On the Principle of Uniform Solution: A Reply to Smith

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All the standard paradoxes of self-reference, semantic and set-theoretic, satisfy the Inclosure Schema, and are therefore of a kind. The Principle of Uniform Solution (PUS) counsels: same kind of paradox, same kind of solution. Since standard solutions to the set-theoretic paradoxes do nothing to address the semantic paradoxes, and vice versa, this shows that each is inadequate. So I have argued.[1]

Smith (2000) disagrees. He suggests that the notion of a kind is relative to a degree of abstraction. That Bill loves Monica, and that Lee shot John, are not the same relative to the level of abstraction *a* loves *b* and *a* shot *b*; but they are with respect to the level of abstraction *a*R*b*. Moreover, the PUS should read: same kind of paradox at a given level of abstraction, same kind of solution at that level. Given this, there is nothing problematic about standard solutions. Relative to the level of abstraction of the Inclosure Schema, both sorts of solution are the same: they are of the form "circumvent the Schema". Whereas at a lower level of abstraction, the paradoxes are not of the same kind: one mentions sets; the other does not. Moreover, the standard solutions are at this level: solutions to the set-theoretic paradoxes mention sets; solutions to the semantic paradoxes do not. Hence, these solutions are quite appropriate at their level.

Let us grant that kindhood is relative to degree of abstraction. In some sense, this is clearly right. It is not clear to me what counts as a level of abstraction, if we are talking of paradox solutions. It is certainly not a matter of simply blanking out symbols of predefined grammatical categories. The Inclosure Schema does that, but it does a lot more than this too. But let this pass.[2] The real problems with Smith's argument lie elsewhere: in the assumption that all levels are equal. Let me explain.

[1] Priest (1994, 1995, Ch.11, 1998). This fact is used as an argument for dialetheism. One might note that it is at least possible to offer a uniform and consistent solution to the paradoxes. That, after all, is what Russell's Vicious Circle Principle purported to do. The term "Inclosure Schema" comes from Priest (1995). In Priest (1994) it is called "Russell's Schema".

[2] Let pass, too, the fact that the PUS shows that the set-theoretic paradoxes on their own are not well solved by standard solutions. See Priest 1995, Ch.11.5.

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The liar paradox comes in many varieties. In some of them, self-reference is achieved by means of a demonstrative ("this sentence", etc.); in others, it is obtained by a description ("the first sentence on such and such a page", etc.); in yet others, it is obtained by some diagonal argument employing gödel-numbers as names. At some level of abstraction, these paradoxes are, presumably, of different kinds. The first depends on context-dependent features of utterance; the second presupposes some account of the way that descriptions work; the third employs numbers. Suppose that I propose three corresponding kinds of solution. The first kind of paradox is solved by some theory of speech acts; the second is solved by some theory of the semantics of descriptions; the third is solved by an appeal to mathematical nominalism. Even if all the solutions were to work for their own kind, it appears to me undeniable that, collectively, there is still something wrong with them. The exact mechanisms of reference are, in a clear sense, accidental to what is going on. The solutions may avoid the paradoxes, but they do so by attacking peripherals, not essentials. In all versions of the liar paradox, there is a sentence, *A*, such that:

$$A \text{ iff } \langle A \rangle \text{ is not true}$$

where $\langle A \rangle$ is some singular term referring to *A*. Call this the Fixed-Point Schema. It is this that needs to be taken on board, not the contingent fact of how the reference is achieved. Nor does it help to point out that at this level, the level of abstraction of the Fixed-Point Schema, all the solutions are the same: they are all "Fixed-Point Schema circumventers". This is just a bit of linguistic legerdemain. It is clear that the solutions are not getting to the essence of things.[3]

It seems to me that things are exactly the same with the paradoxes of self-reference in general. The existence of an inclosure is what explains all of them. Once one sees that a certain operation on any totality of objects of a particular kind generates a novel object of that kind, it becomes clear why applying the operation to the totality of all such objects must give rise to contradiction. Focussing on how the inclosure is achieved is like focussing on how the self-reference is achieved. It mistakes the symptom for the cause. Moreover, saying that the standard solutions to the semantic and set-theoretic paradoxes are the same at the level of the Inclosure Schema, in that they are all "Schema circumventers", is just throwing linguistic dust in one's eyes. As Smith (2000, p. 118) in effect notes, this is an entirely empty claim. That they circumvent the Schema does not show that they address the essence of the phenomenon,

[3] Priest (1998) gives another example of a bunch of paradoxes (of the infinite) which are, at some "level of abstraction", different (some involve time and some do not), but where trying to solve them at that level is clearly incorrect.

any more than the same trick did in the case of the liar paradox. It's not that they are simply different ways of circumventing the Schema. Rather, they entirely miss the essence of the phenomenon.[4]

In other words, and to relate all this to the notion of level of abstraction, the appropriate level at which to analyse a phenomenon is the level which locates underlying causes. This is exactly illustrated by Smith's (2000, p. 119) example of the botanist and the zoologist. The correct level of abstraction at which to analyse the problems in question is not the level of biological processes that have limited branching potential; it is the level of the relevant causal mechanisms. Similarly, just as with the example of the liar paradox above, the correct level of abstraction for an analysis of the paradoxes of self-reference is not one which depends upon the presence of certain words ("set", "true", etc.), but the level of the underlying structure that generates and causes the contradictions: the level of the Inclosure Scheme. Of course, in the case of the paradoxes, we cannot talk of causation, in the sense of physical causation, but even in non-physical situations, it still makes sense to single out the essential features of a situation that are responsible for something or other. This is what explanations in mathematics do. For example, we can explain why the integers have a certain property by pointing out that they form a group under addition, and that all groups have that property. The other properties of the integers, such as how addition interacts with multiplication, are irrelevant to an understanding of why integers possess that property.[5]

If all this is right, then even if it were the case that orthodox solutions to the paradoxes of self-reference work at their corresponding level of abstraction (which, whatever this means, they do not (see Priest 1995, Chs. 10–11)), this would not suffice to protect them from criticism based on the PUS. Some levels are more equal than others.

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[4] As in Smith's example, fn. 4. Someone who did not understand that weight loss/gain is a matter of calories-in minus calories-out would have missed the essence of the problem.

[5] This theme is explained at greater length in Priest (1998). The question of what, exactly, a mathematical explanation is, is a hard one. For a start on the issue, see Steiner (1978), and Resnik and Kushner (1987).

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