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JOHN W. YOLTON

LOGIC. *See:* MATHEMATICAL LOGIC, and PHILOSOPHICAL LOGIC [S]

LOGIC, NONSTANDARD. Logic is that discipline that aims to give an account of what inferences are valid and why. Although it is common to distinguish between two sets of criteria for validity—inductive and deductive—most work in the history of logic has focused on deductive validity. Since the mathematization of logic around the turn of this century, accounts of deductive validity have been given for inferences couched in formal languages. A common practice is to specify validity in terms of some set of axioms or rules and justify it by way of some semantics. To obtain applications of the account, an understanding of the relationship between the formal language and the vernacular (often in the form of some imprecisely specified translation manual) has also to be provided.

The correct characterization of and justification for validity have historically been matters of philosophical contention. There is, however, an orthodoxy (if not unanimity) on the issue that dates back to around the 1920s. The account is essentially that of FREGE's [3; S] *Begriffschrift* and RUSSELL [7] and WHITEHEAD's [8] *Principia Mathematica,* as cleaned up and articulated by subsequent logicians such as HILBERT [3] and Gentzen. It is often known (rather inappropriately) as classical logic and is to be found in virtually any modern logic textbook (see LOGIC, MODERN [4]).

Despite the relative orthodoxy, classical logic faces a number of problems. These have bred dissatisfaction and attempts to articulate rival accounts of deductive validity. Such accounts are commonly referred to as nonstandard logics.

MODAL AND INTUITIONIST LOGICS

Criticisms of classical logic go back to the 1920s. One was given by C. I. Lewis, who objected to paradoxes of "material implication" such as $\alpha \rightarrow (\neg\alpha \rightarrow \beta)$. He constructed systems of logic that contained a notion of strict implication that avoided these paradoxes. This led to an investigation of the logic of modalities, such as necessity and possibility, and modal logic developed (see LOGIC, MODAL [4] and MODAL LOGIC [S]). Lewis's original critique of classical logic is now generally thought to be based on a confusion of the connective 'if' and the relation of entailment. Consequently, modal logic is now usually thought of as an augmentation of classical logic by modal functors rather than as a rival. This development was accentuated by the discovery of world semantics for modal logics in the 1960s by KRIPKE [S] and others. World semantics have, however, provided the basis for the semantics of many new logics. One of these, another augmentation of classical logic, tense logic, was invented by PRIOR [S]. In this, temporal operators—for example, F (it will be the case) and P (it was the case)—are added to the language and given suitable semantic conditions.

A second early critique was provided by Brouwer and other intuitionists (see LOGIC, HISTORY OF [4]). Arguing on the basis of a critique of a Platonist philosophy of mathematics, they rejected a number of principles of classical logic, such as $\neg\neg\alpha \rightarrow \alpha$, $\neg\forall x\alpha \rightarrow \exists x\neg\alpha$. For example, the second of these fails because the mere fact that you can show, for instance, that not all numbers have a certain property does not show how to construct a number that does not have it, which is what is required to ground an existence claim. In the light of these criticisms Heyting formulated an axiom system for intuitionist logic with an informal semantics in terms of provability. After a fairly quiet period the study of intuitionist logic took off again in the 1960s and 1970s. Kripke demonstrated that the logic has a world semantics; DUMMETT [S] introduced new arguments for INTUITIONISM [S] (not just in mathematics) based on the PHILOSOPHY OF LANGUAGE [S]; and applications of the logic in computer science were discovered.

MANY-VALUED AND QUANTUM LOGICS

A third critique dating back to the 1920s was provided by LUKASIEWICZ [5]. Arguing on the basis of the indeterminacy of future events, he introduced a system of logic where sentences can be neither true nor false, and so classical principles such as $\alpha \vee \neg\alpha$ fail. The system was quickly generalized to ones where sentences can have arbitrarily many semantic values, many-valued logics (see LOGIC, MANY-VALUED [4]). The study of many-valued logics accelerated in the 1960s and 1970s. Many logicians suggested that certain kinds of sentences might have no truth value: for example, they are "meaningless" ('It's 3 P.M. at the North Pole'); they are paradoxical ('This sentence is false'); they are vague ('Dry grass is green'). Consequently, we have seen the articulation of various three-valued logics (sometimes called partial logics). Consideration of VAGUENESS [8; S] also makes it tempting to suppose that truth comes by degrees. A natural way of handling this insight is by using a different sort of many-valued logic, where sentences can have as truth value any real number between 0 (wholly false) and 1 (wholly true). Under the influence of writers such as Zadeh, such logics, now usually called fuzzy logics, have found applications in ARTIFICIAL INTELLIGENCE [S]. (*See* FUZZY LOGIC [S])

In the 1930s REICHENBACH [7] and Destouches-Février suggested that various problems in quantum mechanics made it appropriate to use a three-valued logic there. These ideas were not very successful, but similar problems led Birkhoff and von NEUMANN [5] to suggest a more sophisticated approach around the same time. This is now usually called quantum logic (and has, again, received much further attention by logicians such as PUTNAM [S] since the 1960s). They argued that the classical principle of distribution, $\alpha \wedge (\beta \vee \gamma) \vdash (\alpha \wedge \beta) \vee (\alpha \wedge \gamma)$, fails in the microworld. For example, it may be true (verifiable) of a particle that it has a position and one of a range of momenta, but each disjunct attributing to it that position and a particular momentum is false (unverifiable). To construct a logic in which distribution fails, they proceeded essentially as follows. In standard world semantics, sentences can be thought of as taking subsets of the set of worlds as semantic values, and the logical constants can be interpreted as Boolean operations. In quantum mechanics the possible states of the system form a mathematical structure known as a Hilbert space, and it is natural to take the semantic values to be subspaces of this space. Logical constants are then interpreted as appropriate operations on subspaces. For example, disjunction in interpreted as the span of (the smallest space containing) two subspaces.

RELEVANT AND PARACONSISTENT LOGICS

The 1960s and 1970s saw not only the development of many older nonstandard logics but the production of many new kinds. One of these was relevant logic. This grew, like modal logic, from dissatisfaction with the paradoxes of material implication (and those of strict implication, such as $\alpha \wedge \neg\alpha \vdash \beta$). Building on early work of Church and Ackermann, Anderson, Belnap, and coworkers constructed axiom systems for three propositional (and later predicate) logics E, R, and T, which satisfied the criterion that if $\alpha \rightarrow \beta$ is provable, α and β share a propositional parameter. Semantics for the systems came a little later. In particular, world semantics were provided by Routley and Meyer. In the light of these it became clear that there are many more, and possibly more important, systems in the family. One of these is closely related to linear logic, proposed independently in the 1980s by Girard for its applications in computer science.

There are two features of world semantics for relevant logics that distinguish them from those of modal logic. The first is that a ternary relation (instead of a binary one) is used to give the truth conditions of the conditional: $\alpha \rightarrow \beta$ is true at world w if and only if (iff) for all worlds x, y such that $Rwxy$, if α is true at x, β is true at y. The second is that some technique is required to produce worlds that are inconsistent or incomplete. This can be done in two ways. The first is to have an operator on worlds, *, such that α is true at world w iff α is not true at world w^*. (For the worlds of ordinary modal logic, * can be thought of as just the identity function.) The second is to allow sentences to take one of four truth values: true only, false only, both, neither ($\{1\}$, $\{0\}$, $\{1, 0\}$, ϕ). These semantics therefore combine the techniques of both modal and many-valued logic. Standard relevant logics invalidate not only the paradoxes of material implication but also the disjunctive syllogism: $\neg\alpha \wedge (\alpha \vee \beta) \vdash \beta$. Much of the critique of relevant logic has focused on this fact.

A related contemporary nonstandard logic is paraconsistent logic. A logic is paraconsistent iff the inference $\alpha, \neg\alpha \vdash \beta$ fails. (A paraconsistent logic may or may not be relevant.) Paraconsistent logics were developed independently by Jaśkowski, da Costa, and others. Their principle concern was the use of such a logic to make inferences in a sensible way in situations where the information from which conclusions are drawn may be inconsistent—for example, from scientific theories whose principles conflict, or where the information is that in some computational data base. Semantics for paraconsistent logics use techniques such as those used in relevant logic to allow contradictions to be true in an interpretation. Some (though not all) paraconsistent logicians, such as Priest, have endorsed the view that some contradictions may actually be true (*simpliciter*): dialetheism. A major argument for this view is provided by the paradoxes of self-reference (see LOGICAL PARADOXES [5]). Consistent solutions to these are notoriously problematic.

CONDITIONAL AND FREE LOGICS

Dissatisfaction with the material conditional (at least as an account of English subjunctive conditionals) triggered another nonstandard logic around the 1970s: conditional logics. A number of counterexamples were provided by Stalnaker and others to classical principles such as transitivity ($\alpha \rightarrow \beta, \beta \rightarrow \gamma \vdash \alpha \rightarrow \gamma$) and antecedent strengthening ($\alpha \rightarrow \beta \vdash (\alpha \wedge \gamma) \rightarrow \beta$). For example, if you strike this match it will light; hence, if you strike this match and it is under water it will light. In conditional logics these inferences are invalid. The major technique used to achieve this end is a selection function, s, which, given a world, w, and a sentence, α, picks out a set of worlds $s(w, \alpha)$. Intuitively, this can be thought of as the set of worlds relevantly similar to w where α is true. The truth conditions are then as follows: $\alpha \rightarrow \beta$ is true at w iff β is true at every world in $s(w,\alpha)$. Standard conditional logics validate the paradoxes of strict implication, but relevant conditional logics may be obtained by combining the appropriate semantic techniques.

Another kind of nonstandard logic takes issue with the principle built into classical semantics that every name denotes (an existent object). (Consider, e.g., 'Sherlock Holmes'.) Logics that reject this are called free logics. One approach to free logics is to take all sentences containing nondenoting terms to be truth valueless. This idea gives rise to various three-valued logics, proposed in the 1960s by Smiley and others. A sophistication of this idea was proposed by van Fraassen. Given any evaluation of this kind, a supervaluation is any two-valued evaluation that agrees with it except where it makes a sentence neither true nor false. Logical validity is now defined in terms of supervaluations rather than evaluations. This construction allows all classical validities to be preserved.

A rather different approach was suggested by Leonard, Lambert, and others around the same time. This approach modifies the classical rule of existential generalization, $\exists x\alpha(x) \rightarrow \alpha(c)$ (and its dual, universal instantiation), by adding a conjunct to the antecedent to the effect that c exists. An appropriate semantics can be obtained by allowing constants to denote objects outside the domain of quantification. This is a form of Meinongianism (see MEINONG [5]), since it allows nonexistent objects to be named (but not quantified over). Some neo-Meinongians (e.g., Routley) allow them to be quantified over as well. This requires no change to the formal machinery of classical logic. All that has to be changed is the canonical interpretation of the quantifiers in English. Thus '$\exists x\alpha$' is now read, not as 'There exists an x such that α', but as 'For some x, α', where this expression is devoid of any existential commitment.

Whether or not any of the nonstandard logics discussed here are correct, their presence serves to remind that logic is not a set of received truths but a discipline where competing theories concerning validity vie with each other. The case for each theory—including a received theory—has to be investigated on its merits. This requires detailed philosophical investigations concerning existence, truth and contradiction, truth in quantum mechanics, or whatever is appropriate. Detailed discussions can hardly be attempted here. Some can be found in the items cited in the bibliography, which should also be consulted for further historical and technical details.

(SEE ALSO: *Mathematical Logic; Philosophical Logic* [S])

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GRAHAM PRIEST

LOGICAL CONSEQUENCE. Logical consequence is a relation between a set of sentences and a sentence said to follow logically from it. Closely related notions are logical validity and logical truth: an argument is logically valid iff (if and only if) its conclusion is a logical consequence of the set of its premises; a sentence is logically true iff it is a logical consequence of any set of sentences. Other notions definable in terms of logical consequence are logical consistency, logical equivalence, theory, and so forth.

Modern logic offers two distinct concepts of logical consequence: a proof-theoretical concept, "derivability" or "provability," symbolized by $\vdash$, and a semantic concept, "logical consequence (proper)," "logical implication," or "logical entailment," symbolized by $\models$. Given a formal system $\mathscr{L}$ (a formal language together with a proof system and a system of models), the two concepts are defined as follows: if σ is a sentence and Σ is a set of sentences, then $\Sigma \vdash \sigma$ iff there is a proof of σ from Σ, and $\Sigma \models \sigma$ iff every model of Σ is a model of σ, i.e., iff there is no model in which all the sentences of Σ are true and σ is false. GÖDEL'S [S] 1930 completeness theorem establishes the coextensionality of $\vdash$ and $\models$ in standard first-order logic, but in general the two notions are not coextensional (*see* GÖDEL'S THEOREM [3]). The term "logical consequence" is usually reserved for the semantic notion, a tradition followed in this article.

The semantic definition of logical consequence is due to Alfred TARSKI (1936; [8]). An informal version of this definition was implicit in earlier works by Gödel, HILBERT [3], and others, but it was Tarski's treatment of logical consequence and related semantic notions that allowed a rigorous mathematical study of these notions and led to the modern conception of logic as constituted by two equally fundamental disciplines: PROOF THEORY [S] (the theory of $\vdash$) and MODEL THEORY [S] (the theory of $\models$).

Tarski claimed his definition captured the intuitive, everyday notion of logical consequence. He characterized this notion by the following two traits: (i) if σ is a logical consequence of Σ, then "it can never happen" (1936, p. 414) that all the sentences of Σ are true and σ is false; (ii) logical consequences are formal, and as such they are dependent on the form of the sentences involved, not on the particular objects referred to in these sentences. Neither the proof-theoretical definition nor the substitutional definition of logical consequence, Tarski contended, accurately captures the intuitive notion. The proof-theoretical definition leaves some genuinely logical consequences unaccounted for, and it follows from Gödel's incompleteness theorem that no matter how many new (finite, structural), rules we add to the proof-theoretical apparatus, any reasonably rich (higher-order) deductive theory would have consequences which follow logically from it in the intuitive sense yet are not provable from its theorems. The substitutional definition fails in languages with an insufficient stock of nonlogical (substitutional) terms. This definition says that σ is a logical consequence of Σ iff there is no substitution under which all the sentences of Σ are true and σ is false (where substitutions preserve grammatical categories, are uniform, and are restricted to nonlogical constants), but if the language lacks in expressive resources, a failure to satisfy (i) may not be witnessed by an appropriate substitution. Tarski's own definition uses semantic tools (1933). Semantics, according to Tarski, deals with concepts relating language to the "world" (objects in a broad sense), the basic notion being satisfaction—a relation between a formula and a sequence of objects (in the universe of discourse). Model of σ (Σ) is defined in terms of satisfaction: Let σ (Σ) be a sentence (a set of sentences) of a formalized language L. By replacing all the nonlogical constants of σ (Σ) by variables in a proper manner, i.e., preserving syntactic