Non-Transitive Identity

Graham Priest
Departments of Philosophy
Universities of Melbourne and St Andrews

Abstract

The paper defines a notion of identity in the second-order paraconsistent logic *LP*. The notion does not have the properties of transitivity or substitutivity, but these may be regained in consistent contexts. The paper then discusses applications of this notion of identity, especially to entities involved in change, including sorites-generating changes.

1 Problematising Identity

The notion of identity has always been a problematic notion, especially when considerations of intentionality and change are around.[1] And though there is now a standard theory of identity—identity in "classical" first order logic—this can appear as unproblematic as it does only because it is normally presented in a way that is sanitised by the disregarding of such considerations.

For example, suppose I change the exhaust pipes on my bike; is it or is it not the same bike as before? It is, as the traffic registration department and the insurance company will testify; but it is not, since it is manifestly different in appearance, sound and acceleration. Dialecticians, such as Hegel, have delighted in such considerations, since they appear to show that the bike both is and is not the same.[2] A standard reply here is to distinguish between the bike itself and its properties. After the change of exhaust pipes the bike is numerically the same bike; it is just that some of its properties are different. Perhaps, for the case at hand, this is the right thing to say.

[1]In this paper I shall concentrate on issues concerning change, and shall have nothing to say concerning intentionality. A discussion of identity in intentional contexts can be found in Priest (2002a) and ch. 2 of Priest (2005).

[2]See, e.g., Miller (1969), p. 413 ff.

But the categorical distinction between the thing itself and its properties is one which is difficult to sustain; to suppose that the bike is something over and above all of its properties is simply to make it a mysterious *ding an sich*. Thus, suppose that I change, not just the exhaust pipes, but, in succeeding weeks, the handle bars, wheels, engine, and in fact all the parts, until nothing of the original is left. It is now a numerically different bike, as even the traffic office and the insurance company will concur. At some stage, it has changed into a different bike, i.e., *it* has become a different machine: the bike itself is numerically different. (This is a variation on the old problem of the ship of Theseus.)

True sentences of the form $a = a$ and $a \neq a$ are standard fare in paraconsistent theories of identity;[3] but there is more to the matter than this. What is it for an object to be the same object over a period of time in which change occurs? The answer is, plausibly, different for different kinds of objects; for many kinds of objects, the answer is also likely to be contentious. But it is not uncommon so appeal to some kind of continuity condition. Thus, for example, Locke took personal identity to be given by continuity of memory.[4] I am the same person that I was yesterday since I can recall most of what I could recall then, and some more as well. But continuity conditions of this kind are naturally non-transitive. Memories can be lost in trauma, or even in the simple process of ageing. There can therefore be objects, say people, a, b and c, such that there is sufficient continuity between a and b, and between b and c, but not between a and c. Thus, we have $a = b$ and $b = c$, but not $a = c$. Identity fails to be transitive.

Cases of fission and fusion can also give rise to similar problems. Suppose that between t_0 and t_1, an amoeba, a, divides into two new amoebas, b and c; at t_1, b occupies location l_b, and c occupies a distinct location l_c. We may depict the situation as follows:

$$\begin{array}{ccccc} t_1 & l_b & & & l_c \\ & b & & & c \\ & & \nwarrow & \nearrow & \\ t_0 & & a & & \end{array}$$

At least arguably, $a = b$. (If c were to die on fission, this would be clear; and how can the identity of two things depend on what *else* exists?). Similarly, $a = c$. But it is not the case that $b = c$. Moreover, at t_1, b—that is, a—is at l_b; but c is not, even though $a = c$. We have a failure of the substitutivity of

[3]See, e.g., Priest (1987), 5.3.

[4]For references and discussion, see Parfitt (1984), p. 204 ff.

identicals, where the property in question has nothing to do with identity.[5]

There is, of course, much more to be said about all of these examples. But the discussion at least shows that various properties standardly taken to be possessed by identity (consistency, transitivity, substitutivity) are not to be taken for granted philosophically. One can, of course, simply specify *by fiat* that identity has these properties. But this is hardly satisfactory. The notion so produced will then certainly have those properties—and call it identity if you like; but it is all too obvious that the behaviour of the relationship involved in the above examples—and which we used to call identity before the word was usurped—still cries out to be understood.

In what follows, I will provide a theory of a relationship that is naturally enough thought of as identity, but for which the properties that we have just seen to be problematic fail, though in a controlled and recoverable way. In the next few sections we will look at a formal specification of the relation. We will then return to the above examples.[6]

2 Second-Order *LP*

The theory in question is based on a paraconsistent logic, *LP*.[7] For reasons that will become obvious, we will work with the second-order version of this, though there are other ways to proceed, as we shall see in due course. Let us start, then, with a specification of the logic.[8]

The language has the connectives $\wedge$, $\vee$ and $\neg$, and the first- and second-order quantifiers $\forall$ and $\exists$. The material conditional and biconditional are defined in the usual way: $\alpha \supset \beta$ is $\neg\alpha \vee \beta$; $\alpha \equiv \beta$ is $(\alpha \supset \beta) \wedge (\beta \supset \alpha)$. There are predicates and function symbols, but we will suppose, for the sake of simplicity, that they are all monadic. First order variables are lower case, and monadic second-order variables are upper case. I will avoid free variables.

There are various forms that the semantics of second-order *LP* may take; importantly, there are various possible ranges for the second-order variables. I will choose one appropriate way here. An interpretation for the

[5]The example comes from Prior (1968), p. 83. See also Priest (1995).

[6]There are certainly other non-classical theories of identity to be found in the literature, even ones based on a paraconsistent logic. Thus, e.g., in Krause (1992) and Bueno (2000) there is to be found a theory in which substitutivity of idenicals fails. The notion of identity of these papers is still an equivalence relation, however. In particular, identity is transitive. This makes the notion very different from that to be given here, and unsuitable for the major applications at issue.

[7]See, e.g., Priest (1987), ch. 5.

[8]For second-order *LP*, see section 7.2 of Priest (2002b).

language, I, is a triple $\langle D_1, D_2, \theta\rangle$. D_1 is the non-empty domain of first-order quantification. D_2 is the non-empty domain of second-order quantifiers, and is a set of pairs of the form $\langle A^+, A^-\rangle$, where $A^+ \cup A^- = D_1$. I will call A^+ an *extension*, and A^- a co-extension. We require that for every $A \subseteq D_1$, there is a $B \subseteq D_1$ such that $\langle A, B\rangle \in D_2$, but otherwise make no assumptions about how extensive D_2 is.[9] θ assigns every individual constant a member of D_1, every predicate constant a member of D_2, and every function symbol a (monadic) function from D_1 to D_1. If P is a predicate, I will write $\theta(P)$ as $\left\langle \theta^+(P), \theta^-(P)\right\rangle$.

θ can be extended to assign every closed term a denotation by the familiar recursive clause: $\theta(ft) = \theta(f)(\theta(t))$. An evaluation, ν, is a function that maps each formula to $\{1\}$ (true only), $\{0\}$ (false only), and $\{1, 0\}$ (both true and false), according to the following recursive clauses:

$1 \in \nu(Pt)$ iff $\theta(t) \in \theta^+(P)$

$0 \in \nu(Pt)$ iff $\theta(t) \in \theta^-(P)$

$1 \in \nu(\neg\alpha)$ iff $0 \in \nu(\alpha)$

$0 \in \nu(\neg\alpha)$ iff $1 \in \nu(\alpha)$

$1 \in \nu(\alpha \wedge \beta)$ iff $1 \in \nu(\alpha)$ and $1 \in \nu(\beta)$

$0 \in \nu(\alpha \wedge \beta)$ iff $0 \in \nu(\alpha)$ or $0 \in \nu(\beta)$

$1 \in \nu(\alpha \vee \beta)$ iff $1 \in \nu(\alpha)$ or $1 \in \nu(\beta)$

$0 \in \nu(\alpha \vee \beta)$ iff $0 \in \nu(\alpha)$ and $0 \in \nu(\beta)$

To give the truth and falsity conditions for the quantifiers, we assume, for the sake of simplicity, that the language is expanded if necessary to give each member of D_1 and D_2 a name. If $d \in D_1$, I write its name as $\mathbf{d}$; and if $A \in D_2$, I will write its name as $\mathbf{A}$. The conditions may now be stated as follows.

$1 \in \nu(\exists x\alpha(x))$ iff for some $d \in D_1$, $1 \in \nu(\alpha(\mathbf{d}))$

[9] In particular, we do not assume that every pair of the form $\langle A, B\rangle$, where $A \cup B = D_1$, is in D_2. This fact is, in itself, sufficient to give failure of substitutivity for molecular formulas. One might suggest that the only pairs that are in D_2 are those which represent special properties of some kind, such as natural or intrinsic properties. Depending on the how one interprets the notion, it may be natural to add extra closure conditions on D_1, such as closure under negation: $\langle A, B\rangle \in D_2 \Rightarrow \left\langle \overline{B}, \overline{A}\right\rangle \in D_2$.

$0 \in \nu(\exists x\alpha(x))$ iff for all $d \in D_1$, $0 \in \nu(\alpha(\mathbf{d}))$

$1 \in \nu(\forall x\alpha(x))$ iff for all $d \in D_1$, $1 \in \nu(\alpha(\mathbf{d}))$

$0 \in \nu(\forall x\alpha(x))$ iff for some $d \in D_1$, $0 \in \nu(\alpha(\mathbf{d}))$

$1 \in \nu(\exists X\alpha(X))$ iff for some $A \in D_2$, $1 \in \nu(\alpha(\mathbf{A}))$

$0 \in \nu(\exists X\alpha(X))$ iff for all $A \in D_2$, $0 \in \nu(\alpha(\mathbf{A}))$

$1 \in \nu(\forall X\alpha(X))$ iff for all $A \in D_2$, $1 \in \nu(\alpha(\mathbf{A}))$

$0 \in \nu(\forall X\alpha(X))$ iff for some $A \in D_2$, $0 \in \nu(\alpha(\mathbf{A}))$

Finally, validity: I is a model of α iff $1 \in \nu(\alpha)$; if Σ is a set of formulas, I is a model of Σ iff it is a model of every member; and $\Sigma \vDash \alpha$ iff every model of Σ is a model of α.

The first-order part of LP in the above semantics is entirely standard. The second-order part is a natural extrapolation. I merely pause, therefore, to note a few of the properties of the material biconditional that will feature in what follows. In particular, it is easy to check the following. (I omit set braces in the premises.)

$\vDash \alpha \equiv \alpha$

$\alpha \equiv \beta \vDash \beta \equiv \alpha$

$\alpha, \beta \vDash \alpha \equiv \beta$

$\neg\alpha, \neg\beta \vDash \alpha \equiv \beta$

$\alpha, \neg\beta \vDash \neg(\alpha \equiv \beta)$

$\beta, \neg\beta \vDash \alpha \equiv \beta$

$\alpha \equiv \beta \vDash \neg\alpha \equiv \neg\beta$

$\alpha \equiv \beta, \beta \equiv \gamma \nvDash \alpha \equiv \gamma$ (Make β both true and false.)

3 Defining Identity

With this background, we can now come to identity. Taking its cue from Leibniz' Law, identity may be defined in second-order logic in the standard fashion. Thus, let us define $t_1 = t_2$ as:

$Def_=$**:** $\forall X(Xt_1 \equiv Xt_2)$

Because the material biconditional is reflexive and symmetric, it follows that identity is too: $\vDash t = t$ and $t_1 = t_2 \vDash t_2 = t_1$. The material biconditional is not, however, transitive; identity inherits this property. Thus, consider the interpretation, $\mathfrak{I}$, where:

- $D_1 = \{a_1, a_2, a_3\}$
- $\theta(t_i) = a_i$ $(i = 1, 2, 3)$
- $\langle\{a_1, a_2\}, \{a_2, a_3\}\rangle = A \in D_2$
- For every other $B \in D_2$, $B^- = D_1$

Since $\mathbf{A}t_2 \wedge \neg\mathbf{A}t_2$ is true, so is $\mathbf{A}t_1 \equiv \mathbf{A}t_2$; and for every other $B \in D_2$, $\neg\mathbf{B}t_1 \wedge \neg\mathbf{B}t_2$ is true, so $\mathbf{B}t_1 \equiv \mathbf{B}t_2$. Hence, $\forall X(Xt_1 \equiv Xt_2)$, that is $t_1 = t_2$ is true. Similarly, $t_2 = t_3$. But $\mathbf{A}t_1 \equiv \mathbf{A}t_3$ is not true; hence, neither is $\forall X(Xt_1 \equiv Xt_3)$, that is, $t_1 = t_3$ is not true. Thus, $t_1 = t_2, t_2 = t_3 \nvDash t_1 = t_3$. Since transitivity of identity is a special case of substitutivity of identicals, this, too, fails. For another counter-example, note that in $\mathfrak{I}$, both $t_2 = t_3$ and $\mathbf{A}t_2$ are true, but $\mathbf{A}t_3$ is not. Finally, note that identity statements may not be consistent. Thus, in $\mathfrak{I}$, since $\mathbf{A}t_2 \wedge \neg\mathbf{A}t_2$ is true, so is $\neg(\mathbf{A}t_2 \equiv \mathbf{A}t_2)$. It follows that $\exists X\neg(Xt_2 \equiv Xt_2)$, so $\neg\forall X(Xt_2 \equiv Xt_2)$, i.e., $t_2 \neq t_2$.[10]

It might be objected that the account of identity just given is inadequate since what is required in $Def_=$ is not a material biconditional, but a genuine (and detachable) conditional, such as the conditional of an appropriate relevant logic. We would then have transitivity and substitutivity of identity (though maybe not consistency). However, this would be too fast. It is not at all clear that what is required for an expression of Leibniz'La w is a genuine conditional. For example it is not clear that there is a relevant implication between, e.g., 'Mary Ann Evans was a woman' and 'George Elliot was a woman'—at least, not without the suppressed information that

[10]It is perhaps worth observing that if we drop the condition on interpretations that for all $A \in D_2$, $A^+ \cup A^- = D_1$, and so base the theory of identity on FDE, then the Law of Identity, $t = t$, also fails. If we insist that $A^+ \cap A^- = \phi$, and so base the theory on K_3, the Law still fails, but transitivity and substitutivity hold.

Mary Ann Evans was George Elliot. What is required for Leibniz' Law is that for every predicate, P, Pt_1 and Pt_2 have the same truth value; and this is what the material biconditional delivers.

It might still be objected that this is not the case in LP, since $\alpha \equiv \beta$ is true (and false) if α is true only but β is both true and false. But again, this is too fast. Though the semantics are formulated formally as three-valued, there are, in fact, really only two truth values, *true* and *false*. It is just that sentences may have various combinations of these.[11] In particular, $\alpha \equiv \beta$ is true iff α and β are both true, or both false. It is easy enough to check that $\alpha \equiv \beta$ is logically equivalent to $(\alpha \wedge \beta) \vee (\neg\alpha \wedge \neg\beta)$. If α is true only and β is both true and false, *both* are true, hence one should expect the material biconditional to be true—and since one is true and the other is false, one should expect it to be false as well.

4 Identity and Consistency

Call an interpretation *classical* iff for every $A \in D_2$, $A^+ \cap A^- = \phi$. The classical interpretations are simply those where no atomic sentence—and hence no sentence at all—behaves inconsistently. The classical interpretations are, in fact, just the interpretations of classical second-order logic. And, restricted to those, the definition of identity just employed gives the classical account of identity. Thus, though some of the features of the classical account fail, they do hold when we restrict ourselves to classical models. Provided that we are reasoning about consistent situations, then, identity may be taken to behave in the orthodox fashion. I have argued elsewhere[12] that consistency should be taken as a default assumption. If this is right then the classical properties of identity may be invoked unless and until that default assumption is revoked.

The idea may be turned into a formal non-monotonic logic, minimally inconsistent LP. The details for the first order case are given in Priest (1991). How best to modify the idea so that it works in the second-order case, and so for identity, is not obvious. Here is one way. (I do not claim that it is the best.) If I is an interpretation, let $I! = \{d \in D_1 : \exists A \in D_2, d \in A^+ \cap A^-\}$. $I!$ is the set of elements in D_1 that behave inconsistently. If I_1 and I_2 are interpretations, define $I_1 \prec I_2$ (I_1 is more consistent than I_2) to mean that $I_1! \subsetneq I_2!$. I is a *minimally inconsistent* (mi) model of Σ iff I is a model of Σ and there is no $J \prec I$ such that J is a model of Σ. Finally,

[11] This comes out most clearly in the relational semantics for the logic. See Priest (2001), ch. 7.

[12] See Priest (1987), 8.4.

minimally inconsistent consequence can be defined thus:

$\Sigma \vDash_m \alpha$ iff every mi model of Σ is a model of α

If Σ is classically consistent, its mi models are its classical models. Hence, its mi consequences are simply its classical consequences. In particular, since $\{t_1 = t_2, t_2 = t_3\}$ is consistent, $t_1 = t_2, t_2 = t_3 \vDash_m t_1 = t_3$. Similarly, $t_1 = t_2 \vDash_m \alpha(t_1) \equiv \alpha(t_2)$. More generally, $\vDash_m$ is a consequence relation where irrelevant inconsistencies do not prevent classical inferences from being employed. Thus: $t_1 = t_2, Pt_1, Qt_2 \wedge \neg Qt_2 \vDash_m Pt_2$. For if I is a mi model of the premises, $\theta(t_2)$ must behave inconsistently, since $\theta(t_2) \in \theta^+(Q) \cap \theta^-(Q)$. But nothing forces $\theta(t_1)$ to behave inconsistently, so $\theta(t_1) \in \theta^+(P)$ and $\theta(t_1) \notin \theta^-(P)$. But $\forall X(Xt_1 \equiv Xt_2)$ is true, so $Pt_1 \equiv Pt_2$. Since the left hand side of this is true only, the right hand side must be at least true. Hence, Pt_2 is true. The relation is non-monotonic, however. In particular, if we add $\neg Pt_1$ as an extra premise, the left hand side is now both true and false, and the right hand side may simply be false.

In closing this part of the discussion, it is perhaps worth pointing out the following. It is not uncommon for logicians and philosophers to distinguish a class of predicates for which the substitutivity of identity holds and ones for which it fails. Extensional predicates are usually taken to be amongst the former; intentional predicates amongst the latter. For the notion of identity at hand, substitutivity may fail for all sorts of predicates, even extensional ones. What determines whether substitutivity holds is not the *kind* of predicate in question, but simply the *consistency* of the situation.[13]

5 Some Applications

So much for the theory. Let us now turn to some philosophical applications, including the topics in Section 1.

Example 1 Let us start with an object that changes its properties. Consider some object, a; and suppose, for the sake of illustration, that its properties at some time are consistent. Let P be one of these properties. Suppose that at some later time it comes to acquire, *in addition*, the property $\neg P$, all other properties remaining constant. Call the object that results b. Then even after this time, $Qa \equiv Qb$ for every Q. (Recall that

[13]For this reason, the construction will not deal with *prima facie* counter-example to substitutivity involving sentences such as 'Clarke Kent entered to phone box and Superman came out' (considered in Saul (2007)). Being in the phone box is (presumably) quite consistent.

$Pa \equiv Pb \vDash \neg Pa \equiv \neg Pb$.) Hence, $\forall X(Xa \equiv Xb)$, that is, $a = b$. But since Pa and $\neg Pb$, $\neg(Pa \equiv Pb)$; thus $\neg\forall X(Xa \equiv Xb)$. So $a \neq b$. Thus, a and b are both identical with each other and distinct from each other.

Example 2 Now extend the example. Suppose that at a subsequent time again the object loses the property P, maintaining the property $\neg P$. Call the object that results c. Again, all other properties remain constant. Then, as before, $a = b$; similarly, $b = c$. But a has a property that c lacks. Hence, it is not the case that $a = c$. Transitivity has failed.

Example 3 Next, consider the amoeba-fission case. Let B be the predicate 'occupies l_b at t_1'; similarly for C. Take it that—consistently—Bb and $\neg Cb$; and that, similarly, $\neg Bc$ and Cc. Take it also that Ba, $\neg Ba$, Ca, $\neg Ca$. Again, assume that these are the only relevant properties. Then $a = b$ and $a = c$, but it is not the case that $b = c$; moreover, Ba and $a = c$, but we do not have Bc.

Example 4 Finally, let us turn to the motor-bike of Theseus. Let us suppose that the bike goes through seven stages, at times t_0, ..., t_6. Let the motorbike at time t_i be a_i $(0 \leq i \leq 6)$. Consider the predicate 'is identical with a_0'. Arguably, this is a vague predicate. a_0 satisfies it; a_6 does not; and somehow its applicability fades out in between. In a sorites progression of the kind produced by vague predicates, it is common enough to point out that there are borderline cases, and claim that these are cases of truth value gaps. But intuition is satisfied just as well by the thought that these are truth value gluts.[14] Symmetry, after all, is what seems to be required. If we take the borderline cases to be gluts, we may expect the predicate to behave as follows. The predicate 'is identical with a_6' behaves inversely, and is also shown.

$a_0 = a_0$	$a_0 = a_1$	$a_0 = a_2$	$a_0 = a_3$			
			$a_0 \neq a_3$	$a_0 \neq a_4$	$a_0 \neq a_5$	$a_0 \neq a_6$
$a_0 \neq a_6$	$a_1 \neq a_6$	$a_2 \neq a_6$	$a_3 \neq a_6$			
			$a_3 = a_6$	$a_4 = a_6$	$a_5 = a_6$	$a_6 = a_6$

The bike undergoes various modifications, but it retains its identity as a_0 until t_4, by which time it has already become (at t_3) distinct from a_0, and identical with a_6. We also have a failure of transitivity. $a_0 = a_3$, $a_3 = a_6$; but we do not have $a_0 = a_6$. More generally, we would expect to have $a_0 = a_1$, $a_1 = a_2$, ..., $a_5 = a_6$; the failure of transitivity of identity stops us from chaining these together to obtain $a_0 = a_6$.[15]

[14] See Hyde (1997).

[15] The transition stages are to be expected to have other contradictory properties as

The Lockean example of personal identity, note, can be thought of as similar. Two persons are the same if they have a sufficient psychological continuity. But 'sufficient psychological continuity' is a vague predicate. So one should expect personal identity to be vague in just the required way.

6 Vagueness

Of course, there is a lot more to be said about sorites transitions. Vague predicates appear to be no more three-valued than two-valued. What is puzzling about sorites sequences is that there appear to be no semantically significant cut-off points at all. Thus, suppose that $a_0, ..., a_6$ is a sequence of objects in transition from being red to not being red. Then if we treat borderline cases as semantic gluts, the associated truth values may go like this:

a_0	a_1	a_2	a_3	a_4	a_5	a_6
Ra_0	Ra_1	Ra_2	Ra_3			
			$\neg Ra_3$	$\neg Ra_4$	$\neg Ra_5$	$\neg Ra_6$

And the cut-offs between *simply true* and *both true and false* (or *both true and false* and *simply false*) are just as counter-intuitive as any between *simple truth* and *simple falsity*.

In Priest (2003) I argued that versions of the forced-march sorites demonstrate that, one way or another, we are forced to admit the existence of some sort of cut-off points. All that is left for a solution to the sorites to do is to theorise the nature of the cut-off points and, crucially, explain why we find their existence so counter-intuitive. In that paper I suggested a solution in terms of metalinguistic non-transitive identity. We find the existence of a cut-off point counter-intuitive because whatever the semantic values of the relevant sentences on either side of the cut-off point, they are, in fact, the same. The failure of the transitivity of identity prevents the value bleeding from one end to the other.

The theory of non-transitive identity given in Priest (2003) is based on a fuzzy logic. But the one outlined in this paper would do just as well. Consider a language that can describe the semantic properties of the language of the red-sorites. The language has names $\mathbf{Ra}_0, ..., \mathbf{Ra}_6, \{\mathbf{1}\}, \{\mathbf{1}, \mathbf{0}\}, \{\mathbf{0}\}$, and the one-place function symbol, ν ('the truth value of'). Take an interpretation for the language in which $D_1 = \{Ra_0, ..., Ra_6, \{1\}, \{1, 0\}, \{0\}\}$, $\theta(\{\mathbf{1}\}) = \{1\}$, $\theta(\mathbf{Ra}_0) = Ra_0$, etc., and $\theta(\nu)$ is a function, f such that:

well. Thus, if the bike is black at t_0 and red at t_6 then a_3 has the property of having been black (*qua* a_0), but also the property of not having been black (*qua* a_6).

$$\begin{array}{lcll} f(Ra_i) & = & \{1\} & \text{if } 0 \le i \le 2 \\ & = & \{1,0\} & \text{if } i = 3 \\ & = & \{0\} & \text{if } 4 \le i \le 6 \\ f(t) & = & \{0\} & \text{if } t \text{ is a truth value} \end{array}$$

The first three lines give an accurate description of the table for the Ra_is. (The last line is required since f must have values for its other arguments too; what these are does not matter for what follows.)

By a suitable choice of D_2, we can ensure that for each i, the sentence in this language $\nu(\mathbf{Ra}_i) = \nu(\mathbf{Ra}_{i+1})$ is true! This may be achieved in several ways. A simple one is to impose the following constraint on D_2:

For every $A \in D_2$, $\{1,0\} \in A^+$ and $\{1,0\} \in A^-$

(Thus, the object $\{1,0\}$ is a highly paradoxical object.) If $0 \le i < 2$, then the terms $\nu(\mathbf{Ra}_i)$ and $\nu(\mathbf{Ra}_{i+1})$ both refer to $\{1\}$. Hence, for any $A \in D_2$, $\mathbf{A}\nu(\mathbf{Ra}_i)$ and $\mathbf{A}\nu(\mathbf{Ra}_{i+1})$ have the same value, and so $\mathbf{A}\nu(\mathbf{Ra}_i) \equiv \mathbf{A}\nu(\mathbf{Ra}_{i+1})$ is (at least) true. If $i = 3$, then the term $\nu(\mathbf{Ra}_4)$ refers to $\{1,0\}$, so for any $A \in D_2$, $\mathbf{A}\nu(\mathbf{Ra}_4)$ is both true and false, and so $\mathbf{A}\nu(\mathbf{Ra}_3) \equiv \mathbf{A}\nu(\mathbf{Ra}_4)$. When $0 \le 4 \le 6$, the arguments are similar.[16]

The problem with which Priest (2003) ends is how to obtain a metatheory for a vague object-language which has the same underlying logic as the object language. For fuzzy logic, this is still an open issue. But for the theory being deployed here, there are known solutions. In ch. 18 of Priest (2007), it is shown, using what the paper calls the 'model-theoretic strategy', how to formulate the metatheory for a language with underlying logic LP in a naive set theory which itself has underlying logic LP. The logic is not a second-order one, as is the case here, but the availability of sets gives the same effect. In particular, $x = y$ may be defined as: $\forall z(x \in z \equiv y \in z)$.[17] Because of the use of a material conditional, this identity has exactly the same properties as the one we have been using here. Indeed, since the theory is a naive one, in which every condition defines a set, there is very little conceptual difference between this and the

[16] Note that it is true that $\{\mathbf{1},\mathbf{0}\} = \nu(\mathbf{Ra}_3) = \nu(\mathbf{Ra}_4) = \{\mathbf{0}\}$. But even if we extended the language to be able to express the fact that $\mathbf{1} \in \{\mathbf{1},\mathbf{0}\}$, it would not follow that $\mathbf{1} \in \{\mathbf{0}\}$, due to the failure of substitutivity. This provides a solution to the extended semantic paradox given by Smiley, different from the ones given by Priest, in Smiley and Priest (1993). See p. 30 f. and p. 50 f.

[17] As a matter of fact, identity is not defined in this way in that chapter: it is taken as primitive. But essentially the same construction goes through if identity is defined as indicated.

second-order approach. We could, in fact, have avoided using second-order logic by deploying set theory and this definition of identity, instead of the second-order one. I chose not to adopt that course here so as not to raise many important but, in this context, distracting questions.

7 Conclusion

In this paper I have outlined an account of identity and some of its applications. The notion of identity does not have all the properties of the orthodox notion. Especially, transitivity fails. However, the notion may be thought of as a generalisation of the orthodox one, since, when restricted to consistent situations, the orthodox account is obtained. The idea was made precise with the notion of minimally inconsistent consequence. We have also looked at various applications of the notion, especially those that concern change. I have not discussed other approaches to the problems raised, which there certainly are; nor have I tried to mount a case that the approach deployed here is the best. But I do hope to have shown both the technical viability of this notion of identity and its potential philosophical fruitfulness.[18]

References

[1] O. Bueno (2000), 'Quasi-Truth in Quasi-Set Theory', *Synthese* 125, 33-53

[2] D. Hyde (1997), 'From Heaps and Gaps to Heaps of Gluts', *Mind* 106, 641-60.

[3] D. Krause (1992), 'On Quasi-Set Theory', *Notre Dame Journal of Formal Logic* 33, 402-11.

[4] A. F. Miller (trans.) (1969), *Hegel's Science of Logic*, London: Allen and Unwin Ltd.

[5] D. Parfitt (1984), *Reasons and Persons*, Oxford: Clarendon Press.

[6] G. Priest (1987), *In Contradiction*, Dordrecht: Kluwer Academic Publishers; second edition, Priest (2007).

[18]A version of this paper was given at the third World Conference on Paraconsistency, Toulouse, 2003. Versions have also been given at the Universities of Melbourne and St Andrews. I am grateful to the audiences on those occasions for comments and helpful suggestions.

[7] G. Priest (1991), 'Minimally Inconsistent *LP*', *Studia Logica* 50, 321-31; reprinted as ch. 16 of Priest (2007).

[8] G. Priest (1995), 'Multiple Denotation, Ambiguity, and the Strange Case of the Missing Amoeba', *Logique et Analyse* 38, 361-73.

[9] G. Priest (2001), *Introduction to Non-Classical Logic*, Cambridge: Cambridge University Press; a slighly revised form is Part 1 of, Priest (2008).

[10] G. Priest (2002a), 'The Hooded Man', *Journal of Philosophical Logic* 31, 445-67.

[11] G. Priest (2002b), 'Paraconsistent Logic', pp. 287-393, Vol. 6, of D. Gabbay and F. Guenthner (eds.), *Handbook of Philosophical Logic*, second edition, Dordrecht: Kluwer Academic Publishers.

[12] G. Priest (2003), 'A Site for Sorites', pp. 9-23 of JC Beall (ed.), *Liars and Heaps: New Essays on Paradox*, Oxford: Oxford University Press.

[13] G. Priest (2005), *Towards Non-Being*, Oxford: Oxford University Press.

[14] G. Priest (2007), *In Contradiction*, second (extended) edition, Oxford: Oxford University Press.

[15] G. Priest (2008), *Introduction to Non-Classical Logic: From If to Is*, Cambridge: Cambridge University Press.

[16] A. Prior (1968), *Papers on Time and Tense*, Oxford: Clarendon Press.

[17] J. Saul (2007), *Simple Sentences, Substitution, and Intuitions*, Oxford: Oxford University Press.

[18] T. Smiley and G. Priest (1993), 'Can Contradictions be True?', *Proceedings of the Aristotelian Society, Supplementary Volume* 67, 17-54.