Vague Inclosures

Graham Priest
Departments of Philosophy
Universities of Melbourne and St Andrews

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1 Introduction: Vagueness and Self-Reference

Sorites paradoxes and the paradoxes of self-reference are quite different kinds of creature. The first are generated by the fact that some predicates have a certain kind of tolerance to small changes in their range of application. The second are generated by the fact that some things can refer, directly or indirectly, to themselves. Or so it seemed to me until recently. I am now inclined to think differently. The paradoxes of self-reference can naturally be seen as having a form given by the Inclosure Schema. In the Schema, a construction is applied to collections of a certain kind to produce a different object of the same kind. Contradiction arises at the limit of all things of that kind. Sorites paradoxes can be seen as having exactly the same form. In this paper, I will start by explaining how. Given that paradoxes of sorites and self-reference are of the same kind, they should have the same kind of solution. I hold that a dialetheic solution is the correct one for paradoxes of self-reference. It follows that a dialetheic solution is therefore appropriate for sorites paradoxes. The rest of the paper investigates what such a solution is like, and especially so called "higher order" vagueness.

2 The Inclosure Schema

Let us start with the Inclosure Schema and its application to the paradoxes of self-reference.[1] An inclosure paradox arises when for some monadic predicates φ and θ, and a one place function, δ, there are principles which appear to be true, or *a priori* true,[2] and which entail the following conditions. (It

[1]For details of the following, see Priest (1995), esp. Part 3.
[2]Priest (1995), p. 277.

is not required, note, that the arguments entailing the conditions be sound, though dialetheism prominently allows for this possibility.)

1. There is a set, Ω, such that $\Omega = \{x : \varphi(x)\}$, and $\theta(\Omega)$ (**Existence**)
2. If $X \subseteq \Omega$ and $\theta(X)$:
   - **(a)** $\delta(X) \notin X$ (**Transcendence**)
   - **(b)** $\delta(X) \in \Omega$ (**Closure**)

(A special case of an inclosure is when $\theta(X)$ is the vacuous condition, $X = X$, and so mention of it may be dropped.) Given these conditions, a contradiction occurs at the limit when $X = \Omega$. For then we have $\delta(\Omega) \notin \Omega \wedge \delta(\Omega) \in \Omega$.

To illustrate: In the Burali-Forti paradox, $\varphi(x)$ is 'x is an ordinal', so that Ω is the set of all ordinals, On—defined, let us assume, as von Neumann ordinals. $\theta(X)$ is the vacuous condition; and $\delta(X)$ is the least ordinal greater than every member of X. By definition $\delta(X)$ satisfies Transcendence and Closure. The brunt of the Burali-Forti paradox is exactly in showing that $\delta(X)$ is well defined, even when $X = \Omega$. The reasoning shows that On is itself an ordinal—an ordinal greater than all ordinals.

In the liar paradox, $\varphi(x)$ is the predicate Tx, 'x is true', so that Ω is the set of true sentences; $\theta(X)$ is the predicate 'X is definable', i.e., is a set that is referred to by some name; if X is definable, let N be an appropriate name; then $\delta(X)$ is a sentence, σ, constructed by an appropriate self-referential construction, of the form $\langle \sigma \notin N \rangle$. (I use angle brackets as a name-forming device.) Liar-type reasoning establishes Transcendence and Closure. The liar paradox arises in the limit. $\Omega = \{x : Tx\}$, and $\delta(\Omega)$ is a sentence, σ, of the form $\langle \sigma \notin \{x : Tx\} \rangle$, i.e., '$\sigma$ is not true'.

3 Sorites and Inclosures

Let us now see how sorites paradoxes fit the Schema.

In a sorites paradox there is a sequence of objects, a_0, ..., a_n, and a vague predicate, P, such that Pa_0 and $\neg Pa_n$; but for successive members of the sequence there is very little difference between them with respect to their P-ness, so that if one satisfies P, so does the other—the principle of tolerance.

For the Inclosure Schema, let $\varphi(x)$ be Px, so $\Omega = \{x : Px\}$; $\theta(X)$ is the vacuous condition. Ω is a subset of $A = \{a_0, ..., a_n\}$—indeed, a proper subset, since a_n is not in it—and so we have Existence. If $X \subseteq \Omega$ then, since X is a proper subset of A, there must be a first member of A not in it. Let this be $\delta(X)$. By definition, $\delta(X) \notin X$. So we have Transcendence. Now, either $\delta(X) = a_0$ (if $X = \phi$), and so $P\delta(X)$; or (if $X \neq \phi$) $\delta(X)$ comes immediately after something in $X \subseteq \Omega$, so $P\delta(X)$, by tolerance. In either case, $\delta(X) \in \Omega$, so we have Closure.

The inclosure contradiction is of the form $\delta(\Omega) \notin \Omega \wedge \delta(\Omega) \in \Omega$. In the case of the sorites paradox, the contradiction is that the first thing in the sequence that is not P is P. Diagonalisation takes us out of X; and tolerance keeps us within Ω. We see why a contradiction occurs at the limit of P-things.

If the self-referential paradoxes and sorites paradoxes are of the same kind, the Principle of Uniform Solution—'same kind of paradox, same kind of solution'—tells us that we should expect the same kind of solution.[3] I take the correct solution to the paradoxes of self-reference to be a dialetheic one.[4] It follows that the solution to the sorites paradoxes should be so too. A simple-minded thought is this: In the case of the paradoxes of self-reference we endorse the soundness of the arguments. These establish certain contradictions, the trivialising consequences of which are avoided by not endorsing Explosion. We should just do the same in sorites paradoxes: endorse the soundness of the arguments. But this cannot be right. Sorites paradoxes are, in their own right, as near triviality-making as makes no difference. One can prove that an old thing is young, that a red thing is blue, and anything else for which one can postulate an appropriate sorites progression. We must be less simple-minded. What follows is, hopefully, so.

4 The Structure of Sorites Transitions

Come back to the sorites progression of section 3. Pa_0 is true (and true only); Pa_n is false (and false only). If we write the least-number operator as μ then $\delta(\{x : Px\})$ is $\mu h(a_h \notin \{x : Px\}$, that is, $\mu h \neg Pa_h$.[5] Let this be a_i. The Inclosure Schema tells us that $a_i \in \{x : Px\}$ and $a_i \notin \{x : Px\}$, that is, $Pa_i \wedge \neg Pa_i$. So we know that there is at least one h for which Pa_h is both true and false. For all we have seen so far, there may be more than one. If there are, there is no reason, in principle, why these should be consecutive,[6] but the uniform nature of a sorites progression at least suggests this. Assuming

[3] Priest (1995), 11.5, 11.6, 17.6.

[4] Priest (1987), (1995).

[5] In naive set theory, the comprehension schema schema gives: $y \in \{x : Px\} \leftrightarrow Py$, and contraposition gives $y \notin \{x : Px)\} \leftrightarrow \neg Py$.

[6] The Technical Appendix to Part 3 of Priest (1995) constructs models of the Inclosure Schema where some ordinals are consistent and some are not. Section 4 of the Appendix gives a model in which inconsistent ordinals need not be consecutive.

it to be so, the structure of a sorites progression will look like this, where a_k is the last thing that is P, and $i \leq k$.[7]

$$\begin{array}{ccccccccc} a_0 & \ldots & a_i & \ldots & a_k & \ldots & a_n \\ [- & - & P & - & -] & & \\ & & [- & - & \neg P & - & -] \end{array}$$

We can think of the sequence of dialetheic objects as providing a transition from the things that are definitely P to the things that are definitely not P. Many have argued that in sorties progressions there is a borderline area where the relevant statements have truth value gaps. What intuition actually tells us is that in the middle of the progression, things are symmetric with respect to the ends. The statements about the transition objects should therefore be symmetric with respect to the statements about the ends. And from this point of view, being *both* true and false is as good as being *neither*.[8]

Most importantly, however, note the position of a_i. It might be thought that the first thing that is not P should be a_{k+1}, but it is not. The first thing that is *not* P is actually identical with, or to the left of, the last thing which is P! a_{k+1} is not the first thing that is not P, but the first thing of which P is not true. We are in the territory of higher order vagueness here. We will turn to that matter later.

5 Sorites Arguments

What does this tell us about sorites arguments? What tolerance tells us is that for some appropriate biconditional, $\Leftrightarrow$, $Pa_h \Leftrightarrow Pa_{h+1}$ (for $0 \leq h < n$). The sorites argument is then of the form:

$$\cfrac{\cfrac{Pa_0 \quad Pa_0 \Leftrightarrow Pa_1}{Pa_1} \qquad Pa_1 \Leftrightarrow Pa_2}{Pa_2}$$

$$\ddots$$

$$\frac{Pa_{n-1} \quad Pa_{n-1} \Leftrightarrow Pa_n}{Pa_n}$$

The next question is what this biconditional is. The correct understanding is, I take it, that it is a material biconditional, $\equiv$: consecutive sorites

[7] It is clear from the diagram that $\{x : Px\} \cap \{x : \neg Px\}$ is not empty. But since this set is $\{x : Px\} \cap \overline{\{x : Px\}}$, it is empty as well. It is difficult to represent this fact in a consistent diagram!

[8] See Hyde (1997).

statements have the same truth value. This is what, it seems to me, tolerance is all about. Thus, where $\alpha \supset \beta$ is $\neg\alpha \vee \beta$, we have $(Pa_h \supset Pa_{h+1}) \wedge (Pa_h \supset Pa_{h+1})$. This is true if Pa_h and Pa_{h+1} are both true or both false. (If one is true and the other is false, it is false as well.)

Given this understanding of the conditional, every major premise of the argument is true. For every h, Pa_h and Pa_{h+1} are both true or both false. But assuming an appropriate paraconsistent logic,[9] the disjunctive syllogism (DS)—*modus ponens* (MP) for the material conditional—is invalid: $\alpha, \alpha \supset \beta \nvdash \beta$; and of course, exactly the same is true of the material biconditional.

It should be noted that, though the sorites argument itself is invalid, the situation is still inconsistent. The sorites is generated by the sentences:

$$Pa_0$$

$$\neg Pa_n$$

$$(Pa_h \equiv Pa_{h+1}) \qquad (0 \leq h < n)$$

From these, we cannot prove $Pa_h \wedge \neg Pa_h$, for any particular h; but can prove:

$$\bigvee_{0 \leq h \leq n} (Pa_h \wedge \neg Pa_h)$$

To see this, write α_h for Pa_h. Then α_0 and $\alpha_0 \equiv \alpha_1$ give $(\alpha_0 \wedge \neg\alpha_0) \vee \alpha_1$. This, plus $\alpha_1 \equiv \alpha_2$, give $(\alpha_0 \wedge \neg\alpha_0) \vee (\alpha_1 \wedge \neg\alpha_1) \vee \alpha_2$; and so on till we have $(\alpha_0 \wedge \neg\alpha_0) \vee ... \vee (\alpha_{n-1} \wedge \neg\alpha_{n-1}) \vee \alpha_n$. Whence $\neg\alpha_n$ gives the result. In other words, this information tells us that the inclosure is located somewhere along the track; but it, itself, does not tell us exactly where.

6 "Extended" Paradoxes of Self-Reference

We now come to the vexed question of so called higher order vagueness. Let me start, for reasons that will become clear later, by talking about an apparently different issue: "extended paradoxes" in the context of the semantic paradoxes. When people offer solutions to the semantic paradoxes of self-reference, it always seem to turn out that the machinery that they deploy to solve them allows the formulation of paradoxes equally virulent—or maybe better, simply moves the old paradox to a new place. Let me illustrate with respect to the liar and truth value gaps.

[9] In what follows, we will take this to be the logic *LP* of Priest (1987), ch. 5; but matters are much the same in virtually every paraconsistent logic.

The semantic paradoxes deploy the T-schema. If we write T for the truth predicate, and angle brackets for naming, then the T-schema is the principle that:

$$T\langle\alpha\rangle \leftrightarrow \alpha$$

for every closed sentence, α (where $\leftrightarrow$ is an appropriate, detachable, biconditional[10]). Writing F for the falsity predicate, so that $F\langle\alpha\rangle$ is $T\langle\neg\alpha\rangle$, the simple liar paradox is a sentence, λ_0, obtained by some technique of self-reference, of the form $F\langle\lambda_0\rangle$. Substituting in the T-schema, we get:

$$T\langle\lambda_0\rangle \leftrightarrow F\langle\lambda_0\rangle$$

The Principle of Bivalence tells us that for all α:

$$T\langle\alpha\rangle \vee F\langle\alpha\rangle$$

and applying this to λ_0, we infer $T\langle\lambda_0\rangle \wedge F\langle\lambda_0\rangle$: λ_0 is both true and false.

A standard suggestion is to avoid this conclusion is to deny the Principle of Bivalence. Sentences are not necessarily true or false; some are neither (N). So the Principle is replaced by:

$$T\langle\alpha\rangle \vee F\langle\alpha\rangle \vee N\langle\alpha\rangle$$

True, we can no longer infer that λ_0 is both true and false, but now we can construct the "extended liar paradox", a sentence λ_1 of the form $F\langle\lambda_1\rangle \vee N\langle\lambda_1\rangle$. Substituting this in the T-schema, we get:

$$T\langle\lambda_1\rangle \leftrightarrow (F\langle\lambda_1\rangle \vee N\langle\lambda_1\rangle)$$

And all three of the possibilities lead to trouble.

Such a conclusion is obviously fatal to gap-theories of this kind. Some have thought that extended paradoxes of the same kind sink dialetheic ("glut") theories. That $T\langle\lambda_0\rangle \wedge F\langle\lambda_0\rangle$ is obviously no problem for a glut theory. The extended liar is now a sentence, λ_2, of the form: $F\langle\lambda_2\rangle \wedge \neg T\langle\lambda_2\rangle$; or given that there are no gaps, so that anything not true is false, just $\neg T\langle\lambda_2\rangle$. Substituting in the T-schema gives:

$$T\langle\lambda_2\rangle \leftrightarrow \neg T\langle\lambda_2\rangle$$

[10]For the sake of definiteness, let this be the conditional of Priest (1987), 19.8.

and so, given the Law of Excluded Middle, $T\langle\lambda_2\rangle \wedge \neg T\langle\lambda_2\rangle$. But only a little thought suffices to show that this is no problem for a dialetheist. Dialetheism was never meant to give a consistent solution to the paradoxes. (Even in the case of the simple liar, things are inconsistent, since we have $\lambda_0 \wedge \neg\lambda_0$.) The point was to allow contradictions, but in a controlled way. The "extended" argument does show, however, that the very categories deployed in a dialetheic account of the paradoxes are themselves subject to the very sort of inconsistency they characterise. This is, indeed, to be expected. We may show, moreover, that all the inconsistencies generated are under control, by constructing a single "semantically closed" theory, which is inconsistent, but in which the inconsistencies are quarantined. Specifically, we can take a first-order language with a truth predicate, T, and some form of naming device, $\langle . \rangle$. We can then formulate a theory in this language, which contains all instances of the T-schema, and an appropriate form of self-reference. The theory can be shown to be inconsistent, but non-trivial.[11]

7 Higher Order Vagueness

Let us now return to vagueness. Sorites paradoxes occur because the nature of the transition in a sorites progression is problematic. The straight-forward picture:

$$\begin{array}{cccccc} a_0 & \dots & \dots & \dots & \dots & a_n \\ [- & P & -] & [- & \neg P & -] \end{array}$$

jars because of the counter-intuitive nature of the cut-off point between the true and the false. The solution that we have been looking at removes this cut-off point. But though the machinery does so, it produces, instead, two others—one between the true only and the both true and false, and one between the false only and the both true and false:

$$\begin{array}{ccccccc} a_0 & \dots & a_i & \dots & a_k & \dots & a_n \\ [- & - & P & - & -] & & \\ & & [- & - & \neg P & - & -] \end{array}$$

and these would seem to jar just as much. As with the extended liar paradox, the machinery of the proposed solution allows us to produce a phenomenon of the same acuity. What is one to say about this?

The natural thought is that these cut-offs should be handled in exactly the same way. Consider, first, the right-hand boundary. This is located

[11] Specifically, no inconsistencies involving only the grounded sentences of the language (in the sense of Kripke) are provable. See Priest (2002), 8.2.

between those a of which P is true and those of which it is not. Let us now use T, not for the truth predicate, but for the binary truth-of (satisfaction) relation. Specifically if α is a formula of one free variable, say y, let the S-schema be:

$T\langle\alpha\rangle x \leftrightarrow \alpha_y(x)$

where the right hand side is the result of replacing all free occurrence of y with x (clashes of bound variables being handled by suitable relabelling). In particular, for our vague predicate, P, we have $T\langle Py\rangle x \leftrightarrow Px$. When the variable is clear from the context, I will omit it to keep notation simple. Thus, I will write $T\langle Py\rangle$ simply as $T\langle P\rangle$. Then the S-scheme amounts to this:

(*) $T\langle P\rangle x \leftrightarrow Px$

Now, the predicate $T\langle P\rangle$ would seem to be just as vague as the predicate P. In particular, it would seem to be just as tolerant to small changes in its argument as the predicate P. Indeed, (*) would seem to tell us that the tolerances of P and $T\langle P\rangle$ march together. It follows that the predicate is just as soritical; and just as the original sorites was generated by a set of sentences:

Pa_0, $\neg Pa_n$

$Pa_i \equiv Pa_{i+1}$ $(0 \leq i < n)$

So a sorites is generated by the sentences:

$P\langle T\rangle a_0$, $\neg T\langle P\rangle a_n$

$T\langle P\rangle a_i \equiv T\langle P\rangle a_{i+1}$ $(0 \leq i < n)$

The predicate $T\langle P\rangle$ also gives rise to an inclosure. Let $\varphi(x)$ be $T\langle P\rangle x$, so $\Omega = \{x : T\langle P\rangle x\}$; $\theta(X)$ is the vacuous condition. Ω is a subset of $A = \{a_0$, ..., $a_n\}$—indeed, a proper subset, since a_n is not in it. If $X \subseteq \Omega$ then, since X is a proper subset of A, there must be a first member of A not in it. Let this be $\delta(X)$. By definition, $\delta(X) \notin X$, Transcendence. Now, either $\delta(X) = a_0$ (if $X = \phi$), and so $T\langle P\rangle\,\delta(X)$; or (if $X \neq \phi$) $\delta(X)$ comes immediately after something in $X \subseteq \Omega$, so $T\langle P\rangle\,\delta(X)$, by tolerance. In either case, $\delta(X) \in \Omega$, Closure. The contradiction is that $T\langle P\rangle\ \delta(\Omega) \wedge \neg T\langle P\rangle\ \delta(\Omega)$.

Similar considerations apply at the left-hand boundary. Let us write $F\langle P\rangle x$, 'P is false of x', for $T\langle\neg P\rangle x$. A predicate is vague (tolerant) iff its negation is. In particular, $\neg P$ is just as vague as P. And since, as the S-Schema tells us:

$F\langle P\rangle x \leftrightarrow \neg Px$

$F\langle P\rangle$ is a vague predicate, as, then is $\neg F\langle P\rangle$. We therefore have a sorites generated by the sentences:

$\neg F\langle P\rangle a_0,\ F\langle P\rangle a_n$

$F\langle P\rangle a_i \equiv F\langle P\rangle a_{i+1}\ (0 \leq i < n)$

Note that $\alpha \equiv \beta$ is logically equivalent to $\neg\alpha \equiv \neg\beta$.

And as is to be expected, this boundary is another inclosure. Let $\varphi(x)$ be $\neg F\langle P\rangle x$, so $\Omega = \{x : \neg F\langle P\rangle x\}$; $\theta(X)$ is the vacuous condition. Ω is a subset of $A = \{a_0, ..., a_n\}$—indeed, a proper subset, since a_n is not in it. If $X \subseteq \Omega$ then, since X is a proper subset of A, there must be a first member of A not in it. Let this be $\delta(X)$. By definition, $\delta(X) \notin X$, Transcendence. Now, either $\delta(X) = a_0$ (if $X = \phi$), and so $\neg F\langle P\rangle\delta(X)$; or (if $X \neq \phi$) $\delta(X)$ comes immediately after something in $X \subseteq \Omega$, so $\neg F\langle P\rangle\delta(X)$, by tolerance. In either case, $\delta(X) \in \Omega$, Closure. The contradiction is that $\neg F\langle P\rangle\ \delta(\Omega) \wedge \neg\neg F\langle P\rangle\ \delta(\Omega)$—or just $\neg F\langle P\rangle\ \delta(\Omega) \wedge F\langle P\rangle\ \delta(\Omega)$.

8 The General Case

Of course, the situation repeats, generating new boundaries. Thus, the next iteration gives us:

```
[−  −  ¬F⟨P⟩   −    −   −   −]
                            [−  −  F⟨P⟩  −  −    −    −  −]

                            [−  −  ¬P    −  −    −    −  −]

a0  ...   ...   ...  ai  ...  ...  ...  ak  ...  ...  ...  an

[−  −   −   P    −  −  −   −   −]

[−  −   −  T⟨P⟩  −  −  −   −   −]
                                 [−   −  −  ¬T⟨P⟩  −  −]
```

Look below the as. We have just considered the division between P being true and its not being true. We now have the divisions between $T\langle P\rangle$ being true, and its not being true, and the division between $\neg T\langle P\rangle$ being true and its not being true. The first of these is the same as that between P being true and its not being true, since P and $T\langle P\rangle$ are co-extensional. But the second is new. Above the as we have the symmetrical situation concerning F.

And so it goes on. We need to consider all predicates that can be obtained by iteration. Generally, given the vague predicates Q, $\neg Q$, at the next level we have $T\langle Q\rangle$, $\neg T\langle Q\rangle$, and $T\langle\neg Q\rangle$, $\neg T\langle\neg Q\rangle$ (i.e., $F\langle Q\rangle$, $\neg F\langle Q\rangle$). Thus, the hierarchy of predicates looks as follow. To keep notation simple, I will henceforth omit the angle brackets. (Thus, I will write $F\langle T\langle P\rangle\rangle$ as FTP, etc.)

$$
\begin{array}{cccccccc}
 & & & \bullet & & & & \\
 & & \swarrow & & \searrow & & & \\
 & P & & & & \neg P & & \\
 \swarrow & & \searrow & & \swarrow & & \searrow & \\
 TP & & \neg TP & & FP & & \neg FP & \\
 \swarrow \ \searrow & & \swarrow \ \searrow & & \swarrow \ \searrow & & \swarrow \ \searrow & \\
 TTP \quad \neg TTP & & FTP \quad \neg FTP & & TFP \quad \neg TFP & & FFP \quad \neg FFP & \\
 \vdots \qquad \vdots & & \vdots \qquad \vdots & & \vdots \qquad \vdots & & \vdots \qquad \vdots &
\end{array}
$$

By exactly analagous consideration, each pair in the family is vague, and each gives rise to an inclosure contradiction.

9 A "Soritically Closed" Language

How do we know that all these contradictions can be accommodated in a uniform way? With the self-referential paradoxes and their extended versions, we know this because we can construct a single semantically closed language, which accommodates all the contradictions in one hit. Exactly the same is true in this case. We can construct a "soritically closed" language. Specifically, we take a language that has the truth-of predicate T, and a naming device, $\langle.\rangle$. For definiteness, let us suppose that the language contains that for arithmetic, and that the naming is obtained by Gödel coding. We suppose, in addition, one vague predicate, P, and a sorites sequence $(a_0, ..., a_n)$. Let this be $0, ..., n$. Let σ be any string of 'T's and 'F's (including the empty string), and let $\#(\sigma)$ be the number of 'F's in σ. (The parity of this tells us, in effect, whether we are doing a left-to-right sorites, or a right-to left sorites. Even is the fist; odd is the second.)

Our theory comprises the S-schema, plus the following:

$\sigma Pa_i \equiv \sigma Pa_{i+1} \qquad (0 \leq i < n)$

$\sigma Pa_0, \neg\sigma Pa_n \qquad$ when $\#(\sigma)$ is even

$\sigma Pa_n, \neg\sigma Pa_0 \qquad$ when $\#(\sigma)$ is odd

The theory is inconsistent. For every σ, the theory entails:

$$\bigvee_{0\leq i\leq n} (\sigma Pa_i \wedge \neg\sigma Pa_{i+1})$$

(The proof when σ is the empty sequence was already given; in the general case, the argument is exactly the same.)

Moreover, the theory is non-trivial. We can construct an interpretation which shows this, as follows. Start with a language without T. Take an interpretation, $\mathfrak{I}$, which is standard with respect to the arithmetic machinery. Let $0 < m < n$. The extension of P is $\{0, .., m\}$, and the anti-extension is $\{m, .., n\}$.[12] So, in the model, $Pm \wedge \neg Pm$ holds, as does every biconditional $Ph \equiv P(h+1)$, for $0 \leq h < n$. We now construct a model of the S-schema on top of $\mathfrak{I}$ as in Priest (2002), 8.2. (The model constructed there is of the T-schema, but this generalises to one for the S-schema in an obvious fashion.) In this model, we have not just the S-schema, but its contraposed form. Hence, every $\sigma\alpha$ is logically equivalent to α or $\neg\alpha$, and so all the cases of the axioms where σ is non-empty collapse into the case where it is. Moreover, in the construction of the model, it is only sentences involving 'T' that change their value. So the truth values of all other sentences are as in $\mathfrak{I}$. (In particular, then, any purely arithmetic sentence false in the standard model is not provable in the theory.)

What we see, then, is that from a dialetheic perspective, higher order vagueness is essentially the same as the extended paradoxes of self-reference,[13] can be handled in exactly the same way, and is no more problematic.

10 Conclusion

Prima facie, sorites arguments and the paradoxes of self-reference are completely distinct. They are certainly distinct. But what I have tried to establish is that, at a fundamental level, they are the same. Both are inclosure paradoxes, where the underlying form is given by the Inclosure Schema. The two kinds of paradox must therefore have the same kind of solution. Given that the correct solution to the paradoxes of self-reference is a dialetheic one, then so must be a solution to the sorites paradoxes. I have discussed such a

[12]Strictly speaking, $\{x; x \geq m\}$, since every natural number must be in either the extension or the anti-extension of P. But what happens for numbers greater than n in irrelevant for our example.

[13]This important observation is due to Mark Colyvan (2007).

solution at length, and argued that, despite certain superficial differences, it is also essentially the same.

The fact that one has a single family of paradoxes, and a uniform solution, does not, of course, mean that various sub-families cannot have their own specificities. Even within the paradoxes of self-reference, the semantic and the set theoretic paradoxes have differences of vocabulary; more importantly, diagonalisation may be achieved in various different ways (by employing literal diagonalisation, a least number operator, etc.) Thus, it is entirely possible for the sorites paradoxes to have their own specificities, which they do. For example, tolerance plays a distinctive role, and "higher order vagueness" must be accommodated. All this we have seen. But the specificities are superficial, just as the specificities of the set theoretic and semantic paradoxes are superficial when it comes understanding the paradoxes and framing an appropriate solution. Such, at least, has been the import of this paper.

References

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