword and two appendices containing technical material relevant to the body of the book. There is a brief index: a more substantial one would have been very helpful.

Da Costa is the best known and most influential South American logician of the twentieth century. He has worked on many areas, including the

foundations of physics; but it is for his role in the development of paraconsistent logic that he is best known. He has undoubtedly been one of the prime-movers in the development what is perhaps the most controversial of all the heterodox logics of our time. Until now, though, most of da Costa's work accessible to those who do not speak Portuguese has been of a technical nature. His philosophical views have had to be gleaned from occasional remarks in these publications. It is therefore most welcome indeed to have a much more accessible statement of the philosophical views which have both arisen out of, and informed, that technical work.

Da Costa tells us: 'In this book we will deal with the nature of logic. Above all, we will be interested in relations existing between reason and logic, as well as the way that rational activity, which logic reflects in great part, is tied to experience' (p. 19; all translations into English are mine; all italics are original). Da Costa's discussion traverses numerous areas in the history and philosophy of logic: platonism, intuitionism, quantum logic, Aristotle, Hegel, Quine, to name but a few; but it always returns to this central theme. The book is a rich and interesting one, and its contents bear consideration by all those who are interested in the logical enterprise.

2. *The dialectical view of logic*

Da Costa's view of logic is a blend of old and new. He disputes a conception of logic, which he terms 'dogmatic'. This is a view to the effect that to be rational is to be logical, where there is but one correct logic (traditional, classical, logic) which is a priori correct. Against this, he proposes his own view, which he terms 'dialectical', and the central points of which he states as follows (p. 33f.):

1. Logic and rationality are never identical. The exercise of reason can be effected across distinct logico-mathematical systems, systems that differ amongst themselves in the admission or otherwise of certain central principles of the logic called traditional.
2. Reason is not self-sufficient; the logical system that reflects its exercise depends on experience, varying in conformity with the types of object to which it applies. More precisely, a part of the logic is based on the interconnection between reason and experience. This means, in other words, that experience contributes to legitimating rational norms.
3. There is no unique logic. In principle, there are many of them, all legitimate from a rational point of view. The choice between them, in the context of science or a particular body of doctrine, is made more or less as the physicist chooses the geometry that is best adapted to his researches, from amongst the different mathematically possible geometries.

The analogy between logic and geometry is, in fact, a recurrent theme throughout the book.

In a certain sense, da Costa's book is an attempt to theorise the possibility of paraconsistent and other non-classical logics, in the face of the hegemony of classical logic. As the above points make clear, the major strategy employed is one of pluralism. It is the various aspects of this pluralism that I will discuss in the rest of this review.

3. *Pure and applied logic*

Let me start by making a distinction that the book does not make explicitly: that between pure and applied logic. I intend here a distinction similar to that between pure and applied mathematics. A pure logic is a mathematical structure of a certain kind, with a proof-theory, model theory, etc. An applied logic is a pure logic applied to some end. Pluralism may be applied to both pure and applied logic. I will return to the question of pluralism in applied logic later. As far as pure logics go, pluralism is now quite uncontentious. There are many pure logics: classical, intuitionist, paraconsistent, etc.

Da Costa sometimes seems to think that this pluralism in logic has become visible only in the 20th century, and specifically since the development of logics that are non-classical (i.e., not that of Frege and Russell). This is because he tends to identify classical logic with traditional, that is, Aristotelian, logic, as is apparent from the following quotation (p. 56):

> [...] in the course of the 20th century [the belief in the uniqueness of logic] changed. It was established that it was possible to build logics distinct from classical logic. It was this that Brouwer brought to light in developing intuitionist mathematics. Traditional logic, since its origin, was tied to a metaphysical conception whose roots are anchored in platonism. In fact, Aristotle established the logic starting from metaphysical presuppositions of such a kind that one cannot, practically, separate the logical from the metaphysical elements in his work.

But traditional logic and classical logic are not at all the same: they are even inconsistent with one another. For example, traditional logic counts the syllogism *Darapti* as valid, whilst classical logic counts it as invalid; and if classical logic renders it valid by making existential import explicit in the syllogistic forms, the traditional square of opposition breaks down. (For further discussion, see Priest (2000)).

4. *The revisable nature of applied logic*

Let us now turn to applied logic. A logic may be applied for many different purposes, e.g., to simplify electronic circuits. But the prime application of

a (pure) logic is to provide the norms of reasoning about some domain or other. It is, in fact, only in the domain of applied logic that disputes about which logic or logics are correct make any sense. There is no dispute over pure geometries or logics; there is nothing here to choose between.

Da Costa takes it that the choice of the correct logic or logics is to be made on the basis of evidence, part of which may be empirical, and pragmatic criteria such as simplicity, economy, etc. (p. 118f.). In short, an applied logic is a theory, and its choice is, like the choice of any scientific theory, a posteriori and fallible.

This view is more contentious than pluralism in pure logics, but it is recognisable and well known. It is the view made popular by Quine in his more radial days, when he wrote 'Two Dogmas of Empiricism', and which was taken up by Haack in her *Deviant Logic* (1974). On a historical note: the claim that the endorsement of a logic is not a priori certain, and so liable to revision was also endorsed by one of the great dialecticians, Engels. As he put it in the *Dialectics of Nature* (1954, p. 43):

> The science of logic is [...] like every other, an historical science [...]. The theory of the Laws of Thought is by no means an "eternal truth" established once and for all, as philistine reasoning imagines to be the case with the word "logic". Formal logic itself has been the arena of violent controversy from the time of Aristotle to the present day.

I shall say little more about this view here. I take it that the fallible nature of logic is clearly correct. The fact that logic is revis*able* has been amply demonstrated by the fact that logic has been revis*ed*. The transition from traditional logic to classical logic is just such a transition.

5. *Realism or instrumentalism?*

The next question to be addressed is that of the ontological status of logic. Let me explain what I have in mind here by using the analogy with geometry. Which geometry was (presumably rationally) accepted has, as a matter of fact changed. But it would be bizarre to suppose that the geometry of the cosmos itself changed when science shifted it allegiance from Euclidean to Riemannian geometry. The accepted geometry is not, *ipso facto*, the correct geometry. What makes something, as a matter of fact, the correct geometry?

There are at least two standard positions one may take on this matter. The first is instrumentalism: the question of which geometry is really correct makes no sense. Geometry is simply an auxiliary device, and we are free to choose whichever geometry is over-all best according to pragmatic criteria (and will, of course, depend on what our experience tells us). The second is realism: there is a uniquely correct geometry of space, and the correct

(applied) geometry is the one that, in an appropriate sense, corresponds to it. We may use pragmatic criteria to help us judge which geometry this is, but the correctness itself in no way depends on such factors.

A similar issue now arises for logic. Given a domain about which we wish to reason, what makes it the case that one (applied) logic is, as a matter of fact, the correct logic? Certain remarks in the book tend to suggest that da Costa is an instrumentalist (p. 128f.); others that he is a realist ('[...] reason has recourse to categories amongst which there are those which *correspond* in a more or less direct way to the structural categories of reality [...]', p. 54). But his nett position is an unusual one, somewhere between these two. The correct logic, da Costa tells us, is that which is 'best adapted' to the context — the Principle of Adequation. And concerning adaptation, da Costa says (p. 59):

> It is an extremely arduous task to try to define the concept of adaptation, a concept central to the principle of adequation. One must in effect take into account a number of factors: psychological, sociological, aesthetic, historical, epistemological, of formal-logical simplicity, to retain only some of them. Yet it is an evident fact that reason in general chooses its rules according to the domain examined. Let us give an example: in traditional mathematics, the underlying logic is classical logic, because it is the most simple and convenient, in the sense that it moulds itself (*se moule*) best to the mathematics. For quantum mechanics, despite the attempts made by P. Février, Reichanbach and others to modify the underlying logic, one continues to employ classical logic, above all for reasons of simplicity and ease [...].

Thus, the correct logic is determined by both pragmatic and ontological factors. (Given the context from which this quotation comes, which I have omitted, one might be forgiven for thinking that these are simply the criteria for which logic it is *correct to choose*. But correspondence with da Costa confirmed that these are the criteria for which logic is *in fact* correct — though I sometimes wondered whether in the book da Costa had clearly distinguished between these two issues in his own mind.)

Now there is a real issue here about what it is, in the case of logic, for a theory to correspond to, 'mould itself to', its object. (In the case of geometry, since we are dealing with physical objects in the real world — at least if realism is true — the issue is much clearer.) There are remarks bearing on this question scattered throughout the book, though no developed account that I could find. But assuming that sense can be made of this notion in the case of logic, it is not at all clear why other factors should bear on the correctness of a logic — if Riemannian geometry corresponds to the ontological geometry of the cosmos, it would seem perverse to suppose that

there are any other facts which bear on its correctness — and I could find no argument as to why this should be so in the book.

6. *Domain invariance*

I now turn to the issue of pluralism in applied logic. Standardly, logic has been taken to be domain-neutral: there is one logic that applies to all domains about which we might reason. As the previous quotation indicates, though, da Costa is a pluralist. Different domains may require different logics. Given the analogy with geometry, this is an attractive view. It seems uncontentious that different geometries are required for charting the surface of the earth and for charting distant galaxies. But the analogy is problematic.

It is at its most plausible in the case da Costa describes as follows (p. 120f.):

> [...] It is clear that for common objects, such as a book or a person, [...] [$\forall x(x = x)$] applies apparently without a single important difficulty. Any person whatever, say A, even though they undergo multiple modifications in the course of their life, remains in a certain sense identical with themself: $A = A$. That appears even more clearly as concerns abstract objects: for example, the equality $1 = 1$ seems evident and indisputable [...]. However, things are not as simple as naive realism would lead one to believe. In quantum physics, elementary particles, according to all appearances, transgress the principle of identity. Thus Schrödinger affirmed that the relation of identity between particles was devoid of sense: "it is not a problem that depends on our capacity for proving the identity in certain cases and our incapacity for proving it in other cases. It is certain that the issue of 'identity' is, really and truly, devoid of sense". It could be that the position of Schrödinger is acceptable only temporarily and that the future will show us that he is mistaken. Nonetheless the fact is that quantum physics shows the possibility of dialectising the idea of identity, and consequently, the very law that corresponds to it.

Thus, the standard laws of identity may apply when reasoning about macroscopic objects, but not about microscopic objects.

But even here, the pluralist position is not obvious. One might simply claim that, just because the "laws of identity" fail in some domains, they are not logical laws at all: the logical laws are the ones that hold across *all* domains. Of course, we may still invoke the standard principles of identity when reasoning about macroscopic domains, just as the intuitionist may help themself to instances of the Law of Excluded Middle when reasoning about finite domains: intuitionist logic is still globally correct; instances of the Law are simply "contingent truths" about that domain.

In most cases where one might think about applying different logics, however, the differences concern not the objects about which we are reasoning, but our means of reasoning about them, and, specifically, sentential connectives and operators. It is not at all clear why one must suppose the meaning of, say, negation, may change simply because we are reasoning about different kinds of objects. Da Costa, in fact, has a tendency to run together the objects of the domain and our means for reasoning about them. For example, on p. 147 he says: 'Let us not forget that intuitionist logic has a well-defined domain of application: constructive forms of reasoning'. But intuitionist logic is not *about* constructive forms of reasoning, it is itself an account of a particular *way* of reasoning. We may, of course, reason about constructive reasoning; and this domain may well contain constructions such as intuitionist negation. But that does not mean that we have to use intuitionist logic to reason *about* such constructions. As is well known, one may reason classically about intuitionist structures. (I am not saying that it is correct to so reason; merely that arguments to the effect that it is not, must appeal to other considerations, such as those concerning meaning.)

7. *Negation*

Da Costa would no doubt reply that there are, nonetheless, many legitimate negations. Thus (p. 154):

> The advance described [the development of paraconsistent logics] permitted [...] the verification that there are several types of negations which extend in different directions the informal and intuitive negation of everyday experience. We repeat: there are several types of negation, in the same way that there are different sorts of implication. All merit being studied and, in certain cases, it is convenient to use two types of negation simultaneously [...].

Now, it is true that there are many concepts of negation, just as there are many pure logics. But it does not follow that there are many negations. There are, after all, many concepts of matter (Aristotelian, Newtonian, quantum, etc.). It does not follow that there are many different kinds of matter, one corresponding to each conception. We are, in each case, presumably, after the conception that gets things right. And in the case at hand, we are after the conception of negation that correctly captures its meaning.

Da Costa denies, though, that there is any question of a determinate right or wrong here. Negation has a core meaning given by experience; but this under-determines a complete meaning, and the slack can be taken up in many different ways. Thus (p. 45f.):

> For simple statements such as
>
> This rose is red (2)
>
> one knows well what it is that negation signifies. Thus,
>
> This rose is not red (3)
>
> which is the negation of (2), signifies simply that (2) is *false*, i.e., that if we carry out a certain experiment, that of observing the rose in question, we will have a determinate kind of sensation. In the case of statements similar to (2), which express simple and observable facts, the sense of negation, as well as those of the concepts of truth and falsity, can be made clear, thanks to certain pragmatic factors [...]. If one considers statements with a greater complexity, such as, for example,
>
> Every man is mortal, (4)
>
> [...] everything is more complicated. In fact, general statements imply a certain *idealisation* of situations such as those expressed by (2) and of the intuitive usage of logical symbols [...]. The process of idealisation consists in this: negation has an intuitive content, clear and simple, relative to statements such as (2); however, this content does not completely determine the usage of negation for every context, in particular, those which bring into play general propositions of the kind of (4) [...] or others more complex. Consequently, the primitive and initial content of negation is amplified and completed, in such a way as to obtain, for example, the negation of traditional logic.

Now, there is much one might say about this. For a start, it is not at all clear that experience plays any role in determining the meanings of connectives. What role does it play, for example, in the meanings of disjunction and the conditional? If Frege got it anything like right, the meaning of a connective is simply the role that it plays in determining the meanings of sentences containing it. More importantly, a sentence such as (3) does not seem to have any determinate empirical content. What could this be: the experience of a blue rose, a pink rose, a transparent rose, an invisible rose?

But in any case, if all the different accounts of negation (classical, intuitionist, paraconsistent) did succeed in capturing determinate but different meanings for the negation sign, there would be no problem about simply having multiple negation-symbols, one with each meaning. (And da Costa is often wont to advocate the use of multiple negations, e.g., pp. 133, 154.) But there is. It is well known, for example, that in the presence of classical negation, the intuitionist conditional collapses into the classical one; and a paraconsistent set-theory with an unrestricted comprehension schema collapses into triviality. As Prior showed with *tonk* a long time ago, not any old set of rules for a connective succeeds in capturing a well-defined sense.

It is partly for these reasons that intuitionists and some paraconsistent logicians deny that classical negation is a meaningful connective at all. (See, e.g., Priest (1999).)

The issue of which concept of negation is the correct one cannot, therefore, be dismissed so simply.

8. *Conclusion*

The fact that I have disagreed with da Costa on a number of points above does not, of course, reflect ill on the book. The issues in question are hard, and disagreements are to be expected. It is one of the great merits of the book that it brings to the fore such important questions. There are also many other issues in the book, which there is no space to discuss here. I cannot but urge logicians to read the book and ponder its contents. I hope that an English translation will soon be available.

References

[1] Engels, F., (1954), *Dialectics of Nature*, Progress Publishers.

[2] Haack, S., (1974) *Deviant Logic*, Cambridge University Press.

[3] Priest, G., (1999), 'What not? A defence of a dialetheic account of negation', pp.101-120 of D. Gabbay and H. Wansing, *Negation*, Kluwer Academic Publishers.

[4] Priest, G., (2000), 'On alternative geometries, arithmetics and logics; a tribute to Łukasiewicz', *Proceedings of the Conference Łukasiewicz in Dublin, 1996*, ed. M. Baghramian, to appear.

Graham Priest
Department of Philosophy
University of Queensland

M. L. Dalla Chiara, K. Doets, D. Mundici, J. van Benthem (eds.), **Logic and Scientific Methods. Volume One of the tenth International Congress of Logic, Methodology, and Philosophy of Science, Florence, August 1995**, Kluwer, Synthese Library, vol. 259, Dordrecht, 1997, US$275, pp. xviii + 532, ISBN 0-7923-4383-2.

The tenth International Congress of Logic, Methodology and Philosophy of Science was held in August 1995 in Florence, Italy. Invited papers are published in two volumes of which the one reviewed here is the first and contains papers concerned with logical issues. The congress was divided in the following sections, of which starred sections are covered in Volume One: Proof Theory and Categorical Logic*; Model Theory, Set Theory and