Four Corners—East and West

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Abstract: In early Buddhist logic, it was standard to assume that for any state of affairs there were four possibilities: that it held, that it did not, both, or neither. This is the *catuskoti*. Classical logicians have had a hard time making sense of this, but it makes perfectly good sense in the semantics of various paraconsistent logics, such as First Degree Entailment. Matters are more complicated for later Buddhist thinkers, such as Nagarjuna, who appear to suggest that none or these options, or more than one, may hold. These possibilities may also be accommodated with contemporary logical techniques. The paper explains how.

Key Words: *catuskoti*, Buddhist logic, Nagarjuna, First Degree Entailment, many-valued logic, relational semantics

1 Introduction

Western Logic has been dominated by the Principles of Excluded Middle and Non-Contradiction. Given any claim, there are two possibilities, *true* and *false*. These are exhaustive and exclusive. Contemporary Western logic has come to realise that this may be far too narrow-minded. There may well be situations where we need to countenance things that are neither true nor false, or both true and false.[1] Indeed, these possibilities are built into the semantics of various logics (many-valued, relevant, paraconsistent). The technology of deploying such techniques is now relatively well understood.

[1]See, e.g., Priest (2008), ch. 7.

Western logic might well have learned its lesson from India. Though the traditional schools of Indian logic never had the mathematical tools to articulate their positions into anything like modern Western formal logics, a much more open-minded attitude was present from the earliest years. According to a principle of Buddhist logic clearly pre-dating the Buddha, given any claim, there are four possibilities, *true* (only), *false* (only), *both* or *neither*. This was called the *catuskoti*[2] (literally: 'four corners'). Western philosophers and logicians, armed only with their knowledge of bivalent Western logic, have had a hard time of making sense of the *catuskoti*, but by deploying the techniques of modern many-valued logic, this is simple, as we will see.

Matters became more complex as Buddhist thought developed in the early centuries of the Common Era. Here we find the great philosopher Nagarjuna, and those who followed him in the Madhyamaka school, appearing to say that none of the four *kotis* (corners) may hold, or sometimes that more than one—even all—of them may hold. How to accommodate this possibility with the techniques of modern logic is less obvious. However, it also can be done, and we will see this too.[3]

The following paper is therefore another illustration of the possibility of the history of logic and contemporary logic informing each other, to their mutual benefit—and one, moreover, that illustrates the fruitful interplay between Eastern and Western thought.

2 A Little History

The *catuskoti* is illustrated at the very beginning of Buddhist thought, when some of the Buddha's followers asked him to answer various difficult metaphysical questions, such as what happens to an enlightened person after death. The Buddha is explicitly presented with four possibilities, that the enlightened person exists, that they do not exist, that they both exist and do not exist, that they neither exist nor do not exist—the four corners of the *catuskoti*. The Buddha does not balk at the way things are presented. True, he refuses to answer the question, but the normal reason given is that thinking about such things is a waste of time, time better spent on matters more conducive to awakening. Just occasionally, there is a hint that there is

[2]Actually, *catuṣkoṭi*, but I ignore the diacriticals in writing Sanskrit words, except in the bibliography.

[3]It should be pointed out that not all Buddhists subscribed to the *catuskoti*. It was not endorsed by the Dignaga-Dharmakirti school of Buddhist logic. Like the Nyaya, this school of logic endorsed both the Principles of Non-Contradiction and Excluded Middle. See Scherbatsky (1993), pt. 4, ch. 2.

something else going on, possibly a false presupposition to all four possibilities. This thought was perhaps to be taken up later, but nothing further is made of the matter at this point in Buddhist thought. At this stage, then, the *catuskoti* functions something like a *principle of excluded fifth*: there are exactly four exclusive possibilities, *quintum non datur*.[4]

3 Making Sense of the *Catuskoti*

Philosophers who know only classical or traditional logic have a hard time making sense of the *catuskoti*. The natural way for them to formulate the four possibilities concerning some claim, A, are:

(a) A

(b) $\neg A$

(c) $A \wedge \neg A$

(d) $\neg(A \vee \neg A)$

(c) will wave red flags to anyone wedded to the Principle of Non-Contradiction—but the texts seem pretty explicit that you might have to give this away. There are worse problems. Notably, assuming De Morgan's laws, (d) is equivalent to (c), and so the two *kotis* collapse. Possibly, one might reject the Principle of Double Negation, so that (d) would give us only $\neg A \wedge \neg\neg A$. But there are worse problems. The four cases are supposed to be exclusive; yet case (c) entails both cases (a) and (b). So the corners again collapse.

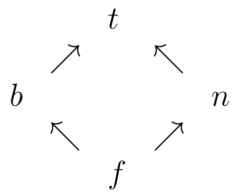
The obvious thought here is that we must understand (a) as saying that A is true and not false. Similarly, one must understand (b) as saying that A is false and not true. Corners (a) and (b) then become: $A \wedge \neg\neg A$ and $\neg A \wedge \neg A$ (i.e., $\neg A$). Even leaving aside problems about double negation, case (c) still entails case (b). We are no better off.[5]

There is, however, a way of understanding the *catuskoti* that will jump out at anyone with a passing acquaintance with the foundations of relevant logic. First Degree Entailment (FDE) is a system of logic that can be set up in many ways, but one of these is as a four-valued logic whose values

[4]For a more extended discussion of the history, including textual sources and quotations, see Priest (2010). See also Ruegg (1977) and Tillemans (1999).

[5]A full discussion of the unsuccessful ways that people have tried to get around these problems within the confines of classical—or at least intuitionist—logic, can be found in Priest (2010). See also Westerhoff (2009), ch. 4.

are t (true only), f (false only), b (both), and n (neither). The values are standardly depicted by the following Hasse diagram:



Negation maps t to f, vice versa, n to itself, and b to itself. Conjunction is greatest lower bound, and disjunction is least upper bound. The set of designated values, D, is $\{t, b\}$. Validity is defined in terms of the preservation of designated values in all interpretations.[6] The four corners of the *catuskoti* and the Hasse diagram seem like a marriage made for each other in a Buddhist heaven.[7]

Proof theoretically, FDE can be characterised by the following rule system. (A double line indicates a two-way rule, and overlining indicates discharging an assumption.)[8]

$$\frac{A, B}{A \wedge B} \qquad \frac{A \wedge B}{A\ (B)}$$

$$\frac{A\ (B)}{A \vee B} \qquad \frac{A \vee B \quad \begin{matrix}\overline{A} \\ \vdots \\ C\end{matrix} \quad \begin{matrix}\overline{B} \\ \vdots \\ C\end{matrix}}{C}$$

$$\frac{\neg(A \wedge B)}{\overline{\neg A \vee \neg B}} \qquad \frac{\neg(A \vee B)}{\overline{\neg A \vee \neg B}} \qquad \frac{\neg\neg A}{\overline{A}}$$

We see, then, how the four corners of the *catuskoti* can be accommodated in ways very standard in contemporary non-classical logic.

4 Rejecting all the *Kotis*

So far so good. Things get more complicated when we look at the way that the *catuskoti* is deployed in later developments in Buddhist philosophy—especially in the way it appears to be deployed in the writings of Nagarjuna

[6] See Priest (2008), ch. 8.
[7] As observed in Garfield and Priest (2009).
[8] See Priest (2002), 4.6.

and his Madhyamaka successors. We have taken the four corners of truth to be exhaustive and mutually exclusive. A trouble is that we find Nagarjuna appearing to say that sometimes none of the four corners may hold.[9] Why he says this, and what he means by it, are topics not appropriate for this occasion.[10] The question here is simply how to accommodate the possibility using the techniques of contemporary (non-classical) logic.

The easiest way of doing so is by taking there to be a fifth possibility:

(e) none of the above.

The most obvious way to proceed is now to take this possibility as a fifth semantic value, and construct a five-valued logic. Thus, we add a new value, e, to our existing four (t, f, b, and n).[11] Since e is the value of things that are neither true nor false (and so not true), it should obviously not be designated. Thus, we still have that $D = \{t, b\}$. How are the connectives to behave with respect to e? Both e and n are the values of things that are neither true nor false, but they had better behave differently if the two are to represent distinct alternatives. The simplest suggestion is to take e to be such that whenever any input has the value e, so does the output: e-in/e-out.[12]

The logic that results by modifying FDE in this way is obviously a sub-logic of it. It is a proper sub-logic. It is not difficult to check that all the rules of FDE are designation-preserving except the rule for disjunction-introduction, which is not, as an obvious counter-model shows. However, replace this with the rules:

$$\frac{\varphi(A) \quad C}{A \vee C} \qquad \frac{\varphi(A) \quad C}{\neg A \vee C} \qquad \frac{\varphi(A) \quad \psi(B) \quad C}{(A \wedge B) \vee C}$$

where $\varphi(A)$ and $\psi(B)$ are any sentences containing A and B.[13] Call these the φ *Rules*, and call this system FDE_φ. FDE_φ is sound and complete with respect to the semantics.[14]

[9] Just to make matters confusing, some people refer to this denial (the 'four-cornered negation') itself as the *catuskoti*. The Buddhist tradition is, in fact, not alone in sometimes denying the four *kotis*. See Raju (1953).

[10] Again, for a fuller discussion of the matter, together with textual sources and quotations, see Priest (2010). See also the pages indexed under 'Tetralemma' in Garfield (1995), and Garfield and Priest (2003).

[11] As in Garfield and Priest (2009). Happily, e, there, gets interpreted as emptiness.

[12] We will see that this behaviour of e falls out of a different semantics for the language in section 6.

[13] Instead of $\varphi(A)$ (etc.), one could have, instead, any sentence that contained all the propositional parameters in A.

[14] Details of the proof may be found in Priest (2010).

5 Accepting More than One *Koti*

Again, so far so good. There is a harder challenge to be faced, though. Forget the fifth possibility for the moment; we will return to it again later. The problem is that Nagarjuna sometimes seems to say that more than one of the *kotis* may hold—even all of them. Again, this is not the place to discuss what is going on here philosophically.[15] The question is how to accommodate the view in terms of modern logic.

In classical logic, evaluations of formulas are functions which map sentences to one of the values 1 and 0. In one semantics for FDE, evaluations are thought of, not as functions, but as relations, which relate sentences to some number of these values. This gives the four possibilities represented by the four values of our many-valued logic.[16]

We may do exactly the same with the values t, b, n, and f themselves. So if P is the set of propositional parameters, and $V = \{t,\ b,\ n,\ f\}$, an evaluation is a relation, ρ, between P and V. In the case at hand, we want to insist that every formula has at least one of these values, that is, the values are exhaustive:

Exh: for all $p \in P$, there is some $v \in V$, such that $p\rho v$.

If we denote the many-valued truth functions corresponding to the connectives $\neg$, $\vee$, and $\wedge$ in FDE, by $f_\neg$, $f_\vee$, and $f_\wedge$, then the most obvious extension of ρ to all formulas is given by the clauses:

- $\neg A\rho v$ iff for some x such that $A\rho x$, $v = f_\neg(x)$
- $A \vee B\rho v$ iff for some x, y, such that $A\rho x$ and $B\rho y$, $v = f_\vee(x, y)$
- $A \wedge B\rho v$ iff for some x, y, such that $A\rho x$ and $B\rho y$, $v = f_\wedge(x, y)$

One can show, by a simple induction, that for every A there is some $v \in V$ such that $A\rho v$. I leave the details as an exercise.

Where, as before, $D = \{t, b\}$, we may simply define validity as follows: $\Sigma \vDash A$ iff for all ρ:

- if for every $B \in \Sigma$, there is a $v \in D$ such that $B\rho v$, then there is a $v \in D$ such that $A\rho v$

[15] A full discussion can be found in Priest (2010).

[16] See Priest (2008), 8.2.

That is, an inference is valid if it preserves the property of relating to *some* designated value.

Perhaps surprisingly, validity on this definition coincides with validity in FDE.[17] This is proved by showing that the rules of FDE are sound and complete with respect to the semantics.[18]

6 None of the *Kotis*, Again

Let us, finally, return to the possibility that none of the *kotis* may hold. In Section 4, we handled this possibility by adding a fifth value, e. The relational semantics provides a different way of proceeding. We simply drop the exhaustivity condition, **Exh**, so allowing the possibility that an evaluation may relate a parameter (and so an arbitrary formula) to none of the four values. The logic this gives is exactly FDE_{φ}.[19]

In fact, if we require that every formula relates to *at most* one value, then it is easy to check that we simply have a reformulation of the 5-valued semantics, since taking the value e in the many-valued semantics behaves in exactly the same way as not relating to any value does in the relational semantics.

7 Conclusion

We have now seen how the ideas of the *catuskoti* and its developments can be made sense of using the techniques of many-valued and relational semantics. FDE does justice to the four possibilities. This has, as we have noted, a many-valued and a relational semantics. If none of the four *kotis* may obtain, we have FDE_{φ}. Again, this has a many-valued and a relational semantics, the second of which allows for more that one of the *kotis* obtaining, as well as none.

Of course, there are important questions about what all this *means*. Some of these questions are familiar from the contemporary philosophy of logic, such as ones concerning the possibility of truth value gaps and gluts. Some of them concern Buddhist philosophy, and specifically the metaphysical picture which informs (and may be informed by) the technical machinery. This is obviously not the place to discuss such matters. Suffice it for the present to

[17] Perhaps not. See Priest (1984).

[18] A proof of this fact can be found in Priest (2010).

[19] Again, a proof of this fact can be found in Priest (2010).

have shown some interesting connections between Buddhist thought and the techniques of contemporary non-classical logic.

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