

Jaina logic: a contemporary perspective

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Jaina philosophy provides a very distinctive account of logic, based on the theory ‘sevenfold predication’. This paper provides a modern formalisation of the logic, using the techniques of many-valued and modal logic. The formalisation is applied, in turn, to some of the more problematic aspects of Jaina philosophy, especially its relativism.

1 Introduction: Indian logic

In Western philosophy, the study of formal logic started in Ancient Greece, with the work of Aristotle and the Stoic logicians. It later developed in scope and depth at the hands of many of the great Medieval logicians. The third great phase of logic, starting late in the 19th Century, and still continuing today, introduced mathematical techniques of great sophistication into the study of formal logic. The modern developments have by no means rendered obsolete earlier studies. We can now view earlier theories through the lens of modern techniques, and understand their natures and consequences in a way that would have been impossible at the time. Conversely, studying the older theories can help to remove the blinkers that a training in modern logic is wont to produce, reminding us that there are other ways of looking at things, and showing us techniques from which we can still learn.¹

The study of logic in India is just as ancient as that in the West. During the time when logic was flourishing in Ancient Greece and Medieval Europe, numerous logical theories were being developed in Hindu traditions, notably by the Nyāya, by Buddhist logicians of the stature of Dignāna and Dharmakīrti, and by Jaina logicians. True, there was no third period in Indian logic, corresponding to the mathematization of logic in the West, but

¹See, for example, Priest and Read 1977, and Priest and Routley 1982.

the mathematical techniques developed in the West can be applied just as well to traditional Indian logic, with the same fruitful outcomes. This, at least, I will try to show in this paper.

We will look at just one of the Indian traditions: that of the Jains. The Jains had a very distinctive approach to logic, countenancing seven semantic values. The number seven sounds a most strange one. Why seven? There is, as we shall see, a perfectly good answer (essentially, that $7 = 2^3 - 1$). We shall also see how the Jaina ideas can be made perfectly rigorous with the techniques of modern logic, and how these techniques throw certain aspects and problems of Jaina logic into relief.

I should say, right at the start, that I make no claim to be a scholar of Indian philosophy; and the Sanskrit texts that I can read and from which I will quote have to be refracted through translation into English. I hope, however, that the present project can be accomplished without straying too far beyond the bounds of my limitations.

2 Anekānta-vāda

Logic is not metaphysically neutral. Any system of logic has various presuppositions of a metaphysical nature built into it. Jaina logic is no exception; and if we wish to understand it, we will have to start with the core of Jaina metaphysics, and in particular the theory of *anekānta-vāda*, or the doctrine of non-onesidedness, as it is sometimes translated (*ekānta*=one-sided). The Jains believed that truth was not the prerogative of any one school. The views of Buddhists and Hindus, for example, may disagree about crucial matters, such as the existence of an individual soul; each has, nonetheless, an element of truth in it. This can be so because reality itself is multi-faceted. Thus, the doctrine of *anekānta-vāda* is sometimes glossed as the doctrine of ‘the many-sided nature of reality’.² Reality is a complex, with a multitude of aspects; and each of the competing theories provides a perspective, or standpoint (*naya*), which latches on to one such aspect. As Siddhasena puts it in the *Nyāyāvātāra* (v. 29):³

Since a thing has manifold character, it is comprehended (only) by the omniscient. But a thing becomes the subject matter of a *naya*, when it is conceived from one particular standpoint.

On its own, each standpoint is right enough, but incomplete. To grasp the complete picture, if indeed this is possible, one needs to have all the

²Matilal 1981, pp. 1, 25. See, also, Ganeri 2001, 5.2.

³Quoted by Matilal 1981, p. 41.

perspectives together—like seeing a cube from all six sides at once.⁴

It follows that any statement to the effect that reality is thus and such, if taken categorically, will be, if not false, then certainly misleading. Better to express the view with an explicit reminder that it is correct from a certain perspective.⁵ This was the function with which Jaina logicians employed the word of ‘*syāt*’. Literally, this means ‘may it be’, and colloquially it is used to mean something like ‘perhaps’, ‘maybe’, or ‘arguably’; but in the technical sense in which the Jaina logicians used it, it may be best thought of as something like ‘In a certain way...’ or ‘From a certain perspective...’.⁶ So instead of saying ‘An individual soul exists’, it is better to say ‘*Syāt* an individual soul exists’. This is the Jain method of *syād-vāda*. The Jaina view about assertion of course raises a number of philosophical questions. We will return to some of them in due course.

3 The theory of sevenfold predication

We are now, at least, in a position to look at the Jaina theory of seven-fold division (*saptabhaṅgī*). A sentence may have one of seven truth values; or, as it is often put, there are seven predicates that may describe its semantic status. The matter is explained by the 12th century theorist, Vādideva Sūri, as follows:⁷

The seven predicate theory consists in the use of seven claims about sentences, each preceded by ‘arguably’ or ‘conditionally’ (*syāt*) [all] concerning a single object and its particular properties, composed of assertions and denials, either simultaneously or successively, and without contradiction. They are as follows:

- (1) Arguably, it (i.e., some object) exists (*syād esty eva*). The first predicate pertains to an assertion.
- (2) Arguably, it does not exist (*syād nāsty eva*). The second predicate pertains to a denial.

⁴See, Ganeri 2001, 5.4. Sometimes, different perspectives are described as being obtained by interpreting a single sentence in various ways. (See Ganeri 2001, p. 133, and Matilal 1981, p. 60.) In this case, the facets of reality are accessed by semantic disambiguation.

⁵Matilal 1981, p. 2.

⁶Matilal 1981, p. 52, Ganeri 2001, 5.5, and 2002, sect. 1.

⁷*Pramāṇa-naya-tattvālokāṃkāra*, ch. 4, vv. 15-21. Translation from Battacharya 1967.

- (3) Arguably, it exists; arguably it does not exist (*syād esty eva syād nāsty eva*). The third predicate pertains to successive assertion and denial.
- (4) Arguably, it is non-assertable (*syād avaktavyam eva*). The fourth predicate pertains to a simultaneous assertion and denial.
- (5) Arguably, it does not exist; arguably it is non-assertable (*syād esty eva syād avaktavyam eva*). The fifth predicate pertains to an assertion and a simultaneous assertion and denial.
- (6) Arguably, it exists; arguably it is non-assertable (*syād nāsty eva syād avaktavyam eva*). The sixth predicate pertains to an assertion and a simultaneous assertion and denial.
- (7) Arguably, it exists; arguably it doesn't exist; arguably it is non-assertable (*syād esty eva syād nāsty eva syād avaktavyam eva*). The seventh predicate pertains to a successive assertion and denial and a simultaneous assertion and denial.

A perusal of the seven possibilities indicates that there are three basic ones, (1), (2), and (4), and that the others are compounded from these. (1) says that the statement in question (that something exists) holds from a certain perspective. (2) says that from a certain perspective, it does not. (4) says that from a certain perspective, it has another status, non-assertable. Exactly what this is is less than clear, but let us return to that matter later.

We may start to apply modern logical techniques at this point. We may think of reality as constituted by a non-empty set of facets, \mathcal{F} . For each $\varphi \in \mathcal{F}$, an assertion has one of three statuses at φ , which we may write as t (true), f (false), and i (non-assertable). Thus, we may suppose that for any facet, φ , there is a map, v_φ , such that for any sentence, A , $v_\varphi(A) \in \{t, i, f\}$.

In understanding the other possibilities we hit a *prima facie* problem. Take (3). This says that from some facet the sentence is t , and from some facet it is f . That's intelligible enough, but unfortunately, it would seem to entail both (1) and (2). If it's true at some facet and false at some facet, it's certainly true at some facet.⁸

The solution is not difficult to find. We have to understand (1) as saying not just that the sentence is true at some facet, but as denying the other two basic possibilities: it is t in some facet, and there are no facets where it is f or i . (3) is now to the effect that there is a facet from which the

⁸Ganeri 2002, sect. 1, seems to miss this. However, he goes on to suggest essentially the idea that I describe in the next paragraph.

sentence is t , a facet from which it is f , and no facet from which it is i . In fact, all the seven cases now fall into place. Each corresponds to a non-empty subset, X , of $\{t, i, f\}$. If $x \in X$, there is some facet, φ , such the sentence has the value x at that facet; if not, then not. The empty set, ϕ , is ruled out, since there must be at least one facet, and so X cannot be empty. If we write $\wp X$ for the powerset (set of all subsets) of X , then the cardinality of $\wp\{t, i, f\} = 2^3 = 8$. Hence, there are $2^3 - 1 = 7$ possibilities. Given \mathcal{F} , the status of any formula is captured by a function, $V_{\mathcal{F}}$, such that for any formula, A , $V_{\mathcal{F}}(A)$ takes one of the seven members of $\wp\{t, i, f\} - \phi$ as a value.

4 Connectives

So far so good. We have made sense of the theory sevenfold predication. The next question which any modern logician will ask themselves is how the semantic values of sentences relate to the semantic values of sentences compounded from them. Let us suppose that we have a structure, \mathcal{F} , such that for every $\varphi \in \mathcal{F}$, and every propositional parameter, p , the semantic values of p (with respect to each $\varphi \in \mathcal{F}$, and so \mathcal{F} itself) are assigned as in the previous section. Suppose also that we have a simple propositional language that allows us to form sentences by means of the standard logical operators of negation, conjunction, and disjunction, \neg, \wedge, \vee . What are the semantic values of such compound sentences? Such a question is not one that Jaina logicians thought to ask themselves, as far as I know. So we are on our own here. There are probably several possible answers, but let us note the two most obvious.

The first, which I will call *Type 1 semantics*, is to take the behaviour of connectives at facets to be determined by the truth tables of some standard three valued logic. Perhaps the most natural are those of the strong Kleene 3-valued logic K_3 , or the paraconsistent logic LP .⁹ In the first, i is thought of as *neither true nor false*; in the second, it is thought of as *both true and false*. The tables, however, are the same in each case, and are as follows:

\neg	
t	f
i	i
f	t

\wedge	t	i	f
t	t	i	f
i	i	i	f
f	f	f	f

\vee	t	i	f
t	t	t	t
i	t	i	i
f	t	i	f

On this account, v is truth functional, in the sense that for any φ , the semantic value of $v_{\varphi}(\neg A)$ is determined by $v_{\varphi}(A)$, and the values of

⁹See Priest 2001, ch. 7.

$v_\varphi(A \wedge B)$ and $v_\varphi(A \vee B)$ are determined by $v_\varphi(A)$ and $v_\varphi(B)$. V , as defined in the last section, is *not* truth functional however. Let \mathcal{F}_0 be the set of facets that we may represent diagrammatically as follows:

φ_1		φ_2	
$p : t$		$p : f$	
$q : t$		$q : f$	
***	***	***	***
$\neg p : f$		$\neg p : t$	
$p \wedge \neg p : f$		$p \wedge \neg p : f$	
$p \wedge q : t$		$p \wedge q : f$	

The values of $\neg p$, $p \wedge \neg p$ and $p \wedge q$ that are generated at each φ are shown below the asterisks. We have $V_{\mathcal{F}}(p) = V_{\mathcal{F}}(\neg p) = V_{\mathcal{F}}(q) = \{t, f\}$; but $V_{\mathcal{F}}(p \wedge \neg p) = \{f\}$, whilst $V_{\mathcal{F}}(p \wedge q) = \{t, f\}$. Type 1 semantics make Jaina logic not so much a 7-valued logic as a 3-valued modal logic.

A second possibility, which I will call *Type 2 semantics*, treats it as a genuine 7-valued logic. It defines $V_{\mathcal{F}}$ for compound sentences differently: directly in terms of the semantic values of their parts. Perhaps the most natural way to do this is pointwise.¹⁰ Thus:

$$V_{\mathcal{F}}(\neg A) = \{\neg x : x \in V_{\mathcal{F}}(A)\}$$

$$V_{\mathcal{F}}(A \wedge B) = \{x \wedge y : x \in V_{\mathcal{F}}(A), y \in V_{\mathcal{F}}(B)\}$$

$$V_{\mathcal{F}}(A \vee B) = \{x \vee y : x \in V_{\mathcal{F}}(A), y \in V_{\mathcal{F}}(B)\}$$

(I use \neg , \wedge , and \vee here, in an obvious way, as the operations on truth values, as well as connectives.) Defined in this way, V obviously is truth functional. Here are the truth tables. I omit set braces. Thus, $\{t, i\}$ is written simply as ti , etc.

\neg		\wedge	t	ti	tf	tif	i	if	f
t	f	t	t	ti	tf	tif	i	if	f
ti	if	ti	ti	ti	tif	tif	i	if	f
tf	tf	tf	tf	tif	tf	tif	if	if	f
tif	tif	tif	tif	tif	tif	tif	if	if	f
i	i	i	i	i	if	if	i	if	f
if	ti	if	if	if	if	if	if	if	f
f	t	f	f	f	f	f	f	f	f

¹⁰As in Priest 1984. The possibility of doing something of this kind is noted in Ganeri 2002, sect. 3—though it is not clear from his discussion that this gives a different result from Type 1 semantics. These truth conditions are not the only ones possible. In 200+ Shramko and Wansing give a different set, derived from a natural ordering on the seven values.

\vee	t	ti	tf	tif	i	if	f
t	t	t	t	t	t	t	t
ti	t	ti	ti	ti	ti	ti	ti
tf	t	ti	tf	tif	ti	tif	tf
tif	t	ti	tif	tif	ti	tif	tif
i	t	ti	ti	ti	i	i	i
if	t	ti	tif	tif	i	if	if
f	t	ti	tf	tif	i	if	f

In \mathcal{F}_0 , $V_{\mathcal{F}_0}(p) = V_{\mathcal{F}_0}(\neg p) = V_{\mathcal{F}_0}(q) = \{t, f\}$, and $V_{\mathcal{F}_0}(p \wedge \neg p) = V_{\mathcal{F}_0}(p \wedge q) = \{t, f\}$.

Which of these two possibilities the Jains would themselves have preferred had they thought about matters, I don't know. A major difference between the two possibilities is that, on the first, we will always have $t \notin V_{\mathcal{F}}(p \wedge \neg p)$, but on the second, it is quite possible to have $t \in V_{\mathcal{F}}(p \wedge \neg p)$. (On both, one can have $t \in V_{\mathcal{F}}(p)$ and $t \in V_{\mathcal{F}}(\neg p)$.) So a crucial issue concerns how one regards conjoined contradictions. This, however, takes us to a more pressing matter.

5 The meaning of i

The next topic on the agenda is validity. What counts as a good argument? This is certainly a topic that exercised Jaina and other Indian logicians. Generally speaking they seem to have endorsed an account of validity in terms of the preservation of, as we would now put it in the context of modern many-valued logics, designated values. (See *Ganeri 2001*, 5.7.) That, at any rate, is the natural path to go down, given the preceding machinery.

What, then, should we take to be the designated values, that is, the values that licence assertion? Start with the three truth values, t , i , and f . t , being *true*, is clearly designated; f , being *false*, is not. What of i ? We now have to face the question of its intended meaning. A natural possibility is that i means *both true and false*. That is essentially how Vādideva Sūri glosses case (4) in the quotation in Section 3. In this case, something that has the value i is true (if false as well), and so is the sort of thing that should be asserted. i should therefore be designated. Unfortunately, Vādideva Sūri also glosses i as *unassertable*. The thought here might appear to be that a simultaneous assertion and denial cancel each other out.¹¹ So the status of i is more like *neither true nor false*. In any case, i should not be designated.¹²

¹¹See Matilal 1981, p. 60. On the cancellation view of negation, see Priest 2006, 1.13.

¹²Designating values that are (at least) true is the most natural policy. There are, of

Which is the most plausible interpretation of *i* in Jain logic, all things considered, is a moot point. *Stcherbatsky 1962*, p. 415, *Bharucha and Kamat 1984*, and *Sarkar 1992* argue that *i* is most plausibly interpreted as *both true and false*. *Ganeri 2002*, sect. 1, and *2001*, 5.6, favours *neither true nor false*. For what it is worth, I do not find his arguments against the *both* position persuasive. In the first place cited, he argues that *both* is implausible, since Jaina logicians, like most Indian logicians, accepted the validity of arguments by *reductio ad absurdum* ('a universally acknowledged way to undermine one's philosophical opponent was to show that their theory contradicted itself'). Now this is not altogether true. A standard view in early Buddhist logic is the *catuskoti*. According to this, statements may be true, false, both, or neither. So *both* is one of the standard possibilities here. And we certainly find philosophers of the status of Nāgārjuna endorsing certain contradictions. Thus:¹³

Everything is real and is not real,
Both real and not real,
Neither real nor not real.
This is Lord Buddha's teaching.

And even if the Jains did not, themselves, accept the possibility of something being both true and false, they must have been familiar with perspectives, such as this, that did.¹⁴ The fact that some contradictions may be true does not, of course, mean that all are. Hence, a philosophical position can be undermined if it is shown to lead to a contradiction of an unacceptable kind. *Reductio* arguments can still, therefore, work.¹⁵

In the second place cited, Ganeri argues that *i* cannot be interpreted as *both*, since the values are claimed to arise 'without contradiction'. (See the first sentence in the quotation from Vādideva Sūri in the quotation above.) This, also, is less than persuasive. What would seem to be meant by two

course, others. For various purposes, one might prefer to designate truth-only, or values that are not at all false. But technically, there are just two possibilities. Either *i* is designated or it isn't.

¹³*Mūlamadhyamakakārika* XVIII: 8. Translation from Garfield 1995.

¹⁴Indeed, Sarkar 1992, pp. 20-21 points out that in defence of their view, some Jains argued, as a *tu quoque*, that their opponents' views were inconsistent. It would seem, then, that the Jains should have allowed for at least some perspectives to be 4-valued. This would replace the logics *LP* and *K₃* with their four-valued generalisation, *FDE*, and the global 7-valued picture would give way to a $(2^4 - 1 =)$ 15-valued picture. Such a possibility is considered in Sylvan 1987.

¹⁵Further, see Deguchi, Garfield, and Priest 200+.

things being contradictory here is that they cannot obtain together.¹⁶ If i is *both true and false*, then A and $\neg A$ are precisely not contradictories in *this* sense.

At any rate, how i is best interpreted, I leave to scholars to argue about. It may well be that different Jains conceptualised i in different ways, or were even just plain confused about the matter. Hence, in what follows, we will consider both possibilities.

6 Type 1 validity

To define validity in Type 1 semantics, we need to know what it is to hold in a structure, \mathcal{F} . Call the set of sentences that are designated in the facet φ the φ *perspective*. Since every φ perspective of \mathcal{F} is an equally legitimate take on reality, the most natural thought here is that a sentence, A , holds in \mathcal{F} iff, for some $\varphi \in \mathcal{F}$, A is in the φ perspective. Write this as $\mathcal{F} \Vdash A$. A valid inference may now be defined in the standard way: where Σ is a set of premises, $\Sigma \vDash A$ iff for all \mathcal{F} , if $\mathcal{F} \Vdash B$ for every $B \in \Sigma$, $\mathcal{F} \Vdash A$. (If Σ is finite, I will usually omit the set braces from around its members on the left of ‘ \Vdash ’.)¹⁷

This version of Jaina logic turns out to be a modification of Jaśkowski’s discussive logic.¹⁸ In this, \mathcal{F} is thought of as a set of worlds of a normal modal logic, and then validity is defined exactly as I have defined it. The

¹⁶See Ganeri’s own discussion (5.3) of the cloth—shot silk, I take it—which is both blue and not blue.

¹⁷A policy which is also egalitarian with respect to the facets is to say that A holds in \mathcal{F} iff A holds in *every* facet of \mathcal{F} , and then proceed as before. This is perhaps a less plausible approach for the Jains, since they would end up not being able to say anything much about reality as a whole. Nonetheless, in this approach, the notion of validity collapses into that of the underlying 3-valued logic of the individual facets, K_3 or LP . Clearly, if an inference preserves designation at every facet, then it will preserve what holds in \mathcal{F} , in this sense. Conversely, if an inference is invalid in this sense, there is a structure, \mathcal{F} , where the premises hold in all facets, and the conclusion fails in at least one, φ . φ is a counter-model to the 3-valued inference. Yet another policy is to define validity as one does standardly in modal logic:

$$\Sigma \vDash A \text{ iff for every } \mathcal{F}, \text{ and every } \varphi \in \mathcal{F}, \text{ if every member of } \Sigma \text{ is designated at } \varphi, \text{ so is } A$$

This definition also (and obviously) delivers the same notion of validity as the underlying 3-valued logic. Such a notion of validity is clearly right if one is reasoning about an individual facet of reality. It would not seem to be right if one is reasoning about *reality as a whole*.

¹⁸As observed by Priest and Routley 1989a, p. 17. See also Ganeri 2002, sect. 3. For a general discussion of discussive logic, see Priest 2002, 4.2 and 5.2.

only difference is that in Jaśkowski’s logic, the underlying logic of each world is classical logic, not a three-valued logic. This, of course, makes a difference to what inferences are valid.

In Jaina logic, thus construed, a single-premise inference is valid iff it is valid in the underlying three valued logic, K_3 or LP , depending on whether or not i is designated. For if the inference is valid in K_3/LP , every facet at which the premise is designated, so is the conclusion. So if, in an interpretation, the premise is designated in some facet, so is the conclusion. Conversely, if the inference is invalid, there is some K_3/LP interpretation where the premise is designated and the conclusion is not. Let \mathcal{F} be the interpretation whose only facet is that interpretation. Then the premise holds in \mathcal{F} , but not the conclusion. Thus, if i is designated, $p \vDash q \vee \neg q$ and $p \wedge \neg p \not\equiv q$ (so the logic is paraconsistent for conjoined contradictions); if i is not designated, the reverse is the case. Both of these inferences, note, hold in Jaśkowski’s logic.

Perhaps the most distinctive feature of Jaśkowski-style logics is the failure of the inference of Adjunction, from A and B to $A \wedge B$. Take \mathcal{F} to contain just two facets, φ_1 and φ_2 , where p is true at φ_1 , but false at φ_2 , and vice versa for q . Then whether or not the underlying logic is K_3 or LP —or classical— $\mathcal{F} \Vdash p$, $\mathcal{F} \Vdash q$, but $\mathcal{F} \not\equiv p \wedge q$. In fact, provided that we stick to the vocabulary at our disposal so far, there are no essentially valid multi-premise inferences. In other worlds, if $\Sigma \vDash A$, then for some $B \in \Sigma$, $B \vDash A$. For suppose that for every $B \in \Sigma$, $B \not\equiv A$. Then, for every $B \in \Sigma$, there is some \mathcal{F} , and some, $\varphi_B \in \mathcal{F}$ such that B is designated in φ_B and A is not. Let $\mathcal{G} = \{\varphi_B : B \in \Sigma\}$. Then for every $B \in \Sigma$, B holds in \mathcal{G} , but A does not hold in \mathcal{G} .¹⁹ (In particular, then $p, \neg p \not\equiv q$, so the logics are all paraconsistent for unconjoined contradictions.) On this account, the facets are all robustly independent.

7 Type 2 validity

Matters in Type 2 semantics are somewhat different. The logic is a standard many-valued logic, and so we have to decide what is to count as a designated value in this context—let us say designated₇. Perhaps the natural approach here is to say that X is *designated*₇ iff for some $x \in X$, x is designated. So, e.g. $\{t, i\}$ is always designated₇, $\{i, f\}$ is designated₇ iff i is designated, and $\{f\}$ is never designated₇. An inference is valid on this conception just if for

¹⁹For the behaviour of Jaśkowski’s logic in these regards, see Priest and Routley 1989b, pp. 160 ff.

all \mathcal{F} , whenever all the premises are designated₇ in \mathcal{F} , so is the conclusion. We will write this notion of validity as \models_7 .

Suppose, to start with, that i is designated. Then \models_7 is exactly *LP*. As an inspection suffices to determine, the matrices for *LP* are sub-matrices of the 7-valued matrices of section 4. Hence, any inference that is invalid in *LP* is invalid for \models_7 ; by contraposition, any inference valid in \models_7 is valid in *LP*. To prove the converse, it suffices to take a natural deduction system for *LP* and show that all the rules are designation₇-preserving. A suitable rule system is given in *Priest 2002*, p. 309. Checking that the rules preserve designation is somewhat laborious, but straightforward. I leave it as an exercise.²⁰

In the second case, where i is not designated, any inference valid for \models_7 is valid for K_3 . The argument is exactly the same as for *LP*, when i is designated. The converse is not true in this case, however. The inference from $p \wedge \neg p$ to q is valid in K_3 . It is not valid for \models_7 , as we have already, in effect, noted. (Let $V_{\mathcal{F}}(p) = tf$, and $V_{\mathcal{F}}(q) = f$.) If one takes a natural deduction system for the logic of First Degree Entailment, *FDE* (*Priest 2002*, p. 309), it is straightforward to check that every inference valid in *FDE* is valid for \models_7 . Hence, \models_7 is at least as strong as *FDE*. It is also a relevant logic, in the sense that if $A \models_7 B$ then A and B share a propositional parameter. (If they do not, assign every parameter in A the value tf , and every parameter in B the value i . It is not difficult to show that A has the value tf , and B has the value i .) But there are no relevant logics with just this vocabulary stronger than *FDE*.²¹ Hence the logic is exactly *FDE*.

It is worth noting, finally, that Adjunction does not *have to* fail with validity of Type 1. Call \mathcal{F} *adjunctively closed* if it satisfies the following condition:

for any pair of facets, φ_1 and φ_2 , if A is in the φ_1 perspective, and B is in the φ_2 perspective, there is a facet, φ , such that $A \wedge B$ is in the φ perspective.

If i is designated then it is possible to construct interpretations that are adjunctively closed. Let \mathcal{F} be any structure. Let \mathcal{F}' be the same except that it has an additional facet, φ' , such that for every atomic sentence, p :

p is in the φ' perspective iff there is a φ in \mathcal{F} such that p is in the φ perspective

²⁰The equivalence is, in fact, a special case of the more general result proved in Priest 1984, where truth values are produced by iterating the powerset construction.

²¹This is not a standard result. Its proof can be found in Humberstone 200b.

$\neg p$ is in the φ' perspective iff there is a φ in \mathcal{F} such that $\neg p$ is in the φ perspective

We can now show by a joint induction, that:

if there is a φ in \mathcal{F} such that A is in the φ perspective A is in the φ' perspective

if there is a φ in \mathcal{F} such that $\neg A$ is in the φ perspective $\neg A$ is in the φ' perspective

It follows that:

(*) if $\mathcal{F}' \Vdash A$ then A is in the φ' perspective.

Adjunctive closure follows swiftly.

Moreover, if \mathcal{F} is adjunctively closed, an argument by a joint induction over the formation of A shows that, where $V_{\mathcal{F}}$ is defined as in Type 2 semantics:

$V_{\mathcal{F}}(A)$ is designated₇ iff $\mathcal{F} \Vdash A$

$V_{\mathcal{F}}(\neg A)$ is designated₇ iff $\mathcal{F} \Vdash \neg A$

(whether or not i is designated). Hence, if structures restricted to those that are adjunctively closed, Type 1 and Type 2 validity therefore coincide.

8 The logic of *syāt*

Of course, there is more to language than conjunction, disjunction, and negation.²² An operator that is obviously important to the Jains is the *syāt* operator. Let us write this as \mathcal{S} , and add it to our formal language, so that if A is any formula, so is $\mathcal{S}A$. What are the semantics of \mathcal{S} ?

I know of no way of providing a very plausible Type 2 semantics. It is natural to think of \mathcal{S} as some kind of possibility operator.²³ (Holding in some facet is rather like holding in some possible world.) The founder of

²²Another important connective is the conditional. A material conditional can be defined in LP and K_3 in terms of \neg and \vee , in the usual way. The conditional is a very weak one in both cases. The logics can be extended to ones with different 3-valued conditionals, such as RM_3 and L_3 . (See Priest 2001, ch. 7.) Most of the considerations of previous sections carry over to such extensions in a straightforward fashion.

²³As Ganeri 2001, p. 141, notes.

modern many-valued logic, Łukasiewicz, suggested the following 3-valued truth table for a possibility operator:²⁴

\mathcal{S}	
t	t
i	t
f	f

We can extend this to the seven-valued semantics pointwise: $V_{\mathcal{F}}(\mathcal{S}A) = \{\mathcal{S}x : x \in V_{\mathcal{F}}(A)\}$. This gives the following truth table:

\mathcal{S}	
t	t
ti	t
tf	tf
tif	tf
i	t
if	tf
f	f

But such semantics have most implausible consequences. It is not difficult to check that whether or not i is designated, $\mathcal{S}A, \mathcal{S}B \vDash_7 \mathcal{S}(A \wedge B)$. This seems wrong: the fact that A and B each holds in a perspective would not seem to guarantee, without further consideration, a perspective in which both hold. And even when i is not designated, $\vDash_7 \mathcal{S}A \vee \mathcal{S}\neg A$. In truth, possibility-style operators are just not truth functional. In virtue of this, I will discuss Type 2 semantics no further.

Type 1 semantics provide for a very natural account of the semantics of \mathcal{S} .²⁵ Taking our cue from the semantics of normal modal logics, we suppose that \mathcal{F} comes furnished with a binary accessibility relation, R . $\varphi_1 R \varphi_2$ is to be understood as meaning that the facet φ_1 recognises the facet φ_2 , in the sense that it is one of the perspectives that it takes into account. Thus, Indian Buddhist philosophers were very well aware of Hindu perspectives on various matters, and explicitly took them into account (and vice versa). But for neither of them was the view of Aristotle or Confucius, for example, on the agenda. Given R , the natural truth conditions for \mathcal{S} , whether or not one takes i to be designated, are as follows:²⁶

²⁴Priest 2001, 7.10.6.

²⁵As noted by Ganeri 2002, sect. 4.

²⁶See Priest 2008, ch. 11a. Interestingly, Hautamäki 1983 has a 2-valued modal logic of perspectives in which there are both worlds and perspectives. The indices of evaluation are pairs, $\langle w, \pi \rangle$ where w is a world, and π is a perspective. In my notation, $\langle w, \pi \rangle \Vdash \mathcal{S}A$ iff for some π' such that $\pi R \pi'$, $\langle w, \pi' \rangle \Vdash A$. See also Hautamäki 1986.

$v_\varphi(\mathcal{S}A) = t$ iff for some $\varphi' \in \mathcal{F}$, such that $\varphi R\varphi'$, $v_{\varphi'}(A) = t$

$v_\varphi(\mathcal{S}A) = f$ iff for all $\varphi' \in \mathcal{F}$, such that $\varphi R\varphi'$, $v_{\varphi'}(A) = f$

$v_\varphi(\mathcal{S}A) = i$ in all other cases

These truth conditions suffice to decide various principles of inference concerning \mathcal{S} . For example, it is not difficult to check that $\mathcal{S}(A \wedge B) \vDash \mathcal{S}A$ and $\mathcal{S}A, \mathcal{S}B \not\vDash \mathcal{S}(A \wedge B)$ —in both cases, whether or not i is designated.²⁷

What other inferences concerning \mathcal{S} are valid depends, of course, on the properties of R . One extreme possibility is to take R to be universal (as in the modal system $S5$): for every $\varphi, \varphi' \in \mathcal{F}$, $\varphi R\varphi'$. This would ensure that $\mathcal{S}A$ has the same truth value at every facet. This certainly seems too strong. Why should every perspective agree on what may hold in other perspectives? Even relatively weak constraints seem problematic. Thus, consider the reflexivity constraint: for all $\varphi \in \mathcal{F}$, $\varphi R\varphi$ (as in the modal system $K\rho$). Why should a perspective even recognise itself? Some people, after all, are self-blind. There may be reasons to suppose that some other constraints are appropriate. I will return to this matter later.

9 What should a Jain assert?

We now come to a couple of sensitive issues, both connected with the issue of what someone who accepts the Jaina picture of reality should be prepared to endorse.

For a start, as we noted in Section 2, according to the Jaina view, to assert anything, A , categorically is, if not false, then at least very misleading. Better to assert $\mathcal{S}A$. But one should not assert this categorically either, for exactly the same reason. Better to assert $\mathcal{S}\mathcal{S}A$. One should not assert this

²⁷If i is designated, the logic certainly has some odd validities. Let \mathcal{A} be the operator dual to \mathcal{S} , so $\mathcal{A}B$ is $\neg\mathcal{S}\neg B$. It is not difficult to check that:

$v_\varphi(\mathcal{A}B) = t$ iff for all $\varphi' \in \mathcal{F}$, such that $\varphi R\varphi'$, $v_{\varphi'}(B) = t$

$v_\varphi(\mathcal{A}B) = f$ iff for some $\varphi' \in \mathcal{F}$, such that $\varphi R\varphi'$, $v_{\varphi'}(B) = f$

$v_\varphi(\mathcal{A}B) = i$ in all other cases

Now provided that i is designated, $\mathcal{S}q \vDash \neg\mathcal{A}p \vee p$. For suppose that the premise is designated at some world, w . Then there must be a world such that wRw' . Now suppose that the conclusion is not designated. Then $\neg\mathcal{A}p$ takes the value f at w , and $\mathcal{A}p$ takes the value t . But then p takes the value t at w' , as, then, does $\neg\mathcal{A}p \vee p$. The inference is not valid if i is not designated, as a simple counter-model shows. (This observation is based on Hughes 1990. See, further, Humberstone 200a.)

categorically either, for exactly the same reason. Better to assert $\mathcal{S}\mathcal{S}\mathcal{S}A\dots$. It would appear that the Jains are caught in a vicious regress: they can say nothing.

Is there any way out of this problem? First, note that:

$$\mathcal{S}A \vDash A$$

For if $\mathcal{S}A$ holds in \mathcal{F} , then for some $\varphi \in \mathcal{F}$, $v_\varphi(\mathcal{S}A) = t$ (or i if this is designated). But then for some $\varphi' \in \mathcal{F}$, $v_{\varphi'}(A) = t$ (or i if this is designated). Hence A holds in \mathcal{F} . If R is an arbitrary relation, the converse does not hold. (Details are left as an exercise.) But consider the following condition on R :²⁸

B for all φ , there is a φ' , such that $\varphi'R\varphi$

This is not an implausible condition.²⁹ Whatever the perspective, there is another that at least countenances it—if not itself, then some other. Given this constraint, the converse inference holds. For suppose that A holds in \mathcal{F} , then for some $\varphi \in \mathcal{F}$, $v_\varphi(A) = t$ (or i if this is designated). But then for some $\varphi' \in \mathcal{F}$, $v_{\varphi'}(\mathcal{S}A) = t$ (or i if this is designated). Hence $\mathcal{S}A$ holds in \mathcal{F} . So for any A , A is logically equivalent to $\mathcal{S}A$; $\mathcal{S}A$ is logically equivalent to $\mathcal{S}\mathcal{S}A$; and so on. All of A , $\mathcal{S}A$, $\mathcal{S}\mathcal{S}A$, $\mathcal{S}\mathcal{S}\mathcal{S}A, \dots$ are logically equivalent. To assert any one is equivalent to asserting the others. To assert categorically is to assert in a way qualified by *syāt*.

One might point out that to assert in a qualified way is, equally, to assert in a categorical way; so the problem is still with us. However, given the construction, the very distinction between a categorical assertion and a qualified assertion collapses, leaving no space in which the objection can be inserted: the distinction on which the objection depends no longer exists.³⁰

²⁸It is not difficult to show that imposing this constraint on the modal logic K has no effect on the valid inferences.

²⁹The presence of a possibility operator in discussive logic can result in the presence of genuine multi-premise inferences. Thus, in Jaśkowski's modal logic, the inference $p, \diamond p \supset q \vdash q$ is valid. (See Priest 2002, 5.2.) However, the argument of Section 6 extends in a simple way to show that the \mathcal{S} operator, with only condition **B**, generates no essentially multi-premise inferences. We merely pool all the worlds of the counter-examples. To ensure that condition **B** is satisfied, we throw in an extra world, which accesses all worlds.

³⁰Alternatively, another plausible condition on R is that it be transitive. If a certain perspective is visible, so is anything of which this perspective takes account. But, as usual, the condition verifies the inference $\mathcal{S}A \vDash \mathcal{S}\mathcal{S}A$; and hence $\mathcal{S}A$ is logically equivalent to $\mathcal{S}\dots\mathcal{S}A$, with as many \mathcal{S} s as one likes. The regress would then appear to be no more vicious than the regress: A is true, ' A is true' is true, etc.

The second issue is trickier. Reality is a particular set of facets, \mathcal{F}_0 . If assertion aims at truth, then the things that a person should endorse (given appropriate evidence, etc.) are exactly the things that hold in \mathcal{F}_0 . That is, A should be endorsed just if $\mathcal{F}_0 \Vdash A$. On the other hand, a Jain should endorse, almost by definition, those things that are correct from a Jaina perspective. Such a perspective is just one particular member of \mathcal{F}_0 , φ_0 . There are, presumably, others—Buddhists perspectives, Hindu perspectives, and so on.

We meet here a tension inherent in any form of relativism. A Jain is committed, presumably, to the view that Jainism is a more accurate perspective of how things are than are others. If not, why be a Jain rather than a Buddhist or a Hindu? On the other hand, Jains hold that reality is multi-faceted, and no one view completely captures how things are: each captures one of the facets. What holds in \mathcal{F}_0 is, after all, what holds in *any* $\varphi \in \mathcal{F}_0$. This puts Jains in a somewhat awkward position when they argue with a Buddhist, Hindu, etc. If they disagree with such an opponent, they must hold that they are right in a way that the opponent is not; but also that the opponent is just as right as they are. Such a tension would seem to be resolvable in one of only two ways: either with the insistence that all views are not, after all, equal, that the Jaina view is privileged in some way, or in a thoroughgoing relativism.

How Jaina logicians actually did address these matters, I leave for people more knowledgeable about these things than myself to hammer out. The present machinery suggests a way of going between the horns of the dilemma, however. We have said, so far, very little about the individual members of any \mathcal{F} . All these are things which assign one of three truth values to atomic sentences, and which may relate to each other via the accessibility relation, R . It is quite compatible with all this that there is one of them, φ , that exactly reflects \mathcal{F} . And when the \mathcal{F} in question is reality itself, \mathcal{F}_0 , the φ in question, φ_0 , gives the Jaina perspective—or so the Jaina might hope. In this way, the Jaina perspective can be both one amongst many, and the ultimately correct one.

Is it possible to construct such a structure? It is. Suppose that we have an interpretation, \mathcal{F} , and a $\varphi \in \mathcal{F}$ such that for an infinite number of propositional parameters, p , p is t at φ , and for an infinite number of parameters, p , p is f at φ . (Maybe those with an odd numbered index are true, and those with an even numbered index are false.) Enumerate the formulas of the whole language, and map them onto propositional parameters, by running through the enumeration. If $\mathcal{F} \Vdash A$, map A to the first parameter not so far used which is t in φ ; if $\mathcal{F} \not\Vdash A$, map A to the first parameter not so far used which is f in φ . Let us write the parameter to which A is

mapped as p_A . Then by construction:

$\mathcal{F} \Vdash A$ iff p_A is true in φ

We might take p_A too express the proposition that A holds in \mathcal{F} . φ shows how one can have one's Jaina cake and eat it too.

Actually, if i is designated one can do something even stronger—construct a structure, \mathcal{F}' , that contains a facet, φ' , such that for every A :

$\mathcal{F}' \Vdash A$ iff A is in the φ' perspective

Take the structure \mathcal{F}' of Section 7. (*) in that section gives us the left to right direction of this. The right to left direction is trivial.

Let me end with one other objection to Jaina logic. It is clear that Jaina logic countenances contradictions in some sense. We can have structures, \mathcal{F} , such that $\mathcal{F} \Vdash A$ and $\mathcal{F} \Vdash \neg A$, for some A ; and if i is designated, then we can have a $\varphi \in \mathcal{F}$ such that $A \wedge \neg A$ is designated at φ as well. The Jaina tolerance of contradictions was the target of major objections by many Indian logicians.³¹ Such objections may obviously be defused by the techniques of modern paraconsistent logic, of the kind deployed in this essay.

10 Sevenfold predication again

In the preceding sections, we have developed a semantics for a simple (modal) propositional language, based on the theory of sevenfold predication. In this section, we turn to the expression of the theory in the language itself. To do this, we have to be able to express the notions ‘is true’, ‘is false’, and ‘is non-assertable’ in the language. We might plausibly express such notions as predicates or as operators. To keep things at the level of propositional logic, and so simpler, here, we will take them to be operators. Hence, we augment the language with three monadic operators, T , F , I , such that if A is any formula, TA , FA , and IA are formulas. The simplest and most obvious truth tables for these operators are as follows:

T	
t	t
i	f
f	f

I	
t	f
i	t
f	f

F	
t	f
i	f
f	t

It is a simple matter to check that:

³¹See, e.g., Matilal 1981, chs. 14, 15, and Ganeri 2001, 5.3.

if for some φ' such that $\varphi R\varphi'$, A is t at φ' , STA is t at φ ; otherwise it is f
if for some φ' such that $\varphi R\varphi'$, A is i at φ' , SIA is t at φ ; otherwise it is f
if for some φ' such that $\varphi R\varphi'$, A is f at φ' , SFA is t at φ ; otherwise it is
 f

The sevenfold theory can now be expressed schematically in the language as follows:³²

$$\begin{aligned}
& (STA \wedge \neg SIA \wedge \neg SFA) \\
& \vee (\neg STA \wedge \neg SIA \wedge SFA) \\
& \vee (STA \wedge \neg SIA \wedge SFA) \\
& \vee (\neg STA \wedge SIA \wedge \neg SFA) \\
& \vee (STA \wedge SIA \wedge \neg SFA) \\
& \vee (\neg STA \wedge SIA \wedge SFA) \\
& \vee (STA \wedge SIA \wedge SFA)
\end{aligned}$$

Call this JA . Consider the semantic value of this at a facet, φ . If φ does not access any facet, then at least one conjunct of each disjunct (a/the negation-free one) is f . Hence, JA is f . If φ does access some other facets, then there are seven possibilities. In the first, φ accesses only facets where A is t . In this case, the first disjunct is t , and all the others f . In the second, φ accesses only facets where A is f . In this case, the second disjunct is t , and all the others f . And so on for the other possibilities. Hence, the value of the whole disjunction is t .

We see that JA need not hold at every facet. That seems right. There should be perspectives which disagree with Jaina logic, and so from which it does not hold. However, for any set of facets, \mathcal{F} , and given that condition **B** of the previous section holds, there will be some facet $\varphi' \in \mathcal{F}$, such that JA is t at φ' . Hence, JA holds in \mathcal{F} , as one would hope.³³

³²If we extended the language to a first-order language, and took T , I , and F to be predicates, we could express this in the form of a single sentence: $\forall x((STx \wedge \neg SIx \wedge \neg SFx) \vee \dots)$, where the quantifiers range over sentences. We could also express things that the Jaina would also endorse (though others might not), e.g. that there are some things in each of the categories: $\exists x(STx \wedge \neg SIx \wedge \neg SFx)$, etc. It would be necessary to be able to establish that there are at least some facets at which the predicates behaved appropriately, so that $T\langle A \rangle$ is designated iff A takes the value t (where $\langle A \rangle$ is the name of the sentence A); and similarly for I and F . Doing so is a distinctly non-trivial exercise.

³³More generally, one might take the operators T , I , and F to be non-truth-functional, so that for every $\varphi \in \mathcal{F}$, v_φ assigns formulas of the form TA , IA , and FA directly, as in the semantics for the modal operators in the logic $S0.5$. (See Priest 2008, 4.4a.) Call φ *regular* if it assigns truth values in accord with the truth tables. Then JA is guaranteed to hold at φ only if it accesses some regular facet. And it will still turn out to be a logical truth provided we insist that \mathcal{F} contain at least one regular facet.

11 Conclusion

We have seen how the Jaina theory of Anekānta-vāda can be taken to form the basis of a semantics for a simple propositional language. (The extension to a first-order language is relatively routine.) We have seen how the semantics validates the Jaina theory of sevenfold predication—both about the language and within the language. Jaina logic can therefore be given a rigorous formulation in terms of modern logical techniques. But we have also used these techniques to interrogate Jaina logic itself, particularly concerning its account of assertion and its relativism. The techniques highlight certain of its problematic features, but also provide possible solutions to some of those problems. At any rate, we have seen, as promised, how the application of contemporary logical techniques to historical theories in Indian logic can be just as fruitful as their application to historical theories in European logic.

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