XVIII
The Philosophical Significance and Inevitability of Paraconsistency

G. Priest and R. Routley

1. Introduction

Paraconsistency strikes at the root of principles which are fundamental to, and entrenched in, much philosophy. It is therefore bound to be philosophically problematic and to have important philosophical ramifications. In this introduction we will try to chart and analyse some of these issues. By its nature, this will require us to deal with a number of separate and not otherwise connected issues. However, we will start by looking at some important points raised by the arguments for paraconsistency in chapter v, section 1 above. We will then go on to investigate some of the philosophical consequences of paraconsistency.

2. Reasons for paraconsistency

In ch. v, sect. 1 we gave two reasons for studying paraconsistency. The first, and weaker, was that some theories are inconsistent but non-trivial; the second was the truth of certain contradictions. Both of these claims are bound to be hotly contested, especially the second; for this reason we now consider them in greater depth.

2.1. There are natural inconsistent but non-trivial theories

No one would deny that we can construct purely formal, uninterpreted calculi which have as theorems formulas of the form 'A' and '~A', but which are not trivial. If paraconsistency is to have real interest, it must be possible to do more than this: we must be able to find real life, philosophical or scientific examples of inconsistent but non-trivial theories. A number of these were suggested in ch. v, sect. 1.1. Let us look at these more closely.

2.1.1. Inconsistent bodies of law and the like. One example of an inconsistent corpus from which non-trivial conclusions are drawn concerns certain bodies of law. Now it is not difficult to find bodies of law which are *prima facie* inconsistent. But it will undoubtedly be claimed by some—the "friends of consistency" we will encounter again and again—that the inconsistencies are only *prima facie*, that when properly understood the law is consistent. The obvious move is to suppose that one or more of the laws which produce inconsistency contain implicit exceptive clauses which prevent them from being applied in the contradiction-producing case. For example, it is often pointed out that laws can be ranked in increasing order of strength through common law, statute law, constitutional law, etc. This may suggest that if a lower ranking law contradicts a higher ranking law, it *ipso facto* ceases to be applicable. Another way in which we may try to make good the idea that a law has implicit exception clauses is this. The preamble of the bill which passes a piece of legislation may make the intentions of the legislators quite clear. It may then be said that although a particular case falls under the act as literally worded, it was never really meant to apply to this kind of case. The intentions of the legislators therefore provide implicit exception clauses. (This is a somewhat problematic point however, since a judge will often uphold the letter of the law, even when it is clear that doing so goes against the intentions of the legislators.)

Moves of the above kind can sometimes be reasonable. However, someone who denies the occurrence of inconsistent bodies of law must do more than claim that this or that manoeuvre is *sometimes* performable. He must claim that it *always* is. And put like this, it seems most unlikely. A case may easily arise where both of the contradiction-producing laws have equal rank, where the intentions of the legislators are lost in the mists of time, are moot, or are even downright inconsistent, where there is no precedent for waiving one law rather than the other, etc. In short there is no objective situation which can be used to underpin the claim that one law has implicit exceptive clauses or takes precedence over another. In such circumstances to insist that nonetheless one law has implicit exceptions is mere whimsy; there may well be much more to the law than what is literally written in a parliamentary bill, but to suppose that something can be a fact of law when it is grounded in no aspect of the legal process is baseless obscurantism.

Of course, when a contradiction of the kind we have pointed to becomes important, there are procedures for resolving it. The matter goes to court where a judge makes a decision. Since there are, *ex hypothesi*, no legal reasons for deciding one way or another, the judge will decide on extra-legal (socio-political) grounds. However, the important point here is that the judge is not trying to find out what the (consistent) law is, but is himself *making* law. What the judge decides just is the law and that is that (until and unless the legislature decides to change it or a higher court, if any, amends it). In this situation there is no way in which a judge can be wrong

i.e. make a ruling which is incorrect.[1] Thus the judge, by making new law, is changing the corpus of the law. His action provides the basis for the law, henceforth, to be considered to have an exceptive clause, and hence after his ruling the corpus of law may no longer have this inconsistency. But this does not change the fact that before the ruling the corpus was genuinely and not just *prima facie* inconsistent. Thus there can be genuinely inconsistent bodies of law.

What holds of law applies also, given appropriate adjustments, to like bodies of (partly prescriptive) doctrine, such as those applied by morality or religion. Again there are evidently or demonstrably inconsistent bodies of doctrine whose inconsistency cannot be satisfactorily explained away. Important examples are offered by irresoluble moral dilemmas.[2]

2.1.2. Inconsistent theories in philosophy and the history of ideas. A major reason for taking the paraconsistent enterprise seriously is that inconsistent but putatively non-trivial theories abound in intellectual endeavours. Indeed much of our intellectual history is composed of such theories. This is particularly true of our philosophical heritage,[3] where it is not entirely implausible to advance the following (classically preposterous) thesis:

TH1. *Any sufficiently complex and interesting philosophy will be inconsistent.*
There are several ways of arriving at, and supporting, this large thesis. One is by direct argument from the character of such philosophies, another weaker but persuasive argument is by induction from the inconsistent nature of major philosophies. The themes used in the induction are of much independent interest, namely:

TH2[M]. All [most of] the major philosophical positions, from the history of philosophy, are inconsistent.

TH3. No philosopher has succeeded in avoiding inconsistencies of a fundamental kind, those encountered in achieving the complex aims involved in working out a fairly comprehensive philosophical position.[4] In each case the themes concern major, complex or comprehensive theories. (Without doubt there are, or can be designed, miniature philosophical theories which are consistent, e.g. simple nominalist theories or theories pegged to a consistent segment of the cumulative hierarchy of sets.)

The latter themes give the appearance of being much more factual than the initial theme, which also makes predictions about future, and indeed merely possible, philosophies. But the appearance is something of an illusion; a variety of less factual and overtly normative considerations enter into attempts to show that the positions of given philosophers are inconsistent. For this reason, establishing TH2 and TH3, and even the weaker TH2M, is far from straightforward and cannot be achieved with any high degree of certainty. Open to the friends of consistency are always too many escape routes from (apparent) inconsistency, such as those that "interpretation" of a philosophy can supply.

Fortunately, then, a much weaker thesis will serve very well for paraconsistent purposes, namely,

TH4. Some major philosophical positions, which are not trivial, are inconsistent. Rather surprisingly, given the dominance of philosophical positions (often ideologies) which entail that all inconsistent theories are trivial, TH4 is a thesis to which many philosophers will assent at once. Indeed, they will frequently go further, with a little or no prompting, and propound theses like TH1-TH3. Yet practically no-one believes that major philosophical positions which are inconsistent are trivial, or thereby trivialized. Accordingly, the situation cannot be accounted for under the usual (classical-type) methodology of philosophical theories. In short, thesis TH4 leads to the further thesis

TH5. The theory of philosophical theories must be paraconsistent; no other type of theory is adequate to cope with the data; in particular, no classical account of philosophy will do.

The main detailed work which follows will concentrate on establishing the thesis TH4, that some major philosophical theories are inconsistent *but* non-trivial, that underpins TH5. Naturally, derivation of TH5 involves further assumptions, such as that philosophical theories are theories; that is to say, are at least deductively closed.[5] That this holds can be argued as follows: x's philosophical theory is given by what x is committed to philosophically. But if x is committed to A, and B is deducible from A, then x is committed to B, whether or not he asserts it. Thus philosophical theories, as encapsulating philosophical commitments, are closed under deducibility.

Similar points serve to distance a philosopher's theory from what the philosopher asserts. The separation is familiar from discussion of criteria for ontological commitment. There are analogous, but even more complex, problems in determining criteria for philosophical commitment. However, assertion, without (later) amendment or withdrawal, is normally sufficient for commitment; this (qualified) sufficient condition is crucial for the exegesis of positions from philosophical texts, and will be applied in what follows.

By no means all inconsistent theories are trivial. But recent philosophical theories which are inconsistent and also incorporate classical or intuitionist logic—or, more accurately, the theories of logical consequence these logics supply—are trivial, and accordingly are worthless *unless* repaired. Such theories, while they are relevant to theses TH3 and perhaps TH2, are not germane to TH4, and so will receive only brief presentation and exemplification. Such theories trivialize because, of course, they supply spread laws such as $A \& {\sim}A \vdash B$. Examples are theories of Frege, Russell, the early Wittgenstein[6] and Quine. Main inconsistencies detected in these theories

are not, however, tied essentially to the underlying (classical) logic, so they have a wider interest.

Frege's theory, and likewise a transitional position of the early Russell, is inconsistent because of the logical paradoxes (to say nothing of the concept *horse* and the like). It is not merely that Frege's theory succumbed to the Russell paradox, but that his suggested amendment to avoid that paradox left him open to the derivation of further paradoxes.[7] Many philosophical theories seem indeed to succumb to logical or semantical paradoxes. Cantor's theory, if that is accounted philosophical, is one; Aristotle's theory with its (apparent) acceptance of the Liar at face value as both true and false[8] is another. Whether these theories are trivial or not turns on the question of what their underlying logics look like. More generally, paradoxes of one sort or another are a prime source of inconsistency not just in logical theories but in philosophical theories.[9]

Dealing with Russell introduces a complication already hinted at which is very important, both as regards determining what a philosopher's theory is, and as regards enabling a philosopher to escape from inconsistency; namely, the change or adjustment of theories over time. Russell, to take a more extreme example, did not develop a single philosophical theory: rather, he went through a sequence of somewhat different theories with significant common elements. In chapter I, section 5.4, we observed the phenomenon of theory change on a smaller scale (in terms of a number of changes) with Wittgenstein. Frequently, of course, inconsistency in a previous position is a major reason for moving on to a new one; and often this would take the form of inconsistency with—sometimes presented as inability to account for—data the position was supposed to comprehend. A somewhat Hegelian account of philosophical motion, or "progress", must be a *proper* part of any adequate theory of philosophical theory dynamics, especially of the theories of one individual or school.[10]

Inconsistency was certainly *a* moving force in Russell's development. He discarded several of the earlier positions he had held because, in large measure, of inconsistencies, e.g. "naive" logicism, the "naive" theory of denotation. He was halted entirely in his attempt in 1913 to work out a theory of intentionality by inconsistency,[11] and forced thereby into extensional reduction in the form of logical atomism. This theory not only proved, rather quickly, inconsistent with much rather evident data, but was internally inconsistent. For example, Russell's logical atomism was inconsistent as to the existence of facts. On the one hand, the world consists, according to the theory, entirely of simples (including relations). On the other, inconsistent, hand, sentences are true only by virtue of appropriate facts, which are not however conveyed by any listing of simples—so that facts also demand, and obtain, basic ontological status:

> ... facts, which are the sorts of things you express by a sentence, ... just as much as particular chairs and tables, are part of the real world.[12]

But also,

> ... facts ... are not properly entities at all in the same sense in which their constituents are. That is shown by the fact that you cannot name them. You can only deny, or assert, or consider them. But you cannot name them because they are not there to be named ...[13]

Russell's later work contains many lesser inconsistencies. One example (developed elsewhere[14]) concerns negation, where Russell both sponsors a classical theory and also elaborates an account which leads to a non-classical model of negation.

Russell's multiple relation theory of belief leads to inconsistency: logical forms both are and are not constituents of judgement complexes.[15] Wittgenstein's first criticism of the theory also pointed to a connected inconsistency, that 'Socrates and Mortality are of the same type ... This directly contradicts Russell's claim ... that universals and particulars are of different logical types' (Griffin, p. 173). As Griffin goes on to remark, this 'is a new version of an old problem: namely the problem in *The Principles of Mathematics* of the verbal noun which occurs as a logical subject, and which Russell wanted to be the same as the verb which occurs as relating relation' (pp. 173–4).

Discovery of contradiction in his earlier position was a major source of motion for Russell from one position to a later. Thus he abandoned 'the relational theory of space *together with* Bradleyan monism' because of contradictions deriving therefrom.[16]

There was a further major inconsistency in Russell's 1913 research program, brought out by Wittgenstein's second criticism of Russell's theory of judgement which was (see *Tractactus* 5-5422) that 'the correct explanation of the form of the proposition 'A makes the judgement p' must show that it is impossible for a judgement to be a piece of nonsense (Russell's theory doesn't satisfy this requirement)'.[17] (Griffin explains in detail the tensions in Russell's 1913 program: for a summary see p. 180, from which it follows that the combination of theories Russell proposed to combine is inconsistent.)

Most larger philosophies contain a good many minor inconsistencies; such appears certainly to be the case with the theories of Russell, Wittgenstein and Quine. A "minor" example from Quine concerns the proposition "God exists", which is rendered true by the theory of descriptions Quine adopts, but false by Quine's overall atheistic physicalism which leaves no existential place for God.[18] Such an example is minor because it can be avoided by a relatively minor change in the underlying theory of descriptions (as it is, Quine advances mutually inconsistent theories of descriptions). The fact that this example is "minor" and thus comparatively easily rectified does not imply that it does not have a devastating impact on the theory as given, since classically any contradiction is a catastrophe.[19]

To obtain examples of inconsistent philosophical theories which are not trivial without change of logical base, it is not necessary to proceed backwards in time to periods before the ascendency of classical logic—especially since non-classical theories are now being developed, as by isolated philosophers who were never caught in the classical trap, and as in this book—but it is advantageous to do so, especially if major philosophies are to provide examples. There are several well marked ways in which a philosophical theory can end in inconsistency, whether unintentionally or not; for example, by inclusion of sufficient self-referential apparatus for the formulation of paradoxes, by other paradoxes of a variety of types, through involvement in an infinite regress, by self-refutation of one sort or another, and so on. But as there is so far no very worthwhile classification of these ways to inconsistency (a thoroughly effective classification is not to be expected however), the preliminary grouping that follows is rather rough and ready. So also the selection of further philosophers we now present as having inconsistent theories is not at all systematic, nor is the particular selection of inconsistencies from the philosophers cited systematically determined. We set down what we fell over that was solid enough to trip us up. More exactly, we list examples of inconsistencies we managed to recall or came across or that others pointed out, where we thought we could make the claims good enough to stand up on their own.

Important sources of inconsistency in recent philosophy closely tied up with both features of mathematical objects and the logical paradoxes are (insufficiently qualified) characterization postulates, that is, postulates which assign objects characteristics which serve to identify and distinguish them, such as that the object characterized as f is indeed f. The most famous modern examples are the inconsistencies concerning the round square and the existent King of France, that Russell located in the theory of objects of the Meinong school at Graz.[20]

More generally, comprehensive theories of abstract objects are especially liable to upsets of one sort or another, arising from characterization postulates (inevitable for such objects) and ending in inconsistency. The stock example is Plato's theory of forms, which is revealed as inconsistent, for instance, by the problem of self-prediction and through the Third Man Argument.[21]

Another connected source of inconsistency to which philosophical theories, including recent theories, are especially prone derives from self-refutation. Inconsistency frequently emerges in this sort of way: According to the theory t, (a) All philosophical [metaphysical, etc.] theories are of kind k. But (b) t itself is a philosophical [metaphysical, etc.] theory, at least according to its own lights, (c) t is not of kind k. One famous example concerns logical positivism, which can be represented as a metaphysical theory to the effect that all metaphysical theories are nonsense. One of the damaging inconsistencies in Wittgenstein's early work is of this type, and issues in the proposed throwing away of the ladder or theory by which one has ascended.[22] Wittgenstein's later theory is in a similar predicament, with its theme to the effect that all philosophical theories are mistaken, or else that there are none.[23] Much the same applies to Collingwood's absolutist theory of metaphysical presupposition, where kind k amounts to involving metaphysical presuppositions. Because of their metaphysical presuppositions no philosophical theory can be taken as true or rejected as false; but this depends on the theory of metaphysical presuppositions itself being taken as true.[24] As regards the history of philosophy, Collingwood's theory leads to derivative inconsistency, such as that some historical theories (e.g. Collingwood's) are both criticizable and not.

The trouble with Collingwood's theory resembles, in its self-refuting aspects, the trouble with Protagoras' 'man is the measure of all things' doctrine of more than 2,000 years before (at least as that doctrine is interpreted by Plato). To engage properly in discourse at all, as he does, Protagoras has to assert something to be the case; yet he renounces the claim to be able to assert true statements, in his teaching that no one can inform one of anything.[25] In this way Protagoras' position leads to inconsistency (in fact of the general form already given above). Other positions of a thoroughgoing relativistic or sceptical cast do also.[26]

Inconsistencies over knowledge or belief, common among philosophers, sometimes arise, as in scepticism, out of self-refuting theses, sometimes not. It is not clear which mechanism is operative in Lao Tse, whom Bose has accused of inconsistency over knowledge. According to Lao Tse, it is both wise (sensible) to know the laws (of nature) and also not wise to know anything.[27] The situation looks a bit like the Socratic "paradox", that Socrates' wisdom consists in his knowledge that he knows nothing. Socrates is also inconsistent. For $K_s(p)(p \to \sim K_s p)$. But if this proposition, q, is true then it follows, since $(p)(p \to \sim K_s p)$ (by $K_s r \to r$), that $\sim K_s q$, i.e. $\sim K_s K_s(p)(p \to \sim K_s p)$. But in the case of such conscious knowledge, it seems certainly true that $K_s r \to K_s K_s r$ (i.e. an S4 principle holds[28]). So $\sim K_s(p)(p \to \sim K_s p)$, establishing inconsistency as regards q.

Bose goes on to charge that 'Locke and Rousseau are "bogged in inconsistencies"', but he unfortunately supplies no details. According to Passmore, however, who does give details, the inconsistencies in Locke are quite blatant. One instance is this: 'An idea, let us say, is whatever lies before the mind, and yet we can have before our mind the idea's capacity to represent what is not an idea.'[29] Locke is, so Passmore contends, not inconsistent in an uninteresting way, for instance out of carelessness, or because he has not entirely overcome an older position he is working away from. Such inconsistency is not deep and is easily rectified, for instance, by minor adjustments to the theory or the restatement of certain themes or claims, so removing inconsistencies while remaining faithful to the original.[30] No, Locke is inconsistent because he has to be, to establish what

he wants to establish: that is why he is inconsistent about ideas and about our power over our beliefs, and why that inconsistency is deep in his theory and not easily excised. Thus

> there can be no doubt that Locke would have liked consistently to maintain two theses, the first that rational human beings will regulate their degree of assurance in a proposition so that it accords with the evidence... The second, that human beings are so constituted as naturally to do this, were created rational, that they go wrong only where some of the evidence is not before them. But he finds it impossible to reconcile this second with his experience of the actual irrationality of human beings...[31].

Consistency of his belief with the evident data is never satisfactorily achieved, with such results also that there is not a uniform picture of belief in Locke but competing inconsistent theories, modelled on knowledge and, differently, on desire. The account of belief as an important surrogate of knowledge[32] is one source of a serious inconsistency in Locke's ethics of belief: On the one hand, he maintains, on several occasions, that 'to believe this or that to be true is not within the scope of the will', is not a voluntary matter. On the other hand, he also maintains, and is theoretically committed to maintaining, that belief ought to be, can be, and is often a voluntary action.[33]

Inconsistency was not confined to seventeenth and eighteenth century empiricists, such as Locke and Hume. It much afflicted the rationalist alternative as well. Spinoza's *Ethics*,[34] in particular, appears to be riddled with inconsistencies, many of which seem sustainable, for example, the following, concerning God and the notion of love. On the one hand God loves himself (an immediate consequence of proposition 35, book 5), whence, by the definition of love (p. 130: love is pleasure accompanied by the idea of an external cause, Definition of Emotion 6: see p. 172), God has emotions, and God is affected with the emotion of pleasure. On the other hand, 'God is free from passions, nor is He affected with any emotion of pleasure of pain' (proposition 17, book 5). Derivatively then, God, an already perfect being, sustains both increases and also, worse, decreases in perfection.

Another deep inconsistency arises from Spinoza's strong determinism. Spinoza is committed, in the *Ethics*, to these theses:

(1) Δp (everything is non-contingent)

(2) $\Box p \supset p$ (whatever is necessary is true)

(3) $\Box p \supset Pp$ (what is necessary is permissible)

(4) $(\exists p)(p \& \sim Pp)$ (non-permissible acts occur)

But (1) and (2) imply

(5) $p \equiv \Box p$

and (3) and (5) imply

(6) $p \supset Pp$;

And (4) and (6) are inconsistent. This simple argument[35] to inconsistency presents a difficulty more generally for philosophers committed to a determinism, with some teeth in it: for

(5′) all (and only) what happens is determined, and

(3′) what is determined is not reprehensible (forbidden).

Therefore nothing that happens is reprehensible. But, by independent evidence, some things that happen are reprehensible.

It has so often been charged that Descartes is inconsistent that a great deal of controversy surrounds the question. Until this is straightened out, if it can be, Descartes is even less guilelessly included as a philosopher with an inconsistent position than the other philosophers we have indicted. Among the more important grounds on which Descartes has been accounted inconsistent are the following non-trivial issues: Firstly, clear and distinct perceptions need to be guaranteed by something beyond themselves, e.g. God; but clear and distinct perceptions do not need to be so guaranteed, because they are self-guaranteeing in virtue of their clearness and distinctness. Secondly, one both can trust one's senses and cannot trust one's senses, as Descartes' argument on scepticism reveals. Descartes finds reasons for rejecting sense evidence as a ground for truth claims. But his reasons require[36] an appeal to sense evidence, e.g. that he has sometimes been deceived by his senses, something determinable only by relying on, what he rejects, subsequent sense evidence. So he both relies upon and rejects sense evidence. It does not matter, for our purposes, that the arguments contain large gaps; more important is that the conclusions reached and maintained in the dialectic are inconsistent. Thirdly, the theory of mind-body interconnections was entangled in difficulties and inconsistencies.[37] Consider the nature of a relation relating mental and physical phenomena, of which there are many; it must be included either in the material or in the mental, yet in view of its relata it cannot belong to either area. Or consider the problems concerning rational purposive and voluntary behaviour. On the one hand these things can be distinguished from their opposites, yet on the other, given the theory, they cannot be so distinguished.[38]

Reporting examples as controversial as the Cartesian philosophy will make things much easier for the friends of consistency. These friends will,

without doubt, claim to be able to remove many, or even all,[39] of the examples of (unintentional) inconsistency in major philosophers that we have assembled; and certainly they will be able to throw up enough dust to obscure the issue. So it is as well that we have several examples of admitted or intentional inconsistency to complete our case.[40]

Our main examples of inconsistent philosophical theories that were recognized as such have already been given. They include tentatively Heraclitus, fairly certainly various Indian schools such as the Jains, and quite definitely Hegel and Marx and Sartre.[41] For example, Hegel's theory is non-trivial because there are definite propositions—many from the history of philosophy—that Hegel rejected and which (on relevance grounds, for example) appear not to be entailed by his theory. And Hegel's theory is certainly inconsistent. A body in motion, for example, is both at, and also simultaneously not at, a given position. The basic categories, such as Being, are both self-identical and not. And so on.

Hume is another philosopher who is inconsistent in an extremely interesting way. Not only are some of Hume's inconsistencies integrally embedded in his philosophical system, but further, Hume is one of the rare major philosophers—apart from Hegel and his dialectical successors—who explicitly acknowledges and discusses inconsistencies in his philosophical system.

The commentator on Hume is faced with inconsistencies on considerable scale, as Passmore, Selby-Bigge and other commentators have said.[42] In a certain sense the whole of Passmore's *Hume's Intentions* is, as remarked by its author, about inconsistencies in Hume.[43] A great deal of scholarly argument is readily viewed as an attempt either to point to inconsistencies in major philosophies or to protect them against that charge. That is particularly so with Hume and Descartes. But what is astonishing about Hume is his confession of irremedial inconsistency in his system:

> I had entertain'd some hopes, that however deficient our theory of the intellectual world might be, it wou'd be free from those contradictions, and absurdities, which seem to attend every explication, that human reason can give of the material world. But upon a more strict review of the section concerning *personal identity*, I find myself involv'd in such a labyrinth, that, I must confess, I neither know how to correct my former opinions, nor how to render them consistent. If this be not a good *general* reason for scepticism, 'tis at least a sufficient one (if I were not already abundantly supplied) for me to entertain a diffidence and modesty in all my decisions. (*Treatise*, p. 633)

Hume was however no dialectician:[44] he was very uneasy about contradictions, and would have removed them if he could, if he knew how. But his attitude to inconsistency in his own system, very definitely is not classical. Classically, Hume's system, if inconsistent, trivializes, which is not a good reason for modesty, though it is then not an unreasonable ground for scepticism. But with the rejection of a classical framework, inconsistency on its own is *not* a good *general* reason for scepticism; this much dialethic theory shows. For if inconsistencies are, like other statements, restricted in their consequence cone, if they do not lead everywhere, then they may not lead to sceptical conclusions, such as that we do not *know* this or that we thought we did know. Generally, arguments to scepticism involve further assumptions than merely isolated inconsistent premisses. Further premisses supplied in Hume's system do give grounds for scepticism concerning personal identity, and given the underlying classical nature of his system, that scepticism may well get more widely distributed. The argument to limited scepticism takes the following lines. If, for some x, x knows facts about personal identity then these facts about personal identity are true, and hence, since facts about the world, consistent. But there is, given Hume's theory no consistent account of the facts concerning personal identity, hence, contraposing, x lacks knowledge. A different argument runs from inconsistency to what Hume pleads, as, 'the privilege of a sceptic', unintelligibility, that the matter is beyond (his) understanding. But this involves the assumption, rightly criticized in detail by Reid[45] that what is impossible is beyond understanding or conception.

A residual problem, however, with Hume's claim to unavoidable inconsistency is that the principles he 'cannot render consistent' though it is not in his 'power to renounce either of them' appear, at least on the face of it, *not to be inconsistent* at all. The principles are:

> *that all our distinct perceptions are distinct existences*, and *that the mind never perceives any real connexion among distinct existences*. Did our perceptions either inhere in something simple and individual, or did the mind perceive some real connexion among them, there would be no difficulty in the case (*Treatise*, p. 636).

However, it is not a difficult feat to combine these principles with others of Hume's theses, to obtain an explicit and 'serious inconsistency in Hume's views'.[46]

2.1.3. Inconsistent theories in science and the history of science. Inconsistency of theories is by no means confined to more spacious philosophical edifices; other examples we appealed to in ch. v, sect. 1.1. were inconsistent theories in the history of science, for example Bohr's theory of the atom and the (early) infinitesimal calculus. There are many other examples, including (as Galileo showed[47]) the Aristotelian theory of motion.[48]

Now as in the legal and philosophical cases, the friends of consistency have to argue that the contradictions are only *prima facie*. The most plausible way of doing this is to suppose that, in practice, the theories were guarded by *ad hoc* auxiliary assumptions, or even slightly modified in an *ad hoc* fashion in such a way as to avoid overt contradiction. Again, it must be

insisted that this was *always* done. However, history does not bear this out.

For example, when Bohr enunciated his theory of the atom he was quite clear that this contradicted Maxwell's equations. He *may* have had it in mind that Maxwell's equations would eventually have to be modified because of this. However, he provides no suggestions, even *ad hoc* ones, of how this is to be done, and goes ahead and uses Maxwell's equations whenever he feels like it. There is a blatant inconsistency which he ignores. Maybe others later tried to patch up the inconsistency. Maybe Bohr himself, at a later date, thought the correspondence principle might be used to solve the problem. However, there is no avoiding the fact that Bohr's theory of the atom as presented in 1913 was inconsistent.[49]

Similar points apply as regards the infinitesimal calculus. This was inconsistent and widely recognized as such. In this case various attempts were made to rework the theory in a consistent way. (For example, Berkeley had a theory of the "cancellation of errors"). However, the attempts did not meet with a great deal of success.[50] Moreover these attempts confirm the fact that the theory and certain of its parts, e.g. the Newtonian theory of fluxions, *were* inconsistent.[51] If they were not, attempted consistentizations would hardly have been necessary.

Thus, there are genuinely inconsistent theories in the history of science. Moreover, even if our claims specifically about Bohr's and Newton's theories were not correct, the friends of consistency could not claim *a priori* that *prima facie* inconsistent theories always had *ad hoc* consistentizations. That would, after all, beg the question against us. What would be required, rather, would be a detailed historical analysis of many and varied cases of such inconsistent theories. And such an analysis, which we are content to leave to suitably informed historians (those acquainted, among other things, with paraconsistency), would, we conjecture, issue in our favour.

2.1.4. The matter of non-triviality. So far we have argued that in many domains of intellectual endeavour inconsistent theories abound. To complete the paraconsistent picture we need to argue that many of them are non-trivial. Of course this is not always true. As we have seen, philosophers such as Frege and Russell who produced inconsistent theories and who explicitly endorsed classical logic had the misfortune to produce trivial theories. However, in most of the other cases we have discussed we wish to claim that it is at least reasonable to suppose that the theories concerned are non-trivial. But it is not easy to argue this. None of the theories we have discussed is a formal theory. (Indeed one aim of our enterprise was to find inconsistent informal theories.) Hence a rigorous non-triviality proof is out of the question, since a very exact formulation is required before a start can be made upon applying modern techniques for establishing non-triviality of a theory. Conceivably some of the theories in question could be formalized,[52] thus posing the possibility of a rigorous non-triviality proof. However, then the question of the adequacy of the formalization would typically pose similar problems. How then are we to proceed? There are (at least) two possibilities.

The first is to pay particular attention to the sort of logic underlying the theory. If it fails to contain the principle *ex falso quodlibet* then the fact that the theory contains contradictions does not *ipso facto* produce a presupposition of triviality. This is perhaps good enough for our purposes. If the theories in question are theories of science or law where there is no self-conscious reflection on the logic being used, then this approach may be only marginally fruitful. For the underlying logic is exactly natural logic —the logic of ordinary discourse. Whether or not this is paraconsistent is precisely what we are arguing about. However, clearly, we think it is. With philosophical theories we are somewhat better off. For often philosophers explicitly or implicitly endorse certain logical systems. Perhaps the best example for our purposes is again Hegel. Despite the fact (or perhaps in virtue of the fact) that he wrote two enormous books on logic, we and many other people would not care to say exactly what his logical theory was. But this much is clear: Hegel's philosophy is explicitly inconsistent. But no man in his right mind would have an explicitly inconsistent philosophy and a non-paraconsistent logic. Hence Hegel's logic was paraconsistent. (Perhaps the weakest part of this reasoning is the suppressed premiss.)

We can also mount an argument from the form of the underlying logic for the non-triviality of the main traditional theories, for example, empiricist and rationalist philosophies of the 17th and 18th centuries. Their received logical theory would have been some form of Aristotelian logic. Now despite the fact that the law of Non-contradiction is a keystone of Aristotle's logic, Aristotle's syllogistic (and indeed the larger traditional theory) is paraconsistent! Specifically the inference

S is P

<u>S is not P</u>

∴ S is Q

is not a valid syllogistic figure, and the inference

S and P

<u>S is not P</u>

R is Q

is a fallacy of four terms.

The second approach is somewhat stronger and allows us to produce a legitimate presupposition of non-triviality. Although there is no general decision procedure for deciding whether something follows in a theory (even in the formal case) we may have well founded and reliable intuitions about this. Neither is this an appeal to some modish irrationalism. The intuitions in question are to be obtained only by a lot of hard rational work, such as intelligent and informed guessing, and deductive and non-deductive reasoning. It is necessary to be thoroughly familiar with the theory; to know what typical proofs are like; to have tried to prove certain things unsuccessfully and understood why attempted proofs or arguments break down; it is necessary to know what sort of interpretations or partial interpretations the theory has, to know major heuristics for proving things in the theory and so on. It is exactly this kind of experience, for example, on which most contemporary logicians would base the judgement that ZF and Peano Arithmetic are consistent. Now in many of the cases countenanced such intuitions are available. For example, experienced lawyers know what sort of cases can be made out and what cases are hopeless on the basis of well understood laws. Yet no lawyer would claim that a body of inconsistent law (such as the one described in ch. v, sect. 1.1) would allow him to be able to make out a good case for anything at all. Similarly scientists working on an inconsistent theory, such as Bohr's or the infinitesimal calculus, would obviously reject the idea that their theories could be used to prove everything. With philosophical theories which are somewhat more fluid, it may be difficult to build up the kind of intuition that is necessary. Yet we think that any Spinoza scholar would firmly reject the idea that from Spinoza's principles follows Cartesian Dualism, and any Locke scholar reject the claim that from Locke's theory follows the mind-brain identity theory.

Let us say again that these intuitions (roughly, intuitive theories) are by no means conclusive. However, they serve at the very least to establish a legitimate presupposition. We may even push the case further. Consider any of the inconsistent theories or situations we have so far mentioned, and suppose that we were able, with some form of reasoning to show that from the principles of the theory followed something absolutely antithetical to the spirit of the theory. Clearly those who accept the theory may give it up, or they may try to reformulate the theory in order to avoid this. However, they may simply reject the reasoning involved. For example, consider the following thought experiment. Suppose we had argued with Hume from the admitted inconsistency of his position, using *ex falso quodlibet*, to the conclusion that we can be certain that the sun will rise tomorrow. Would he have accepted this? Of course not. He *would*, quite rightly, have rejected this form of inference. Obviously what is accepted and what rejected by a theory depends upon the underlying logic of the theory. The thought experiment illustrates that the converse may also be true. If we are certain that a theory (or a person holding it) strongly accepts A but rejects B, then we have good evidence to suppose that the person would reject the inference from A to B. Moreover the fact that something is rejected means *ipso facto* that the theory, whatever its underlying logic is to be, is non-trivial. Since in all the cases we have considered we can be sure that the judges, scientists, philosophers, etc., in question would strongly reject certain things, we have good reason to believe the theories in question to be non-trivial.

When we go on to try to produce a theoretical account of natural logic, the logic of ordinary discourse, we run into the same type of phenomenon. We have an independent fix on neither the logic nor the theories, such as semantics and set theory, embedded in the practice. Hence we must determine the best theory of these jointly. The situation is not unlike that in accounts of radical translation, where we have no independent fix on either speakers' beliefs or their meanings, but must fix the two simultaneously. In the present case we have firm dispositions to accept, e.g., the T-scheme and equally firm dispositions to reject other beliefs about truth. We can not have a theory which endorses both these intuitions and the triviality-producing absorption principle: $A \rightarrow (A \rightarrow B)/(A \rightarrow B)$. Clearly we can reject the T-scheme and accept absorption. However, a simpler and far more plausible course of action is to accept the T-scheme and reject absorption (as we in effect argue at many places in the book). This is particularly so since once we reject the mistaken identification of implication (or $\rightarrow$) with material implication and its mates, it is not at all clear that our intuitions for accepting absorption are very strong.

With segments of naive semantics and naive set theory however, where the intended formal principles can be uncontroversially articulated, the matter of non-triviality can be more straightforwardly resolved. For we can choose, in well-motivated ways, underlying logics in terms of which non-triviality can be proved (see below).

2.1.5. Naive set theory. Another of the diverse examples we gave in ch. V, sect. 1.1. of an interesting inconsistent theory was naive set theory. This is the theory of sets produced and developed in the late nineteenth century mainly by Dedekind, Cantor, and Frege. The inconsistency of this theory is incontrovertible. The claim that the theory came complete with fail-safe devices, of the kind considered in previous sections, to prevent the deduction of naked contradictions has no plausibility. This is because, among other things, some people (such as Burali-Forti and Russell) went ahead and deduced contradictions.

The defence action by the friends of consistency has therefore had to take another direction in this case. The main move has been to suggest that whilst, for example, Cantor's theory was inconsistent it was not essentially

so. That is, the theory of sets is at bottom a perfectly consistent theory. However, the early set theorists blurred a couple of fundamental points producing inessential contradictions. The immediate and crucial question is, therefore, what this reasonable and consistent theory of sets is. (Different answers to this will locate different sites of confusion in early set theorists.) If this cannot be answered, this line of resistance collapses. So what is the reasonable, consistent core of the naive theory? It is a matter of history that for a long time there was no clear answer to this question. Rival answers provided by Russell, Zermelo, von Neumann, Quine, *et al.*, vied for place. However, it is now fairly clear that a consensus has emerged among mathematicians (though *not* perhaps among philosophers). The fundamental notion here is that of the *cumulative hierarchy*, i.e. the hierarchy defined by recursion on the ordinals thus:

$V_0 = \phi$

$V_\alpha = P(V_\alpha)$, i.e. the power set of V_α

$V_\lambda = \bigcup_{\beta<\lambda} V_\beta$, for limit λ

The surprising fact is that virtually every consistent set theory proposed this century can be seen as characterizing an initial segment of the cumulative hierarchy (in the sense that the segment is a fairly natural model for the theory).[53] Thus ZF set theory characteristics V_θ where θ is the first inaccessible ordinal, while the proper classes of Bernay's set theory can be taken to be just the members of $V_{\theta+1}$. Finite type theory based on the natural numbers is essentially $V_{\omega+\omega}$ and so on. The only proposed set theory which cannot be fitted into this picture is Quine's system NF (and its class-extension ML). It is indicative of the hegemony that the cumulative hierarchy has now achieved, that Quine's systems are, by and large, regarded as little more than curiosities.

Let us ask then whether the cumulative hierarchy is the consistent and reasonable core of naive set theory? (That it is an interesting and important structure is *not* in dispute.) The answer to this must be a fairly definite 'No'. One reason is that the notion of set produced by the cumulative hierarchy is very different from that produced by the naive theory. For a start the naive notion is clear and precise, whilst that of the cumulative hierarchy is, as we shall explain, inherently vague. The naive notion of set is that of the extension of an arbitrary predicate, a notion that can at once be spelt out in a pair of axioms, comprehension and extensionality. This is as tight an account as can be expected for any fundamental notion. It was thought to be problematical only because it was assumed (under the ideology of consistency) that 'arbitrary' could not mean arbitrary. However, it does. By contrast the notion of set given by the cumulative hierarchy is only as clear as the notions used in defining it. To begin with, the notion of an arbitrary ordinal *is* a highly problematic one. (The notion of an arbitrary sub-set also poses problems for the consistentist but we will pass this over.) Specifically, the construction of sets presupposes a prior construction of ordinals. However, this raises all sorts of problems about "how far" the construction can be continued, about sizes of infinities, etc. Indeed it is just these kinds of problems that the theory of sets was supposed to solve. We do not deny that once one has a notion of set one can non-circularly produce a notion of ordinal and use this in turn to define a special collection of sets, the cumulative hierarchy. But to suppose that one can use the notion of an ordinal to produce a non-question-begging definition of 'set' is moonshine.

There is a second and stronger argument that the cumulative hierarchy is not the essential core of the naive theory. This is that there are quite essential features of the naive theory which cannot be handled by the cumulative hierarchy. Specifically, the cumulative hierarchy cannot handle such intuitively acceptable sets as the universal set, and such intuitively acceptable set theoretic operations as complementation. Sub-collections of the hierarchy which have members of arbitrarily high rank are just not sets acceptable at all. The hierarchist may dig his heels in and insist that there really are no such sub-collections, but this only illustrates our main point. The notion of such a sub-collection is a quite legitimate naive one. If it is not a legitimate one for the hierarchist, then the hierarchy does not grasp the essentials of the naive notion. It might be thought that the notion of a proper class would help here. It does not. One might conceive of a collection with members of arbitrarily high rank as a proper class. But if one does this, there is no reason for not supposing that these proper classes can be members of hyper-proper classes. And what this shows is that we are still going "up" the cumulative hierarchy. So contrary to our original supposition, our "universe" is not *the* universe at all but merely a proper initial segment of the cumulative hierarchy. If we really did have the *whole* universe to start with, the notion of a proper class would take us nowhere.

The standard reply to the argument we have just advanced is that the sorts of set theoretic constructions we have been referring to are not part of the essence of set theory at all but part of the peripheral confusion. This however is an illusion encouraged by the fact that during the period 1920–1960 it seemed that the hierarchy did provide a sound basis for all mathematics—at least all mathematics for which set theory was just part of the auxiliary machinery. However, the illusion has now been shattered by category theory. Category theorists wish to refer to the category of *all* groups, sets, etc., or even the category of all categories. This cannot be done on the hierarchical view of set. Of course, there have been some attempts to get round the problem. Most of them work on some variant of the strategy of supposing that "all groups" means all groups of rank less than β for some nice ordinal β. However such ploys, which there is no need to discuss

in detail,[54] are just a subterfuge. The unpalatable fact (unpalatable to category theorists who accept the hierarchical view of sets, that is) is that one just cannot perform the constructions with global categories that category theorists need to make. There is no such thing as the category of *all* sets, etc., and that is that.

Thus the adequacy of the hierarchy view for all mathematics can no longer be maintained. Naive set theory has an essential power (essential for real mathematics, that is), which goes beyond that of the hierarchy; the hierarchy is not the consistent essence of naive set theory. Such a stable consistent essence has not been obtained. Nor is it likely to be: there are no doubt various consistent cut-downs of the inconsistent naive theory, but all will sacrifice significant features of the original whole; all will stop the expression of what can be expressed.

2.1.6. Naive semantics. The final example of an interesting inconsistent theory that we gave in ch. v, sect. 1.1. was that of naive semantics. Semantics, as usually understood, is the theory which concerns notions such as truth, satisfaction, denotation, etc. and naive semantics is the theory of such notions embedded in natural language discourse on such matters. Unlike naive set theory it has never been much elaborated, with distinctive theorems, results, etc. Nonetheless, *prima facie*, truth, denotation and satisfaction would seem to be characterized by the following axioms (already given in the introduction to Part Three):

$$\mathrm{Tr}\ulcorner\phi\urcorner\leftrightarrow\phi' \qquad (1)$$

$$\mathrm{Den}\ulcorner t\urcorner x\leftrightarrow t=x \qquad (2)$$

$$\mathrm{Sat}\ulcorner\phi\urcorner\leftrightarrow\phi'_s \qquad (3)$$

respectively, where ϕ is an arbitrary formula (closed in (1)), ϕ' a suitable translation, t is an arbitrary term, s a sequence, $\ulcorner\ \urcorner$ is the naming functor, and ϕ'_s is ϕ' with every free occurrence of the i'th variable replaced by 's(i)'. It is exactly this kind of insight which is enshrined in Tarski's calling (1) a *condition of adequacy* on any definition of truth.[55]

Each of (1)-(3) leads, as is well-known and as we have indicated, to its respective paradoxes. Both hard approaches, which fly in the face of the data, and softer approaches, which try to take some account of the continuing successful operation of natural languages, have been tried. Hard-liners, such as Tarski[56] and many logical positivists, grant that (1)-(3) do characterize our naive notions and have accordingly concluded that they are incoherent. But this effectively concedes what we are arguing for, that our naive notions are inconsistent. That they are incoherent is an unwarranted classical extrapolation. The friends of consistency who try to take a softer line have been forced by the paradoxes to maintain that, despite appearances, (1)-(3) do not characterize our naive semantic notions. It should be said straightaway that no argument has ever been produced for this claim other than that (1)-(3) lead to inconsistencies—which is no kind of argument at all against a paraconsistentist. Moreover, the sorts of thing that have been proposed to replace (1)-(3) are usually not only ugly but of dubious efficacy: they are either strong enough to produce some form of logical paradox or far too weak. But the detailed argument here comes down to the matter of proposed solutions to the semantical paradoxes, something we shall have to take up in detail below (in 2.2.2.2). What we will do now is to adduce one further argument as to why the T-scheme (1), in particular, cannot be jettisoned in semantics.

Perhaps the most fundamental insight in semantics in the last 100 years is that the meaning of a sentence is (given by) its truth conditions, or better, that to give the meaning of a sentence is to give its truth conditions. Though the insight is Frege's, he did not take it very far. However, it has without doubt provided the keystone of almost all the formal semantic theories this century. This means that the T-scheme must be an essential part of any semantics. For this, after all, is the scheme which states the truth conditions of ϕ. Admittedly, the T-scheme may not always occur in its pristine form. It does in Davidson's original theory. But it has to be context-sensitivized for indexicals. Moreover it has to be world-relativized as well in Montague or Routley/Meyer semantics, and it has to be constructivized for Dummett. Yet it is there playing its central role. In fact, without it we would be hard-pressed to know what a formal semantical theory looked like. Thus the friends of consistency, in urging us to junk the T-scheme, are in effect doing nothing less than urging us to junk semantics.

As always in philosophy, there are comebacks. The friends of consistency could claim—a claim yet to be made good—that formal semantics could be worked out with a limited T-scheme, which, like that of the hard-line levels-of-language people, coincides with the T-scheme for restricted languages and fragments of natural languages. And of course there is little problem in preserving the general truth scheme if we are concerned with the semantics of only certain fragments of natural language—in particular those which do not involve semantical notions themselves. This is what semanticists have by and large busied themselves with. However, once we try to map out the semantics of a whole natural language (including its own semantical discourse)—the doing of which has always been the pretension of semanticists despite their fragmented vision—the problem can no longer be shelved. Let us say it again; give up the general T-scheme and there must be some sentences whose truth conditions, and therefore meaning, cannot be given. Goodbye universal semantics.

With this powerful case for paraconsistency—from the semantical analysis of philosophically unavoidable inconsistent theories, such as those supplied by natural languages which quite properly include their own semantical terms—we move, or rather the dialectic moves us, from the questions of inconsistent theories to the question immediately raised by the apparent, or possible, truth of some of these theories; the matter of true contradictions.

2.2. The truth of some contradictions

The second reason we gave for paraconsistency in the introduction to Part Two was the truth of certain contradictions (see 1.2). This claim is likely to seem even more contentious. So let us examine it in more detail. The examples of true contradictions we gave concerned (i) multicriterial terms and (ii) the logical paradoxes. We shall consider each of these in turn. There are other examples that have been canvassed, but we prefer not to hang our main case on them. (We will discuss them later, in 3.2.)

2.2.1. Multicriterial terms. The situation *in abstracto* is this: We have some term t, and two criteria C_1 and C_2 which are empirically determinable. That C_1 holds is logically sufficient for applying t. That C_1 fails is logically sufficient for applying ⌜not-t⌝. Similarly with C_2. C_1 and C_2 are not, however, synonymous. Thus a situation may arise where C_1 holds but C_2 fails, making both t and ⌜not-t⌝ true of some object or situation.[57] Now the friends of consistency must deny the possibility of the sort of situation described. On what grounds is this to be done? There are several.

The first is to deny that there are any criteria in the sense we require, namely such that they are logically sufficient for the application of a term. It might be argued that criteria are connected to the applicability of terms via empirical "correspondence rules" such as 'when C_1 occurs, x is t'. Thus when the situation described arises what we have is, in fact, a falsification of either the correspondence rule for C_1 or that for C_2. However, it is not possible to maintain this line. It is a fact of life that there are criteria of the kind described, where there is an analytic connection between the satisfaction of a criterion and the correct applicability of a term. Thus, for example, having two legs is logically quite sufficient for the correct applicability of 'biped'. Having male genitalia (or perhaps having a certain chromosomic composition) is logically sufficient for the correct applicability of 'male'. Measuring six inches by a ruler is sufficient for the correct applicability of 'six inches long', etc.

The second possibility is to admit that there may be an analytic connection between criterion and applicability of term, but to argue that if the situation we have described occurs, what this shows is just that, despite appearances, either C_1 or C_2 has in fact failed. This kind of approach might be backed by an appeal to the general fallibility of observation, etc. Now it has to be conceded that in the sort of situation described, the move of doubting that either C_1 or C_2 obtained is a possible one. However, it seems to us that we would never take the clash *per se* to show that either C_1 or C_2 failed. We might well investigate the holding of C_1 and C_2 to try to find independent evidence of their failure. However, there is no reason why, in general, this should be forthcoming. And, if it is not, we would find it very unreasonable to insist, nonetheless, that one or other has failed. For all the arguments about theory-ladenness notwithstanding, to deny that, for example, an animal has male genitalia, for no concrete reason and when the evidence is right before one's eyes, takes a lot of balls. In fact, if we tested C_1 and C_2 independently and found them both to hold, we would not insist that one of them failed; what we would do is move on to another tack, which is the third way one might try to argue that the situation described never really occurs.

The third way is this. We might argue that the fact that we have two criteria analytically connected with the applicability of a certain term, one of which is realizable, indeed, realized, whilst the other is not, shows that the term is in fact ambiguous, representing two quite distinct concepts. Let us call these t_1, corresponding to C_1, and t_2, corresponding to C_2. Thus in the case described, t_1 is correctly applicable (but not $\ulcorner$not-$t_1\urcorner$) and $\ulcorner$not-$t_2\urcorner$ is correctly applicable (but not t_2). Hence the contradiction is only apparent. This insistence that there be a (1-1) correlation between concepts and criteria of applicability is, in fact, a well-known position. It was argued by operationalists such as Bridgeman. Equally its shortcomings are well-known.[58] The essential point is that as a matter of fact the senses of terms are not individuated in this way, via criteria of application. This can be seen by the fact that if we did try to pursue science whilst enforcing this kind of individuation, the complex network of theoretical interconnections of science would break down irreparably. Thus we have, in outline, a transcendental argument against this position.

Notwithstanding any of the above, in the kind of situation described, concepts do have a tendency to split. As a response to the crisis provoked by the falling apart of the criteria, the old concept will frequently divide into two, one corresponding to each of the criteria.[59] Be that as it may, this does not show that there was no true contradiction. *Ex post* the contradiction is resolved into two non-contradictory statements. But this does not affect the fact that *ex ante* the contradiction stood: otherwise there would have been no conceptual change.

This exhausts the relevant possibilities; there is no way of avoiding the conclusion that true contradictions are produced by multi-criterial terms

(even *if* in the ongoing dialectic these contradictions are duly resolved or removed, and replaced by others, and so on).

2.2.2. Logical and semantical paradoxes. The logical paradoxes are (as we stressed in ch. v, sect. 1.2. *prima facie* sound arguments with contradictory conclusions. Anyone who wishes to deny the truth of the conclusions must deny the soundness of the paradoxical arguments—of every single paradoxical argument, that is. But they must also do more than this. They must locate, specifically, the place where the argument fails (*and* be prepared to accept all the consequences thereof). However, just proposing a precise location of the unsoundness is not sufficient. That, after all, is too easy: merely list every principle used in a paradoxical argument, select one at random and deny it. Someone who wishes to reject the paraconsistent position must not only locate the source of the unsoundness but must explain, in a non-question-begging and coherent way, what is wrong with it. In fact, even more than this is required. An explanation of why the incorrect principle was found plausible in the first place is also required. Otherwise, the paraconsistent position, that all the principles are correct, still outstrips its rival in explanatory power. These are tall orders, which have never been met, as even many of those who would unreflectingly dismiss paraconsistency effectively acknowledge:

> No one, for example, who has thought at all seriously about the paradoxes will feel at ease with the supposition that they must contain one or more specific errors which, if presented to us, we should be readily capable of recognising as such and excising from our conception of admissible argument and definition. We have learned of a variety of strategies which seem to keep us out of trouble; but none of them has the simple intuitive appeal originally possessed by the 'naive' assumptions concerning class existence and predication, and what constitutes an admissible range of quantification, which featured in, for example, Frege's foundational theory. It is difficult to defend a notion of 'error' in this context for which the criterion is not precisely the potential to generate paradox; and this criterion, naturally, fails to discriminate in point of preferability between the alternative, seemingly successful strategies for avoiding paradox.[60]

These sorts of difficulties, for the friends of consistency, are basically the same whether the paradoxes are set theoretic or semantical ones. Indeed there is, we should argue, no essential difference between these types of paradox, a feature dialethic resolution reflects. Non-paraconsistent approaches have however generally been formed into a bogus distinction of types, and in order to deal critically with these approaches it is convenient to follow this artificial separation. The contemporary extensional bias in logic has moreover led to the widely assumed reduction of all logical paradoxes to set theoretical paradoxes, with which we shall start.

2.2.2.1. Paradoxes in set theory. The almost universally accepted villain of the set theoretic paradoxes is the abstraction axiom: the principle that every condition defines a set which is its extension. This, it is claimed, cannot be true in every case. Moreover, there is also a fairly general mathematical consensus as to which of its instances are acceptable: essentially those that are true in the cumulative hierarchy (see 2.1.5 above). This answer is not completely definite. It suffers from the vagueness we noted before concerning the "height" of the hierarchy. (For example there is no way to use the conception of set to determine whether the condition 'the rank of x is less than the first inaccessible' defines a set.) However, the answer is perhaps just passable, and enough to get to first base. Unfortunately for its supporters it is put out before it gets to second.

For a start no one has ever explained what is wrong with the instances of the abstraction axiom which fail in the cumulative hierarchy. (It cannot even be claimed that they necessarily lead to contradiction when added to ZF.) If it could be shown that the cumulative hierarchy was *the* essential core of the theory of sets, then this would go some way towards answering this challenge; however, as we have seen, this cannot be done. Moreover, even if it could, the solution would still not be adequate. For, why we should ever have thought that every condition defines a set, remains a *complete* mystery. It doesn't even look a *plausible* claim. How could such a mistake be made? No: the genuine conception of set is that given by the unrestricted abstraction scheme, according to which a set is the extension of an arbitrary property or condition. The cumulative hierarchy is exactly what it appears to be with a little historical perspective: *a* consistent substructure of the inconsistent universe of sets, masquerading as the whole thing.[61]

2.2.2.2. Paradoxes in semantics. The matter of the semantical paradoxes is more complex, though *prima facie* it should be simpler. There is no consensus whatever amongst logicians concerning the location of unsoundness in the semantic paradoxes. "Solutions" are two-a-penny, if not cheaper. This should mean that the consistent position doesn't even get to first base. If the opposition cannot even field a team, we win by default. However, in practice this just makes life difficult for the paraconsistentist. For instead of having one major rival to argue against, there is a variety of competing "solutions" to counter—some plausible, some wildly implausible, some endorsed, some merely mooted, some ancient, some modern, and about all they have in common is the ideology of consistency (plus the fact that they are wrong). Moreover, they all appear Hydra-headed: show that one distinction doesn't work and a dozen more appear in its place; show that an account runs counter to a well-supported philosophical theory and a dozen patched-up versions appear to replace it. In virtue of this, it would be understandable if we turned our back on the debacle and waited until at most one (and preferably none) survive the struggle for existence. However,

something needs to be said if we are not to be accused of funking the issue. It would clearly be an impossible undertaking to criticize in detail all the rival theories that have been proposed—or even just the major ones—in an introduction, only part of which is devoted to the topic. It is fortunate, then, that it is unnecessary to do so here, since general arguments can be found that (largely) postpone the need for such detailed criticism.

First observe that given the derivation of a paradox, all the moves in it are at least plausible. This suggests that moves of these kinds are normally correct, though—if the consistency hypothesis is correct—they fail on certain occasions. On which occasions? One striking feature of the paradoxical derivations, often remarked, is that they all use sentences that are in some sense, self reflexive (or at least "ungrounded"). Exactly how to characterize this reflexivity is itself a difficult problem, but let us pass it by. Let us call this class of sentences R (for reflexive). We might suggest that an occurrence of a member of R in a normally valid principle of reasoning may suffice to invalidate it. Assuming that members of R are fairly rare, this at least suggests an answer to how we come to be under the illusion that the principles are universally valid.

However, the crux of the matter now is this. What exactly is it about members of R that may give them the ability to invalidate normal logical principles? To answer this question we need to isolate a certain class of sentences D (for defective) for which normal reasoning (including applications of the T-scheme, etc.) can be seen to fail. But what are these? This is where the picture starts to fragment. Many different answers have been suggested. Without attempting to be comprehensive, we think it is fair to say that the mainstream answers to this question are of two types:

(α) the members of D are not well-formed. They may be well-formed sentences of English (or *prima facie* well-formed) but they are not well-formed sentences of a logically correct language (or of the deep structure of English);

(β) the members of D are neither true nor false, lack a truth-value, fail to make a statement/proposition, or in some way fail to relate in an appropriate way to true/false.[62]

Once D has been isolated, the paradox is "solved" by insisting that the member of R used in the argument is in D.

We are now in a position to formulate some general criticisms. But before doing so we should point out that many of the solutions face internal difficulties concerning D even before R comes into the picture. For example, someone who espouses strategy (β) needs to give a detailed account of truth-bearers and why certain sentences may fail to express (be) them. Many problems lurk here, as a substantial literature attests. Similarly someone who espouses strategy (α) needs to show that the grammar at work is really the grammar of English (or at least the grammar of rational discourse). Notoriously difficult problems lurk here.

Our first criticism of the sort of solutions we have outlined is that they are almost all, without exception, *ad hoc.* It is rarely argued that the member of R in question is in D, except on the circular basis of the paradoxes themselves. It is just assumed. Even when a general criterion for being a member of D *is* formulated, it usually turns out to include a clause whose sole *rationale* is to capture members of R. Because of this, such solutions beg the question against paraconsistency. For some paraconsistentists may well conclude that sentences of the class D invalidate standard principles. However, they will just deny that the member of R in question is a member of D. For example, they may well be pushed into (erroneously) admitting that sentences which don't make statements cannot be logically manipulated.[63] However, they can still claim that the liar sentence, for instance, *does* make a statement, a paradoxical one.

Our second criticism is that it is usually not at all clear that the proposed solutions really do avoid the problem. For whilst simple forms of paradox may be avoided in this way, more complex forms are just around the corner. Specifically, paradoxes of the "extended" variety characteristically arise to trouble proposed solutions. Let us illustrate this with the extended liar paradox. The liar paradox is avoided by insisting that 'This sentence is false' is in class D. Thus the paradoxical derivation is stymied. But now consider 'This sentence is false or is in D'. The supposition that it is either true or false leads to the usual contradiction. Moreover, the escape clause that the sentence is in D now leads to a contradiction also. In this situation, the apologist can make one of two moves. He can isolate a new class of sentences, D′, for which standard principles fail, and insist that the extended liar sentence is in it. This move will not help. Firstly, repeated, it leads to an infinite regress. The regress may not be vicious but it doesn't get us anywhere. For if (D_α) is the, possibly transfinite, sequence of sets generated by this process, we are still faced with the absolutely extended liar paradox:

This sentence is false or there is a β such that it is in D_β.

Secondly, it becomes more and more difficult to find new classes D′, D″, etc., which have any semblance of plausibility. So the whole thing bogs down very quickly. The other move the apologist can make is to insist that although the extended liar sentence is, in fact, in D, this cannot be truly said. This evidently opens the apologist up to an *ad hominem* argument of a devastating kind.

It also illustrates a third argument against "solutions" to the liar paradox. The purported solutions seem, without exception, to run the solver into the ineffable. The solver will be lead to the position that there are certain things which are the case which cannot be said or, more prosaically, the solutions will lead the solver to assert that certain things which can obviously be said, cannot be said. Ironically, one of these things often turns out to be the very solution itself.[64] This exclusion of sayable things as unsayable does not always occur in a uniform manner, but happen it invariably does. Moreover there are deep theoretical reasons why this should be so. The root of the problem is that English has an expressive power which is, in a sense, over-rich. It permits the saying of things whose semantic conditions determine that a contradiction is true (see ch. v, sect. 1.2). What all the solutions amount to in the end are proposals to limit this expressive power. However, this obviously means that there will be things which are the case, which can be expressed in English but which cannot be expressed in the self-imposed idiolect of the solver.[65] What this shows is that the original problem has not been solved but merely avoided. For the original proposal was to give an account of our semantic concepts and to show how, despite appearances, they do not, in fact, lead to paradox. In other words what is required is a semantic analysis of English, or at least those parts of it which themselves concern semantic notions, which shows their structure to be consistent. But what now transpires is that the semantic account offered in the course of "solving the paradoxes" is of notions which are expressively decidedly weaker than those embedded in English. Hence they are not those notions. Thus the semantic analyses are not those of our original concepts and do not therefore show them to be consistent.

These difficulties have suggested an heroic last stand to some friends of consistency. They (finally) concede that the semantic notions of English *are* inconsistent but urge that, in the cause of "science" (or whatever), they be ditched for ones that are. However this will not do. For, first, it concedes anyway, what we have been arguing, that the semantic conditions of English *do* determine certain contradictions to be true. Secondly, it recommends a move whose only benefit is the production of consistency. But once we are persuaded that the spread law, *ex falso quodlibet*, of classical and intuitionist logic is wrong, there is no objective benefit here (though it may, subjectively, make the friends feel better). Thirdly, the move occasions serious losses since it involves significant impoverishment of our expressive power (and so also of our logical powers). The adoption of some positivistic new-speak has therefore nothing to recommend it.

This completes our swift overview of the case against those who would "solve" the logico-semantical paradoxes. For the most part our arguments have to be understood rather as argument-schemes. For, strictly speaking, each proposed solution has to be taken in its own rights, its weaknesses exposed and explored. Then the argument schemes can be instantiated to produce more concrete arguments against any such solution. No doubt many of the finer details will have to be hammered out, but we have no doubt that this can be done. Moreover some concrete positions will have *prima facie* replies to some of the arguments and these too will have to be handled on their individual (de)merits. If induction is a good guide, the replies will pose more problems than they solve. For this has been the whole character of the enterprise of "solving" the paradoxes. Indeed, the enterprise can be seen as a research program, starting in all seriousness at the beginning of this century. Its problematic is that the paradoxes indicate a flaw in some logico-semantic principles and the aim has been to find it. The strategies (α) and (β) given above are the main heuristics that have been used to find a solution. Each has been elaborated (in many ways), its weaknesses exposed, and its auxiliary belt of protective hypotheses multiplied. However, it is characteristic of the debate that rather than making bold progress towards a solution, it has bogged down in trying to solve no problems other than those spuriously created by the research program itself. It is, in Lakatos' terms, a degenerating problemshift.[66] By contrast, the newer paraconsistency program, which does not try to locate a fault in the paradoxical reasoning, is definitely advancing, solving interesting technical and philosophical problems. Eventually, therefore, we would expect to see the program of solving the paradoxes begin to wither away. It will come to be seen as a plausible idea that never worked properly.

3. Ramifications and consequences of paraconsistency, and further reasons for paraconsistency

We have already argued that paraconsistency, and especially dialethism, has a major impact on logic and semantics, an impact which has many important philosophical consequences. The effect is equally devastating on the third of the conventional divisions of semiotics, pragmatics, and spreads out from semiotics to virtually all other reaches of philosophy. Because of the spread concerned we shall have to be very selective; we choose to concentrate, apart from pragmatics, on some facets of metaphysics and the philosophy of mathematics. We certainly make no claim that these are the only philosophical areas of consequence for paraconsistency; they are not.

3.1. Pragmatics

There are issues that come under the rubric of pragmatics—though this standard description is not an entirely happy one—questions concerning assertion, acceptance, and the use of argument to change beliefs rationally, that we can hardly avoid tackling. For there are various arguments to the effect that the ways assertion and belief function rule out paraconsistency. We will try to show that this is not so. In doing so we will show that certain standard conceptions of how they do function are incorrect and suggest better accounts. Thus paraconsistency has philosophical consequences in this area too.

3.1.1. Assertion: the question of content. Consider first someone who asserts a contradiction, say, to take the worst case, an explicit contradiction, such as $A_0 \& \sim A_0$. Obviously dialethicians are among such people. They are quickly confronted by several arguments to the effect that contradictions are not rationally assertable. If these were correct then strong paraconsistency would not be rationally espousable. Fortunately for paraconsistency, then, the arguments—though they do raise serious issues, for instance as to when it is reasonable to stop avoiding contradiction by resort to consistencizing stratagems and rationally to accept an inconsistent theory—are generally rather feeble.

According to the first such argument, contradictions are not only untrue, but manifestly so. Hence it is irrational to assert them, since we should not assert (sincerely, of course) a blatant untruth. This argument simply begs the question; dialethists hold that some contradictions *are* true. In reply to this, it might be argued that even on our grounds since the contradictions concerned are not only true but false, they still ought not to be asserted, since a rational man eschews falsehood. However, this again begs the question. If truth and falsity were exclusive, then the eschewal of falsity would follow from the aiming at truth. However once one sees that truth and falsity are inextricably intermingled, like a constant boiling mixture, the rational man must face the fact that the primary aim of complete truth-achievement can only be satisfied by accepting some falsehoods: the alternative of rejecting dialetheias would leave his grasp of truth partial and his knowledge incomplete. The second argument that contradictions are not rationally assessible is only marginally better. To see what it is, notice that someone who asserts $A_0 \& \sim A_0$ is denying an instance of the Law of Non-contradiction (LNC). It is often suggested that is impossible to rationally deny the LNC, since even to do so presupposes it.[67] Now even granting that it is impossible to deny the LNC without presupposing it (which we find no good reason to believe), this objection need not worry a paraconsistentist. For what it shows is only that the contradiction-utterer is committed to certain secondary contradictions.[68] And to suppose that this is objectionable is again just to beg the question against paraconsistency. This kind of objection doesn't really take paraconsistency *seriously.*

The same points apply to semantical arguments sometimes used to back up the first and second arguments, for example by showing, most simple-mindedly by appeal to classical truth-tables, that contradictions can't ever be true. To appeal to such (classical) considerations is to assume several questions at issue. In any case, such semantical "verifications" can be met by semantical refutations, rival semantics which admit some contradictions as true.[69]

The third argument against the possibility of rationally asserting a contradiction is to the effect that contradictions have no sense or content. There is quite literally, therefore, nothing to assert and *a fortiori* assert rationally.[70]

But in that event contradictions should have no sensible consequences, for the consequence relation would otherwise enable assertions with content to be got out of those with none, something to be got out of nothing. Yet contradictions do have consequences, A & ~A entailing A, for example. So what is the case, if any, for supposing that contradictions have no sense or content? It must depend, in the end, on some definition of sense or content which implies that result. Neither of the usual accounts of *content*—the semantic account in terms of the class of worlds a statement excludes, and the consequence account in terms of the statements the statement entails—have the requisite results; indeed classically and intuitionistically they, mistakenly, assign contradictions total content, and paraconsistently (e.g. relevantly) they assign non-null content.[71] Only in certain connexive logics, where contradictions entail nothing, do contradictions have zero content under the consequence account. But such a special logical base should be rejected as logically and paraconsistently inadequate, as we have seen.[72]

There is a better prospect of success, in bringing contradictions out as senseless (if that really is the objective), with the two accounts of sense which are dual to the accounts of content considered. For these accounts do render contradictions classically senseless. Under the first, which is often taken to be a *Tractarian* account of sense,[73] the sense of an assertion is something like the non-trivial set of worlds (or evaluations) at which it is true (i.e. its range). Then if contradictions were true at no worlds (under no evaluations), the conclusion would follow. However, there are evaluations which make contradictions true at some worlds.[74] Certainly if one restricts the totality of evaluations to classical ones,[75] (e.g. those for which $\nu(A)=\{0\}$ or $\nu(A)=\{1\}$ for all A), then there are no worlds at which contradictions are true. But to insist that one must do this when it is obviously unnecessary is, again, just to beg the question against paraconsistency. Under the alternative dual account of sense, the sense of an assertion is given by the complement of the set of assertions it entails. Should then we suppose that contradictions entail everything, they again come out with zero sense. However, the defining characteristic of paraconsistent logic is precisely the rejection of the counterintuitive view that contradictions entail everything. Hence this line of argument will not work against paraconsistency.

A similar set of points apply, with even more force, against specially rigged accounts of sense—or for that matter of rationality—which serve to bring out the anti-paraconsistent conclusion. Rival, and more natural, accounts can be given, as we have just explained, which undermine the anti-paraconsistent conclusion and show that it is far from obligatory. So why should it, and its grounds be accepted? The further advancement of the anti-paraconsistent case resorts at this stage of the dialectic, and has to resort, to special pleading.

The objections to the rational assertibility of contradictions we have considered so far do little more than beg simple questions against paraconsistency. However, there is an apparently deeper theoretical argument to the effect that if we were to allow people to assert contradictions then *no* assertion would have any content.[76] The argument is this: for an assertion to have content it must exclude certain possibilities, otherwise it carries no information. Now when a paraconsistentist asserts A he thereby excludes nothing. Certainly he does not exclude ~A: that may be realized too. Hence his assertion has no content. The argument is very plausible. Indeed we can even strengthen it. For that the truth of A does not logically exclude the truth of anything else can be proved in the semantics of most paraconsistent logics. *Any* set of formulas has an interpretation in which the formulas are all true. However, the argument fails, and does so at the first step. There is no reason to suppose that for a sentence to have determinate and non-trivial content it must exclude anything. Consider '2+2=4' and 'Perth is in Australia'. If paraconsistency is right, neither of these assertions *logically* excludes its negation, or anything else. Yet each has a different determinate but non-trivial content. This is so because each carries information the other *does not include*. So the second implies that Perth is somewhere, that Perth is in either Australia or Indonesia etc. whilst the first does not. (Only the sentence 'everything is true' has total content: its content is determinate but trivial.) This notion of content can be captured by taking the content of an assertion to be the set of sentences it implies, or what comes to the same thing, its place in the De Morgan lattice of propositions.[77] At any rate it shows that logical exclusion is unnecessary for content.

As we have seen, paraconsistent theory can supply its own accounts of content, both semantical and consequential. (In fact the definitions take exactly the same form as the classical definitions but are based on different theories of worlds and of consequence respectively.) In this way paraconsistency shows that accounts of how assertions convey information in terms of what they classically exclude are misguided. Of course, if we accept such accounts of content we cannot use content-exclusion as a way of defining the sense, or content, of negation. But then there are plenty of other ways of doing this, for example, through a semantical account.[78]

A final thrust against paraconsistency may be based not on the notion of content, or what is asserted, or assertion, but on the notion of rationality. For it is widely assumed that contradictions, whatever they may assert (whether everything, something or nothing) cannot be *rationally* accepted or maintained. While it may be widely assumed, the assumption itself is usually not rationally based. Insofar as arguments are offered, they typically depend on a characterization of rationality that simply makes consistency a necessary condition of rationality. But why do that? Imposition of such a powerful condition requires legitimation—otherwise it can simply be rejected (e.g. as stipulative)—by arguments to the effect that contradictions ought not to be accepted (where the *ought* is one of *rationality*, to be expanded semantically through a rule: In no ideally rational world ...). What arguments there are, mostly deriving from Aristotle, implicitly invoke the principle that ought implies can, and try to show that contradictions *cannot* be accepted, by virtue of their logical character, e.g. they never hold anywhere, they lack content, are senseless, etc. We have already refuted these arguments: we have argued that contradictions do not have such radically defective logical character, that they are capable of being assumed, accepted and believed, and sometimes are, and that the arguments to the contrary, emanating from Aristotle, are one and all fallacious.[79] Furthermore, we have argued, in some detail, that in *certain* cases contradictions ought to be accepted, because there is no really rational way of avoiding them, and because they are true.

3.1.2. Criticism and the change of belief. There is another way of trying to infiltrate traditional and classical rationality assumptions, through the question of the rational change of belief. It is often suggested that if true contradictions are admitted, then the process, indeed the possibility, of rationally forcing someone to change their beliefs by criticism is made impossible.[80] The central point is that if one criticizes a theory held by a dialethist, there is nothing in the domain of logic to stop him accepting *both* his own view and the criticisms of the critic, even though they constitute a contradiction. Hence, it is claimed, his view cannot be rationally criticized.

The premiss of this argument is correct. The conclusion is a blatant non-sequitur. It assumes that just because *some* contradictions are true, any contradiction may be rationally accepted. This is almost as crazy as the view that just because some assertions are true, any assertion may be rationally accepted, and involves the same two fallacious slides: from *some* to *all* and from *true* to *rationally acceptable as true.* But, it will be objected, we need a criterion, a decision-procedure for telling which contradictions are acceptable and which are not. There is reason to suppose that the stronger demand for a decision-procedure cannot be met. There is no decision-procedure for determining when a contradiction is true. Moreover to demand one is unreasonable: we know there is no decision-procedure for truth, even in very simple cases. There is no decision-procedure for true sentences of the form ⌜Rab⌝, ⌜Fa⌝, ⌜p ∨ q⌝, etc. Why should sentences of the form ⌜p & ~p⌝ be any different?

How then are we to determine whether a given contradiction in a given context is rationally acceptable? A preliminary answer is that, at this stage, we need to consider each sort of case on its merits, and even then there may be no immediate clear-cut answer. There is perhaps no answer to the question 'When is an arbitrary contradiction rationally acceptable?' which

is neither pretty vacuous, nor false. For there is, after all, no answer so far to the question 'When is an arbitrary assertion rationally acceptable?' which is neither vacuous nor false. But even though both a criterion and the need for one are in doubt, partial sufficient conditions are not. We have argued that certain contradictions are true. The arguments—whether cogent or not—are laid out (in 2.2) above, for the reader to decide. The important point here is that these arguments are rational considerations driving us towards the acceptance of certain contradictions as true—and can be recognized as such by any rational agent of sufficient competence. Thus the sort of things which drive us to accepting a contradiction as true are exactly the same as those which drive us towards accepting other propositions as true. But if paraconsistency is correct, can we build up a rational case for every contradiction? Of course not. Consider the contradiction "This object is both an elephant and not an elephant", where the object in question is a telephone.[81] What can be said for this? We can certainly argue rationally for one limb of the conjunction. Elephants are grey, organic, have trunks, etc., whereas this object, the telephone is white, inorganic, has no trunk, etc. If anyone can put forward a serious rational case for the other conjunct (without, of course, using *ex falso quodlibet* or other sophistry), we will give up not only paraconsistency but much other rational activity.

In this way paraconsistency shows that rational criticism is not based on assumptions of consistency. A view is effectively criticized if it can be shown to lead to something that is rationally rejectable—be it a contradiction or not.[82] The insistence upon the total unacceptability of any contradiction (or at least for a decision procedure for the total unacceptability of *some*) is the last refuge of the "Euclidean" desire for certitude or conclusiveness,[83] which, once common in the oceans of epistemology, now lives on like some coelacanth in the stagnant waters of (classical) logic. It used to be insisted that rational procedures, especially those of science, provided certainty either in proof (for inductivists) or in refutation (for naive falsificationists). However it has become increasingly clear, particularly through the work of people such as Lakatos, that this is something of an illusion. The job of killing a theory is frequently a long business: there is mostly no "instant rationality", no experiment which is guaranteed to work.[84] Nonetheless a sufficient weight of evidence can eventually succeed. Paraconsistency takes us some steps further towards showing that there is no argument of any kind which is guaranteed to work. Thus it may well be that a person *can* rationally hang on to an inconsistent theory, including an explicit contradiction to which it leads, at least for a time. Perhaps, however, as other evidence and arguments build up, as this consequence of the theory, or others, are found to be too damaging, this may no longer remain rationally possible. Perhaps not. Paraconsistency thus helps dispose of the last vestiges of "instant rationality".

Similar points to those that meet the charge that paraconsistency renders impossible the rational process of forcing someone to change their beliefs by criticism, also serve to meet the allegation that paraconsistency itself is uncriticizable;[85] for the allegation is really a special case of the more general charge. While it may be more difficult to criticize a paraconsistentist, since (as we have seen) one cannot automatically expect him to concede defeat if his position turns out to be inconsistent, still it is very far from impossible. For example, if an analysis of the way assertion and rational change of belief function were to show that these things would not be possible if paraconsistency were correct, we would have a powerful transcendental argument against paraconsistency. However, a correct analysis of these subjects does not show this, so we have argued. More generally, there are *many* ways in which paraconsistent arguments and logical procedures (including certainly our own) are open to criticism, and to reappraisal in the light of criticism, as criticisms of one paraconsistent position by or from another,[86] and resulting amendments of positions, reveals.

3.1.3. Consistency and the metatheory. The thrust of the argument of this section so far is to the effect that, *pragmatically, contradictions are not so very different from other kinds of assertions*: they have a determinate sense, may be true, may be rationally believed, and may be rationally rejected. Are there, however, any special classes of contradictions which do function in a "classical" fashion? Nothing we have said so commits us to a position on this, though we incline to the view that the answer is 'No'. However we wish briefly to discuss one class, which, it has been suggested, has this special status.

Specifically, it has been suggested that no contradictions of the form ⌜s is true and s is not true⌝ should be true. Now since the truth predicate is the main predicate of a semantic metalanguage, what is really at issue here is the consistency of the metalanguage. (If atomic formulas containing the truth predicate behave consistently, so will all usual compounds thereof.) Actually even to express it this way is somewhat obnoxious: for to talk of 'metalanguage' and 'metatheory' is already to start buying into the Tarski hierarchy which we do not want to do. So let us rephrase the issue; should our own semantic theory be consistent? It will be evident from all we have said on semantics (in 2.1.6 and 2.2.2.2) that we think the answer is 'No'. This, after all, is the lesson of the liar paradox, 'This statement is not true'. This statement is both true and not true. We envisage no stratification to avoid this. Natural language (or a formal language which models it) should be catholic enough to formulate its own semantics within itself.

However, there are some arguments to the effect that there must be an (ultimate) metatheory which is consistent. We can now dispose of these

fairly quickly. Such arguments are found in Batens[87] who, like Rescher and Brandom, espouses inconsistency but only at the "object level". One of his arguments (pp. 277, 231) is to the effect that if we cannot locate a domain somewhere which we can guarantee to be consistent, the possibility of rationally criticizing a theory and rejecting it is impossible. We dealt with this objection in the last section. The other argument (p. 277) is less clear. It is to the effect that 'one may describe an inconsistent domain [with a paraconsistent theory] but that something may be called a description only if its metatheory is consistent'. To be honest we find the argument for this not very clear, but the following quotation a few lines later suggests that the problem is that if the metatheory cannot rule out, absolutely, certain things being in the theory, the theory has no content: 'I cannot see how one could disagree with [Popper's] basic insight that only those theories are informative which 'forbid' something'. If we are right the problem is just like the one concerning content and exclusion we have dealt with above. But that response can be supplemented: The mere fact that $A \in T$ and $A \notin T$ does not imply that $B \in T$ for arbitrary B. (Unless of course we reason classically.) Then the fact that the metatheory is inconsistent does not imply that T is trivial i.e. it rules out nothing.

3.2. Metaphysics

In sect. 2.2 above we allowed for the possibility that there might be true contradictions other than those cited there. It is now time to discuss some of them. Designing inconsistent theories has not been the prerogative solely of mathematicians and scientists. Philosophers too, as we have seen, have proposed their considerable share of inconsistent theories, sometimes intentionally, sometimes not. Obviously, should any of these theories be correct, then they would provide further examples of true contradictions. But almost invariably such theories have been rejected by philosophers working in a predominantly empiricist climate, just because they were inconsistent. An important immediate effect, then, of paraconsistency is to give these theories a new lease of life. While a large number of these might be cited, (cf. sect. 2.1.2) two in particular stand out; Meinong's theory of objects, and dialectics, which we discuss in turn:

3.2.1. Meinong's theory of objects. Meinong's fundamental thesis is that every singular term of language signifies (something). The *significata* are objects, only some of which exist. The arguments for this position are numerous.[88] A main one, however, is that it enables a simple and uniform account of the semantic function of individual terms in sentences in which they appear. Classically the semantic function of an individual term is spelt out in terms of the object which is its denotation, and the properties which it has. However, traditionally this account has been thought to work only when the term signifies an existent object. For it is supposed that the notion of a non-existent object is a highly problematic one. Thus the classical account is drastically limited in application, since much common language appears to be about non-existent objects, e.g. fictional objects, objects of belief and other intentional objects, abstract objects such as numbers, properties, functions, etc. Thus in giving an account of the semantics of this kind of language, about what does not exist, some other manoeuvres have had to be made. One is to insist that this sort of term *does* denote an existent, though non-actual object. This is "Platonism", which is particularly common in the standard semantical accounts of mathematical theories. Another manoeuvre, more common with respect to fictional and intentional objects, is to try to paraphrase away the apparent reference to objects. This is reductionism. All of these moves encounter severe problems, though this is not the place to go into them.[89] The Meinongian solution is to maintain essentially the classical account of the semantic function of names but to ditch the assumption that what is signified must exist. Thus, in effect, a denotational account of names applies, quite simply, across the board, and all the problems disappear.

What has all this to do with paraconsistency? There are, as might be expected, numerous objections to Meinong's solution. Perhaps the most common is that it is impossible to make sense of, or form a theory with, objects which do not, at least in some sense, exist. This is a hoary old problem and we do not intend to buy into it now.[90] The objection we are concerned with is this: At least some non-existent objects must have properties. The semantic account of the function of names depends on this. Moreover we must be able to determine their properties, at least in some cases; otherwise we would never be able to distinguish them and tell whether sentences about them are true or false. How we are often able to determine the properties of existent objects is clear enough in outline, since they causally interact with us via our sense organs. However, this sort of answer obviously fails in general for non-existent objects. But if we can find no way of attributing properties to, or determining the properties of, non-existent objects, then they become very like Kantian *Dinge an sich* and just as useless. How then are we to do this? Meinongians have a number of answers, but one of central importance concerns the characterization postulate. Consider a descriptive term 'a . . .', 'the . . .'. Let us use 'τ' for a general description operator. Then the term $\tau x \phi$ has just these properties, ϕ, by which it is characterized. This is the characterization postulate:

$$\phi(x/\tau x \phi)$$

It is analytic by virtue of the senses of the terms involved, and hence known *a priori.* Now application of the characterization postulate soon leads to contradictions, in the case of certain impossible objects. Consider, for example, an object which is strongly contradictory in having F and not having F, $\tau x(Fx \,\&\, {\sim}Fx)$ say. Then $F(\tau x(Fx \,\&\, {\sim}Fx)) \,\&\, {\sim}F(\tau x(Fx \,\&\, {\sim}Fx))$. Hence, should we reject all contradictions, the characterization postulates must also be rejected—an outcome with important (negative) implications for the whole Meinongian enterprise. Indeed, an instance of just this argument was used by Russell in his damaging critique of Meinong's theory of objects, one of Russell's underlying assumptions being that contradictions are, one and all, outlawed.[91] However, once the possibility of true contradictions is conceded what is there against supposing that impossible objects such as $\tau x(Fx \,\&\, {\sim}Fx)$ yield some of them? It may just be a fact of life that some Meinongian objects are inconsistent objects, in the sense of having contradictory properties as supplied by the characterization postulate. The objection, on consistency grounds, to the characterization postulate is thus foiled. As is now becoming better known, Meinong immediately responded to Russell's critique along these lines, emphasizing that of course impossible objects have impossible properties and violate LNC, which holds at best for actual and possible objects.[92]

To be honest, the situation is not quite as simple as we have so far suggested; namely, go dialethic, and Meinong's theory of objects can be unproblematically rehabilitated. For even a dialethician cannot accept the characterization postulate in full generality, since using an unrestricted form, one can prove absolutely anything. It is enough to observe that $F((\tau x(Fx \,\&\, p)) \,\&\, p$ is an instance of the unrestricted postulate. Accordingly, either the postulate is already implicitly restricted, e.g. in a natural way, or else some restrictions have to be imposed on it. This opens another avenue of escape for the Meinongian, who can try to formulate restrictions on the postulate which rule out any applications which lead to inconsistency. However, this makes life a lot harder and messier.[93]

3.2.2. Dialectic. As we have seen in an introduction to Part One, dialectic is not so much a single theory as a cluster of ideas and themes to be found in a number of different thinkers, starting, in the modern period perhaps, with Fichte, going through Schelling, Hegel, Marx, Engels into contemporary thinkers such as Lenin, Sartre and Mao-Tsetung. It would be foolish to suppose that there is a uniform account of dialectics to be found in all these people. What these people share often seems, at least to those unsympathetic to dialectic, to be little more than a form of words, whose meanings differ radically. However, let us start with these words. A central component of dialectics, as construed in the modern period, is that of contradiction.[94]

The main things asserted about contradictions are:

(D1) There are real contradictions: some situations realize contradictions. (This is one form of the law of the unity of opposites.)
(D2) Change is brought about by the resolution of contradictions: in a dynamical system the state S′ succeeding a state S is produced by resolving some of the contradictions in S. (S′ is in the negation of S.)

We grant that different dialecticians have understood the notion of contradiction in different ways.[95] Thus, for example, a contradiction can be a self-contradictory proposition, incompatible concepts, a conception of a situation different from the reality of that situation, a process which moves towards an end which is self-defeating, inverse operations, opposing forces, opposing interests, conflicting tendencies, and so on. Different senses of contradiction will of course give rise to different senses for claims D1 and D2 above. It would be rash to try to find much more than a family resemblance between the various notions of contradiction listed above, and we will not try. Moreover, some of these notions of contradiction have little connection with the way that a logician understands the term, which is primarily the first on the list. For example, the notion of opposing tendencies has comparatively little to do with this. We concentrate on the logical notion, not because we regard the other notions as incorrect or uninteresting, but because if dialethic and dialectic are mutually relevant, this will be the locus. Thus—let us emphasize again—we are not claiming that what we go on to say about contradictions is a correct analysis of all dialectic—far from it. What we do say is that some major dialecticians have often deployed this sense of contradiction,[96] and that this is the focal sense from which the others derive and draw their strength. For these reasons dialethism is a valuable technical aid, in fact absolutely essential, in understanding logically, what is going on in dialectic. By contrast, attempts based on received logics (classical or intuitionist or traditional) to explain what is going on are bound to be rather abject failures, and to either write dialectic off, as Popper does as 'a loose and woolly way of speaking' 'without the slightest foundation',[97] or else turn dialectic into something very different from what it has been historically.[98] However, our aim here is not one of historical exegesis but one of trying to establish the mutual relevance of dialectic and dialethic, and in this way to help to logically rehabilitate dialectic. To this end we consider contradictions in knowledge and contradictions in the natural world. These are not the only places of relevance, but they suffice to establish our main point.

3.2.2.1. Contradictions in knowledge and the corpus of science. According to the law of the unity of opposites, when applied to knowledge, the historical state of knowledge at a certain time is liable to contain contradictory propositions. That this is indeed so is fairly easily seen. First, we have already argued that certain theories within the corpus of knowledge may be internally inconsistent.[99] However, contradictions arise for other reasons as well. As Popper has emphasized, a well-corroborated experimental result may well contradict a well-corroborated theory. The corroboration of both would normally suffice to put them both in the corpus of knowledge.[100] Popper, of course, insists that the theory should be jettisoned in this context. However, as others such as Lakatos have argued against Popper, "falsification" and rejection are not historically contemporary events in most cases.[101] The contradiction in the state of knowledge persists, seeking a resolution. The third reason for contradictions in the state of knowledge is, as Lakatos has again emphasized, that at certain times it may contain competing research programs, whose "hard cores" will certainly contradict one another.[102] Thus D1 is vindicated in this context.

Having seen this, D2 is fairly trivial. These contradictions provide an important motor for knowledge. For the fact that they are included generally provides a good reason for changing the corpus of knowledge in such a way as to resolve them.[103] Here 'resolve' does not mean simply 'eliminate', but 'transcend', in the sense of finding satisfactory explanations for the corroboration of both parts of the contradiction. Thus are the insights of Popper concerning the struggle between theory and experimental falsification, and of Lakatos concerning the struggle of rival research programs to supersede each other, built into a dialectical account of knowledge and the growth of science.[104]

It has been objected that our admission that some contradictions are true, undercuts this account of the dialectics of knowledge—and, more sweepingly, that D1 undercuts D2—for the following reason. Knowledge may well develop by the resolution of contradictions. However, the reason that it does so is because a contradiction in knowledge is unsatisfactory, and it is this because it must be false. Once one denies this then there is no reason why the contradiction should not be allowed to stay and knowledge remain static.

The point is simple. It suggests that there is an incompatibility between progress through contradiction-resolution and belief by the historical actors in true contradictions (not, *nota bene*, true contradictions as such). But this is confusion. For even assuming that the aim of the actors which produces the change in knowledge is the elimination of falsehood, it does not follow that they will rest content with a contradiction. Neither dialethicians nor dialecticians believe that *all* contradictions are true: they are as likely *not* to be true as anything else, indeed, more likely. Given a contradiction in the corpus of knowledge, and given the belief that it, like most contradictions to be encountered, is false, strong motivation to eliminate it from the corpus remains. But what of those contradictions which are true and seen to be such? Will these be resolved? Not necessarily. They may well stay. There is no impetus for a dialethician to exorcize the logical paradoxes, for instance, from the corpus of knowledge. But neither is it any part of dialectics that *all* contradictions are change-producing. In fact, there is a standard distinction drawn in dialectic between antagonistic (i.e. change-producing) contradictions and non-antagonistic ones.[105]

But all this is to assume that the aim of the actors is simply elimination of falsehood. And this of course, is too simple. For falsehoods are not simply eliminated but transcended. As knowledge expands we produce deeper and deeper explanations.[106] The true is explained and its limitations shown, the false is shown to be false and its corroboration explained. Thus a false contradiction may be jettisoned not for itself, but as a result of increasing explanatory depth. Moreover, even true contradictions may be resolved in this way. For deeper theories may well produce meaning-change which resolve contradictions by meaning-fission.[107]

3.2.2.2. Contradictions in the natural world. Let us begin again with the law of the unity of opposites as encapsulated in D1. If the world is the *Tractarian* world of the totality of that which is the case, then it contains contradictions. The logical paradoxes are examples of these. But are there true contradictions concerning the *natural* world, as opposed to the analytic part of the world? We have already answered this question in the affirmative, too, through the analysis of multicriterial terms.[108] This, however, by no means exhausts the possibilities. A paraconsistent Marxist may well argue for true contradictions (in our sense) concerning society, and a dialethic quantum mechanicist (at present a science fictional object) would argue for true contradictions at the subatomic level.[109]

This is still to neglect the much remarked connection between contradiction and change. For when dialecticians such as Hegel and Engels have emphasized the presence of contradictions in the natural world, they have done so in connection with change. For example, Engels is quite prepared to concede that a true description of the world, as it is at any one instant, a static account, may be consistent. However, once we correctly consider it as a dynamical system, in a state of flux, then a true description of what is going on must contain a contradiction.[110] Thus contradictions arise when a system is moving from one state to another and are resolved on terminating the motion. This is D2. It must be confessed that dialecticians have not, by and large, been prepared to *argue* the issue. Usually they have been content to cite the authority of Zeno, and his paradoxes of motion. Certainly some of Zeno's arguments (for example, the arrow) can be represented as arguments for the claim that in a state of change something both is and is not the case. But we have already discussed Zeno's arguments[111] and have not endorsed them. Let us, however, see what we can make of the situation without them.

Suppose a system is in a state S_0, and at time t_0 it changes to state S_1. What state is it in at t_0? *A priori* there are three possibilities: it is in one or

other of S_0, S_1 but not both; it is in neither S_0 nor S_1; it is in both S_0 and S_1. Maybe on different occasions and with different sorts of change, all three possibilities are realized. But, in particular, if S_0 is p's being true and S_1 is $\sim$p's being true, and a change of the third type occurs, then a contradiction is realized at t_0: both p and $\sim$p are true. Of course this possibility is ruled out classically: the very least one can say for paraconsistency is that it opens it up.[112] To determine whether or not this possibility is actualized we need to seek further arguments. Such arguments can be found. For example, one concerns Leibniz' limit principle "whatever holds up to the limit holds at the limit". This has a good deal of plausibility where physical processes are concerned. And if it is correct it implies that both p and $\sim$p are true at t_0 since t_0 is a limit of the intervals of time both before and after it. In this case, such change does involve contradictions, and D1 and D2 are both further confirmed as regards contradictions in the natural world.[113]

3.3. The philosophy of mathematics

Not only can mostly defunct metaphysical theories be rehabilitated through paraconsistency; many programs in the philosophy of mathematics can also be reactivated. For largely historical reasons the philosophy of mathematics has been intimately tied this century to the investigation of logic. And virtually all the philosophy so done has been predicated on the unquestioned assumption that logic, *that* logic, is either classical or intuitionistic. Thus paraconsistency is bound to have consequences for the philosophy of mathematics, some of which we now examine.[114] The most devastating effect of paraconsistency is to undo many of the negative results that have emerged over the last fifty years, especially those arising from or concerning Gödel's theorem, logicism and Hilbert's program, which we discuss in turn.

3.3.1. Gödel's first incompleteness theorem. Gödel's theorem can be stated in the following form: any (ω-) consistent theory which is strong enough to represent all recursive functions is incomplete. With this profound result we need not presently quarrel. Its proof requires only rather minimal assumptions concerning the underlying logic of the theory.[115] Of course, Gödel—and everyone else—again assumed that the underlying logic of the theory must be classical, or else intuitionistic, but at least in the case of Gödel's first theorem this is unnecessary. No, what we do take issue with are the bloated claims which are often made and which are supposed to follow from this result. The more modest of these[116] are usually to the effect that any axiomatic mathematics or arithmetic is incomplete.[117] Put another way, this is the claim that the set of true mathematical (or arithmetical) sentences is not recursively enumerable. Now as will be quite clear, this follows from Gödel's theorem if and only if the set of mathematical (arithmetical) truths is consistent. However in virtue of what we have said concerning the set theoretic paradoxes (in 2.2.2.1), the set of true mathematical assertions obviously is not consistent.

But what of the set of true arithmetic assertions? It could be that Peano Arithmetic, for example is inconsistent. It is not impossible, despite the consistency proofs, but it seems unlikely since the sort of conditions which seem necessary for producing paradoxes do not arise in Peano Arithmetic.[118] However, this does not show that the set of true statements about numbers is not recursively enumerable: It shows only that the set of such statements expressible in the language of Peano Arithmetic is not. For it is of course quite possible for a recursively enumerable set to have non-recursively enumerable subsets. But it is part of current dogma that anything mathematical that can be said about numbers can be said in the language of Peano Arithmetic. This is simply false.

Consider, for example, the theory elsewhere[119] called "semantically closed arithmetic". This can be thought of as Peano Arithmetic extended by new $n+1$-place predicates Sat_n (for all $n \geq 1$) and axioms:

$$Sat_n(\ulcorner \phi \urcorner x_1 \ldots x_n) \leftrightarrow \phi(v_1/x_1 \ldots v_n/x_n),$$

where ϕ is any formula with n free variables $v_1 \ldots v_n$ and $\ulcorner \phi \urcorner$ is its Gödel code. This theory will be able to express facts about numbers that are not expressible within the language of Peano Arithmetic, e.g. $Sat(\ulcorner x = 1+1 \urcorner 2)$. Presumably semantically closed Peano Arithmetic will not be a conservative extension of Peano Arithmetic.[120] If so, Peano Arithmetic is not only theorem incomplete but expressively incomplete, and, we suspect, the concurrence of these things may not be accidental. We have conjectured that semantically closed arithmetic, or at least some natural axiomatic extension of it, is (extensionally) complete; but the solution to this problem is, as yet, unknown.

Anyway these few observations are sufficient to sink, at least for the time being, most of the important purported philosophical consequences of Gödel's theorem (some more of which we will mention in the next two sections).

3.3.2. Logicism. A main aim of logicism, the foundational program proposed by Frege and Russell around the turn of the century, was to show that all mathematical truths are logical truths.[121] The attempt to do so fell, according to recent extensional reconstructions of the program, into two parts: (a)

showing that set theory was a branch of logic; (b) showing that mathematics is reducible to set theory. Both (a) and (b) have run into a heap of problems, problems that are largely removed once we turn paraconsistent.

(a) The set theory that Frege worked with was essentially an elaboration of naive set theory. His theory was, like the naive theory, found to be inconsistent, and so a consistent reformulation had to be sought.[122] The troubles for logicism begin right here. The main problem here is that set theory, as now reformulated, does not look much like logic at all. Frege's set theory could plausibly be seen as a theory about notions which are very general, subject-neutral and well within bounds of traditional logical concern, namely properties, concepts, and their extensions. Moreover its principles appeared (indeed we would claim *are*) analytic, a matter of logic (in a fairly tight sense). Thus Frege's set theory appeared, in all relevant respects, a part of logic. However, the same does not apply to the mangled forms that have passed for set theory in the twentieth century. The now-received theory of sets appears to be a subject whose concern is a quite specific domain of abstract objects, slightly more general than, though essentially no different from, other specifically mathematical objects such as groups, categories, etc. Thus, received set theory is often accounted a branch of mathematics and not logic at all. Moreover the axioms involved can hardly claim the self-evident analyticity of Frege's axioms.

By turning paraconsistent (by adopting a suitable underlying paraconsistent logic) we can however revert to naive set theory, and so, as with Frege's original theory, avoid all these objections, since the theory has, like Frege's, an evidently logical cast. This is not all. Much criticism has been directed at the inclusion of axioms of choice and infinity within the logicist reduction base, on the grounds, once again, that these axioms are not logical in character and not (analytical) theses of logic. These problems too disappear. For both the axioms of infinity and choice, which have purely logical formulations, are provable in absolutely naive set theory.[123]

(b) The question of whether mathematics is reducible to set theory is a little more open. That classical mathematics is largely reducible to, say, ZF set theory with classical underlying logic is now widely acknowledged—though there are some arguments to the effect that it is not completely so reducible, which we will consider in a moment. However, it is not at all clear yet whether mathematics, or even number theory, is reducible to naive set theory with a paraconsistent logical base. For although the purely set theoretic principles are a great deal stronger, the logic is correspondingly weaker. Thus a lot of the moves in the standard reduction cannot be made. This does not, of course, mean that reduction is impossible; it may well be possible in a different way. However to determine whether this *is* possible will require a great deal of work which has not yet been done. What little work has been done on the relevant/paraconsistent formalization of mathematics (e.g. by Meyer and by Brady), shows that very often it is possible to find relevantly/paraconsistently acceptable versions of theorems/proofs that are traditionally formulated/done assuming objectionable classical principles. However, work in this area is only just beginning.

There are arguments that such work would be a waste of time, at least as regards reinstating a logicist program. Let us consider then the standard theoretical arguments against the total reducibility of mathematics to set theory, and ask whether these hold against paraconsistent naive set theory. There are three such objections to be parried.

The first objection is that the embedding of category theory in set theory is impossible because of the problem of "large" categories. While this is a very real problem for standard, putatively consistent, set theories (as we urged when we examined this issue in 2.1.5 above), it is sufficiently clear that naive set theory provides adequate conceptual apparatus for category theory.[124] This therefore ceases to be an objection to reducibility.

Secondly, there is the major objection from Gödel's theorem. Gödel's theorem appears to show that the set of mathematical truths is not recursively enumerable, and hence not capturable by any axiomatic system, naive or otherwise. However, as we saw in the previous section, paraconsistently there is no reason to believe this to be so. Hence this objection to logicism also lapses.

The third objection is related to the second. Gödel's theorem purports to show that any axiomatic set theory is theorem-wise incomplete. This objection is to the effect that any set theory is expressively incomplete in the sense that there are mathematical objects which cannot be defined in it.[125] Basically it is as follows: Let # be a Gödel coding of formulas. Define a function F from the natural numbers to the ordinals thus:

$$
\begin{aligned}
F(\#\phi) &= \alpha \quad \text{if } \phi \text{ is a formula of one free variable} \\
&\qquad\ \text{and } \alpha \text{ is the least ordinal that satisfies it;} \\
&= 0 \quad \text{otherwise.}
\end{aligned}
$$

The supposition that F is representative by a formula of set theory leads to the expected derivation of a contradiction. In fact the derivation is just a version of König's paradox.

All the objection shows, however, is that F is not definable in a consistent set theory. It obviously does not show that F is not definable in naive set theory, which we know to be inconsistent anyway. Moreover we know how to define satisfaction set theoretically. Thus we may suppose that naive set theory can define its own satisfaction predicate. And given it can, then F is definable within naive set theory by essentially the above definition. Furthermore, in the event that—because of the weakness of the underlying logic—naive set theory could not define its own satisfaction relation and prove it to have all the right properties, all this would show is that set theory as such does not exhaust all logical notions. For satisfaction clearly is a

logical notion. Hence we could add it to naive set theory (with appropriate axioms, etc.) and in the extended theory F will be definable, just as above. This extension of the theory in no way undercuts the logicist claim since the extended theory is just as much logic as the original set theory. Thus this objection to logicism lapses, as did all the others. These points, taken together, meet the main objections to logicism.

3.3.3. Hilbert's program. Hilbert's program was philosophically motivated, but technically what it came down to was (a) axiomatizing mathematics or, at least, various of its parts, and (b) proving consistency by finitary means.[126] The notion of being finitary was always vague to a certain extent. However, finitary methods had to be constructive in some sense, and certainly much less than full classical methods of proof. Hilbert's program ran into trouble at both stages because of Gödel's theorems.

(a) The attempt to axiomatize even elementary arithmetic was given up because of Gödel's theorem, which was thought to show that this was impossible. However, as we have already seen, paraconsistency undermines this impossiblity argument.[127]

(b) The attempt to prove the consistency of axiomatic arithmetic by finitary means was abandoned because of Gödel's second incompleteness theorem, which seemed to show that the sentence which canonically asserts the consistency of a theory is not provable within the theory itself. It follows that the consistency of any reasonably strong theory of arithmetic is not provable by finitary means. Now this all lapses once we abandon classical logic. For the proof hinges on the fact that the underlying logic of the theory is classical. And without this the proof of Gödel's second theorem fails. This is most easily illustrated through the system R# of relevant arithmetic, that is, a system of arithmetic comprising suitable versions of the Peano postulates but based on a relevant logic R. The system R# is fairly strong: it can represent all recursive functions. Yet this system has a simple consistency proof, finitary by any standards, which is representable within the system itself.[128] Thus the whole question of "finitary" consistency proofs for various interesting mathematical theories is reopened.

Naturally once we move to inconsistent mathematical theories, of which, as we have seen, there are a number, the question of a consistency proof lapses. However, the subject does not lose its interest. For the role that is played by consistency classically is played paraconsistently by non-triviality. The important question becomes whether in certain mathematical theories everything can be proved. But the subject assumes new dimensions as well: for questions concerning *degrees* of inconsistency (and, correspondingly, extent of consistent subtheories) are raised. More specifically the idea is to characterize what inconsistencies are provable. For example, in naive set theory are there provable any inconsistencies concerning small (e.g. finite) sets? The question of what mathematical methods are necessary to establish these results is also of interest. Thus this kind of investigation, largely initiated by Hilbert, still has much interest. Indeed, questions of degrees of inconsistency are, we suspect, deep and will come to play a very important role in the subject. However this kind of investigation is very much in its infancy. Only a start has been made: Brady has now proved that naive set theory is non-trivial.[129] But even the question of whether the obviously undesirable $\phi \neq \phi$ is provable is open. In these various ways, paraconsistency opens up main parts of Hilbert's program and associated areas that had been classically closed off, and reawakens interest in them.

4. Conclusion: the ideology of consistency

There are, we have argued, no insuperable philosophical problems in supposing that there are true contradictions and, moreover, there are substantial benefits attached to doing so. What mainly prevents the acceptance of this view is the ideology of consistency: the deep-seated and irrational view that the world is consistent.

It is worth inquiring why the view is so deep-seated. A superficial answer is to the effect that the belief in consistency is an unwarranted induction from common experience which, for the most part, is consistent. However, this does not get to the root of the problem. For belief in consistency has not been universal. The belief was rejected by many pre-Socratics and by most nineteenth century German philosophers. Why then should it be so dominant now? Part of the answer lies in the present dominance of Anglo-American analytical philosophy which is squarely in the empiricist tradition. It does not take much perception to observe the lines running between Locke, Hume, Mill, Russell, Carnap, etc. Underlying empiricism has always been an atomistic metaphysics[130] whether it is Hume's "world" of independent experiences, or Russell's logical atomism. Now atomism has always been ill-accommodating to contradictions. For each atom is quite independent of all others. It moves within its own "logical space" and can have no relationships with other atoms. Hence it cannot come into conflict with them and produce a contradiction. Thus, for example, Wittgenstein in the *Tractatus*, who simply echoes Hume, in slightly different terminology:

5.134 One elementary proposition cannot be deduced from another.
5.135 There is no possible way of making an inference from the existence of one situation to the existence of another, entirely different situation.
5.136 There is no causal nexus to justify such an inference.

These separability assumptions ensure consistency of elementary components, from which all else is built up. The dominant metaphor in nineteenth century German philosophy is of course very different. Instead of the collection of atoms, it is the organic whole, that is, a whole the parts of which are internally related to each other. The possibility of essential conflict, and so internal contradictions, is thus to be anticipated.

Placing the dominant philosophical paradigm in its empiricist atomist tradition indicates part of the answer to the question of why the tenor of mainstream Anglo-American philosophy is antagonistic to contradictions. Another piece of the jigsaw is this: atomism has always played, through individualism, a political role as well as a metaphysical one in empiricism. For society also is conceived of in terms of a collection of autonomous individuals or political atoms (contrast again the organicism of Marx and Hegel), and this picture has formed the basis of virtually all bourgeois political and economic theories, and certainly of the dominant politico-economic paradigm. Thus insofar as a consistency hypothesis and repugnance of contradiction are part of a general empiricist/atomist/bourgeois perspective, our allusion to the ideology of consistency is far less fanciful than it may at first have seemed.

Anyway these are deep issues and we shall not pursue them further now. In fact we have done little more than scratch the surface of the philosophy of paraconsistency. Some of the issues are taken further in this section, but we hope to have at least shown that the philosophical ramifications of paraconsistency are wide-ranging and deep. It is impossible to tell where exactly they will lead eventually, but such is the case with any radical new theory.

Notes

[1] Strictly speaking there could be, if the decision could be overturned by a higher court. However for the court of highest jurisdiction, the claim is correct.

[2] On such dilemmas and their place in paraconsistent deontic logic see Routley and Plumwood, 1989. The theme that the natural logic of legal language is paraconsistent and not classical was first suggested by Quesada; see further his paper, this vol., pp. 627–652.

[3] Many non-philosophical theories are in a similar "predicament", as we shall see.

[4] This theme is adapted from a note from J. Passmore.

[5] The point could be rendered analytic by appropriate distinction between *practice* and *theory* and tightening of the latter notion.

[6] There is substantial evidence however, assembled by Goldstein, that Wittgenstein adopted a theory of content in the *Tractatus* which implies the rejection of classical spread laws. What this seems to show however is not, as Goldstein

suggests, that the *Tractatus* is based on a non-classical logic, which lacks such principles as Contraposition—but a further inconsistency in the *Tractatus* between the underlying classical logical theory and the theory of content grafted onto it.

[7] See e.g. Quine, 1955. But Wittgenstein wanted to insist that Frege's theory, though inconsistent, was not trivial, i.e. there were implicit restrictions on what rules could be applied where: see the first introduction to Part One.

[8] See the introduction to Part One above. This is only one of the many apparent inconsistencies in Aristotle's philosophy. Another is as regards the extent to which the principle of non-contradiction (LNC) applies to appearances as well as to substances: see Łukasiewicz, 1971, p. 502.

[9] Inconsistency of philosophers is almost to be expected. But logicians, who are supposed to be especially skilled at seeing the consequences of their assumptions, have a considerable record of inconsistency where they try to design more comprehensive systems. A list is impressive and includes such logicians as Frege, Church, Quine, Lewis, . . .

[10] Inconsistencies, especially in the form of anomalies, are only part of the story; philosophical fashions as influenced or even controlled by underlying socio-economic conditions, are another crucial part of the fuller story. Movement to avoid inconsistency does not always represent progress (we follow the honorific use). Thus Russell's movement from logical paradoxes to the ramified theory of types, Mally's dismantling of the theory of objects in the face of perceived inconsistencies in favour of a much more complicated construction, the epicycling of degenerating philosophical theories such as Plato's later philosophy and the current extensional Davidsonian program, do not strike us as cases of philosophical progress.

[11] See Griffin, 1980.

[12] From Marsh, 1956, p. 183.

[13] Marsh, 1956, p. 270: the passage quoted concludes: "the knowing of facts is a different sort of thing from the knowing of simples". P. Simpson, to whom we owe this example of inconsistency in Russell, points out that a rescue attempt might perhaps be mounted by arguing that facts only "exist" derivatively (or at a different level) in the way that complexes such as particular chairs and tables do. But that supposes that facts are, what they are not in the theory, certain—somehow independently determined—assemblages of simples.

[14] Routley and Plumwood, 1982.

[15] See Griffin, 1980, p. 155.

[16] See Griffin, 1982, appendix.

[17] See further Griffin, 1980, pp. 176–177.

[18] In 1951: for details see EMJB 1.13.

[19] For other examples of lesser inconsistencies in Quine, see Routley, 1982 and especially Gochet, 1986, which also presents deeper tensions in Quine's overall position. One more serious difficulty arises from the attempt to combine physicalism (and individualism) consistently with the orthodox kind of set-theoretic methodology physicalists typically adopt. For an unsuccessful attempt to resolve part of this problem see Smart, 1978. And Quine's very recent changes to his philosophy, which, in abolishing individuals, upsets much of his mature work, can be viewed as an attempt—among other things—to remove this inconsistency.

A related inconsistency in Quine's mature theory concerns the status of classical mathematics, which emerges as both true and false; a similar inconsistency infects Smart's work (see EMJB, p. 620). Armstrong's naturalistic theory is also bogged down in a connected inconsistency concerning mathematics (see again EMJB, p. 750).

[20] See, e.g. Russell, 1905. It is by no means clear that Russell's objections apply against Meinong himself (see EMJB). Inconsistent by virtue of such characterization postulates is not only the Graz theory but also, for instance, Castañeda's 1974 theory (see EMJB, p. 880ff). Castañeda's theory is also rendered inconsistent, and thereby trivial given its classical basis, by virtue of inconsistencies in guise-theory; see Clark, 1981 and also 1978.

[21] The basic argument is in Plato's *Parmenides*, §132; the resulting inconsistency is widely discussed in the literature. The more extensive inconsistencies in Plato's theory of forms—some of them integral to the theory—are well-documented in Griffin and Johnson, 1983, where it is also argued that leading modern attempts to consistencize the theory fail. To set the matter in the present context, see EMJB, p. 639ff.

[22] Other inconsistencies in Wittgenstein's theories have already been recorded, earlier on, and these draw upon only a small portion of his work. (Some of the other respects in which the work is pragmatically self-refuting, and so inconsistent when the further assumptions are applied, are so trite as to scarcely bear recording; e.g. whatever can be said can be said clearly.) Whether the more significant inconsistencies trivialize the later theory is, however, unclear, but it would seem not since classical logic is supposed to be restricted in application in inconsistent situations.

[23] A similar problem may be found lurking in Tao, which is a teaching that denigrates learning.

[24] The evidence for these claims, which are strictly in the nature of promissory notes, is to be found in Collingwood, 1939, pp. 58–75, and 1940, p. 21ff. Collingwood's theory is on a direct collision course with this enterprise.

[25] This is a highly condensed version of Passmore's explanation of the historical charge of self refutation against Protagoras; see Passmore, 1961, pp. 64–70.

[26] See Passmore, 1961, chapter 4, for examples in the case of scepticism and for further discussion of self-refuting theories which often issue in inconsistency.

[27] Bose, 1967, p. 15. But there is some doubt as to whether Lao Tse is genuinely committed to the latter proposition, though other sources seem to supply it also.

[28] See Hintikka, 1962.

[29] Passmore, personal communication, 1982.

[30] The inconsistency in Reid over the admission of ideas *may* be of this shallower type. For it seems, at first sight anyway, that he has no real need to make the following concession to Locke from which the trouble starts. Referring to what he calls 'the appearance of colour', Reid says

> Mr Locke calls it *an idea* and it may be called so with the greatest propriety . . . . It is a kind of thought, and can only be the act of a percipient or thinking being (*Inquiry* VI. iv; *Works* i. 137).

But this concession is inconsistent with Reid's rejection of ideas in his critique of the Theory of Ideas. More generally, as S. Grave to whom we owe these points concerning Reid remarks, Reid's account of perception, by all senses except touch, is inconsistent with part of his attack on the Theory of Ideas. A way of avoiding such inconsistency emerges from the treatment of ideas and sense data in EMJB, but it is very doubtful Reid could, or would, like this way.

[31] Passmore, 1978, see p. 207.

[32] Passmore, 1978, p. 187.

[33] On all the points, and for the quote from Locke, see Passmore, 1978, pp. 185–189. Passmore proceeds (in accord with the prevailing consistency assumptions operating in the history of thought) to try to reinterpret Locke to remove the contradic-

tion: 'faced with so absolute a contradiction ... we have no option but to look again at our interpretation' (p. 189). But as Passmore is well aware (e.g. p. 190, middle), his various proposed ways out run into logical conflict with other parts of Locke's philosophy, and Locke would not have found them at all palatable.

[34] All page references are to Spinoza, 1675. As always, there are ways out of such inconsistencies, by distinctions the theory does make; e.g. by a distinction between intellectual love, which God has, and non-intellectual love, which God does not have. But this conflicts with the definition of *love*. Strictly, since *love* is defined through *pleasure*, and *pleasure* (human chauvinistically) for humans only (pp. 128–130), none but humans can have love; neither animals nor God can. (There are inconsistencies also in the theory and treatment of animals in Spinoza.) C. B. Daniels, to whom we owe this example, suggests there would be another way out if time has a beginning (see proofs of propositions 33 and 34, book 5).

[35] The argument is taken from Routley, 1968.

[36] As several passages from the *Discourse on Method*, e.g. p. 101, and *Meditations* e.g. pp. 145–146, show. (Page references are to Haldane and Ross, 1911–1912.) These references and this inconsistency in Cartesian scepticism we owe to J. Kleinig. Compare also Passmore, 1961, on the self-refuting character of absolute scepticism.

[37] A good feel for the problem is given by Ryle, 1949, p. 12ff.

[38] See Ryle, 1949, pp. 20–21.

[39] They may however be happy to see some of the philosophical competition removed.

[40] Of course there are many other examples of unintentional inconsistency we could have developed, given sufficient time and energy. In an obvious sense, then, our case is, inevitably, incomplete. There are many philosophers we might have looked at (more closely) but haven't. For example, Mill is inconsistent in his account of causation (in *A System of Logic*) as to whether exhibited constant conjunction is sufficient, or hypothetical conjunctions are also required. In Leibniz, apart from the matter of the infinitesimal calculus, there are the inconsistencies that helped lead Russell to propound the double philosophy theory (in 1900). And then there is Kant ...

[41] The theories may also be inconsistent in respects other than those recognized. For example, a main theme in certain kinds of Buddhism is that nothing is self-contained, that everything is attached to other things. Yet the objective recommended is to obtain release (Nirvana) (e.g. from pain and troubles) by detachment, by severing connections (e.g. important attachments to place and people): that is the ideal personal situation of self-containment, which is impossible. Yet Nirvana is attained.

Deliberate inconsistencies in Hegel and Marx have already been documented, e.g. in chapter II, but those in Sartre have not. Contradiction figures essentially in Sartre's accounts of anguish, said to arise from a paradoxical feature of existence, that I am the self that I will be but I am not the self I will be; and in his analysis of self-deception or bad faith, which require the 'forming of contradictory concepts which unite in themselves both an idea and the negation of that idea'. More generally, describing human existence adequately requires use of contradictions: 'we have to deal with human reality as a being which is what is not and which is not what it is'; and the same applies to true descriptions of persons (e.g. of the pederast). For a fuller elaboration of all these points, and for references, see Tormey, 1982.

[42] Passmore, 1952, p. 1, and Selby-Bigge as referred to in Passmore.

[43] For further examples of (unacknowledged) inconsistencies in Hume see this work of Passmore. Yet other examples (we owe to F. White) derive from a systemic

inconsistency, concerning external objects, God, and such theoretical objects as energy and force, to the effect both that these objects do not exist and that they do exist but we cannot know that they do. As regards the negative ontological claim, Hume says much to back up a theme of the *Abstract* of the *Treatise* that we have no idea at all of force or energy, and these words are altogether insignificant, or they can mean nothing but that determination of thought, acquired by habit, to pass from the cause to its usual effect. Yet in the *Enquiry Concerning Human Nature* (IV, I, 29) Hume's case seems to be a negative epistemological one, simply that ultimate causal powers in things cannot be known to us.

Two further inconsistencies in Hume (pointed out to us by D. Stove) concern induction and caused existence. As regards induction, Hume both argues that induction is fallacious (in sections 4–6 of the *Enquiry*) and also accepts induction as not fallacious in his argument against miracles (in section 10 of the same work). As to coming into existence, he both accepts and elsewhere rejects the proposition that something might begin to exist without a cause: for details see Stove, 1975.

[44] Though there is surprisingly dialectical-looking material at places in the *Treatise*, e.g. p. 205.

[45] The point is discussed in detail in EMJB, chapter 12.

[46] Garrett, 1981, see p. 337. The inconsistency is spelt out on p. 350ff.

[47] Galileo Galilei, 1914.

[48] For further examples see, e.g. Feyerabend, 1975, p. 258; 1978 §iv.

[49] Further discussion of the Bohr theory can be found in Lakatos, 1970, §3(C2).

[50] See Boyer, 1949, ch. 6.

[51] "[circa 1720] mathematicians still felt that the calculus must be interpreted in terms of what is intuitively reasonable, rather than of what is logically consistent". Boyer, 1949, p. 232.

[52] This is a real possibility with theories such as parts of the infinitesimal calculus and quantum mechanics. With some philosophical theories, it is quite another thing. It is extremely difficult, if not impossible (for the more ordinary philosopher at least), to obtain a commanding view of a philosophical position like Hegel's, a view in terms of which one could begin formulating the theory in a suitably exact way.

[53] See Fraenkel, Bar-Hillel, and Levy, 1973, pp. 321–331.

[54] Details can be found in Fraenkel, Bar-Hillel and Levy, 1973, p. 143ff.

[55] See Tarski, 1936, pp. 187–188.

[56] Tarski, 1936, pp. 164–165.

[57] A slight generalization of the situation is presented in Pinter, 1980 where the criteria C_1, C_2 are replaced by sets of criteria. However this change makes no essential difference. For concrete examples of this sort of situation see ch. V, sect. 1.2.

[58] See Hempel, 1966, Ch. 7; Papineau, 1979, pp. 8–10.

[59] See Priest, 1980 for a further discussion of this point, and Maund, 1981 for further examples of splitting.

[60] Wright, 1980, p. 297.

[61] How could people make the mistake that it was the whole thing? That people will do (wildly) irrational things if it is demanded by an ideology is well-documented. In this case, the ideology of consistency demanded that an *Ersatz* for the universe of sets be produced and the cumulative hierarchy was, if not exactly an ideal candidate, at least a lowest agreed common denominator which, as it turned out, captured, in a neat synthesis, several apparently rival proposals.

[62] The two main positions outlined are, of course, rather bloodless abstractions. Concrete proposals are always more complex and variegated; they need to be much more specific about the scope and basis of D, for example. Still some abstraction is necessary to make the discussion manageable.
[63] There are many counterexamples, including imperatival, erotetic and significance logics.
[64] For this sort of criticism of the theory of types, see Fitch, 1952, Appendix C, and also Black, 1944.
[65] See Priest, 1984, for a further discussion of these issues.
[66] See Lakatos, 1968, pp. 128–129.
[67] See e.g. Stahl, 1975, p. 45. The argument goes back to Aristotle: see J. Łukasiewicz, 1971. Łukasiewicz devastates Aristotle's argument. For further detailed criticism of LNC as a law of thought and as impossible rationally to deny, see R. and V. Routley, 1975.
[68] See chapter V, sect. 2.2.
[69] See the semantics of various of the paraconsistent logics discussed above in chapter V.
[70] An argument of this sort is presented fairly dogmatically by Rescher and Brandom, 1980, pp. 24–25:

> We have little choice but to regard the ... [contradiction] as self-destructive, as simply self-annihilating ... a blatant contradiction [is unintelligible] ...,

and a similar non-classical cancellation view is an important strand in Wittgenstein's later thought (see chapter I, section 5.4). There are certainly other choices, and furthermore better choices, as is argued in Routley and Plumwood 1982.
[71] These content measures, classical and relevant especially, are investigated in detail in Routley, 1977, and one of them is discussed above in chapter XIII, sect. 4.6.
[72] In chapter V above, section 2. A much more detailed critique of connexive logics may be found in RLR, chapter 2.
[73] The account of Wittgenstein's *Tractatus* is different, in that it brings out tautologies as well as contradictions as lacking sense. Exactly what non-contrived theory of sense underlies the *Tractatus* (if any) is still a matter for debate.
[74] We have explained exactly what these are in chapter V, section 2.3.
[75] This is essentially what Rescher and Brandom do, 1980. Naturally the argument does not terminate where we have left it. The dialectic next shifts back to the questions of what counts as a *world* and what as an *evaluation* or *interpretation*.
[76] This argument is hinted at by Rescher and Brandom, 1980, and is to be found, in effect, as an argument for the existence of a "strong negation" in Batens, 1980, pp. 226–227. It is stated more explicitly in Lear, 1980, p. 112.
[77] See Priest, 1980a.
[78] For example, as given in chapter V, sect. 2.
[79] The arguments that contradictions are not logically defective as objects of acceptance or belief, that they have content, etc., are given as indicated earlier in this subsection. The arguments that contradictions can and are believed by rational creatures are assembled in R. and V. Routley, 1975, where too the arguments deriving from Aristotle are undone. The demonstration that Aristotle's arguments are fallacious is given by Łukasiewicz, 1971. A further (overlapping) case against the mainstream traditional assumption that rationality entails consistency is presented in R. Routley, EMJB, and also in Rescher and Brandom, 1980. But though Rescher and Brandom wax eloquent against the traditional assumption,

they do not really notice that similar arguments tell against the modification of the traditional assumption that they adopt, that rationality entails no (truth-table) contradictions.
[80] Lewis, 1982, suggests this. Batens, 1980, pp. 230–231 also asserts as much; he says that paraconsistency makes all theories "unfalsifiable". And Popper, 1963, states it explicitly p. 317.
[81] If this looks too like a category mistake select some other object, e.g. a goose, a shadow, etc.
[82] This matter is explored further in Priest, 1989.
[83] See Lakatos, 1962.
[84] Men and women of practice have of course always known this.
[85] See Lewis, 1982.
[86] As throughout this work.
[87] In his, 1980. Baten's discussion by no means exhausts the arguments already abroad. For much more on the issue as to whether a consistent metatheory is required, see RLR chapter 3, and Priest, 1984.
[88] See EMJB, especially chapter 1.
[89] They are discussed in EMJB, especially chapter 4.
[90] Again a full discussion can be found in EMJB.
[91] Russell's critique appears in a series of papers in *Mind* around 1905: see especially, 1905.
[92] These points were made by Meinong in several places, e.g. his 1907.
[93] For a fuller discussion of these issues, see EMJB. A consistent theory of objects is also developed in Parsons, 1980.
[94] We envisage, for instance, content analysis of leading modern writings on dialectic, e.g. works of the authors mentioned above.
[95] Indeed, arguably some of the people referred to above used more than one notion of contradiction.
[96] See especially the discussions of Hegel and of Marx in chapter II.
[97] Popper, 1963, p. 316.
[98] Some attempts of this type are collected in Marconi, 1979.
[99] See above, this chapter, sect. 2.1.3.
[100] At least on the usual fallibilist picture of the growth of knowledge, which we are not here challenging.
[101] Lakatos, 1970.
[102] Lakatos, 1970.
[103] Generally, because most contradictions encountered are simply false, but not invariably. The matter is developed below in the text.
[104] See further Priest, 1980.
[105] See Mao Tse-tung, 1977, §4.
[106] See, e.g. Bhaskar, 1978.
[107] We have already concluded that true contradictions may be sited at the nodes of meaning changes in 2.2.1 above. There are a number of other interesting issues here entangled, concerning the transcendence of contradictions and the relation of actors' intentions to their efforts; a discussion of these issues can be found in Priest, 1980.
[108] See again the discussion in 2.2.1 above.
[109] See chapter XIII, section 3.4.
[110] See Engels, 1878, p. 139.
[111] In chapter II, § 1.
[112] And thereby removes problems concerning change, going back to Parmenides and Aristotle, which are generated by classically-based considerations.

[113] The matter depends critically however on the underlying arguments and assumptions. These are further discussed in Priest, 1982.
[114] A complementary discussion appears in EMJB, chapters 10 and 11 and the Appendix.
[115] There are, however, substantive assumptions concerning the meta-theory involved and its relation to the object logic—assumptions which should be questioned, for example by anyone who aims to resolve the logical paradoxes through systematic restrictions on the role of self-reference. Moreover, while the claim is true of the original Gödel theorem, where the hypothesis was ω-consistency, not merely consistency, it may not hold of the Rosser form of the theorem where the hypothesis is reduced to consistency. For Rosser's proof appears to depend upon the use of Disjunctive Syllogism. (What remains uncertain is whether this use is essential).
[116] The wilder claims concern such matters as human creativity, the distinction of man from machines, and so forth. All these corollaries—which do not follow without further very large assumptions—fall with the more modest claims. For a further discussion of some of the epistemological issues which hang on Gödel's theorem, see Priest, 1979.
[117] And what is implied: extensional incompleteness.
[118] Specifically, what gives rise to them are truth conditions, which, though recursive in the sense that the truth conditions of any formula is given in terms of the truth conditions of its parts, are not recursive in the sense that the truth conditional analysis of an arbitrary formula, made by an application of the various rules, is bound to ground out. See Priest, 1983.
[119] See Priest, 1979.
[120] See further Priest, 1979.
[121] Logicism is now usually taken to comprise, as well as a reduction thesis to the effect that mathematics reduces (in some way) to logic, an analyticity thesis, to the effect that the truths of mathematics are analytic.
[122] We have discussed the naive theory and contemporary reformulations already in sect. 2.1.5 above.
[123] See Routley, 1977 and chapter XIII, section 3.2.
[124] As before, it remains to be clarified whether it provides a sufficiently generous proof theory. However the objection was one of conceptual impossibility.
[125] It is elaborated by Pollock, 1970.
[126] For fuller details of the program and its philosophical motivation, see Hilbert, 1964.
[127] Even were arithmetic not axiomatizable, the rest of Hilbert's program would retain no little interest. To be sure, Hilbert's philosophical position would be undercut, but the questions of the consistency of sizeable parts of mathematics, and of the strength of methods needed to prove this consistency, remain interesting and important technical problems—within the paraconsistent enterprise also.
[128] For details see Meyer, 1975, and also Routley, 1977.
[129] Brady, 1989.
[130] On this point see Bhaskar, 1978, Ch. 2. For an explanation of why atomism characteristically underlies empiricism, which proceeds through the even deeper Reference Theory, see EMJB, especially chapter 9.

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