RELEVANCE (RELEVANT) LOGICS

The conditional, "if … then …" (→) has been a contentious topic throughout the history of Western logic, and numerous accounts of its behavior have been proposed. One recurrent account (usually called the *material conditional*) is that $A \rightarrow B$ is true just if the antecedent, A, is false or the consequent, B, is true. This account was built into the logic of Frege and Russell, and so came to assume orthodoxy throughout much of the twentieth century (at least where there are no subjunctive moods in the antecedent or consequent). The account has obvious problems, however. It entails, for example, that both of the following are true—which they do not appear to be: "If Melbourne is the capital of Australia, it is in China" (false antecedent), "If there is life on another planet, Canberra is the capital of Australia" (true consequent).

It is natural to suppose that in a true conditional the antecedent must be relevant to the consequent in some way. This idea is packed into the contemporary definition of a relevant logic. A propositional logic is a relevant/relevance (both words are used) logic just if whenever $A \rightarrow B$ is a logical truth A and B share a propositional parameter (variable). (A quantifier logic is relevant if its propositional part is.)

Relevant logics can be of several different kinds. However, one has come to dominate current work in the area. This is the Anderson/Belnap tradition. Axiomatizations of logics (or fragments of logics) of this kind were proposed by Ivan Orlov (1928), Alonzo Church (1951), and Wilhelm Ackermann (1956). But the subject took off with the work of the Pittsburgh school of Alan Anderson and Nuel Belnap in the 1960s and 1970s. Probably the most important system of relevant logic developed by the school was the logic *R* (though Anderson and Belnap themselves preferred the system *E*). This contained most of the intuitively correct principles concerning the conditional, but not "paradoxes" such as $(A \& \neg A) \rightarrow B$ and $A \rightarrow (B \rightarrow B)$.

Semantics of various kinds for relevant logics were produced about ten years later by, among others, J. Michael Dunn, Alasdair Urquhart, and Kit Fine. But perhaps the most versatile semantics for relevant logics are the world-semantics developed by the Canberra school of Richard Sylvan (né Routley) and Robert Meyer (who had also been a member of the Pittsburgh school).

The world-semantics of relevant logics may be thought of as extending the possible-world semantics of modal logic by adding a class of logically impossible worlds—though validity is defined in terms of truth-preservation at just the possible worlds. (This comes out most clearly in the simplified form of the semantics, as later developed by Graham Priest, Sylvan, and Greg Restall.) At a possible world, w, the truth conditions for → are the same as those for the strict conditional in the modal logic *S5*:

> $A \rightarrow B$ is true at w iff for all worlds, x (possible and impossible), when A is true at x, B is true at x.

At an impossible world, logical truths—for example, of the form $B \rightarrow B$—may fail. This is achieved by giving the truth conditions of → at such a world, w, in terms of a ternary relation, $\boldsymbol{R}$:

> $A \rightarrow B$ is true at w iff for all worlds x, y, such that $\boldsymbol{R}wxy$, when A is true at x, B is true at y.

These semantics give the base member of the family of logics, *B*. Other logics in the same family may be obtained by adding constraints on the relation $\boldsymbol{R}$. The Anderson/Belnap logic, *R*, is one requiring a number of such constraints. At the time of writing, the nature of $\boldsymbol{R}$, and so of plausible constraints on it, are still contentious issues.

Another important feature of the semantics of relevant logics is their handling of negation. If $(A \& \neg A) \rightarrow B$ is not to be a logical truth, there must be worlds at which $A \& \neg A$ holds (bringing out the connection between relevant logic and paraconsistent logic). This may be achieved in (at least) two ways. In the first (due originally to Dunn), formulas may take the values *true* and *false* independently (and so may take both or neither). The truth/falsity conditions for negation at a world, w, are then:

> $\neg A$ is true at w iff A is false at w
>
> $\neg A$ is false at w iff A is true at w

If A is both true and false at w, so is $\neg A$. So (given the natural semantics for &) $A \& \neg A$ is true (and false) at w.

The second way to handle negation is to treat truth and falsity as usual, but to use the "Routley *"—invented by Valerie Routley (later Plumwood) and Sylvan. For each world, w, there is a world w^* (usually taken to satisfy the condition that $w = w^{**}$.) The truth conditions for negation are:

> $\neg A$ is true at w iff A is false at w^*

If w is w^*, then exactly one of A and $\neg A$ holds at w. But if w is distinct from w^*, and A is true at w and false at w^*, then $A \& \neg A$ is true at w. Again, at the time of writing, the philosophical meaning of * is still a contentious issue.

However the semantics of negation is handled, there will be worlds where A and $\neg A$ hold; and so, assuming the standard behavior of disjunction, where $\neg A \vee B$ holds, for arbitrary B. It follows that the disjunctive syllogism (A, $\neg A \vee B \vdash B$) is invalid. This is significant because it shows that the ramifications of relevant logic spread much wider than may have been thought. In particular, the syllogism does not seem inherently dubious in the same way that the paradoxes of the material conditional are. The invalidity of the syllogism has therefore occasioned much of the criticism attracted by relevant logic. Defenders of relevant logic have replied in various ways.

Philosophical critiques aside, relevant logics have turned out to have a number of interesting mathematical properties. For example, *R* and some of the other stronger logics (though not the weaker ones) have the unusual property (for a propositional logic) of being undecidable (as shown by Urquhart). Relevant logics are intimately related with algebraic structures called De Morgan lattices, and can also be shown to fit in to the more general class of substructural logics.

See also Logic, Non-Classical; Modal Logic; Paraconsistent Logics.

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RELEVANT ALTERNATIVES

To know a proposition, is it necessary that one is able to rule out every possibility of error associated with that proposition? Notoriously, infallibilism about knowledge—as defended, for example, in early work by Peter Unger (1975)—demands just this and argues on this basis for the skeptical conclusion that knowledge is rarely, if ever, possessed. Intuitively, however, the answer to this question is "no," in that in everyday life we only demand that knowers rule out those error-possibilities that are in some sense relevant. For example, to know that the bird before me is a goldfinch, I may be required to be able to rule out that it is not some other bird that could be in the area just now, like a jackdaw, but we would not normally demand (at least not without special reasons) that I be able to rule out the possibility that it is not a mechanical goldfinch made up to be an exact replica of the real thing.

If this line of thought is right, then this prompts a relevant alternatives (RA) theory of knowledge that demands that one only needs to be able to rule out all relevant error-possibilities in order to know, not that one is able to rule out all error-possibilities, even irrelevant ones. (A similar view could be applied to other epistemic notions, like warrant or justification. For simplicity, the focus here is on knowledge.) Such a position would thus be a form of fallibilism, which is directly opposed to infallibilism and which thereby counters those versions of skepticism that are based on infallibilist considerations. The task at hand for the RA theorist is to offer a principled account of what makes an alternative relevant.

RELEVANT ALTERNATIVES AND SENSITIVITY

One can find the beginnings of an RA theory of knowledge in the writings of such figures as Ludwig Wittgenstein and John Austin. The first worked out versions of an RA theory, however, can be found in the works of Fred Dretske (1970) and Robert Nozick (1981), who primarily understand knowledge in terms of the possession of beliefs that are sensitive to the truth in the following manner:

> *Sensitivity*
>
> An agent, S, has a sensitive belief in a true contingent proposition, p, if and only if, in the nearest possible worlds in which p is not true, S no longer believes p.

To illustrate this, consider again the example of the goldfinch discussed earlier. Given that the actual world is