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Semantic Closure

Abstract. This paper argues for the claims that a) a natural language such as English is semanticaly closed b) semantic closure implies inconsistency. A corollary of these is that the semantics of English must be paraconsistent. The first part of the paper formulates a definition of semantic closure which applies to natural languages and shows that this implies inconsistency. The second section argues that English is semeantically closed. The preceding discussion is predicated on the assumption that there are no truth value gaps. The next section of the paper considers whether the possibility of these makes any difference to the substantive conclusions of the previous sections, and argues that it does not. The crux of the preceding arguments is that none of the consistent semantical accounts that have been offered for solving the semantical paradoxes is a semantic of *English.* The final section of the paper produces a general argument as to why this must always be the case.

§ 1. Introduction[1]

When formal and natural languages were thought to rather different kinds of animals, and in particular after the publication of Tarski's seminal work on truth ([23]) it was widely held that whereas formal languages such as that of *Principia Mathematica* might be consistent, natural languages, such as English are not. (See, for example [3], § 60b.) More recently with developments in both grammar and logic, we have seen a thoroughly laudable closing of the gap between formal and natural languages, until the idea of English as a formal language does not seem at all utopian. Of course the contrary views concerning consistency have had to give way. And since the only formal logic that logicians knew was not paraconsistent, it was the inconsistency of natural language that gave. Thus, for example Herzberger ([11]) argued that givea a rensonable understanding of what it is for a natural language to be inconsistent, this could not possibly arise. It is my contention that the wrong contrary has gained dominance: English (and any similar natural language) *is* inconsistent, and for just the reasons it was thought to be so. And this means that, when considered as a formal language its underlying logic must be paraconsistent i.e. the semantics of natural language must be paraconsistent and not classical. The purpose of this paper is to try to establish just that. The

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[1]A first draft of this paper was part of a paper entitled "The Logical Paradoxes: a unified account" read to a meeting of the Australasian Association of Logic at Melbourne University 1977. I am grateful to a number of people for comments on it, but particularly Richard Routley.

structure of the paper is as follows. First I shall assume that there are no truth value gaps. On the basis of this assumption I shall define semantic closure and show that semantically closed languages are inconsistent. Then I will defend the view that English is semantically closed. In the next section I will consider the possibility that there are truth value gaps and argue that this does not affect any of my substantive conclusions. The crux of the preceding arguments is that none of the semantical accounts that have been offered for solving the paradoxes is a semantics of *English*. The final section contains a general argument as to why this must be so. For the time being then, let us assume that there are no truth value gaps i.e. that all sentences in question are either true or false.

§ 2. Semantic Closure and the Semantic Paradoxes

In [23] Tarski claimed that the cause of the semantic paradoxes is semantic closure, a semantically closed theory/language being one which can adequately express its own semantic concepts. Tarski's point is easily shown. Any theory/language that can represent its own semantic notions can certainly represent its satisfaction relation. (In fact, as Tarski ([24]) indicates, satisfaction is the most basic semantic notion in that all the others may be defined in terms of it.) I will prove that any theory/language that can represent its own satisfaction relation is inconsistent. For simplicity I will restrict myself to satisfaction of formulas with one free variable. Let us start with the definition of semantic closure for theories.

A theory is semantically closed (with respect to its satisfaction relation) iff

(i) for every formula with one free variable φ, there is a term a_φ, its name.

(ii) there is a formula with two free variables $Sat\ (x\ y)$ such that every instance of the scheme

$$Sat(t\ a_\varphi) \leftrightarrow \varphi(v/t)$$

(1)

is a theorem, where t is any term, φ any formula with one free variable v, and, $\varphi(v/t)$ is φ all occurences of 'v' replaced by 't'. (With the usual precautions concerning the binding of variables free in t). Now to show that any semantically closed theory is inconsistent, we need only take the formula $\neg Sat(x\ x)$ for φ and $a_{\neg Sat(x\ x)}$ for t in (1). We then get

$$Sat(a_{\neg Sat(xx)} a_{\neg Sat(xx)}) \leftrightarrow \neg Sat(a_{\neg Sat(xx)} a_{\neg Sat(xx)})$$

Assuming that

$$\frac{A \leftrightarrow \neg A}{A \wedge \neg A}$$

is a valid scheme of the logic underlying the theory[2], a contradiction results. This is just the heterological paradox, since $\neg Sat(x\,x)$ says that the formula x is not truly predictable of itself.

In [23] Tarski uses the truth predicate 'Tr' rather than the satisfaction predicate to make the same point. For this reason he requires an extra "empirical" premiss to obtain a contradiction, namely, the existence of a formula φ such that a_φ is the name of $\neg\mathsf{Tr}a_\varphi$. This is strictly necessary. For it can be shown that there is a consistent theory which contains its own truth predicate.[3] Let $\mathscr{L}$ be a first order language with Hilbert's ϵ symbol. The first order theory T has the usual axioms for Hilbert's ϵ symbol:

$$\exists x\varphi \supset \varphi(x/\epsilon\, x\varphi)$$

$$\forall x\big(\varphi(y/x) \equiv \psi(z/x)\big) \supset \epsilon\, y\varphi = \epsilon\, z\psi$$

and the additional

$$\neg\, \epsilon\, x(x = x) = \epsilon\, x(x \neq x)$$

$\boldsymbol{T}$ is easily shown to be consistent. Now for a_φ take $\epsilon\, z\varphi$, where z is the first variable (in some enumeration) not occurring in φ, and for $\mathsf{Tr}y$, take $y = \epsilon\, x(x = x)$. It is now a straightforward matter to show that every instance of the scheme

$$\mathsf{Tr}a_\varphi \equiv \varphi$$

is a theorem of $\boldsymbol{T}$, as follows:

$$\begin{aligned}
\varphi &\supset (\varphi \equiv z = z)\\
&\supset \big(\epsilon\, z\varphi = \epsilon\, z(z = z) = \epsilon\, x(x = x)\big)\\
\neg\varphi &\supset (\varphi \equiv z \neq z)\\
&\supset \big(\epsilon\, z\varphi = \epsilon\, z(z \neq z) = \epsilon\, x(x \neq x)\big)\\
&\supset (\epsilon\, z\varphi \neq \epsilon\, x(x = x)).
\end{aligned}$$

None the less, we can define the other semantics notions such as truth and denotation from satisfaction in the usual way and, given a little bit of extra machinery prove the other semantic paradoxes. However, this is not necessary. For Tarski's contention that semantically closed theories are inconsistent has been demonstrated. To put it in the vernacular, the semantic paradoxes result from semantic closure.

Let us now turn to natural languages. Tarski held, and I shall agree with him, that the semantic paradoxes in natural language occur because

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[2]This is certainly a non-trivial assumption. It fails, for example in Fitch's logic [5]. However I will pick it up again subsequently. See notes 5 and 6.

[3]I will show that there is a consistent classical theory. It follows that there are consistent relevant, intuitionist etc. theories.

it is semantically closed. This now has to be argued for. For the above definition of 'semantically closed' will not carry over verbatim to natural languages. The problem of course is that the definition is couched in terms of 'formula', 'term', 'theorem' etc., jargon applying only to formal languages. However it is easy enough to rephrase the definition whilst retaining its spirit.

A natural language is semantically closed (with respect to its satisfaction relation) iff

(i) For every phrase φ there is a noun phrase a_φ, its name.

(ii) There is a phrase *Sat* requiring two noun phrases to be inserted to make a sentence, such that every sentence of the form

$$\ulcorner Sat(t\ a_\varphi) \text{ if and only if } \varphi(t)\urcorner \tag{2}$$

is true, where φ is any phrase requiring one noun phrase to be inserted to make a sentence ant t is any noun phrase. The brackets indicate the appropriate insertion operations within the phrase. (Those who do not like sentences being true may replace 'is true' by 'expresses a true proposition' 'can always be used to make a true statement' – or whatever their favourite theory is – and continue to do so until further notice.)

This definition, though rather cumbersome is obviously the analogue of our previous definition. The only real point of interest is that we have changed 'is a theorem' in clause (ii) to 'is true'. We can now proceed exactly as before to establish that any semantically closed natural language contains true sentences of the form $\ulcorner\varphi$ and it is not the case, that $\varphi\urcorner$. Again we see that paradoxicality is the result of semantic closure[4].

A semantically closed natural language therefore contains true contradictions. Of course it might be doubted that there are any semantically closed natural languages. That there are I will argue in the next section. However before we move on to that let us return to Herzberger's proof [11] that no language can be truth-conditionally inconsistent, and therefore semantically closed. Ha argues that there can be no such language, on the grounds that if there were, there would be true contradictions. Of course the argument works only if one rejects the view that there are true contradictions. This is precisely what I am denying. That there are true contradictions is an idea which is at the very root of the semantics of paraconsistent logic. In fact Herzberger's argument is related to mine as *modus tollens* is to *modus ponens*. For him to use the argument against me would therefore beg the question. However the claim that English is semantically closed obviously requires an independent consideration to this I now turn.

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[4](Strictly speaking we do need an extra assumption in this case, namely the existence of a noun phrase 'itself' such that *Sat*(t itself) has the same truth conditions as *Sat* ($t\ t$). We now take φ to be $\ulcorner$It is not the case that *Sat* (itself)$\urcorner$, and t to be a_φ.)

§ 3. The demise of a hierarchy

To deny that English is semantically closed, one has to deny either that clause (i) or clause (ii) of the previous section, holds for English. The prospects for denying clause (i) seem bleak indeed. Every phrase of English contains a name. Given a phrase, to form its name, we simply enclose it in quotes. (Before quotation marks were a regular feature of the vernacular, the same function was performed by allowing the phrase to denote itself. In medieval jargon, the phrase when used thus had *material supposition.*) There is therefore little scope for denying clause (i).

The other option for someone who wishes to deny that English is semantically closed, is to deny that English satisfies condition (ii) of the previous section i.e. to deny that English has a predicate satisfying the satisfaction scheme. In fact I will discuss this question not for satisfaction and the satisfaction scheme but for truth and the truth scheme.

(2) $$\ulcorner a_\varphi \text{ is true if and only if } \varphi \urcorner$$

where φ is a closed sentence. There is no harm in this since both biconditionals should be treated in the same way, and this approach has the merit of not only keeping the dicussion simpler but also of relating it to the current literature.

Pretty obviously, English has a truth predicate viz. 'is true'. There is also a very strong presupposition that it satisfies the T-scheme (2). For indeed it is exactly that which characterises it as a *truth* predicate (at least extensionally) and not some *ersatz*. It is this point which Tarski underlines when he calls (2) a *condition of adequacy* on any definition of truth. Hence the onus is on those who claim that instances of the T-scheme fail to prove it. What reason for supposing that instances of the T-scheme fail can be given?

Other than the existence of truth value gaps there is only one that I am aware of. This is the claim that the truth predicate of English is not univocal. English, it is suggested is not one language but a hierarchy of semantically open languages. Each language of the hierarchy has its own truth predicate which can be legitimately applied to the sentences of the language below and only those of the language below. If we suppose, as we may, that the names of all the sentences in the hierarchy occur at all levels, this view implies that the truth scheme at level $n+1$ is true if φ is of level n but may be false otherwise. This move if of course due to Tarski. It must be emphasized in fairness to him that he did not think that natural languages are of this form. However this sort of view, or something like it, is the dominant orthodoxy, or at least was unitl only recently, amongst logicians.

There are many things wrong with this view. First, English does not seem to be of this form. What we might call its "surface structure" is certainly not of this form. We are therefore called on to "look upon" English in this way — or what comes to the same thing — to regiment it in this form. This move is *ad hoc*. It is a technical trick for sidestepping what for many people is an uncomfortable fact — semantic closure.

Secondly, and worse, this manoeuvre is far too strong. This arbitrary fragmentation of truth rules out as inexpressible, not only paradoxical sentences but also wholely unobjectionable ones. For example, I claim that all the claims im this paper (including this one) are true. This is something that cannot be expressed on the hierarchy view since the very sentence claimed to be true is in the scope of the truth predicate occuring in the sentence. However, there is nothing wrong with this claim. Assuming what is more than likely, that I have made at least one false claim in the paper other than that one, then it is just plain false.

The third and most damaging objection to the hierarchy view is that it produces a vicious regress. For any semantically open language is forbidden to talk of its own semantics. Yet we can and do wish to talk of its semantics. This requires a semantically open metalanguage. But of course we wish to talk about its semantics; so we need a meta-meta-language. And since this is open, to talk of its semantics we need a meta-meta-meta .... The construction can be iterated ω times but we then wish of course to talk about the semantics of the hierarchy. So we need a meta-language of order ω. And so on. We can continue and obtain a metalanguage of order α for every ordinal α and hence obtain an absolute infinity of languages. (Though Kripke [13] p. 697 indicates that there are technical difficulties involved in constructing the transfinite members of the hierarchy. So much the worse for it.) But there is no reason why we should not talk about the semantics of the whole hierarchy. Yet there is nowhere left to go. The transfinite game of tail catching has got us nowhere. We are still faced with something expressible in English but not expressible in any of the languages of the hierarchy; so we are no better off than when we started. The expressive power of English cannot be captured by any hierarchy of semantically open languages.

The point of course, is that the universality of semantics makes the metalanguage construction inherently unstable. I am not of course denying that such castles in the transfinite air can be constructed. They can be. But they have no more significance than a mathematical game. Whatever they are, they are not English. In giving a semantical account of English the distinction between object and metalanguage is a logical apartheid which must go; which is not to say that we can not distinguish between discourse and discourse about discourse, but the latter should not be isolated from the former in a separate ward. In another jargon, we could say that the metalanguage is the alienated essence of (object language) truth. The alienation should of course be transcended.

A recent attempt to rework this position to avoid some of its problems is that by Burge [2]. Instead of there being a hierarchy of languages each with its own truth predicate he suggests that there is only one language in which the truth predicate is indexical. The paper contains some telling criticisms of other purported solutions to the paradoxes. However, its own solution is equally problematical. There is no independent evidence for the claim that 'true' is indexical. There is no more reason to believe 'true' indexical than to believe 'philosopher' is. However, the major objection against this proposal is that it does not avoid paradoxes. For although an indexical has no context-independent extension, it does have a range of possible extensions. (Compare the word 'here'.) Now let $R = \{x; x$ is a possible extension of 'true'$\}$ and consider the sentence

$$\neg(\exists x \in R)(a) \in x.$$

(i.e. "This sentence is not in any member of R".)
Suppose that $\neg(\exists x \in R)(a) \in x$; then (a) is true. So for some $x \in R$, $(a) \in x$. Contradiction. Alternatively, suppose that for some $x \in R$, $(a) \in x$. Then by the T-scheme for x, $\neg(\exists x \in R)\ (a) \in x$. Contradiction.

If this reasoning can not be carried out in the language in question (which presumably it can not since the theory is demonstrably consistent) this just shows that the language in question is not English since we have just carried it out in English.

To summarise this section: English is semantically closed. The denial of this can be maintained.

§ 4. Falling through the gap between Truth and Falsity

In accordance with the strategy laid out in the introduction, the discussion so far has been predicated on the assumption that there are no truth valueless sentences. It is now time to consider whether the substantive conclusions I have reached are affected by the possibility that there are. Before I discuss this let me make two preliminary remarks. First, one of explanation: the thesis that there are truth valueless sentences comes in two varieties. According to the first, whilst sentences (perhaps relativised to a context) are the kinds of things that are true/false, some sentences are neither. A more complex version holds that it is what is expressed by a sentence, a statement or a proposition, that is true or false (at least primarily) and that some sentences do not express statements or propositions. I do not wish to discuss the issue of whether it is sentences or statements which are the primary bearers of truth. (On this see [10].) I intend my discussion to apply to both versions of the thesis. To this end I shall now write both 'true' and 'false' with initial capitals. Those who think that sentences are true/false can read "True (False) sentence" in the obvious way. Those who think that statements are true/false can read it as "sentence which makes a true (false) statement". The thesis that there are truth-valueless sentences can now be expressed as: there are (indicative) sentences that are neither True nor False. Let us call such sentences 'Valueless'.

The second preliminary point is the question of whether there are any Valueless sentences. Again I wish to remain neutral on this question. That there are Valueless sentences is obviously one option for dealing with certain apparently defective sentences embodying reference failure, category mistakes etc. However the other and often simpler option is just to call them false — at least when they are atomic. Nor is the incorrectness of this option easily shown. Hence, in what follows, I will grant only for the sake of the argument the claim that there are some Valueless sentences.

Let us now return to the main question of whether the existence of Valueless sentences affects my substantive conclusions. My argument so far hinged on the universal validity of the truth scheme (and the satisfaction scheme. I shall continue to discuss the former.) The central question is therefore whether it fails if there are Valueless sentences and, if so, whether this affects the use to which I put it. First, it seems *prima facie* possible that the T-scheme

$$a_\varphi \text{ is True iff } \varphi$$

fails if φ is a Valueless sentence. Fof ir φ is a Valueless sentence then it is certainly not True. It would seem therefore that 'a_φ is True' is False. Since, presumably $\ulcorner A$ iff B$\urcorner$ is not True (whatever it is) if A is False and B is Valueless, this instance of the T-scheme would seem to fail. However, this reasoning is *not* mandatory for two reasons. First, although 'a_φ is True' is not True, it does not follow that it is False. It too may have a truth value gap. In which case this instance of the T-scheme will be of the form $\ulcorner A$ iff $B\urcorner$ where both A and B are Valueless. It may be though that the biconditional still fails. However this is now highly moot. It is easy enough to devise very plausible semantics in which the biconditional comes out as true under these conditions. For example, in the four valued semantics for first degree entailment (see [4]) the fact that both A and B may be Valueless does not refute $\ulcorner A$ iff $B\urcorner$.

But even 'a_φ is True' is False, it still does not follow that (2) fails. After all, since neither side is True, the biconditional is still Truth preserving in this case and this may be sufficient to make it True. Again, it depends on the precise semantics that 'iff' is supposed to have. There are, of course, many possible candidates, and since I do not need to determine this for the present context I will not.

The upshot of the previous discussion then is this. The truth scheme *may* fail if the formula in its right hand side is Valueless. If it does, then the correct way to formulate the truth (or satisfaction) biconditional in the definition of 'semantic closure' is with the condition that the formula on the right hand side of the biconditional be not Valueless.

There is, however, a much more positive point for us to take into the subsequent discussion, which is this: according to the value-gap theorist, a sentence may be neither True nor False. If he is not to be refuted by an *ad hominem* argument, he must maintain that there is some way he can Truly say this. In particular therefore if x is not True, he is committed to the position that 'x is not True' is True. This does not commit him to the view that 'x is True' is False, since the 'not' here may be a strong negation wgich maps Valueless sentences into True ones. However, for future reference let us record the commitment:

(3) If x is not True, 'x is not True' is True.

It might be thought that my admission that the T-scheme may fail damages my case that semantic closure implies contradiction. However it does not. For it is not the general validity of the T-scheme which is at issue, but the specific instance of it which is used in deriving the contradiction. Now if the T-scheme fails, it does so only when the sentence or its righthandside is Valueless. The crucial question now therefore is whether sentences such as 'This sentence is False' and similar sentences which are paradox-producing when substituted in the T-scheme are Valueless.[5] Many have suggested this e.g. van Fraassen [7], Martin [14], Kripke [13], Bar-Hillel [4], Goddard and Goldstein [9]. But it seems to me that the case is not cogent.

The claim is usually suspiciously *ad hoc*. We must be given a reason for supposing that a paradoxical sentence is Valueless or the "solution" is worthless. It is not in doubt that we can defuse the paradox if we are at liberty to make any move we like. For just this reason a move that is not backed up with a suitable *rationale* is an intellectual fraud. Neither is the mere fact that if the sentence were not Valueless a contradiction would ensue, a sufficient *rationale*. For that there are true contradictions is the very thesis I am affirming. The argument therefore would beg the question against me just as Herzberger's did. (See § 2). Some *rationales* have been offered but none of them is very satisfactory. For example Martin [14] tries to show that paradoxical sentences are category mistakes. This appears most implausible. In 'This sentence is False' the subject appears to be the right *kind* of thing for the predicate to be about. The ad hoc nature of *this* move is witnessed by the fact that in Martin's "decision procedure" for category mistakes a special clause is required to capture paradoxical sentences, which would otherwise appear to pass the test. The fact that the demonstrative (or at least a similar device) is essentially eliminable from paradoxical sentences is sometimes cited as a reason for Valuelessness. For example Ryle [22] has suggested that this in eliminability shows that no statement is made in uttering a paradoxical sentence. Hence we have a case of reference-failure. This essential ineliminability is closely related to Kripke's

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[5]Alternatively it may be held that the inference $A \rightarrow \neg A / \neg A$ fails because of a failure of the law of excluded middle for Valueless sentences. Fitch [5] takes this line. Hence my comments apply equally to him.

notion of ungroundedness. Only grounded sentences receive a truth value given a certain way of assigning truth values.

A basic problem with these approaches as rationales is that whilst eliminability or groundedness (or whatever) may be a sufficient condition for having a Truth value. It is never obvious why it should be supposed that it is also necessary. Moreover, it does not seem to be either. Take for example Ryle's position. Suppose that my father asserts the mendacity of all Popes whilst unknown to me some 4th century Pope asserted the veracity of fathers of philosophers. If Ryle is right then either my father or the Pope failed to make a statement; without loss of generality we can assume it to be my father. Yet by all standard tests he did. I understood what he said; I can draw inferences from it, I can act on the information contained in it and so on. (The point is wittily made by Popper [16].) Or take Kripke's position. Let p be any sentence that obtains no truth value at a fixed point. Then obviously, 'p is not true' should be true at the fixed point. Yet it receives no truth value. It seems then that none of these motivations will do what is required.

A further argument against the claim that paradoxical sentences are Valueless is that it conflates the distinction between *neither True nor False* and *both True and False*. Consider for example the pair of sentences:

(4) This very sentence is True

(5) This very sentence is False

There is something odd about both these sentences, but it is not the same thing in both cases. In the case of (4), the semantic rules governing the use of the demonstrative 'This very sentence' and those governing the predicate 'is True' are not sufficient to determine the Truth value of the sentence. In other words, the semantics of the words involved *underdetermine* the Truth value of (4). This *is* a case of a sentence that is neither True nor False. By contrast, in the case of (5) the semantic conditions of the words involved *overdetermine* its Truth value. The rules determine it to be both True and False. This case then should not be confused with the previous one.

I have argued so far that there is no good reason to believe that paradoxical sentences are Valueless and some for supposing that they are not. However even if they were my main contention that semantic closure gives rise to inconsistency still stands. This is because, although simple paradoxical arguments may be blocked by the supposition of Valuelessness, more complex ones are not. Consider the extended liar paradox:

(6) This very sentence is not True

This is either True, False or Valueless. If it is True, it is not True. If it is False it is True. If it is Valueless then it is certainly not True and hence

True. In all cases we have a contradiction.[6] The only step one might doubt is that last one: if (6) is not True, then it is True. If we suppose for example that (6) makes no statement then it should not follow that it makes a true one. (See [9]). Yet the value-gap theorist is already committed to (3) which underwrites the inference in question.

Of course this kind of problem has not passed the notice of those who would "solve" the logical paradoxes in this way. There are basically two proposed ways out of the problem. The first is to settle for a weak language in which (6) can not be expressed. This is the solution of Martin [15] p. 287 and van Fraassen [8] pp. 18,22. This of course just shows that the language they are dealing with is not English (For (6) plainly is expressible in English) i.e. that their semantics fails as an account of English semantics. Other authors e.g. Fitch [6] p. 402, Kripke [13] p. 715 resort to the faithful device of a metalanguage. Although the language has not the expressive power to say that one of its own sentences is Valueless, this can be said in the metalanguage. Of course, this just starts the infinite regress that I have discussed in connection with the Tarski hierarchy (§ 3). For the same reasons it is ultimately as futile.

Hence we see that the thesis that there are Valueless sentences does not undercut the substantive conclusions of sections 2 and 3. Any language that is not semantically closed is not English and semantically closed languages are inconsistent.

§ 5. Incompleteness and paraconsistency

I have argued that all of the attempts to show that English is consistent fail. We have considered the main ploys for trying to avoid this fact. In each case the main reason for the inadequency of the ploy is that according to it something which patently can be expressed in English can not be. This is no coincidence. For we can produce a general argument that any solution to the paradoxes i.e. any consistent semantics for 'true', must be expressively incomplete, in the sense that there are things expressible in English which are consistently ineffable. The semantics are not, therefore, those of English.

Consider the following argument due to [12]. A predicate, P_0, of a language, $\mathscr{L}$, is *ungrounded* if there is a family of predicates of $\mathscr{L}$, not necessarily distinct, $\{P_i; i \text{ is natural number}\}$, such that for all i, P_{i+1} is in the extension of P_i. (e.g. as "monosyllabic" is in the extension of "polysyllabic".) It is *grounded* otherwise. Now $\mathscr{L}$ can have no predicate which has as exten-

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[6] Notice also that this reasoning does not use the law of excluded middle. It does use the law of excluded fourth and this might be thought to make it suspect for Fitchean reasons. However there are plenty more paradoxes which do not use excluded middle or anything like it. I will give one in the next section. Another can be found in Priest [19].

sion the class of grounded predicates. For suppose it does. Let this predicate be G. If G were ungrounded, so would be one of the predicates in its extension. But *ex hypothesi* these are all grounded. So, therefore, is G. But then G is in the extension of G, which obviously implies G is ungrounded. Contradiction.

Herzberger claims that his argument shows that any language is incomplete. Of course what it actually shows is that any language is incomplete or inconsistent, in the sense of containing true contradictions. Herzberger ignores the latter possibility. Yet the irony of the Herzberger paper is that if he were correct the predicate 'grounded sentence of English' would not be expressible in English. Yet Herzberger manages to express it in the first few pages of his paper just as I have done. (The fact that I have used a little symbolism is irrelevant: it could obviously be expressed in longhand). By Herzberger's general argument it follows that English is either incomplete or inconsistent. Since the "missing" predicate is not missing at all, it is inconsistent. Although Herzberger's argument is ingenious, his conclusion is not really very startling. After all it is precisely the content of Tarski's theorem, that a theory can not express its own truth predicate unless it is inconsistent. (Gödel's incompleteness theorem is also to the effect that any sufficiently rich theory is either theorem-incomplete or inconsistent.)

We have seen that there are excellent reasons driving us to the conclusion that English is semantically closed. This means that there are true contradictions in English, sentences such that both they and their negations are true. However, obviously not all English sentences are true. (In fact only a minute fraction of sentences of English are paradoxical.) So it follows that the classical rule of inference *ex falso quodlibet* ($A \wedge \neg A / B$) is invalid. There are cases where the premiss is true and the conclusion is not. In short, the underlying logic of English is paraconsistent. Moreover any adequate account of the semantics of English will have to face semantic closure and the existence of contradictory truths. This is true of Davidson's, Montague's or any other account of the semantics of English. There are no problems here. The semantics of paraconsistent logics show exactly how true contradictions can be handled (See [17] and [18].

Even if replies to the arguments in this paper could be found, an inconsistent theory of the semantics of English can not be ignored. The (dubious) merits of a consistent theory would have to be argued for. Bearing in mind the epicyclic complexity of attempts to solve the paradoxes compared with the simplicity of the paraconsistent naive theory of truth, this is virtually impossible. The other advantages of limited inconsistency (see [17], [21] and [20] make it totally so. The *ad hocery*, Gothic hierarchies and loss of expressive power required by the rejection of semantical closure could seem reasonable only to a logical community living, as Wittgenstein put it ([25], p. 53), in superstitions fear and awe of contradiction. The time has come to put the superstition aside.

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