# Negation as Cancellation, and Connexive Logic

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ABSTRACT. Of the various accounts of negation that have been offered by logicians in the history of Western logic, that of negation as cancellation is a very distinctive one, quite different from the explosive accounts of modern "classical" and intuitionist logics, and from the accounts offered in standard relevant and paraconsistent logics. Despite its ancient origin, however, a precise understanding of the notion is still wanting. The first half of this paper offers one. Both conceptually and historically, the account of negation as cancellation is intimately connected with connexivist principles such as $\neg(\alpha \to \neg\alpha)$. Despite this, standard connexivist logics incorporate quite different accounts of negation. The second half of the paper shows how the cancellation account of negation of the first part gives rise to a semantics for a simple connexivist logic.

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Both conceptually and historically, this account of negation is intimately connected with connexivist principles such as $\neg(\alpha \to \neg\alpha)$. Despite this, standard connexivist logics incorporate quite different accounts of negation. In the second half of the paper I will show how the cancellation account of negation of the first part gives rise to a semantics for a simple connexivist logic.

## 1. Negation as cancellation

### 1.1. *Three accounts of negation*

Accounts of negation fall into three distinct kinds, which can be defined in terms of the behavior of the contradiction, $\alpha \wedge \neg\alpha$, to which the negation gives rise.[1]

The first account is a *total* one. According to this, the content of the contradiction $\alpha \wedge \neg\alpha$ is total. A contradiction therefore entails everything, entailment being cashed out in terms of containment of content. Accounts of this kind are, historically, the most recent of the three kinds, but are also now the most orthodox, the position being entrenched in modern explosive logics such as "classical" logic and intuitionistic logic.

The second account is a *partial* account. According to this, a contradiction $\alpha \wedge \neg\alpha$ has, in general, some content, but not all. A contradiction therefore entails some things, but not others. Such accounts are now familiar from modern paraconsistent logics. For example, in standard relevant logics $\alpha \wedge \neg\alpha$ entails $\alpha$ and $\neg\alpha$, but not an arbitrary $\beta$.

The third kind of account is the least familiar to modern logicians, though it is, arguably, the most venerable. It is the *null* account. According to such an account, a contradiction has no content. Accordingly, $\alpha \wedge \neg\alpha$ entails nothing. Consider the following quotation from Strawson (1952, p. 2f.):

> Suppose a man sets out to walk to a certain place; but when he gets half way there, he turns round and comes back again. This may not be pointless. But, from the point of view of change of position, it is as if he had never set out. And so a man who contradicts himself may have succeeded in exercising his vocal chords. But from the point of view of imparting information, or communicating facts (or falsehoods) it is as if he had never opened his mouth. He utters words, but does not say anything. . . . The point is that the standard function of speech, the intention to communicate something, is frustrated by self-contradiction. Contradiction is like writing something down and erasing it, or putting a line through it. A contradiction cancels itself and leaves nothing.

The quotation illustrates a null account very clearly. The difference between such an account and a total account is also clear. On a total account, a person who asserts

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a contradiction is most certainly in a different position from one who has not opened their mouth. They are committed to *everything*, not nothing. They may also have communicated a falsehood – if they can get their hearer to believe them.

Despite this, the formal account of negation that Strawson gives later in the book is the familiar explosive one of classical logic. This illustrates a further fact: explosive accounts of negation are so entrenched in some people's minds that even when they give an entirely different account, this fact is opaque to them. Both themes are further illustrated in the following quotation from Findlay:[2]

> . . . a contradiction is for the majority of logical thinkers, a self-nullifying utterance, one that puts forward an assertion and then takes it back in the same breath, and so really says nothing. It can readily be shown that a language system which admits even *one* contradiction among its sentences, is also a system in which *anything whatever* can be proved. . . .

### 1.2. *A little history*

The cancellation view of negation is, as I said, the oldest of the three kinds of view of negation. It's germ is to be found in Aristotle. At *Prior Analytics* $57^b3$, Aristotle claims that contradictories cannot both entail the same thing. Now suppose that (in modern notation) $\alpha \wedge \neg\alpha$ entailed both $\alpha$ and $\neg\alpha$, then, by contraposition (which Aristotle endorses immediately before this), each of $\neg\alpha$ and $\neg\neg\alpha$ would entail $\neg(\alpha \wedge \neg\alpha)$. Hence, as must have been obvious to Aristotle, a contradiction cannot entail both conjuncts, and so, presumably, either conjunct. These are the prime candidates for something a contradiction may entail.[3]

Presumably influenced by Aristotle, Boethius certainly seems to have subscribed to a null account of negation.[4] More of him anon. The view must have been a commonplace in the early middle ages since Abelard tells us:[5]

> No one doubts that [a statement entailing its negation] is improper and embarrassing (*inconveniens*) since the truth of one of two propositions which divide truth [i.e., contradictories] not only does not require the truth of the other but rather entirely expels and extinguishes it.

The view certainly became less than orthodox in the later middle ages, for reasons that we will come to, but it survives in places well into the 18th century. For example, Berkeley tells us in the *Analyst*:[6]

> Nothing is plainer than that no just conclusion can be directly drawn from two inconsistent premises. You may indeed suppose any thing possible: But afterwards you may not suppose anything that destroys what you first supposed: or, if you do, you must begin *de novo*. . . . [When] you . . . destroy one supposition by another . . . you may not retain the consequences, or any part of the consequences, of the first supposition so destroyed.

And despite the modern dominance of explosive views of negation, the cancellation view still resonates into the 20th century, as we have already seen.

### 1.3. *The account made precise*

Consequence relations incorporating total or partial accounts of negation are now very familiar. The same cannot be said of null accounts, however, despite the antiquity of the tradition. In this section I will suggest one. It is clear that a null account of negation gives rise to a paraconsistent logic. Most such logics are partial, not null, however. The following is an exception.

We will stick to propositional logic with the connectives $\neg$, $\wedge$ and $\vee$ ($\supset$ can be defined in the usual way). The extension of the idea to quantifier logic is obvious. I will use lower case Roman letters for propositional parameters, lower case Greeks for arbitrary formulas, and upper case Greeks for sets of formulas. I will often omit set braces. I will use "consistent" and its cognates for classical consistency, and use $\vdash$ for classical entailment.

The main problem in formulating a null account of negation, as should be clear, is how to make sense of the idea that a contradiction has no content. We will enforce this in the most simple-minded way. Let us say that:

$$\Sigma \models \alpha \text{ iff } \Sigma \text{ is consistent, and } \Sigma \vdash \alpha$$

Thus, an inconsistent set of sentences entails nothing.[7] A feature of this definition is that it does not validate contraposition. As is easy to check, $p \models p \vee \neg p$, but $\neg(p \vee \neg p) \not\models \neg p$. For reasons that will become clear later, it will be useful to have an account with contraposition built into it. An alternative such definition is:

$$\Sigma \models \alpha \text{ iff } \Sigma \text{ is consistent, } \neg\alpha \text{ is consistent and } \Sigma \vdash \alpha$$

I will call this account the *symmetrised* account. In what follows, $\models$ may be either of the above relations unless otherwise stated.

Both accounts are transparent, and need no long discussion. It is clear that $\alpha \wedge \neg\alpha \not\models \beta$, as required. It will be worth noting a few of the more unusual features of the account, however.

First, $\models$ is non-monotonic. As is easy to see: $p$, $\neg p \vee q \models q$; but: $p, \neg p, \neg p \vee q \not\models q$. This is as it should be with negation as cancellation, since it is part of the very conception that adding premises may, in fact, *take away* information.

Next, like most non-monotonic logics, $\models$ is not closed under uniform substitution. For example, $p \models p$, but $p \wedge \neg p \not\models p \wedge \neg p$. This, too, is in keeping with the notion of negation as cancellation. For a substitution instance of a consistent sentence may well be a contradiction, and so substitution may also cancel consequence. (Note, though, that one could, if one wished, generate a logic closed under substitution, simply by taking the closure of $\models$ in the obvious way. In the case of the symmetrised account, doing this gives the non-transitive paraconsistent logic of Smiley (1959).)

Though the properties of $\models$, then, are rather unusual, they are exactly what one would expect of a consequence relation suitable for negation as cancellation. Let us now move on to the related subject of connexivism.

## 2. Connexivist logic

### 2.1. *Connexivist principles*

There are various principles concerning entailment that are characteristically connexivist. Two of the most important are:

(*Ar1*) $\neg(\neg\alpha \models \alpha)$

(*Ab1*) $\neg((\alpha \models \beta) \wedge (\neg\alpha \models \beta))$

The first says that nothing is entailed by its own negation. The second says that contradictories cannot entail the same thing. These principles are connected. *Ar1* follows from *Ab1* given the law of identity, $(\alpha \models \alpha)$ and the Disjunctive Syllogism. (Substitute $\alpha$ for $\beta$.) Conversely, suppose for *reductio* that $(\alpha \models \beta) \wedge (\neg\alpha \models \beta)$. Contraposing the first conjunct and chaining the two together with Transitivity gives $\neg\beta \models \beta$, contradicting *Ar1*.

Assuming the Law of Double Negation, *Ar1* can be formulated slightly differently as:

(*Ar2*) $\neg(\alpha \models \neg\alpha)$

Assuming also (perhaps more contentiously) Contraposition, *Ab1* can be similarly reformulated. Substituting $\neg\beta$ for $\beta$, and contraposing the entailments gives:

(*Ab2*) $\neg((\beta \models \neg\alpha) \wedge (\beta \models \alpha))$

According to this principle, something cannot entail contradictory conclusions.

Variants of *Ab1* and *Ab2* can be obtained by writing them as conditionals, thus:

(*Bo1*) If $\alpha \models \beta$ then $\neg(\neg\alpha \models \beta)$

(*Bo2*) If $\alpha \models \beta$ then $\neg(\alpha \models \neg\beta)$

A word on terminology: the "Ar" in *Ar1* and *Ar2* stands for "Aristotle," a name frequently given to these principles; the "Bo" in *Bo1* and *Bo2* stands for "Boethius," a name frequently given to those principles; the "Ab" in *Ab1* and *Ab2* stands for "Abelard." Justifications for these appellations will appear in a moment.[8] Where each of the pairs differ, the 1 version has a negation on the left of a "$\models$" where the 2 version has a negation on the right of the corresponding "$\models$."

One thing that makes all these principles highly distinctive is that none of them holds classically. For example, $\alpha \vdash \alpha \vee \neg\alpha$ and $\neg\alpha \vdash \alpha \vee \neg\alpha$, contradicting *Ab1*. And $\neg(\alpha \vee \neg\alpha) \vdash \alpha \vee \neg\alpha$, contradicting *Ar1*. Similarly for the others.

Neither, if we replace "$\models$" (and the "if" of *Bo1* and *Bo2*) with a conditional connective, do we get classical tautologies. Since classical logic is Post-complete, the addition of such formulas to classical logic renders the result trivial. The addition of the corresponding things to a relevant logic, at least as strong as *B*, may not render the logic trivial,[9] but it certainly renders it inconsistent. For example, in *B*, $(\alpha \wedge \neg\alpha) \rightarrow \neg\alpha$ and $(\alpha \wedge \neg\alpha) \rightarrow \alpha$. Contraposing the second, and chaining together with Transitivity gives $(\alpha \wedge \neg\alpha) \rightarrow \neg(\alpha \wedge \neg\alpha)$, which contradicts *Ar2*. Invoking $(\alpha \wedge \neg\alpha) \rightarrow (\alpha \wedge \neg\alpha)$ contradicts an instance of *Ab2*, and, with *Bo2*, also gives a contradiction. Similarly, in *B*, $\alpha \rightarrow (\alpha \vee \neg\alpha)$ and $\neg\alpha \rightarrow (\alpha \vee \neg\alpha)$. Contraposing the second and chaining gives: $\neg(\alpha \vee \neg\alpha) \rightarrow (\alpha \vee \neg\alpha)$, giving inconsistencies with *Ar1*, with *Ab1*, and with *Bo1*.

## 2.2. *A little more history*

Despite their strangeness to the modern eye, the connexive principles are of venerable lineage. As I have already noted, *Ab1* was stated explicitly by Aristotle at *Prior Analytics* $57^b3$:[10]

> . . . it is impossible that the same thing should be necessitated by the being and by the not-being of the same thing. I mean, for example, that it is impossible that *B* should necessarily be great since *A* is white and that *B* should necessarily be great since *A* is not white.

He goes on to argue for the principle. The argument assumes *Ar1*, and is essentially the one given in the previous section.

Aristotle's endorsement of connexivist principles greatly influenced subsequent logicians. Boethius, in particular, endorsed a number of connexivist principles. He states, for example, that "$\alpha$ entails $\neg\beta$" is the negation of "$\alpha$ entails $\beta$." *Ab2* follows, as (given double negation) does *Bo2*. He even went as far as to endorse the *converse* of at least *Bo2*.[11]

Abelard was much influenced by Boethius; and he, too, endorsed connexivist principles including *Ar1*, *Ar2*, *Ab1*, and *Ab2*. We have already noted Abelard's comments on the possibility of something implying its own negation (*Ar2*), which I now repeat:

> No one doubts that [a statement entailing its negation] is improper and embarrassing (*inconveniens*) since the truth of one of two propositions which divide truth [i.e., contradictories] not only does not require the truth of the other but rather entirely expels and extinguishes it.

After Abelard, connexivist principles started to fall into disrepute. One reason for this is their tendency, as we have seen, to lead to inconsistency in the presence of extensional conjunction and disjunction principles, such as $(\alpha \wedge \neg\alpha) \models \alpha$ and $\alpha \models (\alpha \vee \neg\alpha)$ – principles that started to find general acceptance in the middle ages, but that anyone who subscribes to a cancellation view of negation is more than likely to reject. But the connexivist principles are certainly to be found in later writers. For example, we find Kilwardby in his 13th Century commentary on the *Prior Analytics* saying:[12]

> What is understood in some thing or things, follows from it or from them by a necessary and natural consequence; and so of necessity if one of a pair of opposites is repugnant to the premises . . . the other follows from them. If one of the opposites does not follow, the other can stand. *If one of the opposites stands, the other cannot*.

This appears to be an endorsement of at least *Bo2* (and maybe also the converse of *Bo2*). Kilwardby also endorsed extensional disjunction principles, however, and so was in some trouble.[13]

## 2.3. *The connection*

The connexivist principles appear rather odd to the modern eye, and it is not clear what might justify them. The answer is, in fact, simple. They are all justified by the null account of negation. The quotation from Abelard above clearly links the two ideas. The connection is also explained by Routley and Routley (1985), p. 205, as follows:[14]

> Entailment is inclusion of logical content. So if *A* were to entail $\sim A$, it would include as part of its content what neutralizes it, $\sim A$, in which event it would entail nothing, having no content. So it is not the case that *A* entails $\sim A$, that is, Aristotle's thesis, $\sim(A \rightarrow \sim A)$, holds.

The connection between connexivism and negation as cancellation can be shown by the fact that the semantics of the first part of the paper verify the connexivist principles.

- Suppose that $\alpha \models \neg\alpha$. Then $\alpha$ is consistent and $\alpha \vdash \neg\alpha$, which is impossible. Hence we have *Ar2*. *Ar1* is similar.
- Suppose that $\alpha \models \beta$. Then $\alpha$ is consistent and $\alpha \vdash \beta$. Hence it follows that $\alpha \nvdash \neg\beta$. Hence, $\neg(\alpha \models \neg\beta)$. *Ab2* and *Bo2* follow.

The symmetrised account is required to verify the other two principles. It does so as follows:

- Suppose that $\alpha \models \beta$ and $\neg\alpha \models \beta$. Then $\alpha$, $\neg\alpha$ and $\neg\beta$ are consistent, $\alpha \vdash \beta$ and $\neg\alpha \vdash \beta$. Hence, $\vdash \beta$, which is impossible. *Ab1* and *Bo1* follow.

Thus, all the connexivist principles follow simply from the null account of negation, but from none of the other accounts.

## 2.4. *A connexive logic*

There are, in the literature, modern formal logics which incorporate connexivist principles. Perhaps the best known are to be found in Angell (1962), McCall (1966) and (1975).[15] These systems all incorporate a total account of negation, however. (Though we do not have $\models (p \wedge \neg p) \rightarrow q$, we do have $p, \neg p \models q$.) Later systems of connexive logic, e.g., those of Routley (1978) and Mortensen (1984), build on the semantics of relevant logics, and incorporate a partial account of negation (and though we do not have $\models (p \wedge \neg p) \rightarrow p$, we do have $p, \neg p \models p$).[16]

None of the articulated systems, then, is based on a null account of negation. And this is odd. As we have seen, both historically and conceptually, the natural mate of connexivist principles is the null account. We will now see how the account of negation given in the first half of the paper can be made to give the semantics of a connexive logic in a natural way.

Note, first, that the material conditional of 1.3 is not connexive. For, example, $\not\models \neg(p \supset \neg p)$. So let us add a conditional connective, $\rightarrow$, to the language. An interpretation for the language is a triple, $\langle W, g, \nu\rangle$, where $W$ is a set of worlds, $g$ is a distinguished member of $W$, and $\nu$ assigns each propositional parameter a truth value (1 or 0) at each world, $w$. Given an interpretation, we now define by recursion what it is for a formula, $\alpha$, to hold at a world, $w$, of that interpretation, $\models_w \alpha$.

| | | |
|---|---|---|
| $\models_w p$ | iff | $\nu_w(p) = 1$ for propositional parameters, $p$ |
| $\models_w \neg\alpha$ | iff | $\not\models_w \alpha$ |
| $\models_w \alpha \vee \beta$ | iff | $\models_w \alpha$ or $\models_w \beta$ |
| $\models_w \alpha \wedge \beta$ | iff | $\models_w \alpha$ and $\models_w \beta$ |
| $\models_w \alpha \rightarrow \beta$ | iff | $\exists w' \in W, \models_{w'} \alpha$, [$\exists w' \in W, \models_{w'} \neg\beta$,] and $\forall w' \in W$, if $\models_{w'} \alpha$ then $\models_{w'} \beta$ |

Note that the truth conditions for $\rightarrow$ are exactly the natural generalisations of the one-premise inference relation of 1.3. And just as there are two versions of this, there are two versions here. In the plain one, the clause in square brackets is omitted; in the symmetric one, it is included.

To complete the picture, we require a definition of consequence. This is also the natural generalisation of the account of 1.3. Say that an interpretation, $I = \langle W, g, \nu\rangle$, is a model of sentence $\alpha$ iff $\models_g \alpha$. It is a model of a set of sentences if it is a model of each member. Then:

> $\Sigma \models \alpha$ iff $\Sigma$ has a model, and every model of $\Sigma$ is a model of $\alpha$

Or, for the symmetrised account:

> $\Sigma \models \alpha$ iff $\Sigma$ and $\neg\alpha$ have models, and every model of $\Sigma$ is a model of $\alpha$

In both cases, we can define logical truth in the usual way:

> $\models \alpha$ iff every interpretation is a model of $\alpha$[17]

Again, in what follows, $\models$ may be taken as referring to either the plain account or the symmetrised account, unless otherwise stated. Material in square brackets is peculiar to the symmetrised account.

For a suitable proof-theory, note that we can translate any formula, $\alpha$, of the language, into one of *S5*, $\alpha^+$, in an obvious way. In particular, $(\alpha \rightarrow \beta)^+$ is $\Diamond\alpha^+ \wedge \Box(\alpha^+ \supset \beta^+)$ [$\wedge\Diamond\neg\beta^+$]. Let $\Sigma^+ = \{\alpha^+; \alpha \in \Sigma\}$. Then $\models \alpha$ iff $\models_{S5} \alpha^+$; and $\Sigma \models \alpha$ iff $\Sigma^+ \not\models_{S5} \perp$, $\Sigma^+ \models_{S5} \alpha^+$ [and $\neg\alpha^+ \not\models_{S5} \perp$] (where $\perp$ is an arbitrary contradiction). To determine whether these consequences obtain, any decision procedure for *S5* can be used.

## 2.5. *Its properties*

Let us now look at the properties of the above semantics. For the truth conditions of $\neg$, $\wedge$ and $\vee$, the possible-world business is otiose; and an inference concerning only truth-functional connectives is valid iff it is valid in the corresponding semantics of section 1.3.

Turning to $\rightarrow$, it is easy enough to check, e.g., the following:

$$p, p \rightarrow q \models q$$
$$p \rightarrow q, q \rightarrow r \models p \rightarrow r$$

Consider the second. (The first is left as an exercise.) Suppose that $I$ is a model of $p \rightarrow q$ and of $q \rightarrow r$. Then for some $w$, $\models_w p$ [and for some $w \models_w \neg r$]. Moreover, for all $w$, if $\models_w p$ then $\models_w q$ and if $\models_w q$ then $\models_w r$. Hence, if $\models_w p$ then $\models_w r$. Thus, $I$ is a model of $p \rightarrow r$. It is easy enough to construct a model of $\{p \rightarrow q, q \rightarrow r\}$ [and a model of $\neg(p \rightarrow r)$]. The result follows.

The symmetrised account also verifies:

$$p \rightarrow q \models \neg q \rightarrow \neg p$$

Details are, again, left as an exercise. The plain account does not verify contraposition. To see this, merely consider the interpretation $\langle\{g\}, g, \nu\rangle$, where $\nu_g(p) = \nu_g(q) = 1$. This is a model of the premise but not of the conclusion.

Turning to connexivist principles, all the following hold:

$\models \neg(p \rightarrow \neg p)$
$\models \neg(\neg p \rightarrow p)$
$\models \neg((p \rightarrow q) \wedge (p \rightarrow \neg q))$
$p \rightarrow q \models \neg(p \rightarrow \neg q)$

Take, for example, the first. Suppose that $I$ is a model of $p \rightarrow \neg p$. Then for some $w$, $\models_w p$ and for all $w$, if $\models_w p$ then $\models_w \neg p$, which is impossible. Or take the last. It is easy enough to construct a model of the premise [and the negation of the conclusion]. Now, suppose that $I$ is a model of $p \rightarrow q$, and, for *reductio*, of $p \rightarrow \neg q$. Then for some $w'$, $\models_{w'} p$. But for all $w$, if $\models_w p$ then $\models_w q$, and if $\models_w p$ then $\models_w \neg q$. Hence, $\models_{w'} q \wedge \neg q$. Contradiction. (The others are left as exercises.)

The symmetrised account also verifies the other two connexive principles we noted:

$\models \neg((p \rightarrow q) \wedge (\neg p \rightarrow q))$
$p \rightarrow q \models \neg(\neg p \rightarrow q)$

For example, for the second, it is easy enough to construct a model of the premise and a model of the negation of the conclusion. Suppose that $I$ is a model of $p \rightarrow q$, and, for *reductio*, of $\neg p \rightarrow q$. Then for some $w'$, $\models_{w'} \neg q$. But for all $w$, $\models_w p$ implies $\models_w q$, and $\models_w \neg p$ implies $\models_w q$. Hence, $\models_{w'} q \wedge \neg q$. Contradiction. In the plain semantics, both of these principles fail. For the second of these, consider the interpretation $\langle\{g, w\}, g, \nu\rangle$, where $\nu_g(p) = \nu_g(q) = \nu_w(q) = 1$, but $\nu_w(p) = 0$. This is a model for the premise, but not the conclusion. Other details are left as exercises.

As with the semantics of section 1.3, these semantics are not monotonic or closed under uniform substitution, and for exactly the same reason. In particular, none of the inferential principles (i.e., those with something to the left of the turnstile) just cited is valid for arbitrary substitutions (though the logical truths are). Consider *modus ponens*, for example. Substitute $p \wedge \neg p$ for $p$. The premises are then $\{p \wedge \neg p, (p \wedge \neg p) \rightarrow q\}$. It is clear that this has no model; hence the inference fails. It is also clear that this is as it should be for a logic that incorporates negation as cancellation. For example, the first premise has no content; even if one had the second (which, in fact, itself has no models), we could not, therefore, infer $q$.[18]

These are, perhaps, the major differences between the connexive logics given here, and those extant in the literature. It is worth noting that one could obtain a closely related logic that was closed under substitution by simply closing $\models$ under uniform substitution. And one could obtain a logic that was both monotonic and closed under substitution by defining semantic consequence in the more familiar way:

$$\Sigma \models \alpha \text{ iff every model of } \Sigma \text{ is a model of } \alpha$$

Such a logic would be more like orthodox connexive formal logics, though not faithful to the spirit of the connexive tradition, as I have argued.[19]

### 2.6. *Generalisation*

To finish, let me note that the whole construction can be generalised in a natural way. Supposing that one has any semantics for a language with a modal possibility operator, $\diamond$, satisfying $\neg\diamond(\alpha \wedge \neg\alpha)$, and a conditional connective, $\Rightarrow$, interacting in an appropriate way with the operator $\diamond$, we may define a connexivist conditional, $\alpha \rightarrow \beta$, as $\diamond\alpha \wedge (\alpha \Rightarrow \beta)$ [$\wedge\diamond\neg\beta$]. For example, $\Rightarrow$ can be any strict conditional or the conditional of many relevant logics.

If we wish contradictions to entail nothing, we must define validity in such a way that a valid inference must have premises with a model, and not in the more usual way. Ironically, this will not work if the underlying logic is a standard relevant/paraconsistent one. This is because anything, and, *a fortiori*, any contradiction, has a model in such semantics.

* * *

My aim here has been to defend neither the null account of negation nor connexive principles. I wish to do neither of these things. I do not think that negation is correctly characterised by cancellation. Take, for example, someone who is a strong fallibilist. They endorse each of their views, but also endorse the claim that some of their views are false. Their views are inconsistent, but hardly contentless.[20] And if the null account of negation is wrong, this undercuts the motivation for connexivism too.

The aim of the paper has been: first, to reiterate the historical nature of two themes, the null account of negation and connexivism; second, to show the connection between them; third, to give a modern theory of inference that incorporates both; and, fourth, to show its distinctiveness. I trust that I have succeeded in these aims.

## Notes

[1] These are explained and illustrated in Routley and Routley (1985), on which this paper draws heavily.

[2] Findlay (1958), p. 76; italics original. Lear (1988), p. 263ff., appears to be in the same boat. See Priest (1998), sections 12, 13.

[3] Despite this, Aristotle's account of negation is not a null one. At *Prior Analytics*, 64[a]15, Aristotle tells us that "it is possible that contradictions may lead to a conclusion, though not always or in every mood [of a syllogism]" (Translation from Ross (1928)). Hence, his account is a partial one.

[4] See Sylvan (1989), ch. 4, to which this essay is also heavily indebted.

[5] Abelard (1956), p. 290, as quoted in Sylvan (1989), p. 102.

[6] Berkeley (1951), p. 73; as quoted in Routley and Routley (1985), p. 205.

[7] Equivalently, $\Sigma \models \alpha$ iff $\Sigma \vdash \alpha$ and $\Sigma \nvdash \neg\alpha$. There are more sensitive policies here. For example, in the manner of Rescher and Manor (1970–1971), we might take $\Sigma \models \alpha$ to mean that every maximally consistent subset of $\Sigma$ classically entails $\alpha$. However, this account gives no significant advantage for present purposes. Also, according to this account, $\alpha \wedge \neg\alpha \models \beta$, for any tautology $\beta$. Hence, contradictions do entail some things, but only those entailed by the empty set of premises. In a sense, then, they still have empty content.

[8] In virtue of the quotations in the next section, it might make sense to reverse *Ar* and *Ab*; but the names have now stuck. Note that some people call the result of replacing the "$\models$"s in *Ab2* with conditionals, "Strawson" after the following passage from Strawson (1952), p. 85: "The formulae '$p \supset q$' and '$p \supset \sim q$' are consistent with one another, and the joint assertion of corresponding statements of these forms is equivalent to the assertion of the corresponding statement of the form '$\sim p$'. But 'If it rains, the match will be cancelled' is inconsistent with 'If it rains, the match will not be cancelled' and their joint assertion in the same context is self-contradictory."

[9] Though the relevant logic *R* plus *Ar1* or *Ar2* is trivial. See Mortensen (1984).

[10] Translation from Ross (1928). The following is also worth noting. As is well known, one cannot translate the Aristotelian syllogistic forms into classical quantifier logic in any natural way so as to preserve both the valid syllogisms and the square of opposition. (The problem is that of existential commitment.) McCall (1967) shows that it is possible to do this using a quantified logic incorporating *Bo2*.

[11] See Kneale and Kneale (1962), p. 191, who call these "mistakes." See also McCall (1966). Sylvan (1989), p. 98, calls the converse of *Bo2* "hyperconnexive."

[12] See Bochenski (1961), p. 199. My italics.

[13] See Sylvan (1989), p. 107.

[14] There is further discussion of this and other issues concerning connexivism in Routley et al. (1982), pp. 82–95.

[15] A rudimentary one is to be found in Nelson (1930).

[16] The unsatisfactory nature of this is pointed out in Routley (1978). One consequence is that it forces a highly non-standard account of conjunction.

[17] This is simply equivalent to $\phi \models \alpha$ in the plain case, but not in the symmetrised case. If we define it this way in the symmetrised case, there are no logical truths. For in this case, if $\phi \models \alpha$ then $\alpha$ must be true in all models, but $\neg\alpha$ must be true in one.

[18] It is also worth noting that the symmetrised version has no logical truths of the form $\alpha \to \beta$. For suppose that $\models \alpha \to \beta$. Consider any interpretation $\langle\{g\}, g, \nu\rangle$. Then $\models_g \alpha$, $\models_g \neg\beta$, but if $\models_g \alpha$ then $\models_g \beta$, which is impossible. The plain version does have logical truths of this form, e.g., $(\alpha \vee \neg\alpha) \to (\alpha \vee \neg\alpha)$.

[19] The logic is identical to none of the standard systems, however. It is easy to construct a counter-model for $p \to p$. This is valid in all standard connexivist systems.

[20] This is only an indication of one sort of consideration that I would raise against the view. For a fuller discussion, see Priest (1998), section 13. See also Routley and Routley (1985), p. 212.

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