KAREL LAMBERT. ***Free Logic: Selected Essays***. Cambridge: Cambridge University Press, 2003. Pp. xii + 191. ISBN 0-521-81816-8.

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Classical logic (at least according to its orthodox interpretation) endorses the claim that:

Every term of the language denotes an existent object. (1)

By classical logic, here, I mean first-order logic, as derived from Frege's *Begriffsschrift* and Russell's and Whitehead's *Principia Mathematica*. (1) is certainly not the standard view in the history of logic. Terms that appear to have no denotation, such as '1/0', are all too obvious. And even if a term denotes something, many held that this need not exist. Aristotle wrote: 'Even non-existents can be signified by a name' (*An. Post*. 92^b 29–30). Most later medieval logicians held that the denotation of a term need not exist; thus, for example, Paul of Venice: 'The absence of the signification of a term from reality does not prevent the term's suppositing for it' ([1978], p. 13). And Russell himself—at an earlier period—held that

> whatever may be an object of thought, or can occur in a true proposition, or can be counted as one, I call a term . . . Existence . . . is the prerogative of some only among [terms] ([1903], pp. 43 and 449).

The origins of the classical view can perhaps be traced back to Kant's discussion of existence in the *Critique of Pure Reason*. However, it was not until the work of Frege and the later Russell that the new orthodoxy emerged in logic.

Beginning in the 1950s, however, logicians came to realise that modern formal logics were by no means committed to (1), and they constructed logics in which it fails: free logics. Perhaps the first paper in this direction was Henry Leonard's [1956] 'The logic of existence'. However, undoubtedly the logician who made the most sustained and important contribution to the project was Karel Lambert. (Indeed, it was he who coined the term 'free logic' in the late 1950s.) This book is a collection of Lambert's papers on the matter.

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Two of the chapters have not appeared before. One of these, chapter 4, is a discussion of Hilbert's and Bernays's theory of definite descriptions and some of its cousins. The other, chapter 5, presents a general semantic approach to free theories of definite descriptions without extensionality ($\forall x(A(x) \equiv B(x)) \supset \iota x A(x) = \iota x B(x)$). Of the other papers, the earliest (ch. 2) appeared first in 1963; the latest (ch. 7) in 1997. These papers discuss the semantics and proof theories of various free logics, and related topics such as Russell's theory of descriptions, set abstraction, predication and extensionality, truth-value gaps, and quantum logic. The longest chapter in the book (ch. 8), 'The philosophical foundations of free logic', is a nice survey of free logics, their rivals, and some of their ramifications. Having these papers accessible in one place is greatly to be welcomed.

A little more editorial work would have improved the book. There is no consolidated bibliography, no index, and there are quite a few typos. Most are not such as to the faze the reader, but one caused me to ponder for a while; in (33), p. 28, the occurrences of 'y' should be 't'.

I think it fair to say that, for philosophical logicians at least, the main interest of the present book is historical. Generally speaking, the techniques of free logic are now well known and understood. But if this is so, it is because Lambert and his co-workers have been very successful in what they set out to do. The logical weapons they forged have become part of the standard armoury of contemporary philosophical logic.

One natural way to avoid (1) is to take the denotation function to be partial, and to invoke truth-value gaps. This strategy features in some essays in the collection. However, another way (not necessarily incompatible with the first) which also features, perhaps more frequently, partitions the domain of objects into two parts, 'inner' and 'outer'. The inner domain comprises existent objects; the outer comprises non-existent objects. Terms that have denotations may have them in either one of the domains. Quantifiers, though, are taken to range over the inner domain only. Thus, the inference of particular generalisation:

$$A(t) \vdash \exists x A(x) \tag{2}$$

will fail if t does not have a denotation in the inner domain. (It is the failure of this inference which is the most characteristic feature of free logic.)

In the context though, not quantifying over the whole domain would appear somewhat arbitrary. One way to see this is in connection with a theory of definite descriptions that Lambert favours (ch. 2). If there is a unique object that satisfies $A(x)$ and this exists, then the description $\iota x A(x)$ refers to it. Yet if there is a unique object that satisfies $A(x)$, and this does not exist, the description need not refer to it at all. In many ways it would seem much more natural to allow quantifiers to range over the whole domain. '∃' can then, of course, no longer be read as 'there exists' or even as 'there is' (existence and being coming to the same thing); but 'something' will do nicely. Definite descriptions now behave in the way that one would expect, and quantifiers that range over only the inner domain can, in effect, by defined in terms of the existentially unloaded quantifiers and a monadic existence predicate. (And of course, (2) still fails for the existentially loaded quantifiers.)

Lambert is certainly aware of this possibility but is not particularly attracted by it. He cites with approval a remark by Dana Scott: 'If we come to value the virtual entities [*i.e.*, the members of the outer domain] so highly that we want to quantify over them, then we have passed to a *new* theory with a *new* ontology' (p. 114, italics original). Now this is exactly what a free logician should, I think, dispute. One's ontology is what one takes to exist, or have being (ὀντος, being). And the objects in the outer domain do *not* have being. (Recall the remark by Paul of Venice above.) To insist, *à la* Quine, that the domain of one's quantifiers marks one's ontological commitment, is simply question-begging in this context.

The book has little to say about the philosophy of mathematics as such. But perhaps the most obvious application of free logic to the philosophy of mathematics is to make it possible to take mathematical and other abstract objects to be non-existent objects. (And of course we need to quantify over such objects in mathematics.) This view was, in fact, that of the late Richard Sylvan (Routley). (See Sylvan [2003] and Priest [2003].)

How viable such a view is, of course, another question. But without free logic, it would be impossible even to formulate, let alone investigate views of this kind. We are therefore fortunate that the work of Lambert and other free-spirited logicians reminds us that the traditional picture concerning existence was hijacked by Frege and Russell. Though the techniques of free logic may now be well known, their philosophical ramifications still leave much to ponder.

References

LEONARD, HENRY [1956]: 'The logic of existence', *Phil. Studies* **7**, 49–64.

PAUL OF VENICE [1978]: *Logica Magna*. F. Del Punta, ed. M. M. Adams, trans. Oxford: Oxford University Press.

PRIEST, GRAHAM [2003]: 'Meinong and the philosophy of mathematics', *Philosophia Mathematica* (3) **11**, 3–15.

RUSSELL, BERTRAND [1903]: *Principles of Mathematics.* Cambridge: Cambridge University Press.

SYLVAN, RICHARD [2003]: 'The importance of non-existent objects and intentionality in mathematics', *Philosophia Mathematica* (3) **11**, 20–52.