

# Meinongianism and the Philosophy of Mathematics<sup>†</sup>

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## 1. Introduction

'If meinongianism isn't dead, nothing is', Gilbert Ryle is reputed to have said, in the heyday of Oxford Philosophy.<sup>1</sup> I think that Ryle was exactly right. No idea in philosophy is ever past its use-by date, at least, no idea of any substance. We may always come back and find new depths in it, new applications for it, new answers to objections that were taken to be decisive. Thus, for example, platonism has re-emerged many times in the history of Western philosophy, most recently in a perhaps unexpected place: in connection with technical results in the foundations of mathematics. Aristotelian virtue ethics has reappeared recently after a long period in which ethics has been dominated by kantianism and utilitarianism. And so the list goes on.

Of course, this is not how Ryle intended his words to be understood. What he meant was that meinongianism was dead for all time. It would perform no Lazarus-like return. For many years I shared Ryle's view. Educated about thirty years ago in Britain, I took it for granted that Russell had shown that meinongianism was little more than superstition (though one that he himself had subscribed to for quite a long time), and that Quine had shown that it was all just simple obfuscation. That which exists is that over which one can quantify; and that's that.

Thus it was that I was outraged when I met Richard Routley (Sylvan as he later became) in the mid-1970s, and found him stoutly defending a version of meinongianism. (Richard never defended a view in any other way!) I could not understand how the view could possibly be taken seriously. It was my good fortune not just to have met Richard, but to have been able to

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<sup>1</sup> I have not been able to track down the source of this quote; so it may just be hearsay.

talk with him about the matter over many years. He persuaded me that all the knock-down arguments that I thought I had were lame or just begged the question. He persuaded me that meinongianism is a very simple, natural, and common-sense view. He persuaded me that the theory has many applications to areas of philosophy where more orthodox views creak at the seams. I am still not sure whether or not I believe it; but I certainly lean towards it in certain areas.

## 2. Characterisation

Part of the beauty of meinongianism—or at least of Richard's approach to it, spelled out at length in *Exploring Meinong's Jungle* [1980]—is its technical simplicity. To do the idea full justice you need to have inconsistent and incomplete worlds, but these you have anyway, at least if you subscribe to some version of relevant logic. But the main technical trick is just thinking of one's quantifiers as existentially neutral. '∀' is understood as 'for every'; '∃' is understood as 'for some'. Existential commitment, when required, has to be provided explicitly, by way of an existence predicate, *E*, which, *pace* the way that Kant is often—and erroneously—interpreted, is a perfectly normal predicate. Thus, 'there exists something such that' is '∃*x*(*E**x* ∧ ... *x*...)'; and 'all existing things are such that' is '∀*x*(*E**x* → ... *x*...)'. The action of the theory is mainly, therefore, not at the technical level, but at the philosophical level.

There is one technical problem that was never really solved in the 1000-odd pages of *Exploring Meinong's Jungle*, however. This was the characterisation problem. Meinong insisted that the *sein* (being) of an object is independent of its *sosein* (properties): what properties an object has is quite independent of whether or not it exists. What, then, determines the properties that a non-existent object has? Quite simply, according to both Meinong and Routley, these objects have the properties that they are characterised as having. We specify an object by a certain set of conditions. These might be: *was a detective, lived in Baker St, had unusual powers of observation and inference, etc.* Let us write the conjunction of these conditions as  $\varphi(x)$ . Then if we call the object so characterised 'Sherlock Holmes', *s* for short, then *s* has its characterising properties,  $\varphi(s)$ , plus whatever properties follow from these. The idea that an object has those properties which it is characterised as having is called the *characterisation principle* (CP).

Now the trouble with this idea is that the CP cannot be correct in full generality. If it were, not only could one run the ontological argument to prove the existence of God—and everything else—one could, in fact, prove everything. For let  $\alpha$  be any sentence, and consider the condition  $x = x \wedge \alpha$ . Let *t* be the object characterised by this condition. Then the CP gives us:  $t = t \wedge \alpha$ , from which  $\alpha$  follows. It would seem, then, that only a restricted

class of contexts  $\varphi(x)$  can be used in the CP. The problem is: Which? This is the characterisation problem. There are various gestures towards a solution to the problem in *Exploring Meinong's Jungle*, but Routley never achieved there—or anywhere else as far as I am aware—a solution which he regarded as fully adequate.

There does seem to me to be a plausible solution to the characterisation problem, however. Let  $\varphi(x)$  be *any* condition, and let  $a_\varphi$  be the object that this condition characterises. Then  $\varphi(a_\varphi)$  is true—maybe not at this world, but at other worlds. Which? Cognitive agents represent the world to themselves in certain ways. These may not, in fact, be accurate representations of this world, but they may, nonetheless, be accurate representations of different possible worlds. For example, if I imagine a world in which there is a detective who lives in Baker St, etc., the way I imagine the world to be is not an accurate representation of our world, but our world *could* have been like that; that is, there is a possible world that *is* like that. More precisely, there are many such worlds, since the representation is incomplete with respect to many details, e.g., whether the detective was left-handed or right-handed. The object characterised by a representation has its characterising properties, not necessarily in the actual world, but in the worlds (partially) described by the representation (which *could* be the actual world—if the agent's characterisation is verisimilar). Thus, the object,  $t$ , characterised by the condition  $x = x \wedge \alpha$  is such that  $t = t \wedge \alpha$  in a certain set of possible but maybe non-actual worlds, and  $\alpha$  is true at those worlds.<sup>2</sup>

If the CP is to hold in full generality in this way, then for any characterisation,  $\varphi(x)$ , there must be worlds in which this characterisation is satisfiable, that is, in which there is some  $a_\varphi$  such that  $\varphi(a_\varphi)$ . In particular, there must be inconsistent worlds, since we can consider the characterising condition  $Fx \wedge \neg Fx$ . But such are present in the standard world semantics for relevant logics, so this is no problem.<sup>3</sup> This approach to the CP can be articulated in more detail. Much of this is done in Priest [2000], and I will not repeat it here. The above will suffice as background to the main subject of this paper, which is a meinongian philosophy of mathematics. To this I now turn.

<sup>2</sup> This raises the question of the status of worlds themselves. As far as I can see, what I shall go on to say is neutral on this issue. Routley himself held that worlds other than the actual are non-existent objects. This strikes me as a very sensible view.

<sup>3</sup> We also have to be careful about the semantics of modal operators. For example, there is an object characterised by  $x = x \wedge \Box\alpha$ . Call this  $t$ , then by the CP,  $t = t \wedge \Box\alpha$  is true at some world. It had therefore better not follow from  $\Box\alpha$  being true at that world that  $\alpha$  is true at this world. But this is straightforward too.  $\Box$  may have standard S5 truth conditions at normal (= possible) worlds, but different truth conditions at non-normal (= impossible) worlds.

### 3. Meinongianism and Mathematics

Meinongians hold that certain sorts of objects do not exist. Purely fictional objects are standard examples that most meinongians would agree on; but they may disagree about others. Routley held a fairly extreme view: the only objects that exist are concrete individuals (here and now). All abstract objects do not exist. In particular, then, all mathematical objects, being abstract, are non-existent objects. This is the view outlined in the paper of Routley reprinted here.<sup>4</sup>

The status of mathematical objects is a notorious problem in philosophy. All accounts seem to face difficulties. If, therefore, a coherent meinongian account could be given, this would be a notable achievement. And once one is over the meinongian hurdle, a meinongian account of mathematical objects has a certain plausibility. It at least accounts for the fact that there seems to be a very great difference in kind between ordinary concrete objects and mathematical objects. The difference between existence and non-existence would seem to be a very substantial one. The question is whether the meinongian account stands up to closer inspection.

One may have many objections to the view. A number are discussed, and to some extent disarmed, by Routley in Section 4. I think, though, that someone of a more orthodox persuasion is likely to have four very real worries about a meinongian account of mathematical objects. These are as follows.

- (1) If the objects do not exist, they cannot enter into causal connections with us. How, then, can we know anything about them?—which we certainly appear to do.
- (2) The truths about mathematical objects would seem to be a *priori*. This would not seem to be the case with the truths concerning fictional objects: we can make them up as we go along. So how can mathematical objects be non-existent objects?
- (3) We often apply mathematics to tell us about concrete objects, like shopping, bridges, microchips. How can non-existent objects possibly tell us anything about things that do exist?
- (4) The meinongian and the platonist hold there to be abstract objects. They disagree about whether or not they exist, though. But in the end this is just a difference of terminology. When the meinongian says that

<sup>4</sup> Routley [2003]. References in what follows are to this. I shall have little to say here about the first part of this paper, whose major thesis is a denial of the claim that mathematics is extensional. I note, however, that if a meinongian account of mathematical objects is correct, then the treatment of the CP given in the previous section reinforces Routley's position concerning extensionality. For if mathematics is about meinongian objects, and these have their characteristic properties in worlds other than the actual, then other-worldliness is built into mathematics. But at least as understood in standard modern logical semantics, it is precisely the essential employment of different possible worlds that is the defining moment of intensionality.

something is an object, the platonist says it exists; when the meinongian says an object exists, the platonist says that it is concrete (and exists). The meinongian is just, therefore, a platonist in disguise.

In the next four sections, I will examine each of these worries, and say what I can in defence of meinongianism.

#### 4. The Epistemology of Mathematics

The first, epistemological, problem is, of course, a problem for some other philosophies of mathematics as well. In particular, since abstract objects cannot enter into causal relations, a mathematical platonist faces exactly the same problem. It would seem that virtually any sensible answer that a platonist can give, a meinongian can give too. For example, if a platonist may invoke some special sort of intuition, as did Gödel, so can a meinongian.

However, a meinongian has other possible, and perhaps more plausible, answers. A non-existent object has those properties attributed to it by the CP, and those that follow from this. We know those properties precisely because we know the CP and can infer from it. Thus, Sherlock Holmes was characterised in a certain way by Doyle. We know that he had those properties since they are part of the characterisation. Further, we know that Holmes had a friend who was a doctor, not because Doyle tells us this, but because he tells us that Watson was Holmes's friend, and Watson was a doctor; we infer the rest.

How do we know the CP? It is, in a sense, true by definition. Care is needed here, though. According to the account of the CP given above, an instance of the CP,  $\varphi(a_\varphi)$ , is not necessarily true by definition about *this* world. Rather, it is true by definition about some worlds. Which ones? Those in which  $\varphi(a_\varphi)$  holds *ex hypothesi*. That it holds at such worlds is therefore completely trivial: there is nothing of substance to know. When one knows how characterisation works, the rest is automatic.

Applying this to mathematics: suppose that we have a mathematical object,  $c$ . Object  $c$  is characterised by some mathematical theory,  $T(c)$ .  $T(c)$  is true in a whole bunch of worlds—but not this one, since  $c$  does not exist.<sup>5</sup> Since our grasp of the CP is to explain our knowledge of the facts about  $c$ , then  $T$  should, presumably, be something that can be grasped. Hence, it is natural to require that the characterisation be axiomatic, that is, in effect, that  $T$  be an appropriate set of axioms. Suppose, for example, that  $L$  is the language of arithmetic, formulated in the usual way, with a single constant, 0. Let  $T$  be a set of arithmetic axioms, say the Peano axioms. Then  $T$  is a set of claims about 0—and various other entities—that

<sup>5</sup> Nothing that I have said so far, in fact, entails this. But assuming, as I think to be the case, that objects that do not exist at this world have only intensional properties (like being thought of, etc.) there, it does.

are true in some worlds. They characterise its behaviour at those worlds. Similar comments apply to systems other than arithmetic.

At this point, it is natural to object that this cannot explain our grasp of the properties of mathematical objects, since, in the case of arithmetic, set theory, and similar theories, no axiom system is complete, as we know by Gödel's first incompleteness theorem. Incompleteness *per se* is not a problem, however. If an axiom system for arithmetic is such that it can prove neither  $\psi(0)$  nor  $\neg\psi(0)$ , this just shows that 0 is an incomplete object: 0 simply fails to satisfy both  $\psi(x)$  and  $\neg\psi(x)$ , just as Sherlock Holmes fails to satisfy both *was left-handed* and *was right-handed*.<sup>6</sup>

However, there is also the stronger version of Gödel's theorem, according to which certain sentences are not only not provable in the axiom system, but can be shown to be true. In which case, our grasp of the properties of, say, 0, goes beyond any axiomatic characterisation. In answer to this, there are two possible replies.

The first—a fairly orthodox reaction—is to insist that the characterisation be a second-order one, and that a second-order logic be employed in determining the consequences of the axioms. As is well known, the second-order characterisation of arithmetic is categorical, and so the problem does not arise.<sup>7</sup> Naturally, there are other problems, such as the non-axiomatic nature of second-order logic itself, but I shall leave these to those who favour second-order logic.

The other reply—a much less orthodox one—is to point out that Gödel's first incompleteness theorem claims only that *consistent* (first-order) theories of arithmetic are incomplete. But inconsistent meinongian objects are quite possible, so to speak, as I have already observed. It is also well known that there are complete inconsistent theories of arithmetic.<sup>8</sup> Moreover, given that mathematics is a humanly learnable activity, there are arguments to the effect that our arithmetic is both axiomatic and inconsistent. Since these arguments may be found elsewhere, I will not pursue them here.<sup>9</sup> If inconsistency is on the cards, as it always is for meinongian objects, incompleteness is not, therefore, a problem.

<sup>6</sup> Or, maybe better, that object satisfies  $\psi(x)$  [is left-handed] at some worlds and satisfies  $\neg\psi(x)$  [is right-handed] at others.

<sup>7</sup> Well, things are a bit more complex than this if the second-order logic in question is a relevant/paraconsistent one. For paraconsistent second-order arithmetic is not categorical. For example, all paraconsistent theories have trivial models (*i.e.*, models where everything holds). However, if one juggles in the right way, one can arrange a set of second-order sentences containing the second-order Peano axioms such that every model of these is either isomorphic to the standard model or is trivial. This theory is then complete.

<sup>8</sup> See Priest [1997].

<sup>9</sup> See Priest [1987], ch. 3.

### 5. The Mathematical *A Priori*

Concerning the second objection: a similar problem, in fact, appears to apply for platonism. Some truths about abstract objects are *a priori* and some are not. For example, it is *a priori* true that the concept *red* (an abstract object) is subsumed by the concept *coloured*. But it is not an *a priori* truth that the concept *third planet from the sun* is coextensional with the concept *planet supporting life*, though this is just as much an (abstract) relation between abstract notions. A platonist must maintain that some of these matters are *a priori* and some are not. Presumably, this is a feature of the particular relations in question. Maybe it is just a brute fact about those relations. A meinongian can say the same. Some relations between non-existent objects—for example, the ordering between numbers—are *a priori* knowable; some—for example, the relationship between Sherlock Holmes and Baker St—are not.

Such an answer is, naturally, a little disappointing. Can a meinongian do better? As we have already observed, the CP is analytic in a certain sense: its truth is ensured simply by definition, though not at this world, but at others. Hence the fact stated by an instance of the CP can be known *a priori* (about those worlds); and what can be inferred from what is known *a priori* can also be known *a priori*.<sup>10</sup>

Not all facts about objects characterised by the CP need be *a priori*, though. Suppose that the agent's representation of the world is accurate and that  $\varphi(x)$  is uniquely satisfied. Thus, let  $\varphi(x)$  be '*x* is a planet in our solar system supporting life', then  $a_\varphi$  will be the Earth, and this has many properties that one cannot know *a priori*. In this way it is possible to explain why some properties are *a priori* and some are not. In particular, the properties of mathematical objects which follow from the CP are *a priori*.

But what, to come to the main objection, of fictional objects such as Sherlock Holmes? This is certainly a non-existent object, but it would appear that its properties are not *a priori*, since we can make them up as we go along. Arguably, however, the appearance is misleading. The properties of Sherlock Holmes may be just as *a priori* as those of 0. In both cases, we characterise an object purely by fiat. We know *a priori* that the object so characterised has those properties (at certain worlds), and this is so whether the characterisation is provided by what is told in Doyle's novels, or by the Peano axioms. Doyle made up the characterisation of Holmes by fiat. But the Peano characterisation also holds by fiat. Presumably, of course, a fiat that took place a long time ago, and only implicitly: in the

<sup>10</sup> Similar ideas are mooted by Routley, in connection with Problem 7, pp. 43 ff. Routley also claims that the mathematical truths are necessary. This cannot be right, at least on the present account. Facts that hold in virtue of the CP cannot be true at all worlds, or the real world would be trivial, as observed in Section 2.

practice of counting.

Let us look at the matter a bit more carefully. It is important to distinguish clearly between two sorts of activity. The first is specifying a characterisation; the second is figuring out what follows from it. It is the first of these that we normally think of in connection with fiction (making up a story). It can be done entirely *ad lib.*, and it is this fact that gives fiction its contingent feeling. But in certain contexts, we do exactly the same in mathematics. For example, Gödel initiated the study of large-cardinal axioms in set theory. Being a platonist, he assumed that some of these axioms are true and some of them are false, independently of our knowledge. But from a meinongian point of view, when we postulate a large-cardinal axiom, this is just like extending the Holmes stories. We are, in effect, creating a new characterisation, whose consequences we may then infer. And there is no right or wrong way to extend the characterisation of sets, any more than there is a right or wrong way to tell a new Holmes story: any way will do (at least, any consistent way, since we are using an explosive logic if we are doing classical set theory—if we are using a paraconsistent logic, *any* way will do).

The second sort of activity, the drawing out of consequences, is what we normally think of first in connection with mathematics. The characterisations of mathematical objects are normally fixed: mathematics comprises the deduction of what follows from these. There is nothing contingent about this: the consequences are governed by the laws of logic. It is this that gives mathematics its non-contingent feeling. But it is clear that we engage in the second sort of action with respect to fiction as well. When we come out of a film, we argue about the characters, inferring from what was shown or said. And the phenomenology of this process is, in fact, very similar, to arguing about mathematical objects, though the predicates concerned in arguing about fictional objects are mostly vague, and so interesting cases are rarely cut and dried in the same way that they are in mathematics.

Actually, things are not quite that simple. When we argue about works of fiction, we characteristically invoke information that is not part of the characterisation. For example, when arguing about what Holmes did in a story, we might infer that he couldn't have been in Edinburgh on a certain afternoon, since he was in London in the morning. There was no means of transport fast enough to get him from the one place to the other. In particular, there were no planes. It is fair game to invoke this, though Doyle never tells us that there are no planes, so it is not part of the characterisation. At least, not an explicit part; could it be implicit? Not really. It might well be implicit that Holmes's England was late Victorian England, but we need to invoke a *posteriori* information (like the fact that there were no planes around then), to know what follows from this. If this is right then we may have to invoke things other than the CP in establishing facts about

fictional objects, and so such facts may be a *posteriori* after all.

At any rate, the supposed differences between mathematical objects and fictional objects disappear under closer scrutiny, or are perfectly explicable.

### 6. Applied Mathematics

Let us turn to the third objection. How can non-existent objects tell us anything about existent ones? Our treatment of the CP would seem to exacerbate this problem. For according to this, truths about mathematical objects are truths about *another* world. How can these tell us anything about this one?

Routley<sup>11</sup> gestures at a meinongian solution to this problem. Facts about non-existent objects can inform us about existent objects since the facts about actual objects may *approximate* those about non-existent objects. Think, for example, of a frictionless plane, an ideal, but non-existent, object. A real plane is not frictionless, but it can be approximately frictionless. Hence, with suitable provisos, if  $\alpha$  is true of the ideal plane,  $\alpha$  is approximately true of the real plane. Thus, if  $\alpha$  is a claim to the effect that an object slides a certain distance across the ideal plane in time  $t$ , we can infer that an object will slide the same distance across the real plane in a time  $t \pm \varepsilon$ , where  $\varepsilon$  is a contextually determinable real number.

Even if something like this is right, the answer can be only a partial one. For on many occasions we use numbers, non-existent objects on this account, to tell us *exactly* how an existent object will behave. Thus, for example, suppose there is a particular particle, say an electron. Suppose that it is moving with a constant velocity  $v$ , and that it moves for a time  $t$ , through a distance  $d$ . Here,  $v$ ,  $t$ , and  $d$  are particular physical, not mathematical, quantities. But each of them can be assigned a certain numerical magnitude,  $v$ ,  $t$ , and  $d$ , respectively, by some measuring procedure (using clocks, rulers, etc.). Thus, for example, there is some family of observable properties,  $P_n$ , of the distance such that:

$$(1) \quad P_n d \leftrightarrow d = n.$$

This establishes a correlation of a certain kind between  $d$  and  $d$ . Call biconditionals of this kind *bridge laws*. Now, a law of motion tells us that:

$$d = v \times t.$$

Thus, if we establish by observation, via the bridge laws, that  $v = 3$  and  $t = 6$ , we infer that  $d = 3 \times 6 = 18$ , i.e., that  $P_{18}d$ . We have used pure mathematical facts to infer something about a physical quantity. Nor are we dealing with ideal objects here; the particle in question is a real-life particle.

<sup>11</sup> Problem 10, pp. 47f.

How, then, is one to explain the fact that properties of non-existent objects can tell us something about existent objects?

Actually, exactly the same question can be posed for platonism, and the answer in both cases is the same. The physical quantities in question have certain properties, and the mathematical quantities have other properties. But we can move between the one and the other because these properties have the same structure, and, specifically, because the correlation established by the bridge laws is an isomorphism.

This sort of explanation is quite general. A science, or a branch of it, concerns certain physical quantities,  $q_1, \dots, q_m$ . These have associated numerical magnitudes,  $q_1, \dots, q_m$ , determined by bridge-principles of the kind (1).<sup>12</sup> In virtue of certain physical states of affairs, and the bridge principles, we have some mathematical relation,  $F(q_1, \dots, q_m)$ —typically in physics, this would be a differential equation—and working with this we can establish various facts about the  $q_i$ , and hence, via the bridge principles, certain physical states of affairs.<sup>13</sup>

Thus, we can use facts about mathematical objects to infer facts about physical states precisely because the two have the same structure. That a certain relation obtains between the mathematical objects is an *a priori* fact; but which physical relations are isomorphic to which mathematical relations is an *a posteriori* fact. Its discovery is that of a law of nature. This explanation, which depends simply on there being certain correlations between properties of physical magnitudes and properties of mathematical magnitudes, in no way depends on the numerical magnitudes being existent (though of course, they are if platonism is correct). All it depends upon is their having the right *sosein*.

## 7. Platonism

Let us move to the last objection. This is to the effect that meinongianism is just platonism in disguise.<sup>14</sup> According to this objection, the following translation manual shows that a meinongian is simply a platonist with an unusual vocabulary.

<sup>12</sup> The properties,  $P_n$ , employed need not all be observable. Some may be establishable only by inference.

<sup>13</sup> If one is not a realist about space and time (which I am), one may suppose that there are no actual quantities of space and time, but that talking of such is just a way of talking about certain relationships between objects in space and time. One might therefore object to the particular example I used above. If one does, however, a general account of the same form can still be given. The physical quantities in question are just different (depending on how, exactly, talk of space and time is cashed out).

<sup>14</sup> An objection of this kind is made in Lewis [1990].

Meinongian	Platonist
is an object is a existent	exists is a concrete object

The objection might well be reinforced by the fact that so often, in answer to the previous objections, we have found ourselves saying exactly the same thing on behalf of the meinongian and the platonist.

There are many things to be said about this objection. The first is that translation manuals are symmetric. Hence, to suppose that the manual establishes that meinongianism is reducible to platonism is quite question-begging—at least without further argument. We might just as well say that platonism reduces to meinongianism. Without such considerations, we can just as well say that Plato was a meinongian as that Meinong was a platonist.<sup>15</sup>

Next, there are, in any cases, differences between the two positions. Crucially, the meinongian subscribes to the CP; the platonist, at least as usually understood, does not. The meinongian claims that any instance of the CP characterises a perfectly good (though maybe non-existent) object. The platonist does not normally say that an arbitrary characterisation characterises an existent object. Numbers, sets, geometrical lines and points, all these exist. But there is no reason to suppose that any old axiom system specifies existent objects.

There is a version of platonism that does claim this, however: *plenitudinous platonism*.<sup>16</sup> The plenitudinous platonist holds exactly that there is nothing privileged about the axiom systems for numbers, geometric objects, etc. Every axiom system characterises equally good abstract objects. The thought that every (consistent) axiom system has a model gives some credence to plenitudinous platonism. The fact that a sentence has a model does not show that it is really satisfiable by certain objects. For example, there is a first-order model of ' $\exists x(x \text{ is married} \wedge x \text{ is a bachelor})$ ', though there is no existent object,  $x$ , such that  $x$  is married  $\wedge x$  is a bachelor. Still, models are very much like realities, and the fact that every (consistent) characterisation has a model at least gives us a model (so to speak), of what it would be like for every characterisation to characterise existent objects (from a platonist point of view).

The confluence between meinongianism and plenitudinous platonism is

<sup>15</sup> Actually, this is one place where the translation manual is certainly not fully adequate; for Plato held that the forms were not only existent (real), but that they were *more* existent (real) than concrete objects. Meinong most certainly did not claim that non-existent objects were more objects than concrete ones.

<sup>16</sup> I take the name from Field [1998]. The view is advocated by Balaguer [1995], where it is called 'full-blooded platonism'. Balaguer defends this platonism against the epistemological objection of Section 4 on grounds very similar to those on which I defend meinongianism in that section.

still not quite right, though. A thorough-going meinongian holds that every characterisation characterises an object. And here, 'every' means *every*. Even inconsistent characterisations do this. This diet is probably too rich for even a plenitudinous platonist. Platonists are characteristically very much attached to consistency. So this is an important difference between the meinongian and the plenitudinous platonist. Of course, there is still another position out there. This belongs to what we might call the *paraconsistent plenitudinous platonist*. This is a platonist who has foresworn classical logic, and is prepared to endorse a paraconsistent logic.<sup>17</sup> Such a platonist can hold, quite generally, that every characterisation characterises existent objects.

Of course, even this sort of platonist cannot hold that every characterisation characterises an existent object at this world. As we observed in Section 2, if the CP is true at this world, the world is trivial. Hence, the paraconsistent plenitudinous platonist must hold that many of the objects characterised by the CP exist, not at this world, but at others.

At this point, the differences between meinongianism and platonism are disappearing fast. And it must be said that it is the platonist who is making all the concessions. This is a reason to say that the sort of platonism that is left is really meinongianism in disguise, and not *vice versa*.

But speaking of different possible worlds, there is yet one more difference between meinongianism and platonism. The translation manual may work well enough at this world, but not at others. For a meinongian, though some objects do not exist at this world, they exist at others. Sherlock Holmes is such an object. The meinongian must, in fact, claim that some objects, though mathematical, exist at some worlds. (I do not assume that an object must exist at a world at which it has its characterising properties.) Plato's story about the existence of a realm of abstract objects, where the forms and the mathematical objects reside, is a perfectly good story. It is therefore true at some world. Or, appealing to the CP, if  $T(0)$  represents the Peano axioms, and  $E$  is the existence predicate, there is some world in which  $T(0) \wedge E0$  is true. A platonist, by contrast, even a paraconsistent plenitudinous platonist, is unlikely to claim that at some worlds, mathematical objects are *concrete* objects. The thought that the number 3 could be the sort of thing that could be kicked would appear to be beyond the pale for anyone worthy of the name 'platonist'.<sup>18</sup>

<sup>17</sup> This position is mooted in Beall [1999].

<sup>18</sup> Though I have certainly heard the view defended in discussion. If one is prepared to accept this, as well as all the other modifications to standard platonism, then I can see no difference between the platonism in question and meinongianism (at least as concerns the philosophy of mathematics). However, this is certainly a version of platonism that looks more like meinongianism than *vice versa*.

### 8. Conclusion

There may, of course, be other objections to a meinongian account of mathematical objects. Equally, a number of the moves made above in defence of such an account are certainly contestable (even by a meinongian, who might not like the account of the CP I have employed). But Routley remarks (p. 34) concerning the philosophy of mathematics, that 'the area is a frontier one' for meinongianism, and that his view is 'in a pioneer state'. I hope, at least, that this paper shows a little more of the lie of the land.

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ABSTRACT. This paper articulates Sylvan's theory of mathematical objects as non-existent, by improving (arguably) his treatment of the Characterisation Postulate. It then defends the theory against a number of natural objections, including one according to which the account is just platonism in disguise.