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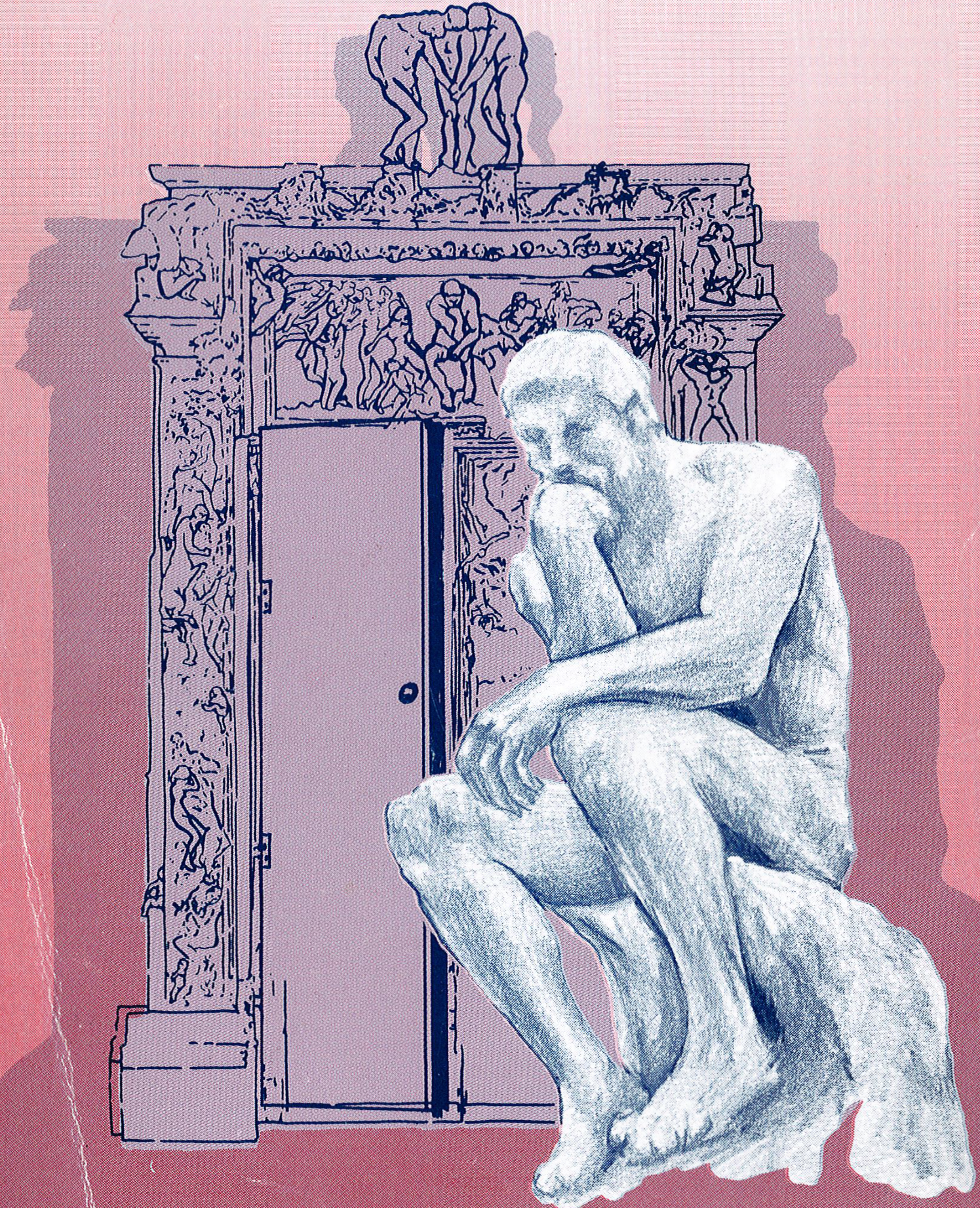
ON PARACONSISTENCY

G. PRIEST and R. ROUTLEY



ON PARACONSISTENCY

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ERRATA for On Reasoning

Bottom p.2 For in general, *if-* constructions serve to present reasons of a very broad sort; for instance, ordinarily motives would serve, prudential advice and practical reasoning certainly would. Truth and falsity, then, are not always particularly appropriate semantic values for the antecedents and consequents of

p.3 Since the logical work is accomplished primarily through specialised forms of reason-giving, we adopt a familiar style of notation in which the connective is embedded between antecedent and consequent. Once again we do not, however, require that the antecedent or consequent should be propositional. The key determinable connections in the bag of notions involved can now be presented and symbolised as follows:

> A is *reason for* B iff A's being the case or occurring counts towards or for the truth of B, i.e. in symbols, iff $A \supset B$. Put differently, $A \supset B$ iff A counts positively as regards B (or A lends weight to, or strengthens B).
>
> A is *reason against* B iff A's being so or occurring counts against the truth or occurrence of B, i.e. in symbols, iff $A \supset B$ (or, put differently, A weakens B).

Normally, A is a reason against B (pushing B away) when, and only when, A is a reason for the reverse (pulling B in), i.e. for the negation of B. Thus in symbols, $A \supset B$ iff $A \supset \sim B$, given a decent (reversal representing) negation.

> A is (consideration) *relevant to* B iff $A \supset B$ or $A \supset B$, i.e. if $A \supset B$.[1]

Top p.4 It is almost immediate that material implication, $\supset$ of classical logic provides but a hopeless account of reason. For since $A \supset B \vee . A \supset \sim B$, everything is relevant to everything

p.81. Delete first four lines.

REFERENCE

R. Routley and others, *Relevant logics and Their Rivals*, Ridgeview, California, 1982; referred to as RLR.

CONTENTS

INTRODUCTION

The essays in this volume are introductions culled from a collection of papers on paraconsistent logic which is soon, we hope, to appear.* This collection was, originally, no more than a collection of papers on various aspects of the subject written by many of the people who made some developments in the field of paraconsistency. However since the group of people for whom such a collection would have much significance is not much greater than that intrepid band itself, we felt an editorial onus to include some essays which would introduce the previously blissfully ignorant reader to the subject. The essays that we wrote are intended to fulfil this function. However we did not restrict ourselves to a mere state-of-the-art summary; (we are probably incapable of doing just that anyway). In many places the essays break new ground, though sometimes only sketchily. And, characteristically, we have made no attempt to pretend disinterest in either the question of the correctness of a paraconsistent approach versus a "classical" approach, or the question of which paraconsistent approach is right. Consequently even seasoned paraconsistentists should find things of interest in the essays.

At any rate, these are the essays which are to introduce our state-of-the-art collection on paraconsistent logic. It seems to us that the separate and more speedy (semi-) publication of the introductory essays would benefit the development of the subject both by providing an accessible and cheap introduction to the subject and by provoking criticism of the positions we take. As a result, too, we may be able to improve the essays (for example by last minute removal of some of the falsehoods which are not also true) before they appear in their places in the collection of essays, irrevocably.

The essays include two on the history of paraconsistency, with dialectics treated to a separate essay, one on the more technical aspects of paraconsistent logic, one on the applications of paraconsistent logic, and one on its philosophical aspects. The separation of the material into five parts, necessary for logistic reasons, is to a certain extent, academic, since there are important issues, arguments, etc., which cut across these lines. As far as possible we have tried to make each essay self-contained. However a reasonable amount of cross-referencing was

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* Paraconsistent Logic, eds G. Priest, R. Routley and J. Norman (Philosophia Verlag).

unavoidable if pointless repetition was to be avoided. Hence virtually every chapter contains material relevant to the central themes of the other chapters. We do not apologise for this; for the organic unity is inherent in the subject. Indeed it is just this organic unity which marks off paraconsistency from a number of cobbled together subjects, and shows it to be a new development of fundamental importance on the scene logico-philosophicus.

* * * * *

References to "this volume" and to other introductions can be interpreted by consulting the following table:

Chapter in this volume	Chapter and Part in Paraconsistent Logic
1	Chapter 1, first introduction to Part I
2	Chapter 2, second introduction to Part I
3	Chapter 5, introduction to Part II
4	Chapter 14, introduction to Part III
5	Chapter 19, introduction to Part IV

CHAPTER 1: FIRST HISTORICAL INTRODUCTION

A PRELIMINARY HISTORY OF PARACONSISTENT AND DIALETHIC APPROACHES

Although the notion of <u>system</u> was brought into prominence by Leibnitz[1], it is only in contemporary times that a clear conception of a formal or semantical system has developed. Thus recent definitions of paraconsistency[2] through such systems - in terms of systems which can tolerate some inconsistency without trivialising - are not strictly or directly applicable in a historical quest. Evidence of paraconsistent approaches in earlier times has accordingly to be more circumstantial.

There are however several good indicators of paraconsistent approaches of one sort or another which can be reliably used. For example, admission, or insistence, that some statement is both true and false, in a context where not everything is accepted or some things are rejected, is a sure sign of a paraconsistent approach - in fact of a dialethic approach. It involves not merely recognition of a non-trivial inconsistent theory, as with a (weaker) paraconsistent position, but the assumption that that is how things are, that, in effect, the world is inconsistent. A concession that both a statement, A say, and its negation, ~A, hold, works in a similar way, clearly revealing a strong paraconsistent approach. So does the concession that some statements A and ~A hold in a nontrivial theory or position, thereby revealing a weaker paraconsistent approach.

But often evidence is less direct. For instance, an author may not explicitly say that both A and ~A hold, or hold in a given theory, but what is said obviously implies that they do, and the author can be assumed to be aware that they do, or a case can be made that the author is aware of this. In such cases the approach is still explicitly paraconsistent. But an author may not be (clearly) apprised of what his or her position (obviously) implies, in which event the position will be either implicitly paraconsistent or else trivial, depending on the underlying logic adopted.

However determining the underlying logic assumed by an author, especially before contemporary times, but sometimes even now, is a particularly difficult and testing business. Fortunately it is often unnecessary to work out the underlying logic in much detail at all. From medieval times it is frequently enough to know where the author stood as regards paradoxes of implication (i.e. in effect, paradoxes of the consequence relation); and otherwise it may suffice to find out what the author thought about watershed principles such as consistency assumptions, for instance, that the world is perforce consistent, or, more strongly, that all that can be spoken of or described (nontrivially) is consistent.

This universal consistency hypothesis - to the effect that all nontrivial theories are consistent - emerges early in Greek thought and is embedded in the Eleatic tradition (sometimes explicitly described as "Eleatic logic", though no logic is ever outlined) emanating from Parmenides and running through the giant survivors (at least so far as representation of their work is concerned) of classical antiquity, Plato and Aristotle and later Plotinus, and into mainstream Western thought. If inconsistency is found in the work of such a mainstream author (as with little doubt inconsistency mostly can be[3]) then the philosophical theory is damaged perhaps severely, and requires not merely patching but some excision to restore consistency.

In classical antiquity, as subsequently, consistency theses take various, stronger or weaker forms - all of which we shall try to demolish. In strongest form, the one-world Eleatic position, reference (through which truth is determined) is confined to the one actual world (Being, or, in modern semantical symbolism, G), assumed consistent, and to certain (sub-)theories, ipso facto consistent, composed from its components. By the time of Plato, the classical exclusion picture of negation has emerged, ruling out the possibility of non-trivial inconsistency, but not restricting (consistent) theories to subtheories of, or to the confines of, the actual world G. In Aristotle however, a different connexive position, commonly confused with the classical position, also appears. According to this position, based on a cancellation view of negation, all propositions, and so derivatively all theories, are consistent.[4]

The attitude taken to inconsistency or contradiction is perhaps the clearest indicator of paraconsistent stance. Strong paraconsistent (i.e. dialethic) positions admit contradiction in the actual world, whereas weaker paraconsistent positions, though not conceding that much, allow inconsistency in some nontrivial theories, or their analogues, such as language or thought. In these terms, the official Soviet position seems to have moved from, first of all, a strong paraconsistent position, before the twenties, admitting contradiction in both the world and thought, to, secondly an anti-paraconsistent position, excluding contradiction both in the world and in theories, to in the third place, post-war, a weak paraconsistent position allowing inconsistency in thought, and "thought-constructions" such as theories, but not in the world. There are some signs now that the full cycle may be completed and the first strong position resumed. These shifts in official position can be determined, furthermore, without knowing what Soviet logic looks like in any detail.[5]

I. WESTERN THOUGHT UNTIL THE FALL OF THE ROMAN EMPIRE: PARACONSISTENT ELEMENTS.

Eastern philosophy has generally been more tolerant of inconsistency, more amenable to paraconsistent approaches than Western. However, despite apparent dominance of classical Greek thought (from our present biased viewpoint) by consistency hypotheses, paraconsistent approaches were by no means exceptional in classical antiquity, but were assumed or espoused by several lesser philosophical schools.

In the first place, overtly paraconsistent approaches were adopted in classical antiquity, for example by the Megarians who held that some statements are both true and false. So it was that Chrysippus, the Stoic, devoted a book 'to replying to those who hold that Propositions may be at once False and True'.[6] Regrettably there are no further details of this book. But we can reasonably conjecture that Chrysippus's opponents were Megarians, who defended their position on the basis of such semantical paradoxes as the Liar. However, we really do not know exactly who the opponents were. And unfortunately this sort of lack of knowledge or uncertainty is virtually the norm as regards paraconsistent approaches in classical antiquity. Much, almost all, of the evidence has been lost or destroyed, not uncommonly at the hand of politically more powerful establishment positions, it would appear.[7] The entire history of paraconsistent approaches from this important period has to be pieced together from a few fragments, and - especially bad[8] for such alternative logical approaches as paraconsistent ones - from what the opposition had to say.

Paraconsistent thinking begins in the West, so far as we can determine, with Heraclitus of Ephesus. This was certainly the view of idealist historians (such as Hegel and more recently Stace), but the matter has been in doubt since antiquity.

Thus Aristotle: 'It is, therefore, impossible to ever conceive that something is and is not, as some think Heraclitus said' (Metaphysics IV, 1005f.). Since then there have been repeated attempts to fit Heraclitus into the orthodox Western framework, which has long been underpinned by consistency hypotheses;[9] that is, attempts to render Heraclitus's position coherent, in the over-restrictive sense of consistent.

But Heraclitus's work - or rather the small but tantalising set of fragments that remain - resists such relocation, and is much more easily and naturally interpreted paraconsistently or dialethically (as Hegel was to interpret Heraclitus). Central in Heraclitus's thought is the theme of unity of opposites,[10] that opposites are united, at least in always being connected, but sometimes more strangely in being identical. Heraclitus is often saddled by commentators with the stronger, extravagant, thesis that all opposites are identical and indeed that all things are identical. The stronger thesis is justified by the principle, sometimes explicitly ascribed to Heraclitus,[11] that connection between the opposites implies their identity, i.e. in symbols $xRy \rightarrow. \; x = y$. While such a fallacious reduction of connection of all sorts to identity was tempting to the Greeks (and apparently encouraged by their language) - as indeed a reduction of relations to properties (or to properties together with identity or instantiation) has remained tempting virtually to present times - there is insufficient evidence that Heraclitus adopted such a reduction of connection to identity (unless being united is simply construed narrowly as being identical, an inference English semantics does not justify). All that many examples of the unity of opposites reveal is unity in the sense of appropriate connections (as in United Nations or United Church),[12] and this is what analogies (such as the bow and the lyre) also suggest.[13] Moreover were Heraclitus committed by the unity of opposites to the identity of all opposites, he would be committed by the main truth expressed in the Logos, that all things are united,[14] to the thesis that all things are identical, and so (since there is something) to the monist thesis that there is exactly one thing - a thesis he did not hold but rejected, since evidently on his view there are many things in tension.[15]

The principal truth expressed in the Logos is none other than the one of the main "laws" of ecology, that everything is connected to everything else,[16] not that there is only one thing. The Logos has in fact multiple roles, in somewhat the way that worlds do (especially the factual world T) in recent semantical theory: the Logos supplies both the general truths about things and the intelligent principles upon which they function (the basic information or axioms); and it takes the form of Fire.[17] The principle truth of the Logos, that everything is united, is supported by, but of course not entailed by, the unity of opposites. It is a part of the weaker theme, that some opposites are not merely connected but identical, a theme that suffices for dialethism given only familiar assumptions.

The central argument that Heraclitus's position is dialethic goes, when duly filled out, as follows:-
i. Some (suitable) opposites are identical. Let f and $\bar{f}$ be among such opposites (a predicate representation of opposites is convenient but not essential). Then $f = \bar{f}$. Now let x be some item that has f; then fx. For all ordinary predicates this follows from a suitable theory of objects (for let x be $\xi z f z$, i.e. an arbitrary object which is f). However, a less exotic route will serve. By excluded middle, fx, or else $\sim$fx, for any object x. Whichever alternative is assumed the argument continues in the same fashion. Now since fx and $\bar{f} = f$, also $\bar{f}$ x, so fx & $\bar{f}$x. While this gives dialethism of a sort, it could be contended that it is only "predicate dialethism", which is compatible with non-paraconsistent positions, indeed with an extension of classical logic.[18] To reach dialethism proper it needs also to be granted
ii. Among the suitable opposites is some pair, h and $\bar{h}$ say, such that $\bar{h}$z iff $\sim$hz, i.e. for which predicate and sentence negation coincide (for suitable z).
Then indeed dialethism proper follows: for hz & $\sim$hz holds.

The best evidence that the premisses, and indeed the conclusion, can be ascribed to Heraclitus derives perhaps from the "river fragments", where the opposites are those involving change (and measure and motion). Consider in particular, the proposition 'In the same river, we both step and do not step, we are and we are not' (Freeman [1948], p.28, #49a). The obvious symbolism - also easily disputed - is in each case hz & $\sim$hz.

A standard argument for such contradictions (presumably known to Heraclitus but not explicit in Heraclitus's work) is that in change, for example in motion which is change of position, there is at each stage a moment where the changing item is both in a given state, because it has just reached that state, but also not in that state, because it is not stationary but moving through and beyond that state.[19] However, precisely the same sort of argument the Eleatics took as a reductio ad absurdum of the hypothesis that change occurred; for whatever yields a contradiction, such as motion does, is impossible.

Sharply opposed, then, to the Heraclitan position, with its unity of opposites, and account of change through identity of opposites, was the Eleatic position deriving from Parmenides, an extreme position from which, however, both the classical establishment positions of Plato and Aristotle and the contemporary establishment position (incorporating classical logic) may be seen to descend.[20] Appreciation of some features of the orthodox (op)position - features that appeared in an early state in the dialectic of paraconsistency versus consistency, of Heraclitus versus the Eleatics - is important in gaining a fuller grasp of what the paraconsistent enterprise is about and what it is up against. The central point is not that the Eleatic position with its themes that nothing changed, that motion was impossible, directly opposed Herealitus's view that everything was in change,[21] that all was in flux, though this opposition is real enough; it is firstly that the Eleatic position was sustained by a series of hard arguments, most notoriously by the paradoxes of Zeno, a collection of generalised reductio ad absurdum arguments, all designed to show that the assumption that motion occurred led to contradiction, from which it was concluded, applying a consistency assumption, that motion could not occur; and secondly because the only way out appeared to be through rejection of the consistency assumption of "Eleatic logic". The impact of arguments that change implies contradiction - important for their fundamental place in the history of dialectic (and accordingly reserved for detailed presentation and discussion in the next chapter) - can thus be interpreted in two quite distinct ways: as a Parmenidean Modus Tollens against change or a Heraclitan Modus Ponens for dialetheism. The latter became the view of Hegel; indeed Hegel was to take motion and change as affording paradigmatic examples of contradictions in nature.

Aristotle devoted a considerable part of his metaphysical theory to refuting would-be Greek paraconsistentists,[22] first of all by trying to show that consistency assumptions, and the connected Law of Non-Contradiction could not be given up, rationally, or indeed without foregoing discourse altogether (the ultimate failure of these very important arguments is discussed below, in part V), and secondly, by offering an analysis of change which appeared to avoid contradictions.[23] Key parts of this analysis involved the use of time to avoid contradiction - instead of saying that a changing thing was both in a given state and also not in that state, it was said that the thing was in that state at time t_1 but not in that state at a different time t_2 - and the theory of potentiality - required to reunify these now temporarily isolated states as parts of the one (and same) change.[24] The appeal to different temporal quantifiers illustrates the <u>method of (alleged) equivocation</u> used since ancient times to avoid contradiction and reinforce consistency hypotheses; namely, where both A (e.g. x is f) and ~A appear to hold, find a respect or factor or difference r such that it can be said that A holds in respect r_1 and ~A in respect r_2. It can then be said that a contradiction resulted only by equivocation on respect or factor r. Often however the method of alleged equivocation does not work in a convincing way, and it breaks down in an irreparable way with the semantical paradoxes, as the Megarians were the first to realise.

In fact Parmenides and Zeno had, earlier on, to resort to the method of equivocation in order to escape contradictions their data seemed to deliver, that things both moved (as a matter of observation) and did not, that Achilles overtook the Tortoise and also could not do so. They drew a fundamental distinction of respects, between how things seem and how they are, between Appearance and Reality. Then it can be said, without contradicton, that Achilles overtakes the Tortiose as regards appearances, but not in reality. This crucial distinction undermined, however, another basic thesis of Parmenides, the One-world thesis, for it seemed to lead to a distinction of worlds, of the world of Appearance from the world of Reality. Plato and also Aristotle (not merely through his very un-Eleatic theory of potentiality) accordingly rejected Parmenides' world-monism and adopted instead "many-world" positions. Such theories need not of course lead in paraconsistent directions, because the many worlds can all be consistent ones (as in model theory and modal logic semantics).

Some of Heraclitus's aphorisms have been taken to suggest, or imply, an extravagant dialethism, that Heraclitus espoused not merely that some contradictions are true, but that all are true.[25] There is obviously a gross difference between these positions, a difference mainstream logics cannot adequately recognise. While the textual evidence available does not indict Heraclitus of extravagant paraconsistentism, so we have argued, some of the Sophists - who make up the next wave of Greek paraconsistentism - do not escape indictment easily. Protagoras, in particular, with his thesis that everything is true, seems to be committed to just such an extravagant position. For if every statement is true, then so is every contradiction; while conversely for any statement A, because A & ~A is true since contradictory, A is true, by simplification.

It is not difficult to see, in principle at least, why some of the Sophists should have arrived at a paraconsistent stance. For according to them, both sides of various (important) views can hold, or hold equally well. That is, in a given situation, perhaps even in the factual world T, both A and ~A hold (and are, for instance, supported by sound arguments) for some A. So long as not everything holds in the given situation, a paraconsistent position results. Such appears to have been the position of some Sophists (as we shall see). How was it that one of the first and most important of the Sophists, Protagoras, went further and adopted what is, at first sight, the trivial position that everything is true?[26]

Commonly an attempt is made to get Protagoras out of this fix (this stupidity, it is often thought) by having him say something different from what he did say, e.g. that everything is relative, that every proposition is supportable, etc. But supportability is not truth, and relativity does not on its own explain how Protagoras could have held what he did. Modern semantical theory can however explain the situation. For it is a theorem of universal semantics that every statement is true according to its own lights.[27] In short, one can so relativize the truth-determining framework, the model structure, as to bring out any given statement as true. Man - or at least men who have discovered the semantical trick - is the measure of truth and falsity, and so of what there is and is not; and Man can simultaneously affirm the themes of correspondence theory to the effect that the things that are are [so] and the things that are not ... are not [so].[28] But, it will be protested, relativism is not paraconsistentcy: the two are very different.[29] And so they are; however such semantical relativism presupposes a paraconsistent stance. For in order that an isolated contradictory proposition is brought out as true (in its own lights) it is essential that there be included in the modelling involved situations where contradictions hold true. If, for example, only classical models were admitted it would be impossible to confirm Protagoras' themes, to refute logical laws or to conclusively support contradictory statements. More directly, it can simply be concluded from the semantical relativism[30] that everything is true according to its own lights (e.g. in its canonical model structure), that contradictory theories can be true.

Given this paraconsistent setting much of the rest of Protagoras's reported work - which is not after all an extravagant dialethism - falls into place. It is a relatively straightforward matter then to see how there can be arguments against what are in fact necessary truths and for what are in fact[31] logical falsehoods. Since there is little problem (on even a modal view) in seeing how contingent statements can be both supported and rejected (in different possible worlds), the setting brings out Protagoras's further theme that every statement is both supportable and refutable, that there are contradictory arguments about everything,[32] - a theme that persisted into the later scepticism of Pyrrho and Carneades.

What the semantical reconstruction may appear to throw into doubt is the claim of Diogenes Laertius that Protagoras's anticipated Antisthenes' argument that contradiction is impossible:

> As we learn from Plato in the Euthydemus, he [Protagoras] was the first to use in discussion the argument of Antisthenes which strives to prove that contradiction is impossible, and the first to point out how to attack and refute any proposition laid down.[33]

But Protagoras could very well have used such an argument - it accords, after all, with his theme that every statement can be supported as well as refuted - without being committed to the position that contradiction is impossible. Indeed he could not coherently hold such a position, since he can bring contradictions also out as true and so as realisable.

The work of Antisthenes, to the small extent that we know it,[34] does however also admit of a - very different - paraconsistent construal. According to Antisthenes, it is impossible to speak falsely,[35] so whatever is said is true (as, in a sense, with Protagoras), so in particular contradictions (if uttered) are true. It is not too hard to see why Plato argued that the position was self-refuting.[36] First, the thesis that the thesis itself is false is true; so it is false. Secondly, if contradiction is impossible, then contradiction occurs truly in the judgement of impossibility, so contradiction is possible. But it is not according to Antisthenes. Of course Plato's clever arguments, as they involve consistency assumptions, are readily turned by a seasoned dialethician, especially the second.

But Antisthenes appears to have been a paraconsistentist only obliquely. His motivation and argumentation seem decidedly different from those that typically underlie paraconsistent approaches. He is said to have denied the possibility of contradiction or logical falsehood, but his arguments, were they to work, would show similar difficulties with falsehood; so it is not too surprising that it is common to attribute to him (as we have done) the (stronger) view that it is impossible to speak falsely. One of his arguments, reminiscent of Parmenides, started from the assumption that all statements are subject-predicate in form, saying of an object what it is. Now such statements can only fail to be true by referring to nothing at all or by referring to something different from the object which has the predicate. But neither case is possible. The first alternative is ruled out (by the strong Ontological Assumption): it is impossible to speak of what does not exist. And the second alternative is ruled out by an analogous assumption: it is impossible for one to speak of something different from what one intended to.[37] Both assumptions have seemed plausible to many philosophers: both are however false. The argument does not provide a solid basis for contending that contradictions are sometimes (in some situations) not false. The main argument for which Antisthenes is known begins by claiming further that all statements are, or are reducible to, apparent identities, specifically to statements of the form x is y. Thus Antisthenes is sometimes accused, of confusing statements of identity and predication - a major tangle in ancient thinking, which Plato gets the credit for cleaning up satisfactorily.[38] But t is not so clear that Antisthenes was confused, or that Plato's clean-up was required (or was desirable). A shallow reason is that subject-predicate statements such as x is f appear to be equivalent to given identities like x is an f, i.e. x is an object which is f, which is of the form x is y, where y is an object which is f. A deeper reason is given by theories like Lesniewski's, which take as basic the undifferentiated is, symbolised 'ϵ'. With modern logical technology it is much harder to resist giving Antisthenes the first stage of his argument (but thereby even easier to resist granting further stages). His further contention is that no object can admit equation with an object differing in conception from it. While such combinations [statements] of the form x is y, would normally be merely written off as false (nonsignificant, etc., as the case may be), Antisthenes wants to reject them altogether as statements. Such combinations must be rejected 'because the conception of one is different from the conception of the other, and two things with different conceptions can never be ... the same',[39] or, if the argument is to go through, even said to be the "same". Why would anyone want to say this? In the first place, a strong version of the Ontological Assumption, the Parmenidean form already seen in the initial argument, appears to be at work again. If x and y are not the same, then the sameness (of conception) the statement "x is y" is trying to ascribe to x and y does not exist, and so cannot be spoken of.[40] In the second place, because of the philosophical pay-off. One corollary of the argument was supposed to be a Parmenidean doctrine of the oneness of being:

> 'for if nothing can be predicated of anything else, ... being can alone be predicated itself'.[41]

Another corollary Antisthenes aimed at, according to Zeller,[42] was proof of the impossibility of speculative knowledge. Presumably such knowledge would require what cannot be said (or thought either?). At this point, as at others,[43] Antisthenes appears to anticipate some aspects of Wittgenstein's work.

Evidently, if Antisthenes was a dialetheist, he was a decidedly extreme one. Antisthenes was at one time a pupil of Gorgias, another of the Sophists who deserves mention in a history of paraconsistent approaches. Gorgias can be seen as initiating yet another direction of paraconsistent thought, that which eventually found much fuller expression in the theory of objects (discussed in part V below). Major themes of the theory of objects were instantiated in Gorgias' writings; notably, the rejection of the Ontological Assumption (and therewith the Reference Theory) are implied by Gorgias' thesis that no universals exist, and in particular that neither Being nor Non-Being exist.[44] It is evident that Gorgias thought we could think and argue perfectly well about what does not exist, without furthermore implying that it does exist in some way, and he proceeded to argue very skilfully in such a fashion. Moreover components of Meinong's freedom theses are clearly illustrated elsewhere in Gorgias' work, e.g., in the theme that 'conjecture is open to all in everything'.[45] While there appears to be little evidence that Gorgias would have adopted a dialethic position, a paraconsistent approach is required to accommodate in a direct way the components of the theory of objects and what Gorgias assumes concerning intensional functors such as those of belief and conjecture.

Isocrates, a member of the Establishment, explicitly critical of Gorgias as well as of Protagoras and of Zeno and Melissus, in his Helena (an oration apparently intended, among other things, to send up Gorgias) also there attacked three contemporary "classes of triflers". The first of these triflers were presumably the Antistheneans; they comprised 'those who maintain that it is impossible to speak falsely, or to utter a contradiction or to "deliver two contradictory discourses" about the same matter' (Helena, §1). The other classes of triflers are now taken to be Megarians and Academics (cf. Freese [1894], p.290), but they also included, it is reasonable to conjecture, the author of Dissoi Logoi. The pre-Aristotelian fragment, now called the Dissoi Logoi, is discussed as follows by the Kneales:[46]

> It is obviously part of the protracted debate on the possibility of falsehood and contradiction. ... the author seems to be arguing that it is possible not only to make contradictory statements but even to maintain in a variety of contexts two plausible theses which contradict each other. To this end he sets out a series of antinomies, each one with thesis and antithesis.

Thus the Dissoi Logoi would certainly seem to be a dialethist tract. However the matter is not quite so clear. The thesis of the Dissoi Logoi, Taylor conjectures,[47] is

> to reinforce the Eleatic doctrine that ταπολλά, the contents of the world of sensible experience, are unknowable, and that no belief about them is any truer than its contradictory.

Hence at this stage paraconsistency is beginning to merge with what was to become a major element in later classical thought, scepticism. Orthodox scepticism can be formulated without paraconsistency. However the sceptical positions of Carneades, Pyrrho and others appear, like the position of Protagoras, to require paraconsistent underpinning and the support of paraconsistent approaches; but, once again, as so much of the relevant work is lost, the matter must remain in considerable doubt.

Very strong evidence that dialethic positions were taken in classical antiquity comes not only from Heraclitus and the Sophists, but from the work of the Megarians, and in particular from their treatment of (semantical) paradoxes, especially the Liar, said to have been discovered or devised by the Megarian, Eubulides.[48] As remarked, according to some Megarians, the paradoxes yield propositions which are at once both false and true. The evidence may not be entirely conclusive, but it is not so flimsy as to justify Mates' contention regarding the Liar, that 'We do not know how any of the competent logicians of antiquity attempted to solve antinomy'.[49] Admittedly it depends on who is counted competent, and some might doubt that paraconsistent logicians make the grade! In any case, Bochénski considers two important ancient efforts - distinct from the Megarian approach - at solving the Liar.[50] While there was subsequently much dispute over what these attempts amounted to, the shape of the attempts, by Aristotle and Chrysippus, is recorded.

In the Sophistic Refutations,[51] Aristotle specifically presents the paradox as 'the problem whether the same man can at the same time say what is both false and true'. Thus the Liar paradox was initially perceived in the obvious way. Aristotle's treatment of the problem was however less obvious and took the following rather cryptic form: 'There is, however, nothing to prevent it [the paradox statement] from being false absolutely, though true in some particular respect or relation, i.e. being true in some things though not true absolutely'.[52] This seems to fit exactly within the second "resolution" of the Liar proposed by Peirce[53] and discussed in more detail below since it seems to have paraconsistent possibilities. In fact it is more plausibly seen not as a paraconsistent resolution, but as a further application of the method of equivocation - though it tries to take some account of paraconsistent data through the notion of "truth in some respect".

The solution of Chrysippus, as recorded, has even less dialethic potential, but suggests a nonsignificance approach, that paradox-generating sentences lack meaning. It is important here, however, in that it explicitly rejects both Aristotelian _and_ dialethic resolutions, thereby confirming the claim that a dialethic resolution had some currency. According to Chrysippus, so it is reported,

> the (fallacy) about the truth-speaker and similar ones are to be ... (solved in a similar way). One should not say that they say true and (also) false; nor should one conjecture in another way, that the same (statement) is expressive of true and false simultaneously, but that they have no meaning at all.[54] And he rejects the afore-mentioned proposition and also the proposition that one can say true and false simultaneously and that in all such (matters) the sentence is sometimes simple, and sometimes expressive of more [as solutions].

What the Megarians and Stoics said about the semantical paradoxes should be integrated of course with what they said about truth and falsity, and also with what they said about implication. For these are _not_ independent issues. Unfortunately again, as to the Megarians' views on such matters, comparatively little is known. Much more is known about the theory of truth in Stoic logic.[55] Whereas the Megarian theory appears to have been four-valued, with values true, false, neither true nor false, and both true and false, the Stoic theory, of the truth-values of propositions or _lecta_ (the basic carriers of truth-values), was (at most) three-valued, lacking the value, _both_. However in their theory of the truth-values of _presentations_ the Stoics did assume a four-value pattern: '... some presentations, are both true and false, and some are neither'.[56] Examples they gave of presentations which are both true and false include the following: first, "when a man imagines in his dreams that Dion is standing beside him (when Dion is alive)"; and secondly, the "image of Electra as a Fury visualised by Orestes in his madness.[57] By contrast, the example of the bent oar was not accepted as a presentation which is both true and false. Why not? An answer - obvious now in the light of intensional semantics - is that the latter example would have induced inconsistency in the factual world T, whereas the two accepted examples involve joint truth and falsity _off_ T, in worlds picked out by intensional functors such as those concerning dreaming and insanity. Now it might be claimed that this shows little about propositional truth and falsity which however is what an argument to paraconsistency requires. But in fact the Stoics considered propositional truth as fundamental, all other types reducing to it.[58] So joint truth and falsity of presentations reflects back onto propositional terms. But whether it delivers a paraconsistent position depends upon how the reflection is effected. We are told that 'a presentation is true iff a proposition accurately describing it is true'.[59] and let us suppose, as is not unreasonable, that a similar scheme holds for the predicate 'false'. Let p be a presentation and p# a proposition that accurately describes it. Then where p is a presentation which is both true and false, a dialectic result is forced, contrary to other information we have as to Stoic views. The problem lies, not with the falsity reduction scheme, but with both schemes, which require situational-relativisation to yield the result that where p is both true and false there is some situation s_p where proposition p# is both true and false. Given that such situations are ones that we can reason about, as we have been doing and is in any case an outcome of the propositional ascriptions, the Stoic position which emerges is a paraconsistent one. For since not every proposition is true in situation s_p, s_p is appropriately inconsistent but not trivial. In sum, the Stoics were committed to contradictions holding off T, but not their holding at T.

The conclusion we are pushed towards is, then, that whereas the Megarians took a dialethic stance, some at least of the Stoics adopted (only) a weaker paraconsistent position. But it looks as if the Stoic ranks may have been split over the issue, as they were split over the closely related matter of the correct theory of implication and conditionals. Indeed there is further evidence from the Stoic theory of "connexive implication" that the Stoics (including Chrysippus) who held that theory were commited to a paraconsistent position. For under this theory, the implicational principle of <u>ex falso quodlibet</u> was rejected, since an arbitrary proposition q may not be connected to p & ~p.[60]

It may be that paraconsistent positions, of a Megarian or Stoic cast, persisted into late antiquity.[61] But 'with the end of the old Stoa there begins a period into which hardly any research has been done'.[62]

II. ELEMENTS OF PARACONSISTENT THINKING IN EASTERN PHILOSOPHY

In some ways the situation is at present even more vexed here than in the case of classical antiquity. For although there appears to be more data - less crucial material has been destroyed or lost - there are severe difficulties in obtaining a clear view of the situation. In part, but perhaps only in small part, this is due to the nature of the material itself. It is also due to the linguistic inaccessibility of much of the material, and the seriously distorting lenses of translators and commentators on the material that has been rendered more accessible. Often this distortion can be directly ascribed to a narrow training in Western analytic thinking and frequently uncritical assimilation of anti-paraconsistent assumptions.[63] This surfaces, for example, in incomprehension in commentators as to how philosophers, such as the Jains, can reject the Law of Non-Contradiction. It appears, differently, in descriptions of Nāgārjuna's negative dialectic in terms, and in a fashion, evidently borrowed from Hegel (thus e.g. Murti [1955] pp.127-8, and Streng [1967] pp.148-9 where the same account is used).

Furthermore, the way paraconsistency enters in Chinese thought, as in Indian, is not through a set of theses that can be simply pointed to as evidence of paraconsistent approaches, but in the way contradictions are tolerated and used to illustrate points.

This is particularly true of the Tao, which contains definite elements of paraconsistency: 'Laotse is full of paradoxes', e.g. 'Do nothing and everything is done'.[64] But a theory may contain paradoxes, or apparent contradictions, without necessarily containing any unresolved or unresolvable contradictions. Consider, for example, Meinong's 'There are things of which it is true that there are no such things' and Jesus's 'He who loses life shall find it'.[65] However there are at least consistent subtheories of the theory of objects and presumably of Christianity which can consistently assimilate these respective paradoxes. So additional evidence is required to show that Tao exhibits genuinely paraconsistent elements. Some of the main evidence for this is rather like some of that adduced in the case of Heracleitus: the relativity of opposites, and the levelling of all opposites, of all things, into one. All things are one. Unity is achieved through complements, or opposites. But also (more in the fashion of Parmenides) the One is eternal and unchanging. Thus the One certainly appears to exhibit inconsistent features and to violate classical logical principles. And according to Needham it was obvious to the Taoists that the Law of Contradiction was constantly being flaunted: 'The natural sciences were always in a position of having to say "It is and yet it isn't". Hence in due course the dialectical and many-valued logics of the post-Hegelian world. Hence the extraordinary influence of the traces of dialectical or dynamic logic in the ancient Chinese thinkers, including the Mohists ...'.[66]

Both the Mohists[67] and the Dialecticians have some claim to be accounted early paraconsistents. In particular Hiu Shih (?380-305 B.C.) who lived during the Warring States period, and belonged to a school of philosophers that was known as the 'Dialecticians' or the 'School of Forms and Names', considered contradictions to be the great insights of the world, the keys to the universe, as it were. (He is also said to have been bent on proving the impossible possible.) Although the many books of Hiu Shih have been lost, some of the paradoxes he propounded have been recorded in the <u>Chuang-Tzu</u>[68] (chapter 33). The sixth paradox there presented is 'The South has no limit and has a limit', which has the apparent form p & ~p.[69]

With Indian thought the position is rather clearer. At least two important positions had paraconsistent commitments: Jainism and Mādhyamika Buddhism. The Jains from

> a very early date flatly den[ied] the law of Contradiction. At a time when the battle raged between the founders of Buddhism and the Sāṅkhyas,[70] when the latter maintained that "everthing is eternal", because Matter is eternal, and the former rejoined that "everything is non-eternal", because Matter is a fiction, the Jainas opposed both parties by maintaining that "everything is eternal and non-eternal simultaneously". According to this theory you could neither wholly affirm, nor wholly deny any attribute of its subject. Both affirmation and denial were untrue. The real relation was something half way between affirmation and denial. Like the doctrine of Anaxagoras in Greece, this denial seems directed much more against the law of Excluded Middle, than against the Law of Contradiction. However in the problem of Universals and Particulars the Jainas adopted an attitude of a direct challenge to the law of Contradiction. They maintained that the concrete object was a particularized universal, a universal and a particular at the same time. Such is also the attitude of one of the earliest Buddhist sects, the sect of the Vāsiputrīyas. ... They maintained that the Personality ... was ... something existing and non-existing at the same time.[71]

Challenging the law of Contradiction as the Jains do does not necessarily commit them to a paraconsistent position: it depends further on how negation is interpreted. The situation is complicated because the Jains apparently talk about contradictions in two different ways.[72] On the one hand there is a cancellation way: the joint assertion of Yes and No is (it is said) ineffable in language, for 'yes' and 'no' being in perfect equilibrium, will automatically cancel each other. On the other hand they allow joint assertion of Yes and No, which is expressible in language, and hence on more classical construals must be somehow resovlable through some indigenous device. The device is that of pluralism (which <u>shades</u> into a certain relativism).

The Jains held as a central theme the non-onesidedness of truth: - 'The Jainas contend that one should try to understand the particular point of view of each disputing party if one wishes to grasp completely the truth of the situation. The <u>total</u> truth ... may be derived from the integration of all different viewpoints'.[73] In this respect the Jains anticipate contemporary discussive logic, initiated by

Jáskowski, and they may similarly be interpreted in terms of integration of different worlds, or positions, reflecting partial truth (see part V). Naturally such a theory risks trivialisation unless some (cogent) restrictions are imposed on the parties admitted as having obtained partial truth - restrictions of a type that might well be applied to block amalgamations leading to violations of Non-Contradiction.

Unlike the Jains, the Mādhyamikas apparently affirmed the Law of Contradiction.[74] But this did not prevent a certain unity of opposites, e.g. in the negative dialectic of Nāgārjuna, a concept, such as Being, can become indistinguishable from its opposite, Non-Being.[75]

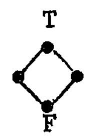
The negative, or destructive, dialectic was based on a four-valued scheme, apparently deriving from Buddha. There were the fundamental metaphysical questions on which Buddha was mysteriously silent. Why? There were well-known questions which the Buddha declared to be avyākrta, by which it is commonly (but likely erroneously) said he meant the answers were inexpressible. For the moment we shall simply say they have value A (for alternative or avyākrta). Among the questions concerned were the following:

(1) Whether the world is (a) eternal (b) or not, (c) or both, (d) or neither -
(2) Whether the world is (a) finite, (b) infinite, (c) or both, (d) or neither -
(3) Whether the Tathāgata (a) exists after death, (b) or does not, (c) or both (d) or neither.[76]

The four alternatives, or values, formed the basis of the tetralemma (Cataṣkoti) of Nāgārjuna's dialectic. The values are, in effect, true, false, both true and false, and neither true nor false; e.g. it is either true that the world is eternal, or false that it is, or both or neither (using the T-scheme).

For the four alternatives yield four values, one for each answer, and conversely the four-valuedness transfers to items as follows. Consider a statement such as "d exists" which may have any of the four values. Then (by T-schemes) there are four cases: d exists and does not, d neither exists nor does not, d exists, d does not exist.

Thus the four-values may be represented on the logical lattice (L_4)[77]

B(i.e. {T, F}) 

N(i.e. { }) •A

- to which both the Buddha and Nāgārjuna in effect added the further value A, for The Rest, for everything that did not fit into the too neat and clean logical lattice.

Buddha, incidentally, was perfectly clear that the classical two-valued barbell lattice T•—• F was quite inadequate: The 'yes' or 'no' answer to [the] fundamental questions could not do justice to truth. Buddha called such speculations mere 'ditthivada' (dogmatism), and refused to be drawn into them.[78] Whereas the two-valued position was accounted dogmatism, the four-valued was construed as rationalism. This too was inadequate, as Buddha had realised, and as Nāgārjuna proceeded to show by his destructive dialectic. Unfortunately for the rationalist picture, there were isolated exceptions, where none of the four values or cases obtain, e.g. where d above is emptiness.[79] Representing the position of the Buddha appears to require (at least) a five-valued logic. But such a framework is as much grist to the paraconsistency mill, as the four-valued rationalist picture from which it emerged.

Nāgārjuna's argumentive procedure was to draw out and expose the disconcerting or antinomic consequences of each of the four rationalist alternatives.

> The technique of the dialectic consisted in drawing out the implications of the view of the opponent on the basis of the principles accepted by himself and thus showing the self-contradictory character of that view. The opponent was hoisted with his own petard. He was reduced to the position of absurdity when the self-contradictory consequences of his own assumptions were revealed. The dialectic was thus a rejection of views by reductio ad absurdum argument. Technically this was known as prasanga.
>
> The purpose of the dialectic was to disprove the views advanced by others, not to prove any view of one's own. He who advances a view must necessarily prove it to others whom he wants to convince; he who has no view to advance is under no such necessity [Singh [n.d.]].

But although primarily negative in its thrust, the procedure yielded (like modern "no theses" positions) some positive insights.[80] Among those were suggestions as to the limits of reason or of expressibility - it is not entirely clear which (what the Buddha hinted at suggested one, and the rejection of rationalism suggests the other), but it had better not be either, at least in any straightforward way. For the structure of the dialectic and the details of the arguments are expressible without any serious loss, and the arguments conform to rational standards, and do not force us into any (imported) Kantian "super rationality".[81] This can be confirmed by examination of Nāgārjuna's application of his dialectic, for example to the notions of causality and of nirvana.[82]

The theory of Nāgārjuna is often said to represent the high point Buddhist philosophy. Thereafter the dialectic potentiality, at any philosophy appears to decline.

III. Paraconsistent Thought in Christendom

Returning to Western Europe and moving on in time, we find that orthodox philosophy in Christendom was strongly anti-paraconsistent. The reason for this is undoubtedly the prominent influence of Aristotle and his logic. Despite this, paraconsistent thought continued as a minor, and sometimes heretical, tradition, which is not surprising when one notes how close to the wind Christian theology sails.

For a start, there were isolated figures such as Peter Damiani (1007-1072). Damiani resolved the paradox of God's omnipotence by retaining the unrestricted omnipotence of God, but admitting violations of the law of Non-Contradiction. God could presumably both produce stones too heavy for anyone to lift and lift such stones, tie a knot that could not be untied and untie such knots, and so on.

> Commenting on the words of the psalmist *Omnia quaecumque voluit fecit*, Damian claims an absolute omnipotence for God. The Almighty has not subjected nature to inviolable laws; and if He wishes He could

> bring about that that which happened in the past did not happen. Certainly such an assertion seems to violate the principle of contradiction; but this principle is valid only for our poor human reasoning...and does not apply to the Majesty of God and sacred knowledge.[83]

Unfortunately, Damiani combined his rejection of Non-Contradiction with a rejection of dialectics (as a useless superfluum) and a disdain for philosophy (as worthless and impotent) and for reason.[84] Thus, his position is not entirely coherent. For as de Wulf goes on to point out, 'All the same he makes use of reasoning in his De Divina Omnipotent...in which he shows that the rules of human knowledge cannot be applied to God' (p.155).

More important than such isolated figures, however, was the whole tradition of Neo-Platonism, which contained significant paraconsistent elements. It is usually thought that whilst Plato may have been willing to concede the inconsistency of the empirical world (Parmenides 129 B), he insisted upon the consistency of the world of forms. Although this is the standard interpretation, the theory of forms quickly leads to inconsistencies,[85] and there is textual evidence (especially in the Parmenides) to suggest that Plato may have thought that the One (the form of forms) had inconsistent properties. Though this is not the most plausible interpretation of Plato's thought (in virtue of Plato's discussion of the issue in the Sophist) it was certainly the way that a number of Neo-Platonists interpreted it.

For example, the founder of Neo-Platonism, Plotinus, was wont to attribute contradictory properties to the One, such as that it is everything and nothing, everywhere and nowhere, and so on.[86] It is sometimes suggested that Plotinus thought that the One was ineffable and, hence, that his attributions of contradictions to it are attempts to express this.[87] Certainly he did think that the One was ineffable. However, this does not resolve the contradictions but multiplies them. To call something ineffable and then write many pages describing it, explaining how it creates the empirical world, how it can come to be known, etc. is a contradiction of which Plotinus can hardly fail to have been aware. Moreover, Plotinus is ready enough to ascribe contradictions to the soul, which is quite effable.[88]

Many of the contradictory aspects of Plotinus' thought were taken up by later Neo-Plationists such as Proclus, Damascius,[89] Pseudo-Dionesious and Scotus Eriginus. These in turn (especially the last two) were very influential in the Christian, Mystical, Neo-Platonic revival of the late Middle Ages and early Renaissance which also had significant paraconsistent elements. Meister Eckhart, for example, asserted that God was being, and yet, beyond being, and, therefore, not being.[90] However, the apogee of this tradition was, with little doubt, Nicholas of Cusa. For Cusa, the One, identified - as by the other Christian Neo-Platonists - as God, is the reconciliation of all contradictories (Of Learned Ignorance, 1, XXII). All things are true (and false) of God. Thus:

> ...in no way do they [distinctions] exist in the absolute maximum [the One]...The absolute maximum...is all things and, whilst being all, it is none of them; in other words, it is at once the maximum and minimum of being (Of Learned Ignorance, 1, IV).

> What is more, whilst God is Father, Son and Holy Spirit, Infinity, Truth and Substance, he is quite ineffable and beyond description (Of Learned Ignorance, 1, XXIV, XXVI).

Quite consistently 'Cusanus criticised the Aristoteleans for insisting on the principle of non-contradiction and stubbornly refusing to admit the compatibility of contradictions in reality'.[91] Many people influenced by Cusanus were to take up similar positions: Boehme, Fludd, Campanella, and the unfortunate Giordano Bruno.[92] Though the influence of Neo-Platonism waned after the Renaissance, it was to exact a profound influence on German idealism. Hegel's Absolute is the One.

Nowhere in medieval times, it seems, was a paraconsistent approach to *insolubilia*, i.e. semantical paradoxes, seriously considered - though it was commonly admitted that '*insolubilia*, as their name implies, cannot be solved without evident objection'[93] The catalogues of "solutions" to insolubilia given in later scholastic writings do not include such an approach. It is true that the twelfth solution to the antinomies in scholastic times, on Bochenski's classification of these, looks a lot like a paraconsistent approach - and it is said to be 'commonly held by all today'!

> ...An insoluble proposition is a proposition which is supposed to be mentioned, and which, when it signifies precisely according to the circumstances supposed, yields the result that it is true and that it is false.[94]

But it is a matter of appearance only; for what is said goes on to avoid the evident conclusion that the proposition is both true and false - by a familiar strategem.

While dialetheic (strongly paraconsistent) positions were uncommon in medieval and post-medieval logic, (weakly) paraconsistent positions were adopted, so it is now beginning to emerge. One striking later example is provided by the Cologne work of 1493, where an idea underlying recent semantics for relevant logics is in part anticipated. The argument of *Ex falso quodlibet* was broken, as in relevant logics, by rejection of Disjunctive

Syllogism (in inferential form licensing inference from $\sim A$ and $A \vee B$ to B) on the grounds that where both $\sim A$ and A are assumed $\sim A$ cannot *also* be used to rule out A in the disjunction $A \vee B$. The Cologne work, and other post-medieval work by de Soto, Javellus and others,[95] may in some measure reflect and build upon the earlier literature on *obligationes* only now beginning to be studied; for this work seems to reveal a much broader trend in the same direction of admitting situations other than the actual, where assertions both hold and not, that is, inconsistent situations.

According to the *obligations*-literature, which appears to concern counterfactual reasoning among other things, one is sometimes explicitly allowed to reason from contradictory statements or impossibilities. In such cases the rule *Ex falso quodlibet* was suspended; in short, a basic requirement for paraconsistency was met. However, only certain impossibilities were admitted (e.g. theological claims concerning the Trinity). As may be expected with medieval logic, there were competing theories of *obligationes*. Roger Swineshed, for example, appears to have anticipated the recent non-adjunctive approach. In his theory, while one is sometimes required to concede two contradictory statements in a disputation that begins with a noncontradictory hypothesis, nonetheless, one must always deny the conjunction of these contradictories.

IV. THE MODERN REVIVAL: PARACONSISTENT APPROACHES THROUGH IDEALISM AND COMMONSENSE

In the modern period, beginning with the Renaissance and Enlightenment (both so-conventionally-called), and running through until the beginning of the present century, two further[96] major philosophical positions emerged which were congenial to paraconsistent approaches and took paraconsistent shape in some of their elaborations, namely idealism, especially as elaborated by Hegel, and, very differently, the philosophy of commonsense, especially as presented by Reid.

Idealism developed in two different forms, transcendental idealism, which was integrally linked with the revival of dialectic and adaption of Greek idealism, and as a movement was centred in Germany, and non-transcendent idealism, which was typically coupled with empiricism (and took the form of phenomenalism) and had its main base in England. The first of these forms we consider in the next chapter when we take up the history of dialectic. The second form appears to have been given paraconsistent shape by some of its less-known practitioners, notably Collier.

In Clavis Universalis, published in 1713, Collier[97] argues that the external world is impossible. In doing so he anticipated the idealism of Berkeley and also the first two of the antinomies of Kant. Now idealism, though it can be congenial to paraconsistency, does not necessarily lead to it, and may well adhere to strong consistency assumptions. For example, the demonstration of the impossibility of a mind-independent external world may be taken just as a reductio argument, of classical form, and as showing no more than that there can exist no such world, certainly nothing to the effect that some contradictions hold true,[98] or may be or be considered true. Collier is however prepared to take substantial steps in dialethic directions; For example (on p.61), he meets an objection that certain themes 'appear to be contradiction' with the response that they are indeed so but 'nevertheless true; nay, ... I could easily show them a hundred such contradictions, which they themselves will acknowledge to be true'.

Collier's work owes what little currency it has to Reid who chanced upon one of the rare copies in a library. Reid in turn is important in the development of paraconsistent thinking for his repudiation of the traditional assumption that conception, and mental operations pretty generally, are restricted to what is possible, and that indeed conception provides a test of possibility. Reid argues, on the basis of commonsense, that the objects of conception may be impossible, so the test is no test but a mistake concerning the logic of conception.[99] Suppose now we consider the deductive closure of what some person, Reid say, conceives at a stage at which he conceives some impossibility and so contradictory statements. Then in this situation which is deductively closed, both p and ~p hold for some p, but not every q since Reid is certainly not committed to the conception of everything (or every proposition). Thus the formalisation of Reid's position would lead to at least a weakly paraconsistent theory, including nontrivial inconsistent situations, other than the actual situation. But by contrast with the apparently dialethic form of Collier's idealism, the commonsense philosophy of Reid took only a weakly paraconsistency direction.

There was a further live historical position from which a paraconsistent approach might have emerged - despite the heavy consistency and classical logic underpinnings of the position as customarily presented - namely pragmatism. For as with German idealism, so with American pragmatism, the classical underpinnings now typically infiltrated are dispensible, without sacrifice of main themes: indeed the positions are evidently more viable without classical handicap. It is easy to see that a position as flexible and adaptable to practice - including reasoning practice - as pragmatism is adjustable to absorb the advantages of paraconsistency. It has looked to some as if just such an adjustment was occurring in Peirce's pragmatism after 1868.[100]

There are grounds for claiming that Peirce proposed a dialethic solution of the Liar paradox, maintaining that

S1. This very proposition is false; and

S2. What is here written is not true

are both true and false.[101] Unfortunately - since Peirce would be a welcome addition to the dialethic band - the grounds shake under further investigation. Peirce appears to have tried two different resolutions of Liar paradoxes (such as S1 and S2), an initial solution (during 1864-5), with much in common with a no-proposition solution and a revised solution (after 1887) deriving from Paul of Venice. According to the initial solution, such statements as S1 and S2 'are about nothing'[102] and so are logically meaningless and not objects of logical laws. So far Peirce is in the camp of those who propose incompleteness or non-significance solutions to logical paradoxes: the resolution is not a paraconsistent one since it removes the paradoxes from the domain of logic. But he gives the resolution a strange twist, contending that, though logically meaningless, the statements are truth-valued and indeed both true and false! Statements which 'stand upon the boundary of the true and the false' are 'in both', much as the boundary of red and green regions 'is both red and green'. The severe semantical difficulties this initial solution leads into, with S2 both true and false and neither because meaningless, are avoided by the revised solution, according to which statements such as S2 are meaningful. Thus 'This proposition is false', far from being meaningless is self-contradictory. That is, it means two irreconcilable things. While it may look as if Peirce is certainly embarked on a paraconsistent course with the revised "solution", there are new considerations that throw this into doubt (apart from the fact that he makes no moves to amend classical logical principles he elsewhere adopts). Firstly, he adopts new accounts of truth and falsity, according to which a proposition is true only if it is true in all respects, in everything said, and false otherwise; and secondly, he insists that 'every proposition besides what it explicitly asserts, tacitly implies its own truth'. He then argues that what S1 strictly implies is false - as paraconsistent logic shows, it is also true - hence as S1 is not true in this respect, it is false, period. More generally, Liar paradoxical statements involve contradiction and are simply false and not also true. Peirce has obviously woven a tangled web here. Although there are clearly paraconsistent stands in it, it is not clear that any coherent theory, consistent or otherwise, can be extracted from it.

V. CONTEMPORARY PARACONSISTENT DEVELOPMENT AND APPROACHES

We now turn to modern developments in paraconsistent logic. There is no strictly continuous development to be found here. This century, paraconsistency has been discovered by many different people working in isolation, who, only afterwards, if at all, became aware o6 the work of others and the tradition into which they fit.

Although formal investigation of explicitly paraconsistent logics apparently did not begin until after World War II, 1910 conveniently marks the beginning of contemporary work on paraconsistent theories. For 1910 saw three events of paraconsistent significance; publication of Łukasiewicz's (subsequently) seminal paper 'On the principle of contradiction in Aristotle', production of the second, revised edition of Meinong's Über Annahem - the basic text on Meinong's theory of objects, enlarged among other things to meet Russell's objections that the theory violated the Principle of Non-Contradiction - and the appearance of the first of Vasil'év's short series of papers on nonclassical logic. None of these historically important approaches, of Meinong, Łukasiewicz, and Vasil'év, makes use of modern symbolic logic: all are set more or less within the framework of traditional logic of the period. Moreover, all these approaches, with the possible exception of Meinong, appear to be at best weakly paraconsistent; they allow at most for off-T inconsistency, i.e. for inconsistent situations beyond the factual world, T. And Łukasiewicz, though he clears the way for the repudiation of the Law of Non-Contradiction (LNC) and so opens one route to paraconsistency, does not himself take that route. Indeed, all these approaches, which we shall consider in turn, sacrifice significant grounds for and elements of paraconsistency.

1. A Theory for Contradictory Objects: Meinong. Meinong is important in the development of paraconsistent approaches because of his theory of objects, a theory

which included inconsistent objects and also, in its more comprehensive form, defective objects (like the Russell class) such as the logical and semantical paradoxes supply. Under the theory, which grows directly out of common sense, contradictory objects, along with other sorts of nonexistent objects are genuine, perfectly good objects. They are objects of thought and other intensional attitudes, they are amenable to logical treatment, and they have features - including many of the features they seem to have.[103]

As a consequence of the theory then, there are objects, contradictory objects, which, in virtue of their nature, have contradictory features, but which are amenable to logical treatment. Plainly the usual logic is not adequate to the task (at least without considerable reorientation). Meinong was well aware of this:

> B. Russell lays the real emphasis in the fact that by recognizing such objects the principle of contradiction would lose its unlimited validity. Naturally I can in no way avoid this consequence. ... Indeed the principle of contradiction is directed by no one at anything other than the real and possible ([1907], p.16).

Contradictory objects, such as the round square, have contradictory properties: the round square is both round and also not round (because square). So the Principle of Non-Contradiction (LNC) in one form is violated, at least in that for some p, both p and also not-p. That this was taken by Meinong, as by Russell, to show that LNC had only limited validity, indicated clearly enough that Meinong was still operating in a consistency framework (something for which there is a good deal of independent evidence[104]). Otherwise he could simply have said that LNC was universally valid and that instances of its negation also held.

Although Meinong must have seen the theory of objects as set within some modification of traditional logic, he did not work out that logic to any conspicuous extent. Formal development of an appropriate logic for the theory of objects, paraconsistent logic, was to take another 40 years even to get under way. How such logics can, in turn, help in formalisation of the theory of objects will be explained in a subsequent introduction.

Even though it did not yield a logic, Meinong's theory had an important conceptual role. His contradictory objects, taken to counterexample LNC, played an important role in Łukasiewicz's realization of the vulnerability of that fundamental principle and of the traditional arguments for it. And Łukasiewicz's conceptual work in turn motivated Jaśkowski's formal theory.

2. The Overturning of Conventional Logical Wisdom: Łukasiewicz: In his penetrating 1910 article, Łukasiewicz opens the way for paraconsistent enterprise, but does not follow the road opened, and at the end of the article veers away from such heresy.

In his subsequent logical work Łukasiewicz became far removed from paraconsistent positions: for example, he accepted, and exploited, spread principles, he argued that material implication was an adequate medium for the formulation of Aristotle's theory of syllogistic, and even his many-valued logics (though subsequently refined for paraconsistent purposes) fail to meet conditions for paraconsistent logics. But then in 1910 the influence of Meinong was strong (see p.488, p.506 ff.): subsequently that logically-liberating influence waned.

Łukasiewicz opens the way for paraconsistent enterprise by showing that arguments for the Principle of (Non-)Contradiction, LNC, all derived from Aristotle, are built on sand; that, in effect, nothing excludes the design of nontrivial contradictory theories and furthermore such theories may even be true. And he tentatively conjectures that, by analogy with the development of non-Euclidean geometries, 'a fundamental revision of the basic laws of Aristotle's logic might perhaps lead to new non-Aristotelian systems of logic' (p.486).[105] in particular,[106] systems lacking, or admitting violation of, LNC.

The bulk of the article (sections 1-17 inclusive) comprises 'historical-critical exposition' of Aristotle's formulation of, and attempts to establish, LNC. But the exposition is of much more than merely historical interest, since as Łukasiewicz says by way of introduction,

> Aristotle's intuitions regarding the principle of contradiction [LNC] are, for the most part and clear down to present day, the usual and traditional ones; and arguments for and against the principle can be found together in the Stagirite in greater completeness than in any one text book of logic (p.487).

In short, the 'historical-critical exposition' also deals with 2000 years of (traditional) argument.

Łukasiewicz unravels three different formulations of LNC from Aristotle:[107] the ontological (or better thing or object) formulation: 'It is impossible that the same thing belong and not belong to the same thing at the same time and in the same respect'; the logical (or semantical) formulation: 'The most certain of all basic principles is that contradictory properties are not true together'; and the psychological (or belief) formulation: 'No one can believe that the same thing can (at the same time) be and not be' (p.487). Although Aristotle equates the logical and ontological formulations, 'none of the[se] three formulations of the principle of contradiction is identical in meaning with the others...' (p.489).

As Łukasiewicz shows in detail, Aristotle's attempt to prove the psychological form on the basis of the logical form fails. For the proof (summarised pp.490-1) 'is incomplete because Aristotle did not demonstrate that acts of believing which correspond to contradictory properties are incompatible' (p.491); and his further discussions of the point are 'inconclusive' - inevitably so, in virtue of counterexamples: 'there are sufficient examples in the history of philosophy where contradictions have been asserted at the same time and with full awareness' (p.492). More generally, Łukasiewicz's criticisms of Aristotle's attempts to establish or entrench forms of LNC are conclusive against them, even though he imports several rather unnecessary, dubious or false assumptions in elaborating his points.[108]

'Although Aristotle proclaims the nondemonstrability of the [nonpsychological forms of the] principle of contradiction ... he does not [attempt to] prove this claim [and] ... he strives in spite of that to give demonstrations for the principle' (p.494 with insert from p.493). Whether this is in order or not,[109] the purported elenctic and ad impossible demonstrations are either circular or else inadequate in one way or another and open to counterexamples (as Łukasiewicz shows in detail in the crucial central section 13 of his paper, pp. 498-9).

A 'specially ... note-worthy ...shift of proof' occurs in 'all of Aristotle's proofs ad impossible' namely, he 'proves not that the mere denial of the principle of contradiction would lead to absurd consequences, rather he attempts to establish the impossibility of the assumption that everything is contradictory' (pp.499-500, where several examples of this some to all shift in Aristotle are cited). 'However, he who denies the principle of contradiction or who demands a proof of it, surely does not need to accept that everything is contradictory ...' (p.499). Certainly not: yet a similar fallacious shift is subsequently made by Łukasiewicz, as will emerge.

Łukasiewicz claims, but does little to show, that there is 'good reason' for the shift 'in certain of Aristotle's positive convictions' (p.500). Be that as it may, these alleged positive convictions are of the first importance for determining the qualified status the LNC is alleged to have in Aristotle's position. For Aristotle, like Meinong and Vasil'ev later, 'limits the range of validity of the principle of contradiction to actual existents only' (p.501) and does not extend it to appearance (p.502). The 'sensibly perceptible world, conceived as becoming and passing away, could contain contradictions' at least potentially (p.501), but beyond this ephemeral world is 'another, eternal and non-ephemeral world of substantial essences, which remains intact and shielded from every contradiction' (p.502).[110] That Aristotle did not here openly reveal his "true position" is part of his diplomacy in trying to enforce the LNC, diplomacy required 'to hold high the value of scientific research' against the flood of falsity which would have destroyed science in its infancy (p.509). But the need for such diplomacy (or dishonesty) depends too on the fallacious shift of proof, confusing the admission of some falsity 'open[ing] door and gate to every falsity' (p.509).

Finally in his historical-critical exposition Łukasiewicz rejects the widespread view, which is Aristotle's view, that LNC is the most final and highest logical principle (p.502-4). He argues firstly that this is not so even according to Aristotle, who recognised that 'the principle of the syllogism is independent of the principle of contradiction' (p.503). The syllogism Łukasiewicz extracts from Aristotle, which remains valid when LNC does not, takes the form:

B is A
C, which is not-C, is B and not-B
∴ C is A [111]

Secondly he points out, correctly, that many other basic logical principles are independent of LNC (p.504).

In the 'positive part' of his paper (p.504 ff.) Łukasiewicz endeavours to show, more generally,[112] that there is no (logical) basis for adoption of the principle pf (Non)Contradiction, that 'a real proof of the principle ... cannot be carried out' - summed up in the theme that 'The principle has ... no logical worth, since it is valid only as an assumption' (p.508). But the argument which divides into two parts (sections 19 and 20) is not decisive. The first part, which aims to show that there is no justification of the principle, fails because, among other things, it does not exhaust the ways of attempting to establish LNC. For example, the principle can be shown valid semantically (as in the semantics for relevant logic). Granted such a proof does use semantical apparatus not available to Łukasiewicz, and does make use of notions under examination in the metatheory, so at least approaching circularity. The second part of the argument, which interestingly is based on 'the fact that there are contradictory objects' (p.506), such as those Meinong pointed out, does not provide counterexamples to LNC in the fashion Łukasiewicz intends without appeal to a consistency assumption bound up with the LNC itself. Otherwise, even if a contradictory object does ensure that for some p, $p \wedge \sim p$, this does not prevent $\sim(p \wedge \sim p)$ from also holding.[113]

So though he has opened the way for the paraconsistent position, Łukasiewicz has not glimpsed the dialethic extension. And indeed he proceeds to indicate some of the time-honoured arguments against it, against 'a contradiction existing in reality', arguments that are no better than those for LNC that he has decisively dealt with in the negative part of this paper. His claim 'there is known to us no single case of a contradiction existing in reality' (p.507) - a very weak claim compared to those philosophers such as Hilbert in the Kantian tradition are inclined to make, since he goes on to contend that 'one will never be able to assert with full definiteness that actual objects contain no contradictions (p.508)[114] - depends firstly upon a type distinction between abstract objects, such as those of logic and mathematics, and actual objects. The former, which may well prove 'contradictory upon closer examination' and sometimes have (as with the Russell class), do not 'represent reality' and are somehow blocked from affecting it.[115] The argument for this weak consistency claim is, even so, not decisive because it depends on a dubious perceived/inferred distinction and because it omits important (inferred) cases such as micro-objects. Łukasiewicz argues that 'it is impossible to suppose that we might meet a contradiction in perception', on the "literalist" ground that 'the negation which inheres in contradictions is not at all perceptible' (p.507). There need however be no such (picturing) correspondence as that presupposed, between what is perceived and how a description of the perceived items would be expanded to lay bare the contradiction. Perception of an impossible object, such as some of the items depicted in Escher drawings, need not, and does not, involve anything like "perception of a negation". Against inferred contradictions in reality, Łukasiewicz appeals, without any argument, to the traditional assumption that 'one will always find ways and means eventually to dismiss inferred contradictions' (p.508), an assumption most commonly supported by the ancient method of manufacturing distinctions which then conveniently reveal equivocations in the inference, which applies equally against contradictions "in" any objects. Much of the subsequent dialectic (in the introductions) will be concerned, in one way or another, with exposing the serious weaknesses of the traditional assumption and methods underlying it.

'As a consequence' of its logical worthlessness, LNC 'acquires a practical-ethical value, which is all the more important', Łukasiewicz tries to persuade us (p.508), using a conspicuous non-sequitur.[116] His brief pragmatic defence of the LNC - a defence he tried initially somewhat tentatively to transfer to Aristotle: 'it appears that even Aristotle at least sensed the practical-ethical worth of the principle' - is based on the mistaken theme that 'the principle of contradiction is the sole weapon against error and falsehood' (p.508).[117] The argument is that otherwise - should 'joint assertion and denial ... be possible'[118] - 'we could not defend other propositions against false or deceitful propositions' (p.508). For example, the falsely accused would find no way of proving his innocence, for without LNC the false accusation could not be removed! But here we have the same fallacious shift of proof as Łukasiewicz observed in Aristotle's arguments. It is enough to meet Łukasiewicz's argument[119] concerning the falsely accused that LNC holds in a range of ordinary circumstances, not that it holds as regards, say, the Russell paradox. The fallacious shift is repeated in Łukasiewicz's attempt to attribute a pragmatic argument for LNC, now as the ultimate lynch-pin for science, to Aristotle, said to issue, when Aristotle 'felt the weakness of his argument' for LNC, in his presentation of it as 'a final axiom, an unassailable dogma' (p.509). 'Denial of the principle of contradiction would have opened door and gate to every falsity and nipped the young blossoming science in the bud' (p.509).

Not so: it might (depending on the given connection of negation and falsehood) have admitted some falsehood - also however true - but this would not spread everywhere unless erroneous spread principles were also supplied. If the protection of science were indeed the ground, Aristotle need hardly have turned against alternative theories of paraconsistent inclination with such 'internal fervour'.

The practical-ethical value of LNC was not evident to Meinong and subsequent exponents of the theory of objects; nor was it evident to Vasil'év.

3. The Russian forerunners: Vasil'év and Bochvar. With his 'imaginary logics' Vasil'ev has been seen as anticipating many-valued logic, and as a forerunner of paraconsistent logic, but perhaps he is more accurately placed as one of the founders (along with McColl and Lewis) of intensional logics. As Arruda[120] points out, 'in no place does Vasil'év speak of any other truth-values than true and false', and the notion of dimension, on which the claim that his logics are many-valued is based, has to do not with the number of different truth-values involved, but 'the number of ... different qualities of a judgement, which is not only two as in traditional logic' (Arruda [1977] p.x). In the narrower, correct, sense then in which many-valued logics have an intended semantical or matrix theory in terms of many values (usually finitely many values), Vasil'év proto-logics are not many-valued: the matrix method, for instance, and the many values on which it is based, are not glimpsed. However in the misleading wider sense, used by Łukasiewicz [1951] for example, in which intensional logics are accounted many-valued, because, for example, they are not two-valued, and may be given representation in terms of many values (commonly infinitely many values), Vasil'év's proto-logics are many-valued, for they certainly seem to violate extensional two-valued requirements. Arruda's claim that Vasil'év is 'a forerunner of paraconsistent logic' (in [1977] and elsewhere) is, as will become evident, not that much more substantial.

Vasil'év's logical endeavours consist largely of a reworking of traditional Aristotelian logic. The traditional square of opposition, for example, though valid for judgements about facts, is said to break down for judgements about concepts, where an amended theory is required. The traditional theory of syllogism is said to be inadequate to accommodate judgement of neither affirmative nor negative but indifferent quality, and to require generalising to allow for higher dimensions of quality. But the new theory of syllogism is not worked out properly. Reworkings and attempts to extend traditional logical theory were by no means uncommon through the nineteenth and twentiety century, but most of them lack much interest for alternative logics. What distinguishes Vasil'év's efforts from more orthodox run-of-the-mill ventures are two features:

1) His rejection of the "law of contradiction" - as distinct from the semantical

LNC, and

2) His introduction and treatment of indifferent judgements, of the form "S is P and not P", both set within the framework of

3) His Imaginary (non-Aristotelian) Logic.

The model and inspiration for Imaginary Logic is the Imaginary (i.e. non-Euclidean) geometry of Lobatchevski (and others). As Aristotelian logic, like Euclidean geometry, concerns the real world, so Imaginary logic, like Hyperbolic geometry, concerns imaginary worlds, i.e. worlds mentally created or imagined.[121] The idea of worlds other than the real or factual - which is the genesis of intensional logic - runs right through the history of thought.[122] What is different, and exciting, in Vasil'év, is the idea that the logical laws may vary in such worlds. Both Vasil'év's main arguments in support of the feasibility and rationality of his Imaginary Logic consider changes in basic logical laws. Firstly, Vasil'év argues, rather like Łukasiewicz, that it is impossible to support, in a non-question-begging way, the uniqueness and immutability of the basic traditional laws of logic (Identity, Sufficient Reason, Contradiction, and Excluded Middle). In a way strangely reminiscent of Łukasiewicz's remarkable suggestion that Aristotle was aware that the Principle of Non-Contradiction could be denied but kept this quiet for political reasons and in order to undercut opponents, so Vasil'ev contends that Excluded Middle 'appeared in Aristotle's mind in order to refute his adversaries, and not for logical reasons' and that his attempt to

> prove [the] law starting from his own definition of a judgement, which always affirms or denies, which is always true or false, so the middle term would neither be true nor false, and would not represent a judgement ... contains a petitio principii, since the law of excluded middle is already subsumed in the definition of judgement ([1911] p.33).

Secondly, Vasil'év argues that the system of Aristotelian logic involves several axioms, and systems resulting by eliminating or replacing one of these axioms remain logic, and worthy of the name, just as the hyperbolic geometry counts as geometry.

The change of basic law of especial paraconsistent interest is that of the law of Contradiction: it is precisely this that the Imaginary logic abandons. But a first serious problem in getting clear about Vasil'év's achievement lies in determining what the "law of contradiction", LC, that Imaginary Logic does without, is. Vasil'év states LC in two forms, first, "A is not not-A" and, second, 'an object cannot have a predicate which contradicts it (or attribute which is incompatible with it)'. The first makes at best dubious sense where A is a term or object,[123] and the contrast made between LC and the semantical LNC strongly suggests that A is what it appears to be, a statement or judgement. In this case, the first might be formalized by either $A \leftrightarrow {\sim}{\sim}A$, LC0, or ${\sim}(A \leftrightarrow {\sim}A)$, LC1, for some suitable equivalence, $\leftrightarrow$. However neither of these seems to be what Vasil'év has in mind. (While the rejection of LC1 might have an important place in the removal of paradoxes, e.g. semantical ones, there is no evidence Vasil'év had paradoxes in view: his "counter-argument" to LC is of a very different cast.) For the second, allegedly equivalent, formulation of LC, also terse to the point of obscurity, appears different. The 'it' cannot strictly refer back to object: what seems to be intended is that object a cannot both have a predicate f and this (i.e. fa) be contradicted by ~fa, by its negation, (i.e. demodalising and generalising) ~(A & ~A), LC2 say.[124] The formulation of semantical LNC Vasil'év gives, in contradistinction to LC, suggests that LC2 is a correct representation; for what it is to be compared with is the principle, 'One and the same judgement cannot be simultaneously true and false', i.e. $\sim\Diamond(TA \ \& \ FA)$, whence ~(A & FA), demodalising and applying a truth (but not falsity) scheme.

According to Vasil'év, although LC is integral to Aristotelian logic and inevitable in the real world (where we do not have negative sense perception),[125] it can be rejected because it is a material (empirical, or real world only) principle. Semantical LNC is entirely different, and cannot be rejected, 'because if someone eliminates this law, he will be making a confusion between truth and falsity, and consequently he is not thinking logically' ([1912] p.217). Even if Vasil'év can be made to look like a paraconsistent logician, he is certainly no dialethic logician; and it looks as if he could be forced into the awkward, and ultimately incoherent, position - the position Arruda effectively places him in - of adopting a nonclassical and perhaps paraconsistent logic in combination with a classical metalogic and concomitantly rejecting the Tarski schema, at least the falsity scheme, $FA \leftrightarrow \sim A$.[126]

What is the evidence then that Vasil'év's position is a paraconsistent one? So far, rather slight. Rejection of any or all of LC0, LC1, and LC2 in a logic hardly establishes its paraconsistent character, since contradictory pairs of statements may still induce triviality. A simple example illustrates the point.[127] In the 3-valued Łukasiewicz logic $Ł_3$ both LC1 and LC2 are rejected. But spread principles such as $A \rightarrow. \sim A \rightarrow B$ remain in place. Arruda tends to assume that Vasil'év's removal of LC is enough to establish him as a forerunner of paraconsistent logic: it is not. To assess Vasil'év's claim to such a distinguished position requires further investigation both of his Imaginary Logic - of which unfortunately (for the task at hand) there are few further details at the sentential level - and of his imaginary worlds.

Vasil'év took it for granted that in the real world we do not have negative sense-perceptions, represented in judgements of the form "S is not P", but that these judgements are only obtained from affirmative judgements of the form "S is P" by inference (specifically, with a further major premiss of the form 'N is incompatible with M'). In imaginary worlds, however, we may have direct negative sense-perceptions yielding judgements in the same way as affirmative ones in the real world,[128] independently of affirmative ones. Accordingly it is not ruled out that both occur at once. That is, where α is ground for the affirmative judgement "S is P" and α^* ground for the negative judgement "S is not P", both α and α^* may obtain. In this case the indifferent judgement "S is P and not P" is true. What exactly this means is obscure. Vasil'év allows substitutions of colour predicates for P. Presumably he has in view situations where the light or the glass (S) is, for example, green and not green, because positive sense perceptions inform us it is green and negative ones it is not green. More familiar examples such as the bent oar help make such scenarios quite intelligible and even a little tempting: the oar is bent, so visual sense-perception informs us (or seems to), and is also not bent, so tactual perception informs us. The fairly accessible claim "The oar is bent and not bent" might be taken as working example of Vasil'év's "S is P and not P". It is not difficult to see that admission that "The oar is bent and not bent", and so that "The oar is bent" and "The oar is not bent" are both true, whilst such statements as "The oar is at the South Pole" are false, forces some important logical changes from tradition and, when pushed, leads to paraconsistent logic.

However in treating Vasil'év as a forerunner of paraconsistent logic one must tread with great care, first in what his rejection of LC amounts to. For although one might in a logical reconstruction take "S is P and not-P" ("S is P†") to be equivalent to "S is P and S is not-P", Vasil'év maintains that there are cases where "S is P†" is true but both "S is P" and "S is not-P" are false. This suggests that the negation in "S is P†" is merely predicate negation, and this would make his position quite compatible with classical sentential logic as we have seen in the case of Meinong. But even if it is something more like sentential negation that is involved (and hence the conjunction is nonstandard) there are problems. For since, as we have seen, the Tarski falsity scheme ($FA \leftrightarrow A$) is abandoned, it is no longer clear what negation amounts to. (Similar problems beset da Costa's paraconsistent logics as we shall see.) But most importantly, there are reasons to suppose that by "S is P†" Vasil'év often means something quite consistent in anyone's language (see further the alternative interpretations of indifferent judgements that Vasil'ev offers, discussed in Arruda [1977]). For he sometimes reads "S is P and not P" alternatively as "S may be P", which is certainly not contradictory; and what seems to lie behind the alternative reading is the following generality interpretation: "S is P" means that S is always P, "S is not P" that S is never P, that in all cases S is not P, and "S is P†" accordingly means that S is sometimes P and sometimes not P, i.e. that S may be P (in a familiar enough construal of the modal, adopted for instance by Russell). All of Vasil'év's proposed interpretations of his Imaginary Logic are of a similar generality type (including those in terms of similarity and difference and in terms of relative and absolute negation); all can be accommodated, more or less, in traditional logical theory; and none call for paraconsistent revision.

But for the sake of the argument let us suppose that Vasil'év has shown that LC2 is not valid in Imaginary Logic because the "logic" includes indifferent judgements. Do we then obtain a paraconsistent theory? It seems that we should; for there are situations when both C and ~C hold though not everything does (since e.g., the purely affirmative and negative judgements which sustain C and ~C are false). So a logic based on the theory should reject the rule C,~C $\vdash$ B; and if it contains a proper implication, $\rightarrow$, truth-preserving over the situations the theory encompasses, then the spread principles C $\rightarrow$. ~C $\rightarrow$ B and C & ~C $\rightarrow$ B should both be rejected. But this is speculative, because Vasil'év did not get around to considering such issues (nor given the historical setting of his work can he reasonably have been expected to). What he did try to show is that it is legitimate to operate logically with indifferent judgements, and how this leads to a revised theory of syllogism. But underpinning theory must lead to predicate paraconsistency in the shape of the rejection of S is P and not P $\vdash$ B, and so should lead, given that indifferent judgements retain the intended LC refuting features, to an underlying paraconsistent logic. In this tenuous sense Vasil'év can be accounted a forerunner of paraconsistent logic.

The revolutionary thrust of the work of Nineteen Tens was not pursued, or widely perceived, and had little real impact. Vasil'év wrote no more logic, but turned to other things; Meinong was progressively diverted into value theory and, in any case, he died in 1920 leaving no dedicated disciples in object theory; only Łukasiewicz had a long continuing career in logic, but the influence of Meinong on his work soon waned, and he concentrated on other non-paraconsistent topics and his evident philosophical talents were not greatly exercised.

Work on paraconsistent theory did not begin again until the late Forties, apart, it seems, from the logical ideas of Bochvar; and from a progressive shift towards paraconsistency in the thought of Wittgenstein (but that thought has only recently been much publicly disseminated, and its influence is again, like that of the work of the 1910s, now).

Whether or not Vasil'év introduced many-valued logics and applied them in the analysis of contradictions, the idea of doing so appeared in Russia; it was proposed (again) by Bochvar in 1939 in his paper 'On a three-valued logical calculus and its application to the analysis of contradictions'.[129] Again however, it is not at all clear that Bochvar was proposing a strongly paraconsistent treatment of the contradictions said to be "analysed", namely the paradoxes of Russell and Grelling. That depends crucially on how the third value is interpreted and whether it is designated. The issue is not straightforward because Bochvar appears to suggest various interpretations. In the main interpretation proposed, the third-value, N, is read as 'nonsense', and is not designated. And this is how the 3-valued matrices turn out, with the matrices for connectives &, ∨, ~ the classical significance ones. Accordingly, the treatment of the paradoxes is a nonsignificance one, not a dialethic one.[130] But Bochvar also proposes to construe N as 'undecidable' in the sense of 'having some element of undecidability about it', and as 'paradoxical'.[131] The classical significance matrices that Bochvar arrives at are not however appropriate for these interpretations.[132]

Although Bochvar does not then adopt a genuinely paraconsistent approach, in particular does not seriously consider a theory in which the contradictions (i.e. logico-semantical paradoxes) may hold, still it may be that the logical system he presents is (weakly) paraconsistent. It is not. For the system contains theses of the form $T(A \wedge {\sim}A) \supset TB$, i.e. there is an "external" implication connective conforming to detachment for which the spread principle $A \wedge {\sim}A \rightarrow B$ holds.[133] For related reasons, that they contain spread principles, familiar many-valued systems, such as the systems of Łukasiewicz, are not paraconsistent logics.[134] Even so with the very weak "internal" subsystem of his larger system Bochvar perhaps offers us the first "logic of paradox" or "calculus of antinomies". Such logics were to be periodically rediscovered over the next 40 years, something that was (then) necessary, for they were continually being lost sight of.

As with Bochvar so with the Chinese logician Moh Shaw-Kwei: although steps towards paraconsistency are taken, a (genuinely) paraconsistent treatment of the paradoxes is not attained. Like Bochvar, Moh is mainly working the other, incompleteness (nonsignificance), side of the street, though some of his results are important for paraconsistent theory. For in his 'Logical paradoxes for many-valued systems' [of 1954] Moh extends Curry's paradox to apply against systems containing higher order rules of Absorption. The argument shows among other things that very many finite-valued logics, including all finite-valued Łukasiewicz logics, trivialise if an unrestricted abstraction axiom is added to them, and so are unsuitable for major paraconsistent purposes. Moh raises the important question as to 'whether we could develop the theory of sets with unrestricted abstraction from the system $L_{\aleph_0}$ (p.39), i.e. from the infinite-valued Łukasiewicz logic? The answer is Yes, it has recently been shown, though the resulting consistent theory of sets has some serious drawbacks.[135] However, the logic is, once again, like finite-valued Łukasiewicz logics, not a paraconsistent one, since it has as a thesis the spread law $A \wedge {\sim}A \rightarrow B$.[136] For similar reasons, Moh's final point, 'that Łukasiewicz's interpretation of the system $Ł_3$ is not satisfactory' and that the third value should be interpreted as paradoxical, where 'we define a paradoxical proposition as one equivalent to its own negation' (p.40), does not lead to a paraconsistent system, though the interpretational idea can and was later to do so when designated values were appropriately adjusted to allow paradoxical assertions as designated. Moh had part of the right idea, that paradox-generating assertions are paradoxical, but with $Ł_3$ had the wrong logical framework for paraconsistent treatment of the paradoxes.

4. *An isolated figure in the contemporary history: Wittgenstein.* Wittgenstein's position[137] changes substantially in the course of his life-time. As regards his treatment of matters of paraconsistent concern such as negation, contradiction and paradoxes, three distinct periods have to be distinguished (since there are decisive differences in his positions in these phases): *early*, *transitional* and *late*. In his early, Tractarian, period he was committed to a very restrictive classical logical theory which entirely excluded paraconsistent approaches.[138] Though in his transitional phase Wittgenstein moved outside the confines of this narrow position, and was already prepared to concede that contradictions could (to some extent)[139] be allowed to arise in a theory (cf. [1975] p.345), he still thought that paradoxes, logical antinomies in particular, *need* to be resolved, and could be resolved by removing ambiguities and equivocations through analysis of meanings of the expressions used in their formulation.

> ... the antinomies did not arise in the calculus but in our ordinary language, precisely because we use words ambiguously. Hence the resolution of the antinomies consists in replacing the hazy way of expressing ourselves by a precise one (by recalling the real meaning of our words). Thus the antinomies vanish by means of an *analysis*, not by means of a *proof* ... A proof cannot dispel the fog, [the] *unclarity*. (McGuiness [1979], p.122).

That was in 1930. But by 1939 his position had changed markedly. The requirement of analysis, and that the antinomies be excluded, are both abandoned. He implies that contradictions like the Liar don't matter: 'it is of no use; it is just a useless language-game (Wittgenstein [1976], p.207). And similarly: 'contradictions. Whether we're to say they have a meaning I don't know - but it's clear they don't have a use' (Wittgenstein [1976], p.223, causing tension for the "meaning is use" equation). The themes that the Liar and other paradoxes are unusable, and that since they are unusable they can stand without removal and without harm in a language-game or calculus occur frequently (e.g. Wittgenstein [1964], p.51). There is no need for the theory of types then, for with the Liar 'nothing has been done wrong' (Wittgenstein [1976], p.207).[140] Wittgenstein indeed sometimes rejects distinction-of-meaning ways out of the paradoxes ([1964], p.102) and the type theory approach of the *Tractatus* ([1964], p.182). Sentences violating type theory, such as 'the class of liars is not a liar' are 'proper sentences' and 'there are language-game(s) with [such] sentence(s) too' ([1964], p.182).

Wittgenstein even asserts sometimes that he makes a statement with the Liar sentence, for instance [1976], p.207, though two pages later he offers instead the option: either 'you may say that it's not a statement. Or you may say that it is a statement, but a useless one' ([1976], p.209). It makes a considerable logical difference - much more than Wittgenstein seems to realise - which of these choices is made. The first option that the sentences do not yield statements, leads to some sort of significance theory, especially given that Wittgenstein equates making a statement or having content with making sense (e.g. [1964], p.171), which filters out paradox-generating sentences as statement-incapable or as not making sense. The second option leads however to either an incompleteness (many-valued) approach or, very differently, to an inconsistency (paraconsistency) approach. (While there are limited intermappings between these three approaches, they are quite distinct.)

Insofar as he mostly adopts the first option, Wittgenstein is not an exponent of paraconsistent logic or in any way prepared to sanction true contradictions (though he is clearly aware that motion could be said to involve a contradiction). Thus, for the most part, Wittgenstein took it that there are several sentences, which do not yield propositions, both paradoxes such as '"heterological" is heterological' ([1964] p.178; cf. also [1964] p.102) and the Liar sentence ([1964] pp.130-31) and quasi-paradoxes such as analogues of the 'Gödel proposition' ([1964] pp.176-7). Such sentences have 'the form of a proposition', 'a propositional-pattern', but they do not express propositions and Wittgenstein often says that they lack, or don't make, sense ([1964] p.177, and also pp. 117-8). Thus Wittgenstein wants to explain away paradoxes like that of heterologicality to give only a different, and benign, sense to 'The contradiction is true', in place of the obvious one[141]: namely, that a certain proposition is a contradiction and that that proposition is true. According to Wittgenstein what the expression means is: 'this really is a contradiction, and so you cannot use the word "'h'" as an argument in '$\xi \in$ h'' '"'h'" is one of these words which do not yield a proposition when inserted into '$\xi \in$ h'' ([1964], p.178). Indeed only 3 pages after he offers an option as to what to say, Wittgenstein asserts that 'in a sense [p & ~p] is bosh' ([1976], p.213).

A double inconsistency emerges in Wittgenstein's later position (which makes it a fit object for paraconsistent investigation). Firstly, if paradox-generating sentences do not make sense, then, since on the face of it they do make sense, some sort of meaning analysis is called for, at least to explain why appearances are misleading. But this contradicts the claim that no meaning analysis is required. (It does not conflict with the assertion that type theory is not needed, for this only offers a rather special meaning analysis.) In fact the theme that certain contradictions[142] do not express propositions, and so despite appearances are neither true nor false, receives little of the explanation or support it obviously requires in Wittgenstein's work. Secondly, Wittgenstein is simply inconsistent as to whether contradictions such as the Liar and Russell paradox do make sense. For example, as well as saying that they don't make sense, or at least allowing that one can say this, he also says: 'There is one mistake to avoid: one thinks that a contradiction must be senseless: that is to say, if e.g. we use the signs 'p', '~', '.' consistently, then 'p.~p' cannot say anything'. ([1964], p.171). Similarly he both allows that language-games can contain contradictions and says that 'a language-game can lose its sense through a contradiction, can lose the character of a language-game' ([1964], p.103).

There are then competing inconsistent strands in Wittgenstein's later work. And Wittgenstein has not really decided which option to take, though insofar as he usually tends to fall back on a nonsignificance approach, to the paradoxes at least, his later position is definitely not a paraconsistent one. It is worth pursuing however the strand that is beginning to emerge in Wittgenstein's later work which does have much in common with a paraconsistent approach. According to this strand, contradictions, including paradoxes, do, as we have seen, make sense; they are, or express, propositions. They can occur in and do not destroy language-games. They need not get one into any trouble ([1976], p.212). But in the on-going dialogue Wittgenstein is soon is forced, by Turing's example of a bridge falling owing to a calculation in an inconsistent logic, to say, inconsistently with the no-trouble theme, that a contradiction may lead into trouble but that it is not more likely to do so than anything else ([1976], p.219).

Wittgenstein gives no examples of paraconsistent logics or calculi. He seems to think that no such examples are needed: 'our task is, not to discover calculi, but to describe the present situation' ([1964], p.104). While this is undoubtedly part of the business, our task us by no means confined to this. Moreover his remark leads to conflict with his insistence elsewhere on an adequate diet of examples, including revealing games. The examples Wittgenstein does select of inconsistent calculi are problematic - as paraconsistent theories are not - because they are trivial. An example he frequently considers is 'Frege's calculus, contradiction and all. But the contradiction is not presented as a disease' (e.g. [1964], p.104). The trouble with calculating with this calculus is that it will lead to anything at all. It would lead to calculations under which, if applied, bridges would collapse, since coefficients could be arbitrarily, and so unsatisfactorily, determined. There is no control, unless implicit restrictions on what is done with it are somewhere or somehow imposed. Wittgenstein appears in fact to be operating with the idea that implicit restrictions are in force and that though contradictions can trivialise ([1976], p.224) they do not trivialise Frege's inconsistent calculus. There is considerable evidence for these claims. Firstly, it isn't true that with Frege's logic 'people went through doors into places from which they could go any damn where ... if they did this Frege's logic would be no good, would provide no guide.

But it does provide a guide. People don't get into these troubles' ([1976], p.228). The reason is that people like Frege are controlled by 'normal rules of logic'; Frege was 'led also by our normal use of words' and so stayed out of trouble. Unfortunately the "normal rules" suggested resemble those of type theory, which is considerably removed from ordinary usage (see Goddard and Routley [1973]); and worse, the rules would delete the contradiction which is supposed, in some sense, to hold, though it remains pretty inaccessible. Secondly, Wittgenstein takes it that Frege's calculus is a usable calculus, but 'if we allow contradictions in such a way that anything follows, then we would no longer get a calculus, or we'd get a useless thing resembling a calculus' ([1976], p.243, cf. also p.228). Thirdly, he says (what is false) that 'no one draws conclusions from the Liar' ([1964], p.170; cf. [1976], p.213). He considers the situation where a contradiction, like Russell's, has been found but we are 'not excited about it and had settled e.g. that no conclusions were to be drawn from it' ([1964], p.170). Such a stance may be alright, depending on how the restriction is applied. Mostly the restrictions Wittgenstein suggests are quite inadequate to make a calculus workable. For instance, drawing conclusions from certain steps that lead to contradictions has also to be excluded to avoid triviality; for explicit contradictions can normally be bypassed. Witggenstein is made keenly aware of this problem by Turing, who pointed out that the rule not to 'draw any conclusions from a contradiction' is not 'enough. For ... one could get around it and get any conclusion which one liked without actually going through the contradiction' ([1976], p.220). Subsequently Wittgenstein was worried by the problem of 'how to avoid going through the contradiction unawares' ([1976], p.227); but he offered no solution to it.

Much the same set of points applies to the other main example in Wittgenstein's meagre diet, division by (n-n) perhaps best developed at [1964], p.168-9). It too trivialises unless classical operations are restricted, unless it observed that they are only valid for a given region. So it is also with naive set theory and inconsistent arithmetic: 'if a contradiction were now actually found in arithmetic - that would prove that an arithmetic with such a contradiction in it could render very good service' ([1964], p.181). But what would want modification is not, as Wittgenstein suggests 'our concept of the certainty required',[143] but the operations that could be applied in the vicinity of the contradiction if useability is not to be sacrificed; that is, the application of classical logic must be restricted.

Yet Wittgenstein, though he strictly formulated no paraconsistent logics as calculi (and apparently lacked any clear appreciation of nonclassical logic, except for, what is equally unsuitable paraconsistently, intuitionistic theory), made considerable allowance for 'investigation of calculi containing contradictions' and predicted a time when 'people will actually be proud of having emancipated themselves even from consistency' ([1975], p.332; cf. also [1964], p.312, 376). He outlines how a different attitude to contradictions could occur, where people want to produce a contradiction, a lot of people try, and at least one person succeeds ([1964], p.105). Wittgenstein does not himself succeed in producing a 'plausible purpose' for this behaviour: one (metaphysical) purpose would be to show, not as Wittgenstein suggests that 'everything in this world is uncertain', but that the world is inconsistent. Such people will 'be glad to lead their lives in the neighbourhood of a contradiction' ([1964] p.105), so to speak. He rightly rejects the ideas that a contradiction automatically destroys a calculus ([1964], p.170) or a livelihood; for contradictions can be 'sealed off' (cf. [1964], p.104), and so allowed to stand (e.g. [1964], p.168).

While his own efforts at furnishing interesting inconsistent calculi are more suggestive than satisfactory, he does include, and appreciate, some of the basic requirements for paraconsistent theories, e.g. as we have seen, rejection of the spread law $A \& \sim A \vdash B$ ([1976] p.209). But what he proposes instead is the ineffective and inadequate rule that no conclusions be drawn from a contradiction. The The considerations he repeatedly adduces in favour of such a rule are
(i) that 'there is always time to deal with a contradiction when we get to it' ([1976], p.209, [1976], p.210, [1964], p.105, [1975], pp.345-6) - as if calculi were always dynamic systems when often they are static - and
(ii) 'when we get to it shouldn't we simply say, "This is no use - and we won't draw any conclusions from it"?' ([1976], p.209), but it can stand (e.g. [1975], pp.345-6). In like vein Wittgenstein wants to say 'something like, "Is it usefulness you are out for in your calculus'? In that case you do not get any contradiction. And if you aren't out for usefulness - then it doesn't matter if you do get one"' ([1964], p.104). The approach through usefulness is wrong, in several respects. Plainly usefulness is not necessary for consistency, since many useless games are consistent. Nor is usefulness a guarantee of consistency. An inconsistent theory may be of considerable use, even in describing the world, as the infinitesimal calculus certainly seems to have been, and quantum theory (which is likely inconsistent) is. Similarly such theories may be used in making predictions, even if the explicit contradictions in them are not (cf. [1964], p.52). Furthermore, it may matter if a contradiction is encountered, even where the objective is not usefulness: there are other objectives, such as (substantial) nontriviality, elegance, etc., that an inconsistent calculus may fail. Use, usefulness, useability are not the uniquely important tests Wittgenstein (cf. [1964], p.105) and the pragmatists consider them to be.

Nor can contradictions simply be dealt with as they arise. By the time a paradox is found it may be too late, as with Frege's logic; the cancer may be beyond treatment. Characteristically contradictions, like cancers, do spread, even if their spread can often be, nonclassically, contained. Wittgenstein's own proposal for a containment, stopping, is inadequate as we have seen: so is the basis for this proposal. He is obliged to say that paradoxes like the Liar are useless and lack an application, that 'Russell's "~f(f)" lacks above all application, and hence meaning' ([1964] p.166). For if such contradictions were technically useful we should want to do things with them, especially to draw out consequences. And we do. In fact Russell's paradox may have extremely important applications, e.g. if Arruda's conjecture is correct, in showing within set theory without special axioms of infinity that some numbers are inaccessible. Similarly semantical paradoxes and their analogues have important roles in establishing features of theories, e.g. of semantically closed arithmetic, in determining the viability of solutions, e.g. to the paradoxes themselves, and in obtaining limitative results, as to what can be proved in systems, what known, etc.

Although Wittgenstein is certain that the paradoxes are useless, he is quite uncertain, as remarked, as to what to say about their semantical status. He shrank from admitting that they could be true, and thus from a strongly paraconsistent position. He oscillated between sometimes allowing Russell's paradox a propositional role, more often rejecting it as not a proposition (e.g. [1964], p.166), and occasionally embarking on the hopeless task of trying to find it an intermediate niche. Thus for instance, he considered how Russell's contradiction 'could be conceived as something supra-propositional, something that towers above propositions and looks in both directions like a Janus head' ([1964], p.131). The unnecessary vacillation is in large measure because he failed to see that paradoxes could be sealed off logically - indeed in many ways - without use of devices like type theory which undermine their propositional status. If he had, then he could easily have adopted what in his later work he reaches for but never attains in a stable way, a weakly paraconsistent position, which allows for a variety of nontrivial theories (or language-games) distinct from the true theory (the true-false "game").

In this way he would have realised, in fair part, his stated aim of altering attitudes to contradiction and inconsistency, at least as regards the following:

(a) A calculus with a contradiction in it is in some way essentially defective.

(b) When a contradiction comes to light, some sort of remedial action is rationally demanded of us; we cannot coherently just let the thing be.

(c) There is such a thing as the correct logic, or set theory; and the paradoxes show that we have not found it. The problem of "solving" the paradoxes is a determinate one; it is that of finding the mistakes in the assumptions which lead to them.

(d) For any particular branch of mathematics, it is desirable that it be set up in such a way that contradictions can be avoided mechanically; that is, so that a slavish, unintelligent and totally aimless application of the rules of inference can never lead to any difficulty.

(e) Consistency-proofs are needed - or at least desirable. A system for which such a proof is missing, or unobtainable, is somehow insecure. 'Only the proof of consistency shows me that I can rely on the calculus' [[1964]p.107.]

(f) A hidden contradiction is just as bad as a revealed one. A system containing such a contradiction is totally spoiled by it. The contradiction is, as it were, a pervasive, general sickness of the system (Wright [1980] pp.296-7).

It has been claimed that conventionalism renders Wittgenstein's otherwise rather intractable position on contradiction coherent. For then mathematics and logic become games or like games, the rules of which are conventionally chosen, and games with inconsistent rules can still be interesting to play.[144] In terms of [the game] analogy, Wittgenstein's questioning of

> some of the ordinary attitudes to contradictions which we listed are extremely easy to understand. There is, for example, no reason why a game with a contradiction, or some other flaw, in the rules must be regarded as essentially defective; nor is there any reason to insist that if the defect comes to light, some sort of remedial action is demanded of us (Wright, [1980] pp.299-300).

But firstly, this sort of conventionalism (though it blends smoothly enough with paraconsistent positions) is neither necessary nor sufficient. It is not necessary because reconstruction of Wittgenstein's positon as weakly paraconsistent will do as well. And it is not sufficient because this conventionalistic construal does not really avoid, or settle, the vexed issue of the semantical status of paradox-generating statements. Secondly, as Goldstein[145] argues, (1) mathematics and logic are not simply games - similarly Wright (p.303) '... the assimilation of mathematics to a game ...seems a travesty' - and (2) Wittgenstein did not believe them to be such ([1976] pp.142-3; [1964] p.163; [1974] pp.289-95). As to (1), there are familiar applications of mathematics in engineering, architecture, areodynamics, etc.: the rules applied in the design of a bridge or a space shuttle are not a matter of conventional choice. 'Logic too, insofar as it is a codification of valid inference, cannot sustain a free choice of rules, for the rules we adopt must faithfully reflect our inferential practices' (Goldstein p.2). Wittgenstein is said to assume the same (in [1978], p. 257, p. 397, and pp. 303-353): these practices,[146] which involve the making of inferences which are truth-preserving, belong to what Wittgenstein calls the 'true-false game' (in McGuiness [1979], p.124).

Simple conventionalism alone will not render Wittgenstein's position coherent. What does help is Wittgenstein's larger picture of logic and mathematics as comprising (like a city of such suburbs) very many different sorts of games or calculi. In some of these, may bear only a family resemblance to the true-false game, contradictions are not forbidden. This type of many logics or many worlds view in no way requires conventionalism however. Such a view has been incorporated in relevant (weakly paraconsistent) and paraconsistent positions. Such a view, open, with but little refinement of his position, to Wittgenstein, is moreover correct, inasmuch as there are very many different, and different sorts of, logics and "worlds"[147], including inconsistent ones.

A dialethic position supports most of the themes (quoted above) with respect to which Wittgenstein wants to change attitudes, again without appeal to conventionalism or the connected assimilation of mathematics to games. Commentators on Wittgenstein have claimed however that if the assimilation is not made, if mathematics does for example genuinely describe structures, then our "ordinary" 'attitudes to contradictions' are soundly based; thus, for example Wright ([1980] p.298) whom we shall take as our target. The problem with respect to inconsistent systems is, Wright alleges, twofold, as regards (1) their applicability and (2) their truth. As to (1), 'if a system is inconsistent, then the inferences permitted within it will not in general be truth-preserving when applied to contingent contexts' (p.298). This is subsequently transformed (e.g. p.303, p.310) into the theme that inconsistent systems permit the derivation of false conclusions from true premisses. The application of relevant logics to inconsistent theories shows that this theme is mistaken. As the semantical analysis of relevant logics (developed, e.g. in RLR) reveals, implication, and the grounds for derivation, remain truth-preserving even in inconsistent situations and theories.

As to (2), 'if we had thought of it as a systematic description of some abstract conceptual structure, then again, not all of its theorems can be regarded as correct descriptions of the intended structure' (p.298). In fact this need not concern truth; for the structures correctly described may not be actual, but, e.g., purely hypothetical. But, in any case, it is again refuted by relevant theories; strong paraconsistency is not called for. It is enough that there are nontrivial inconsistent situations or structures, which inconsistent models will provide.[148]

Wright goes on to claim, in the ordinary way which takes no account of paraconsistency, that

> if ... the essential business of pure mathematical systems is to describe determinate conceptual structures and ... the notion of truth for ... theorems corresponds accordingly ... then it seems inescapable that contradiction is a total disaster, and demands remedy if there is to be any pure mathematics for the structure in question.

The work with relevant theories shows that that is not at all obvious and indeed simply assumes that even weaker paraconsistency is excluded. Why so?

> For not only does an inconsistent system not truly describe the intended structure; it does not truly describe anything at all (p.298).

Firstly, the way this is put confuses strong and weak paraconsistency. To see this in sharp focus, replace 'truly' (twice) by 'adequately' in the preceding quote. Then the resulting claim is false. An inconsistent relevant theory may adequately describe the intended structure; soundness, and even completeness, theorems may be forthcoming. It is simply a mistake, a common enough but serious mistake, that such theories or systems do not describe anything at all. What they do not describe is a consistent structure: but an inconsistent structure is not nothing. But, secondly, even if the term 'truly' is taken literally (in the way that italicisation of the second occurrence suggests), then the argument is not only incomplete, since it dogmatically suppresses the assumption that the real is inconsistent, but unsound since this assumption is false, as we will argue at greater length in the introduction to part four of this book.

Wright's contention 'that inconsistent systems are at best useless; that they can have no practical application' (p.303) falls with his earlier claims. For it depends again on the mistaken assumptions that the rules of such systems will not be truth preserving, and that real choice of right theorems among the theorems yielded in the system will be required. He does throw in, in effect, the further point that the theorems of an inconsistent theory cannot be true under interpretation in an empirical domain. Whether this is so turns on whether empiricalness implies consistency or not[149]. If it does not, as many dialecticians have thought, the point fails.

Wittgenstein is an isolated figure in the development of paraconsistent thought. Although his work had, inevitably, an historical context, especially in fact the matter of the logical paradoxes and their repercussions, contemporaneous developments in the paraconsistent enterprise, and earlier ones (except for German idealism), appeared to have little or no influence on his work. Nor did he have much immediate impact on the paraconsistent enterprise, which evolved largely independently of his work. Only recently has he become an almost-establishment figure, to appeal to - in a rather qualified way - on paraconsistent themes, or, as often, someone to make good, in previously written-off parts of his "theory", by applying paraconsistent results.

As we have seen Wittgenstein did not seriously attempt to specify what a formal logic suitable for inconsistent situations and theories would be like, this was left to his contemporary, Jaśkowski[150].

5. The Polish continuation: Jaśkowski. Jaśkowski introduces his fundamental [1948] paper by repeating some of the historical points Łukasiewicz had made, especially those concerning 'convincing reasoning which nevertheless yield[ed] two contradictory conclusions' (p.143). Jaśkowski goes on to mention the logico-semantical paradoxes, and the heavy price exacted by restrictions that (appear to) restore consistency. He also remarks on how the levels-of-language theory (as it is now called)

> is at variance with the natural striving synthetically to formulate all the truths we know in a single language, and thus renders a synthesis of our knowledge more difficult (p.144).

But that is all. He does not explicitly propose simple acceptance of the paradoxes as truths, established by sound arguments, and earlier indicates that theories admitting these paradoxes cannot now be considered as correct.

However subsequently he does consider representation of the Liar antinomy in the paraconsistent logic he arrives at, indicates that other paradoxes such as Russell's can be similarly treated, and remarks that ordinary procedures leading from inconsistency to triviality fail. He goes on to the important observation that the apparent breakdown of such proof procedures does not establish the nontriviality of inconsistent systems representing paradoxes such as the Liar. In fact he sketches in barest outline, probably for the first time for inconsistent theories, the problem of proof of nontriviality.

Jaśkowski also dismisses vagueness of terms, which 'can result in a contradiction of sentences' 'in every-day usage'; for with increased precision the inconsistency is removed. Finally Jaśkowski mentions what again he considers as a transient feature, inconsistencies in working hypotheses at given stages in the evolution of sciences such as physics. This too he pushes back, to the following:

> ..In some cases we have to do with a system of hypotheses which, if subjected to a too consistent analysis, would result in a contradiction between themselves or with a certain accepted law, but which we use in a way that is restricted so as not to yield a self-evident falsehood (p.144).

That is to put the matter almost as the less hostile among the enemies of paraconsistency might. It is not highly sympathetic to paraconsistency; it is at best a very weak paraconsistent position and definitely not a dialethic position: the possibilities have not been seen.

Nonetheless Jaśkowski formulates, again apparently for the first time, the problem of determining the class of paraconsistent logics, at the sentential level. He clearly distinguishes two classically-conflated properties of systems; being contradictory (i.e. having theses A and ~A which contradict one another) and being over-complete (i.e. trivial). He then presents the problem of the logic of contradictory systems...in the following manner: the task is to find a system of the sentential calculus which: 1) when applied to the contradictory systems would not always entail their overcompleteness, 2) would be rich enough to enable practical inference, 3) would have an intuitive justification. Obviously these conditions do not univocally determine the solutions since they may be satisfied in varying degrees, the satisfaction of condition 3) being rather difficult to appraise objectively (p.145).

Much the same applies to condition 2), which is one reason why these conditions are not included in the definition of paraconsistency; another is their technical intractability (as well as inexactitude). In his subsequent practice Jaśkowski entirely ignores requirements 2) and 3). So begins the formal investigation of contradictory, or paraconsistent, systems.

Among "solutions" Jaśkowski mentions minimal logic but concentrates upon what he calls discursive logics. Though minimal logic satisfies the letter of paraconsistency law, it violates the spirit, in virtue of the minimal thesis $A \rightarrow . \sim A \rightarrow \sim B$. It is not evident, and Jaśkowski makes no effort to show, that minimal logic meets requirements 2) and 3). In fact these requirements are forgotten in all that Jaśkowski goes on to; so in fact Jaśkowski is concerned with paraconsistent logics as a whole. He does not, however, get far with their classification. Many-valued logics are tentatively set aside as not providing solutions. But, as it has turned out, it is only certain many-valued logics (in particular functionally complete ones) that do not afford solutions. Finite-valued relevant logics, for instance, do.[151] Jaśkowski really obtains one rather limited class of solutions to his problem - discursive logics - from among the many rich types there are, and almost all attention is rivetted on a single system D2 obtained by translation from modal system S5.

However the underlying ideas in reaching D2 are both interesting and more general, and link discursive logics not only to the long tradition of philosophical pluralism,[152] but also to other classes of logics very recently discerned, e.g. the nonmonotonic logics of interest to computer scientists.[153] Discursive logic is intended as a formalisation of a logic of discourse,[154] where different participants, e.g. in discourse, advance theses or pool opinions, all these being included as assertions in a single system. Jaśkowski's definition is vague: 'Let such a system which cannot be said to include theses that express opinions in agreement with one another, be termed a discursive system'. What Jaśkowski goes on to say helps, however, to clarify matters:

To bring out the nature of the theses in such a system it would be proper to precede each thesis by the reservation: "in accordance with the opinion of one of the participants in the discourse" or "for a certain admissible meaning of the terms used". Hence the joining of a thesis to a discursive system has a different intuitive meaning than has assertion in an ordinary system. Discursive assertion includes an implicit reservation of the kind specified above, which

Jaśkowski adds in a remarkable, crucial, and unjustified, slide

- out of the functions so far introduced in this paper - has its equivalent in possibility Pos. Accordingly, if a thesis A is recorded in a discursive system, its intuitive sense ought to be interpreted as if it were preceded by the symbol Pos, that is the sense: "it is possible that" (p.147).

Jaśkowski's introduction supplies one basic component of discursive logic; or, put differently, a first requirement that a logic L, with an &-∨-~ sublogic and containing a functor $\Diamond$, which can be read "someone maintains that", yields a discursive logic DL, namely

DL1. A is a thesis of DL iff $\Diamond A$ is a thesis of underlying logic L.[155]

A logic DL so yielded, may lack fundamental logical operations, such as, to focus on the examples Jaśkowski takes as decisive, implication and equivalence functors. Hence, for example, the second requirement

DL2. There is a definable in L a functor $\rightarrow_D$, of discursive implication, which demonstrably satisfies the requirements in DL for being an implication; in particular, it is closed under the Modus Ponens rule, i.e. $A, A \rightarrow_D B \twoheadrightarrow B$, and also maybe has theses such as $A \rightarrow_D A$.

The third requirement, for equivalence, in place of implication, is similar:

DL3. There is a definable in L a further functor $\leftrightarrow_D$, of discursive equivalence, which demonstrably satisfies in DL at least the following conditions: $A, A \leftrightarrow_D B \twoheadrightarrow B$ and $B, A \leftrightarrow_D B \twoheadrightarrow A$.

DL is characterised as the logic with primitive connective set $\{\&, \vee, \sim, \rightarrow_D, \leftrightarrow_D\}$ i.e. where A and B are wff so are $(A \& B)$, $(A \vee B)$, $(A \rightarrow_D B)$, $(A \leftrightarrow_D B)$, etc. DL tells us which wff are theses.

There are modal logics L whose own implication and equivalence functors satisfy requirements DL2 and DL3 and which accordingly furnish discursive logics, as so far defined, through DL. One such logic is Łukasiewicz's Ł-modal system (of [1951]), for which it is provable, for instance, that $\Diamond(A \supset B) \& \Diamond A \supset . \Diamond B$, so guaranteeing Modus Ponens in DŁ. However DL does not satisfy the original motivating considerations in terms of which Jaśkowski introduced discursive systems in the first place. For DŁ is not paraconsistent. It will trivialise inconsistent additions, since it contains the spread law $\Diamond A \supset (\Diamond\sim A \supset \Diamond B)$.

A final requirement on a Jaśkowskian discursive logic is then

DL4. DL is paraconsistent.

Thus DŁ is not a Jaśkowskian discursive logic. In fact Jaśkowski does not strictly show that his main and only serious candidate system D2 is paraconsistent though he clearly assumes that it is, where he explains (p.153) that even if material implication is added to the system the rule γ of Material Detachment is not valid.[156]

Where L is a modal logic, the Jaskowskian discursive logic DL based on, or associated with, L is determined through the following definitions of $\rightarrow_D$ and $\leftrightarrow_D$:

Defn. 1. $A \rightarrow_D B =_{Df} \Diamond A \supset B$; and

Defn. 2. $A \leftrightarrow_D B =_{Df} (\Diamond A \supset B) \& (\Diamond B \supset \Diamond A)$.

Where L is system S5, DL is Jaśkowski's system D2. These definitions are not the only ones that serve to determine Jaśkowskian discursive logics; Jaśkowski mentions, but sets aside, another definition of $\rightarrow_D$,[157] and it is easy to see that there are alternatives to the asymmetrical definition of $\leftrightarrow_D$.

Apart from being the first explicitly paraconsistent logic, there is another important trend D2 helped to set, namely the rejection of Adjunction. Jaśkowski puts it in terms of rejection of the wff $A \rightarrow_D . \ B \rightarrow_D . \ A \& B$, but the rejection, and the ground for it, is more far-reaching, and tells against the Adjunction Rule: $A, B \twoheadrightarrow A \& B$:

> ... from the fact that a thesis A and a thesis B have been advanced in a discourse it does not follow that the thesis A & B has been advanced, because it may happen that A and B have been advanced by different persons. And from the formal point of view, from the fact that A is possible and B is possible it does not follow that A and B are possible simultaneously (p.154)

Though the rejection of Adjunction leaves some strange gaps, (e.g. the rejection of equivalence decomposition, $(A \leftrightarrow_D B) \rightarrow_D (A \rightarrow_D B) \ \& \ (B \rightarrow_D A))$, it is essential to Jaśkowski's discursive logics. For it requires only a fairly minimal modal logic L to prove in DL both LNC, $\sim(A \& \sim A)$, and Conjunctive Spread, $A \& \sim A \rightarrow_D B$. In virtue of LNC, discursive logics violate da Costa's conditions upon inconsistent logics, which are widely (but erroneously) insisted upon, namely that an inconsistent logic must reject LNC. But the real trouble with DL lies not with LNC but with the irrelevant Spread which forces the abandonment of Adjunction. Were Adjunction to hold, the sequence, $A, \sim A \twoheadrightarrow A \& \sim A$

$\twoheadrightarrow B$

would show triviality of any contradictory extension of DL.

Not only then are Jaśkowski's discursive logics not Adjunctive; further, they are irrelevant, because, for example, of Conjunctive Spread. And these vices are connected. Given a choice of rejecting one or other of Adjunction and Conjunctive Spread to avoid paradoxes and catastrophic spread from an inconsistency, the rejection of Adjunction is the wrong choice.[158] For Adjunction itself spreads nothing, but merely assembles, conjoins, data already supplied. That is also why its rejection is unwarranted for leading intended applications. Consider, for instance, a paradox such as Russell's or the Liar, and let p_o be the paradox-producing statement (e.g. $R \in R$). Then by the paradox arguments, and presumably in discursive logic both p_o and $\sim p_o$. In English we can adjoin them (likewise in the metatheory we can conjoin them) - and reasonably enough, since both hold, both are intended to be true - but discursively we are not permitted to go on to p_o & $\sim p_o$. It is not, or not just, that & has departed from its normal interpretation as a conjunction, from meaning and,[159] but that discursive logic is not the right setting for capturing the way we do reason concerning the paradoxes. For what we say given p_o and also $\sim p_o$ is: Yes, p_o & $\sim p_o$, but that is not a ground for going on to anything at all.

But doesn't the discursive interpretation compel the rejection of Adjunction? If it did, that would show that discursive logics offer an unsatisfactory approach to some of the problems they have been presented as handling, e.g. reasoning concerning paradoxes, and hence that discursive logics are only a quite proper class of paraconsistent logics. But in fact one can go either way, in discursive logic itself, on whether Adjunction holds. Even when DL is (inadequately) translated from L through a modal possibility functor, matters could go either way. For in system DŁ, Adjunction holds (and $A \rightarrow'_D . B \rightarrow'_D A \& B$ is provable). Indeed Łukasiewicz is quite adamant that the ordinarily rejected principle $\Diamond A \& \Diamond B \rightarrow. \Diamond(A \& B)$ is nonetheless correct, and devotes some space, in his defence of the Ł modal system, to rebutting the usual counterexamples to it.[160] Appeal to the intended interpretation of the translation functor - which is not really $\Diamond$[161] - can also support Adjunction instead of telling against it. It depends on whether the pair A, B conjoined in Adjunction are seen as separately supplied and requiring joint validation in a single underlying framework, e.g. under one of the systems, in the opinion of one of the participants - in which case Adjunction does fail - or, though supplied by different frameworks do not require joint validation in some one framework. Put differently, under the second construal, closure under certain logical operations is built in, in particular closure under Adjunction. In fact Jaśkowski builds in closure under Modus Ponens - even though opinions are commonly not closed under entailment, so there is a (first) perspective where Modus Ponens fails - but does not notice that the same thing can be done for Adjunction, that there is a second perspective where Adjunction holds.

Though the generation of discursive logics from modal logics is a clever and elegant formal idea, the intended interpretatiom does not sustain the system Jaśkowski studies or anything much like it. For logical possibility as encapsulated in S5 does not bear a good logical resemblance to the discursive operator. For instance, it is extremely doubtful that any discursive functor, C, conforms to much in the way of modal reduction theses, certainly not those S5 and S4 supply, such as

$C{\sim}CA \equiv {\sim}CA$ and $CCA \equiv CA$ respectively. Really, discursive logic presupposes a much weaker underlying logic than Jaśkowski allows. Moreover the interpretation of C has itself to be treated with more care than it has mostly received in discursive logics. If C merely reflects what a participant in discourse or a discussion asserts or what his or her opinion is, then C will not distribute in the requisite way over entailment. For a person does not assert all that his assertions entail. Thus C needs to be interpreted in terms of commitment or some similar notion which is initially closed under entailment. Thus C can be read, for instance 'Someone (in the discussion) is committed to (the statement that)' or 'Someone (participating in the discourse) is (logically) obliged to maintain that'. With C so interpreted, at least the S2° (and S0.5°) principle $A \rightarrow B \rightarrow CA \rightarrow CB$ is correct, and possibly the stronger S3° principle $A \rightarrow B \rightarrow. \ CA \rightarrow CB$. But the principle converting S2° and S3° respectively to S2 and S3 does not seem to be correct, namely $A \rightarrow CA$. For there may be truths to which no one in the group is committed. Nor is the fix which Jaśkowski's strategy may suggest, namely $C(A \rightarrow CA)$, adequate, since no participant may be committed to it either. In any case $A \supset \Diamond A$ plays a crucial role in the straightforward proof of the principle of DL, $\Diamond[(\Diamond p \supset q) \supset. \ \Diamond p \supset \Diamond q]$[162] (and also in the alternative principle, $\Diamond[(\Diamond p \supset \Diamond q) \supset. \ \Diamond p \supset \Diamond q])$, that justifies Modus Ponens in DL. In view of its role, it might be thought worth a good deal of trouble to retain $A \rightarrow CA$, for example by the dubious ploy of requiring that every discussion includes an ideal participant committed to the truth. Such an ideal participant is also wanted on other grounds if a modal-type logic is to remain basic. Otherwise even the principle, CT where T is a tautology, may fail, since no participant may be committed to such principles as $A \vee {\sim}A$, for instance. The ideal participant can guarantee not only this principle but, given its commitment to the truth, the much stronger principle of "Necessitation", CA where A is a theorem, should that be required. Likewise, the ideal participant can (re)instate Adjunction, e.g. by underwriting $CA \supset. \ CB \supset C(A \ \& \ B)$. This principle is of course not demonstrable in usual modal logics, though it holds in the $\mathcal{L}$ modal logic. But it can be added to weaker modal logics, and at a cost appropriately modelled.[163]

It is evident that C can also easily be given alternative renditions, e.g. as 'It is rationally believed that'.[164] It is in this way, in particular, that discursive logics can be linked to other much more recent logical developments such as doxastic logics and nonmonotonic logics. But this was not the direction in which discursive logics first led or the way the contemporary history of paraconsistent logic went. Rather Jaśkowski's seminal work on discursive logic as a paraconsistent logic served to bolster elaborations of paraconsistent logics in South America.

6. The Latin American development: da Costa's theories. The remarkable growth of paraconsistent logic and theory in South America, though it had roots in European thought, apparently began independently of movements in Poland and elsewhere. Only

when the movement was already initiated were the more elaborate historical connections discovered (primarily by Arruda and da Costa).

In fact Asenjo's thesis of 1954[165] marks the beginning of paraconsistent logic in Latin America, though the thesis (which was not accessible) appears to have had little impact in South America, or elsewhere. The technical core of the thesis, which was a matrix calculus of antinomies, was published only much later (in Asenjo [1966]), but the interesting philosophical motivation of the thesis was omitted.[166] What Asenjo's nicely motivated calculus of antinomies comes to (though Asenjo did not notice this) was almost what the Chinese logician Moh also proposed in [1954], a reinterpretation of Łukasiewicz's $Ł_3$ matrices with the third value as paradoxical or antinomic (see above). In short, the proposal is that of Bochvar but with superior matrices. Both Asenjo and Moh saw the third value as applying to paradoxical statements, and as being assigned to wff which are both true and false (or perhaps true iff false in Moh's case). But Asenjo's logic should differ from $Ł_3$, since he evidently intended (though he nowhere says) that the third value should be designated along with truth. That logic is not however derived;[167] the most that is offered in Asenjo [1966] is the claim that da Costa's systems C_ω is sound with respect to the matrices; it is certainly far from complete, as is evident from Asenjo and Tamburino ([1975], p.21) where the logic is finally axiomatised.[168] In particular it contains, as well as an infinite sequence of explicit contradictions of the form $B_i \mathbin{\&} {\sim}B_i$, theses incompatible with motivational arguments of the earlier work appealed to, notably LNC, ${\sim}(A \mathbin{\&} {\sim}A)$.

With da Costa's work we arrive at something strikingly different from what had gone before, deliberately fashioned paraconsistent logical systems - not overtly matrix logics or translations of modal logics - designed to retain systemic strength, and throw out merely what is paraconsistently defective in classical logic.[169] What was (erroneously) thought to be mistaken was (as in Asenjo [1966]) the Law of Non Contradiction in particular, but also Reductio - so that the paraconsistent objective could be achieved simply by removal of the reduction scheme $A \to B \to. A \to {\sim}B \to. {\sim}A$ (responsible also for LNC) from Kleene's axiomatisation of classical sentential logic. Simple and brilliant. So resulted system C_ω, a strong natural system for paraconsistent purposes, one might almost say. But the system, which amounted to Hilbert's positive logic (i.e. the positive part of intuitionism) supplemented by the negation axioms, ${\sim}{\sim}A \to A$ and $A \vee {\sim}A$, was evidently exceedingly weak in its "negation" part, and admitted of considerable further supplementation. Hence the C_n systems with $n \geqslant 1$ obtained by adding a curious sequence of negation postulates. Worse, however, the motivation rested on a mistake, that (as in Asenjo) what had to be got rid of were LNC and also Reductio. This assumption runs right through da Costa's pioneering work.

Although da Costa's initial work on paraconsistent logic was, like Asenjo's, in the form of a thesis [1963] (again not generally accessible), it was almost immediately spun out in a series of papers, introducing in quick succession sentential logic for paraconsistent systems (in 1963), then - what appears again to have been entirely new - predicate logics, and predicate logics with equality for such systems, theories of descriptions for such systems and a set theory based on such a system (all in 1964).[170] Much of this development involved collaborative work. For shortly after the notes reporting on parts of his thesis, da Costa began what proved to be a long and fruitful collaboration with Arruda on paraconsistent logics and theories. Da Costa's work, and that with Arruda, undoubtedly represents the fullest early flowering of paraconsistent logic.

There are three basic groups of systems on which da Costa, Arruda, and their Brasilian school have built a wealth of superstructures (primarily quantification and set theories), namely the C systems, devised by da Costa, the P systems, due to da Costa and Arruda, and later certain Jaśkowski systems varying and generalising Jaśkowski, and jointly investigated with Polish co-workers.[171]

The C systems, the best known and most fully investigated of these systems, constitute da Costa's main solution to the problem of constructing formal inconsistent systems at the sentential level. (These systems, $C_1,\ldots,C_n,\ldots,C_\omega$ are described in the next part.) Already in 1963 important conditions of adequacy were imposed on solutions of this problem. Such calculi should - to paraphrase da Costa - contain the most important theorems and rules of deduction of classical sentential logic, while satisfying also the following conditions. I. In these calculi the principle of contradiction should not be generally valid; II. From two contradictory statements it should not be possible in general to deduce any statement whatever; III. The extension of these calculi to quantificational calculi should be immediate.[172] These conditions, which are reminiscent of Jaśkowski's requirements,[173] persist, with minor variations, in da Costa's later work.

Thus in his important survey paper of [1974], 'On the theory of inconsistent formal systems', conditions I-III reappear in virtually the same form, except that III is (illicitly) specialised to the C_n systems $(1 \leq n \leq \omega)$, and the lead-in is taken up in a further condition, IV, again specialised to C_n systems, but in appropriately more general form amounting to the following: IV. These calculi should contain (for the most part) the schemata and rules of classical sentential logic so far as these do not interfere with the earlier conditions. This condition, in particular, is, as da Costa remarks, 'vague'. Furthermore it is doubtful that many solutions to the general problem of design of inconsistent formal systems will satisfy it, only certain classically-maximal solutions will (and perhaps no C systems are among these). Nor is the condition desirable, unless classical logic is, as it were, correct apart from very minor deviations. But classical logic is no ideal and we should aim to keep only what is correct in it.

Of the remaining conditions, II is basic to the very characterisation of paraconsistent logic; quantificational and other extensions of a suitable sort, as under III, are evidently desirable (though it is not necessary that they be immediately obtained, and there may well be debate, as with relevant logics, over the correctness or adequacy of certain postulates); but condition I is a mistake. It is certainly true that significant paraconsistent logics can be designed in which the principle of contradiction is generally valid, main relevant logics being of this sort. But, further, insistence upon condition I is, so we later argue, a hang-over from classical consistency assumptions, which is squarely among the things paraconsistent logics are concerned with removing.

Even if condition I were conceded (and also for that matter II), a variety of systems other than the C systems can satisfy it and the other conditions: extensions of the relevant system R-W provide just one class of examples.[174] Nor (we argue in the Introduction to Part II) are the C systems a good choice. However they remain an important early choice, and are perhaps the best studied among properly paraconsistent logics.

Upon the C systems, in particular, a wealth of supersystems have been built, quantificational theories (the C* systems), description theories (the D systems) and various set theories paraconsistently varying classical set theories (e.g. the NF_n systems, the ZF_n systems, etc.).[175] The C systems do not however lend themselves to nontrivial extension by expected set theoretic axioms (e.g. relatively unimpeded comprehension) without a batch of rather ad hoc restrictions resembling those tacked onto classical set theories.[176] With C systems, that is, the liberation to be expected by going paraconsistent is not achieved.

The problem was recognised early on by Arruda and da Costa themselves. They remark that, as Moh Shaw-Kwei had shown, suitable unrestricted comprehension (i.e. abstraction) principles cannot be obtained in systems which contain both the rule of Modus Ponens and Absorption (i.e. Contraction) principles of some order.[177] Accordingly they began to investigate systems which escaped the difficulty, systems lacking Modus Ponens (namely the J systems already referenced) and the P systems, which are very weak relevant systems which lack all absorption principles.[178] The class of P systems has since been much extended. The systems have been shown to be of much interest for other purposes, and it has been proved that they can indeed provide the basis for nontrivial (if very weak) dialethic set theories with entirely unrestricted comprehension principles.[179]

7. *The position of relevant logics and the contrasting attitudes of their proponents.* Although some of the earliest systems of relevant logic (e.g. the 1912 logic of Lewis - see RLR 5.1 - and the weak implicational logic of Church [1951]) were paraconsistent the historically more important systems Π' and Π'' of Ackermann [1956] were not. For they contained, as a primitive rule, the rule γ of Material Detachment (viz. $\sim A,\ A \vee B \rightarrow B$) from which the Spread Rule, $A, \sim A \rightarrow B$, followed. Hence the systems trivialise contradictory theories based upon them. For if a theory based on Π' contains A_o and $\sim A_o$ for some wff A_o, then by Spread, it contains every wff.

One of Anderson and Belnap's main achievements in reaxiomatising Π', to obtain the theoremwise-equivalent system E, was the removal of as a primitive rule.[180] But their motivation was far removed from anything to do with paraconsistency, and was based rather on a questionable normalisation principle,[181] to the effect that to every primitive rule should correspond an entailment thesis. But this would require the dreaded Disjunctive Syllogism, $A \ \&\ (\sim A \vee B) \rightarrow B$, corresponding to γ as an entailment thesis (on standard extensional normalisation of rules). Thus, given the normalisation requirement, γ had to be removed as a primitive rule.

The evidence is, on the contrary, that Anderson would not have been sympathetic, at the time he was concerned with recasting Ackermann's systems, to a paraconsistent position or paraconsistent grounds for changing the systems. For instance, Anderson (in [1958]) chastises Wittgenstein for, in effect, flirting with paraconsistency, for his '"so-what" attitude towards contradictions in mathematics' (alleged, p.488, to be difficult to reconcile 'with his own view of language-games'),[182] for his recommendation that 'we stop playing the consistency-game altogether' (p.489), for viewing 'as somehow perverse' 'the fact that avoidance of contradictions *is* held essential by mathematicians' (p.488), Anderson's own view being, evidently, that the avoidance of contradictions is essential (see especially p.489). Anderson's view is the mainstream classical view that contradiction renders a system or theory useless for intended or serious logical and mathematical purposes, even if for some bizarre purposes, such as aesthetic taste, contradictions may be turned out by a theory.

Nor does the work leading up to and embodied in Anderson and Belnap's monumental *Entailment* exhibit any softening toward paraconsistency. Instead a very considerable amount of effort was devoted to the recovery of the discarded rule γ, as an admissible rule of system E. In *Entailment* very little is in fact said about contradiction (it does not even rank a separate index listing).[183] But while the paraconsistent character of systems like R is recognised ('that an extension of R is negation inconsistent does not imply that it is Post inconsistent', i.e. trivial), still anything worth the name of "logic" that extends R is, when negation inconsistent, also trivial.[184] That R retains this power to properly reduce inconsistent "logics" to worthless triviality is seen as a virtue of R, a meritorious feature, not as the drawback it is.

Even now American proponents of relevant logic, such as Belnap and Dunn, are careful to insist that they are not proposing other than epistemic interpretations for their apparent assignments of joint truth and falsity to some statements in some situations (e.g. they can be told true and told false); they are not, they say, making the (absurd) suggestion that some statements are or may be literally both true and false (cf. Belnap [1977] and Dunn [1976]).

The Australian approach through relevant logic has been very different. It has been semantically oriented almost from the outset in the late 60s, and almost from the beginning it has made allowance for inconsistent and incomplete worlds. There has been just one major shift, which began in the early 70s, from a paraconsistent position towards a dialethic position, though not all Australian relevant logicians have made it.

8. The Australian movement. Initial Australian developments in the paraconsistency enterprise were sporadic and of a semantical cast. Goddard (in [1959]) described ingenious situations where the Law of Non Contradiction (LNC) failed, where for certain p, p & ~p held. The counterexamples to LNC proposed in fact bear a striking resemblance to those the dialecticians had used in showing that motion involved a contradiction (though Goddard made no such comparison); but whereas for the dialecticians the counterexamples held for the real world, for Goddard they held only for certain alternative worlds, strange discontinuous worlds remote from the real. What hold, however, in these inconsistent worlds are statements such as that (at the one time) a stone both 'is at B and is not at B' (p.38). Since we can imagine and to some extent describe (pp.38-9) a universe in which LNC fails, such "laws of thought" as LNC are not, so it is argued, laws of thought in the strong traditional sense; they hold conditionally upon certain requirements being satisfied, e.g. our universe having a certain continuous spatio-temporal structure (p.39). There are more direct corollaries for paraconsistency: since there are universes where contradictions hold which are not trivial, a comprehensive logical theory must be a paraconsistent one. Goddard did not have the terminology to state the corollary in this way, but the idea is implicit.

In the early 60s Mackie got the idea (earlier tried, in one way or another, by Bochvar, Moh, Asenjo and others) that as well as true and false statements there were paradoxical statements, namely certain statements generated by the (semantical) paradoxes. Though this might easily have issued in a three-valued logic with values T, F and P - and is sometimes taken to have[185] - it did not appear to;[186] and in fact Mackie remained a rather staunch devotee of classical two-valued logic.

Elsewhere in Australia however, notably in New England (at Armidale) many-valued and intensional logic approaches to the paradoxes, and to much else, were soon underway in the early and mid sixties. Routley, who was interested in significance and had been researching paradoxes from a statement-incapability and incompleteness (and content loop failure) aspect teamed with Goddard, now in Armidale, who had been working on paradoxes from a significance angle: and jointly and, also separately, they produced a range of logics based on 3-valued (the third value being nonsignificance or statement-incapability) and 4-valued (the fourth value being incompleteness interpretations). Several of the systems that resulted were paraconsistent, as was observed (though in more old fashioned terminology) in Goddard and Routley [1973], which summed up (many years later) some of the work of the New England school. There the possibility of paraconsistent logics was proved, by matrix procedures (p.285), and then several examples of dialethic logics were exhibited and discussed, including a paraconsistent connexive logic (see pp.291-2). The theory developed was taken to confirm Wittgenstein's themes that inconsistency need by no means destroy a calculus.

A second line of research at New England also lead in paraconsistent directions, namely the work on non-existent objects, including impossible objects, and quantificational theories that could include such objects in their domains. Reflection on the character of such objects resulted in the investigation of various Characterisation Principles, and belated discovery of the studies of Meinong and the Graz school on the theory of objects. Furthermore application of expected Characterisation Principles lead directly in turn to the need for paraconsistent logics.[187]

But it was a third and apparently (at the time) independent line of investigation which led to the distinctively Australian approach to paraconsistency through relevant logics. This was the logical, and especially semantical, study of paradox-free implication and conditionality brought to New England by Routley, who had been looking at the problems virtually since 1960. Some of the systems devised in New England, such as the I systems, though paraconsistent and paradox-free at the first degree, proved to be irrelevant at the higher degree.[188] Some of the systems, investigated jointly with Montgomery from New Zealand (with whom the rudiments of a general picture of implicational systems was being built up), have yet to be written up in the literature; many of the systems, such as the connexive systems studied, though lacking spread principles and sometimes paradox-free, were not really paraconsistent, but many were. Perhaps the most important paraconsistent directions in which the investigations tended were to the method of counterexamples to logical laws[189] and thence in the late sixties, by which time the New England group had dispersed again, to the semantical theory of inconsistent and incomplete worlds discerned by R. and V. Routley, in the first place for the first degree of logic of entailment.[190]

The semantical theory was extended in the early seventies by Routley and Meyer, to many sentential relevant logics,[191] which proved to be paraconsistent in an interesting semantical way, namely that the base worlds of the models initially arrived at, though generally complete, could well be inconsistent. Thus relevant logics could be straightforwardly and nontrivially rendered dialethic, an outcome which looked extremely promising in accounting logically for much philosophy in the dialectic tradition[192] as well as for paradoxes and their relatives. Soon after this work was initiated, the Australian and Brazilian groups discovered each others' work (through Makinson) and subsequently exchanges began.[193]

Closely connected with the logical paradoxes are such limitative results as Gödel's incompleteness theorems. In the early 70's these were investigated by Priest, who, being in Britain, was working in ignorance of paraconsistent research elsewhere. By 1972 he had come to the conclusion that a paraconsistent approach to problems in the area was required (see Priest [1974] chapter 4); the results of this investigation, which arrived at full-blooded dialetheism, were published in [1979]. This paper was read in Canberra in 1976 when Priest moved to Australia, and Priest and Routley found, to their mutual astonishment, that they had been working along related lines. Since then their work has tended to converge, with their joint work so far culminating in the present volume.

Appendix: note on recent activity elsewhere.

Apart from the main centres of research on paraconsistent theory in Australia, Latin America and Poland and more recently in Bulgaria,[194] investigation elsewhere has for the most part been by fairly isolated workers producing the occasional piece. And some of these pieces are marginal as regards belonging to or furthering the paraconsistent enterprise. This applies especially to work (allegedly) directed at clarifying or elaborating Hegel's logic or Marxist logic.

A worthwhile survey of recent activity on the paraconsistent front in Australia, Brasil, Poland, USA, Argentine, Belgium, Ecuador, Italy and Peru is given by Arruda.[195] There are however points that should be adjoined or amplified. In the first place, there is now a small group working on paraconsistent theories in Canada. Independently of S. Thomason [1974] (which, though it had some circulation, was never published except in abstracted form[196]), Jennings and Schotch sometimes in collaboration with others, have begun detailed direct investigation of Nonadjunctive logics. Secondly, also in the Nonadjunctive tradition emanating from Jaskowski, is Rescher and Brandom's recent text, The Logic of Inconsistency [1980].

Thirdly, while paraconsistent studies are now a major component of logical activity in places in South America and Australia and at some centres in Poland and Bulgaria, they occupy such a privileged position nowhere else. Indeed, on the contrary, they are elsewhere very much a minority activity and commonly regarded with varying grades of scepticism. Particularly in the UK and USA, strong and unquestioned consistency assumptions impede all but peripheral paraconsistent investigations.

FOOTNOTES

1. According to Rescher [1967] chapter 2.

2. Such as are given in the introduction to part II.

3. In the case of Plato and Aristotle, examples are given in the introduction to part IV. Some of these inconsistencies are minor and would require little repair. Some however are major.

   It is perhaps worth making a start on outlining the contrast of the Classical Greek Establishment with Outsiders or Foreigners. Generally, the lifetime members of the Establishment were born in the right place, were citizens, were political conservatives, were propertied, did not need to work or teach for a living. But the establishment had the usual more transient periphery of hangers-on; those who, while not meeting the criteria for direct membership, curried favour with lifetime members and accepted main Establishment values and furthered its cause. Plato and his family, and Isocrates, for example, directly qualified as Establishment members (cf. Davies [1971] on their wealth and holdings). Aristotle, however, occupied a peripheral position for the main part of his teaching life in Athens, a position he eventually lost. Plato and Aristotle, now seen as the intellectual giants of classical philosophy, in fact, represent rather the Classical Establishment - which strenuously opposed paraconsistent approaches. The Sophists, by contrast, were invariably outsiders, were not citizens, were not propertied, had to work for a living (and were looked down upon for so doing), and were not (ideologically or otherwise) part of the status quo or politically conservative. In other historical periods too there is a weak but significant correlation between political conservatism and the rejection of paraconsistency.

4. The one classical inconsistent theory, the trivial one, is ruled out as a theory. The connexive theme concerning propositions - not so bizarre classically if propositions represent theories - is expressed in Aristotle's law $\sim(A \rightarrow \sim A)$, i.e. $A \circ A$, every wff A is self-consistent. The theme concerning theories is derivative because theories can be seen as propositions or conjunctions of propositions.

. What Soviet logic does look like in formal detail is a much more sensitive matter: see the initial considerations in Routley and Meyer [1976].

. So Diogenes Laertius reports [1951] VIII, p.309; and there seems no good reason to doubt his reliability on this occasion.

. Thus, e.g., Protagoras' works by the authorities of Athens. But it was not always at establishment hands by any means. Our "barbaric ancestors" who sacked and destroyed libraries of the ancient world have much to answer for also.

. Consider, for example, the distorted view we would have of Meinong (or even Frege) if we could read only Russell.

. The orthodox framework now takes the form of Anglo-American pragmatist-adjusted empiricism, which is fundamentally committed to classical logic. But since classical antiquity mainstream Western thought has subscribed to what arguably underlies empiricism, the Reference Theory (according to which, in capsule form, truth is a function of reference), which does characteristically lead to consistency theses excluding paraconsistency (see further note 20 and for details see EMJB). Recent examples of attempts to squeeze Heraclitus into the consistency framework include Wheelwright [1959] and Cleve [1965], both authors bewitched by the Ontological Assumption.

). The centrality of this theme (not stated quite as such by Heraclitus) is generally agreed upon by commentators.

l. Thus, e.g., Stokes [1967] p.478.

!. Types of unity, in particular unity as continuity and unity as identity, were distinguished by Aristotle.

l. See the case for construing harmony through connection made out in Stokes [1967], p.478.

. See Freeman [1948], p.28, #50.

. See, e.g., Freeman [1948] p.28, #51; also, differently, #55.

. This is called the 'First Law of Ecology' in Commoner [1971]; it has been much cited elsewhere. On this basis a case can be made out - no more far-fetched than many of the reconstructions of Heraclitus - for seeing Heraclitus as an early (if no doubt somewhat primitive) ecologist. There is to be found in Heraclitus a fragmented picture of an ecologically well-ordered universe; and it is not only the First Law that can be ascribed to Heraclitus. In his claim 'wisdom is ... to act according to nature' (Freeman [1948], p.32 112) Heraclitus captures the thrust of the 'Third Law', 'Nature knows best' (Commoner [1971], p.37). It can be argued that the Second Law, that 'Everything must go somewhere', which is essentially a conservation principle, is represented, even implied, by Heraclitus's exchange principle: 'there is an exchange: all things for Fire and Fire for all things, like goods for gold and gold for goods' (90, cf. also #126).

. This is now often construed as the best approximation available to Heraclitus through which to say that the Logos is incorporeal or even abstract.

. See the classical representation of Meinong's theory of objects through predicate negation, in EMJB.

. This type of argument is examined in much greater detail in Priest [1983] and [198+].

. The descent is traced in EMJB, chapter 1. The common component is, as there explained, the Reference Theory, according to which truth is a function of reference. This Theory yields the main elements of classical logical theory including the Ontological Assumption, to the effect that one can only speak truly of what exists, an Assumption that appears in the extreme form in Parmenides' thesis that what does not exist cannot be sensibly spoken about or discussed (cf. p.11,ff.). Given that what exists is consistent (a common, though now disputed, assumption), it follows that the world as reflecting what is true and comprising the totality of what exists, must be consistent, and that truth cannot be contradictory.

. Thus Aristotle refers to 'the view of Heraclitus that all things are in motion' (Topica 1,2, 104b20).

. It is unclear who, apart from Heraclitus (this on the basis of his "river" fragments), these people were. But Aristotle speaks, for example, of 'Those who ... have arrived at their view (that contradictory and antithetically opposed characteristics can obtain simultaneously) on the basis of sensible perception in that they notice that from one and the same thing proceed contraries' (cited in Łukasiewicz [1971] p.401).

. On the problems with such analyses, see Priest [198+].

. There is some evidence, adduced by Łukasiewicz and considered in Part V of this chapter, that Aristotle's theory of becoming led him into paradoxes, and effectively into a paraconsistent position as regards potentiality.

. To say that all contradictions contain an element of truth is a much less drastic thesis, and indeed should be no more controversial than a principle of excluded middle. For where C is a contradiction, C implies A & ~A for some A, and one of A and ~A is true.

. Protagoras held this extravagant theory according to Diogenes Laertius [1951], p.465, and also to Plato's Theatetus (152A sq.), to which Diogenes refers.

. See EMJB, p.334. The matter is discussed in more detail in Routley [1976].

. From Progagoras's Truth: see Freeman [1948] p.125. The things Progagoras is speaking of, of which Man is the measure, appear to be truths (arguments and the like, i.e., primarily propositional items); but the theme holds derivatively for everything (for consider statements of the form 'a exists', 'b is').

. Relativism also differs significantly from pluralism; and though pluralism is often achieved by pinches of relativism, there are limits to how far this can go. Pluralism, like semantical relativism, has a logical representation within the framework of paraconsistent theory, namely through discussive logics (see especially the section on Jaśkowski below.)

. Semantical relativism fits in not only with popular relativism, that everyone is right in his or her own way, but also with what Lakatos terms Einstein's sarcastic insistence (against Bohr) that 'every theory is true provided one suitably associates its symbols with observed quantities' (quoted by Lakatos [1970] p.143).

- That is on the factual model: see EMJB, p.34. Many such arguments are assembled in RLR.

- Freeman [1948] p.126; Diogenes Laertius [1951].

- Diogenes Laertius [1951] p.485. The passage in Plato is Euthydemus 286c.

- On this see Guthrie [1969]. Note that Guthrie sees Antisthenes as 'deeply involved in the argument about the use of language and the possibility of contradiction which formed part of the theoretical background of fifth-century rhetoric, and in which Protagoras played a leading part' (III, p.304).

- Stronger and weaker versions of Antisthenes' position are in circulation: the weaker position is that is is impossible to speak inconsistently. Aristotle states it as 'the view that contradiction is impossible' (Topica 1, 2, 104b20), a formulation also given by Diogenes [1951] ix, p.465. In a rather different way Aristotle may have been committed to a related theme, to the connexive theme that every proposition is self-consistent, i.e., $A \circ A$.

- According to Diogenes Laertius [1951].

- It may be that a correct principle can be extracted from this assumption, but the argument then requires more than that principle to succeed.

  How, in an Irish way, the given assumption rules out contradiction between discussants is nicely explained by Gillespie (quoted by Ross [1924], p.347, which compares the logic of Antisthenes with what is said to be 'Hobbes' similar normalistic view'):

  > A and B are supposed to be talking about the same thing ... A and B in their discussants make various assertions about the thing, which they no doubt call by the same name; but they do not necessarily attach the same or the right formula to the name. Still in no case can they be said to contradict each other; if both have in mind the right formula, they agree; if one has the right formula and the other a wrong one, they are speaking of different things; if both have wrong formulae in mind, neither is speaking of the thing at all.

- Kneales [1962], p.22.

- Zeller [1877], p.277. Zeller says the same argument was also presented by Stilpo. A main source for reconstructions like Zeller's is the following passage from Aristotle: 'Antisthenes foolishly claimed that nothing could be described except by its own conception - one predicate to one subject; from which it followed that there could be no contradiction and almost that there could be no error' (Metaphysics, 5, 1024b).

- In case the reader thinks this is entirely stupid, he should consider Russell's celebrated third puzzle concerning denoting, which raises an analogous problem for difference: for a discussion of this puzzle, see Routley [1980].

- Zeller [1877] p.277. Zeller tries to arrive at Antisthenes' theme by having us stop at names for (simple) things (p.301) and ruling out predications, but such draconian measures are not required.

- Zeller [1877], p.296.

43. Perhaps the most interesting anticipation is the rejection of essence. According to the Antisthenians 'it is impossible to define what a thing is (for the definition, they say, is a lengthy formula), but it is possible actually to teach others what a thing is like; e.g., we cannot say what silver is, but we can say that it is like tin' (Aristotle, Metaphysics, 8, 3, 1043b24).

It is also, incidentally, worth noting that Antisthenes (though not a cynic but rather a 'precursor of cynicism') continues the ecological tradition of Greek alternative thinkers: what is known of his views on self-sufficiency is interesting in this regard.

44. See On Nature included in Freeman [1948], pp.128-29.

45. Freeman [1948], p.136.

46. Kneales [1962], p.16.

47. Taylor [1911], p.128.

48. The ascription is explicit in Diogenes Laertius: see Bochénski [1961], p.131, where some of the impact of the paradox is outlined, including the remarkable anecdote concerning Philetas of Cos.

49. Mates [1953], p.84.

50. Bochenski [1961], p.132-33.

51. De Sophisticis Elenchis 25, 180 b 2-7.

52. Ibid.

53. There was, however, a prolonged medieval debate over what exactly Aristotle meant by what he said. Bochénski proceeds, without very much evidence, to ascribe a levels-solution to Aristotle ([1961], p.132).

54. According to Bochénski from whom this passage is quoted ([1961], p.133; the original source is Rüstow [1910], p.50), the Greek phrase is ambiguous and could just mean 'that whoever states the Liar attributes a false assertion to the proposition'.

55. The main study of this work is Mates [1953], p.33ff.

56. Mates [1953], p.34. For example truth value gaps are needed for generic objects. Generic Man is neither good nor barbarian, and so an incomplete object.

57. This was said, too cryptically, to be true since caused by something that existed, viz. Electra, but false because the presentation was of a Fury. It is not difficult however to appreciate part of what is being required here.

58. Thus Bochénski [1961], p.14; also Mates [1953], pp.34-35.

59. Mates [1953], p.34.

60. This point is elaborated in DRL. The fact that the Stoics, like Ackermann, appeared to accept principle γ of Material Detachment, shows only that this position was not a dialethic one, not that it could not be paraconsistent.

- In particular, the position of Boethius may well be paraconsistent, since a radically non-classical theory is required to accommodate logical principles espoused by Boethius, especially his law $A \to {\sim}B \leftrightarrow . {\sim}(A \to B)$.

- Bochénski [1961], p.134.

- Similar problems stand in the way of tracing the history of nonstandard thought in other areas. For example, the dominance of the pervasive Reference Theory renders the tracking of thought that does not conform to it much more difficult (cf. Routley [1980]). The predominance of classical logic makes the location of theories that repudiate parts of it so much harder, especially for those who (like us) commonly have to rely on secondary sources.

- See Yutang [1948], pp.48, 52 and 204.

- This example is given, along with others, in Yutang [1948].

- Needham [1969], p.201.

- The Mohists were apparently, like the Taoists, anarchists, but of an interesting and perhaps <u>curious</u> sort, since they were specialists in military (violent) defence. As to what is, rather more relevant here, their logic, see Graham [1978].

- A text allegedly written by the Taoist Chuang Chou (?369-286 B.C.); for a translation, see Giles [1926]. See also Yu-Lan, [1952].

- However it could be reconstrued as a way of representing indeterminacy. The matter is further obscured by the historical setting, as the commentary in the <u>Chuang-Tzu</u> begins 'You cannot speak of ocean to a well-frog ...'.

- Stcherbatsky subsequently suggests that Heracleitus' theory is akin to the Sankhya position (p.426), which seems to involve, in the thesis of ideality of cause and effect, a unity of opposites. In this event, the Sānkhya position is also open to paraconsistent construal.

- Stcherbatsky [1962], p.415.

- These different ways are in fact among those explained in Routley and Plumwood, [1983].

- Matilal [1977].

- See, e.g. Robinson [1957], Stcherbatsky [1962], p.425ff.

- Murti [1955], pp.127-128.

- Singh [n.d.], p.16.

- See further Routley [1983].

- Singh [n.d], p.16.

- See Streng [1967], p.146.

- See Singh, [n.d.], p.19, for suggestions as to what these are.

81. As Indian philosophers influenced, and over-impressed, by Western texts are inclined to suppose.

82. For details see Singh [n.d.], p.20ff.

83. M. de Wulf, [1952] p.155. This reference we owe to S. Haack, who goes on to conjecture that Descartes might also have thought that God could make it the case that the "laws of logic" are false (perhaps by way of an evil demon). Had Descartes thought this he would have been right at least as regards lesser lights than God (such as demons), as semantics now helps to show (see e.g. RLR).

84. Not merely in the modern Marxist fashion for formal logic.

85. See e.g. Griffin [1984].

86. See, for example, Ennead V, 2,1, and also Gilson [1972] p.43 ff.

87. Wallis [1972] pp.57,8.

88. On which see Trouillard [1970] p.240.

89. Trouillard op cit.

90. Smart [1967] p.450.

91. Maurer [1967] p.497.

92. For the references to the Neo-Platonists, we are indebted to P.V. Spade and, especially, Lorenzo Peña. A modern discussion and development of Neo-Platonist ideas from a paraconsistent perspective can be found in Peña [1979] and [198+].

93. Heytesbury [1494], fol. 7rb.

94. Bockenski [1961], p.244.

95. See Ashworth, [1974]. A much fuller account of the medieval and post-medieval anticipation of relevant logic is attempted in Routley and Norman [198+].

96. For the influence of medieval logic by no means vanished in the 16th and 17th centuries, contrary to popular assumption that scholastic logic was swept away with the advent of the Renaissance: see e.g. Ashworth [1969].

97. Arthur Collier (1680-1732) who wrote Clavis Universalis ([1909]) should be clearly distinguished from his contemporary Anthony Collins (1676-1729) who wrote on free-thinking. An account of Collier's life and work by Leslie Stephens may be found in the Dictionary of National Biography.

98. There are quite different positions that can be taken as to the consequences of true contradictions holding in the world, e.g. that contradictions, and such a world, exists, versus the (noneist) conclusion that no such proposition or world exists.

99. Reid, [1895], p.376. The matter is discussed in EMJB, p.688.

. Certainly such a paraconsistent adaption (from a firmly classical meta-stance) is now occurring - more than a century later - in one strand of contemporary American pragmatism, notably in Rescher's thought: see especially Rescher and Brandom [1980]. The serious limitations of this work as a logic of inconsistency will be considered elsewhere. A fuller documentation of very recent paraconsistent developments would take some further account of Rescher's work. For although The Logic of Inconsistency represents rather an about-turn compared with much of Rescher's earlier work (e.g. that criticised in Routley and Meyer [1976]), still his generous philosophical framework can be readily adjusted to allow for paraconsistency, in non-adjunctive form for example, and some of his earlier suggestions, for example on truth-value gluts (an idea also entertained by relevant logic researchers at Pittsburgh), point in paraconsistent directions. Fortunately, then, Rescher's contributions have also been adequately presented elsewhere.

. Michael's exposition of Peirce, in [1975], which we follow, at several points suggests such a solution. All quotations from Peirce are taken from Michael.

. The argument that S2 is about nothing is that an attempt to find what it is about leads to an infinite regress. Peirce then anticipates Meinong's and Ryle's namely-rider location of self reference.

. The most accessible introduction by Meinong to this theory in his 'The theory of objects' in [1960]. Recent presentations and elaborations of the theory of objects appear in EMJB, Plumwood and Routley [1982] and Parsons [1980].

. See EMJB, 'Three Meinongs', where it is also argued that what Meinong rejected was a predicate form of LNC. Through the predicate/sentence negation distinction a reconciliation of a sort between a theory of "contradictory objects" and classical logic can be effected: see e.g. Parsons [1980]. The same distinction of negations can correspondingly be put to work to effect a reconciliation, again of a sort, between dialectical and classical logic. An attempt is made to carry out this latter feat by Wald [1975], p.116 ff.

. As we shall see, Vasil'év independently makes a similar suggestion, which in contrast to Łukasiewicz, he tries to develop. As Arruda has remarked in [1977], there are striking parallels between Łukasiewicz and Vasil'év, including some Arruda does not record, such as the idea that perception is always of things positive, and negation is only inferred (cf. Łukasiewicz p.507). Also common ground is the point that traditionally LNC was much confused with other principles such as Double Negation (p.493).

. It is sometimes said (e.g. by Wedin in introducing his 1970 translation of Łukasiewicz's article) that with the analogy Łukasiewicz may have already conceived the possibility of many-valued logics. This is not evident. What he had conceived is the possibility of logics which altered the basic laws of thought, a class of logics only properly overlapping many-valued logics. And what he does raise (p.486) are several new and fundamental questions concerning independence and interderivability of basic logical principles. Wedin's subsequent remarks (p.486) about Łukasiewicz's amenability to altering his paper, effectively to make it more amenable to the friends of consistency, strike us as wrong. In particular, the nontriviality requirement is not 'something of a meta-logical correlate to the logical principle of contradiction'.

. There are of course further formulations of LNC, e.g. modern syntactical formulations, other psychological forms concerning what can be conceived or imagined, and linguistic forms restricting descriptions of things. Indeed all forms can be seen as imposing restrictions of related sorts. Łukasiewicz goes on to formulate the versions extracted from Aristotle more precisely, p.488.

For example, the accusation of "psychologism", in 6(b) p.491, is but poorly made out; the claims, in 7(a) and (b) p.492, are dubious or unnecessary, for the principle concerned could be part of the logic of belief; the claim in (a') p.499 that the principle of contraposition presupposes the LNC is false. The contrast between the sharply delineated concepts of logic and the 'scraps of fluid and vague speech used in everyday life' is largely technologicians' prejudice.

On the one hand Łukasiewicz suggests the procedure contains a contradiction (p.495), on the other he says it is clear that Aristotle commits no contradiction (p.496).

Łukasiewicz's two-world interpretation brings Aristotle out as occupying a position very similar to Bradley, with a consistent world of reality behind the inconsistent world of appearances. Łukasiewicz's interpretation itself involves some significant shifts, e.g. not merely from how things are potentially to how they are in a world representing these potentialities, but that the latter world is a sensibly perceptible world (p.502). Against Łukasiewicz it could be argued that Aristotle has not conceded that LNC, in object form, holds only for actual objects; rather that 'potentially, the same thing can have antithetically opposed characteristics at the same time, but not actually' (p.501) commits Aristotle only to the potentiality theme $\Diamond fa$ & $\Diamond{\sim}fa$, not to (what Łukasiewicz's modal theory would imply) the theme $\Diamond(fa \; \& \; {\sim}fa)$ Łukasiewicz attributes to Aristotle.

It is closely related to what we will find in Vasil'év.

Aristotle notwithstanding as the negative part has indicated.

Łukasiewicz shifts (p.506 bottom) from "$x \in f \wedge x \notin f$" being certainly false to LNC's not being true except for 'objects ... free from contradiction'. But the argument is garbled because the main italicized passage is grammatically deviant. Although the argument (strictly from instances of $x(\in \wedge \notin)f$ provided by contradictory objects) makes room for the predicate negation/sentence negation distinction that looms larger in Meinong, and can be important for a consistent treatment of inconsistent objects, that issue does not arise.

The reason given for this scepticism is in part almost mystical: 'Man did not create the world and he is not in a position to penetrate its secrets; indeed he is not even lord and master of his own conceptual creations' (p.508). The final qualification to 'conceptual' could be removed.

Łukasiewicz does not try to explain the matter, which has puzzled many philosophers including several critics of Meinong's enterprise. An explanation is offered in EMJB, chapter 9.

Even swifter is his other argument emphasizing the practical worth of the principle at the expense of its logical worth. We are supposed to 'see', from the role of LNC in combatting falsehood everywhere, 'that the necessity of recognising the principle ... is a sign of the intellectual and ethical incompleteness of man' (p.508). This is rubbish (human chauvinist rubbish at that) - perhaps taken over uncritically from some more eminent philosopher - of a type fortunately not often encountered in Łukasiewicz.

. The LNC is hardly the sole weapon in any case, as operating with logics that lack it soon reveals. There are other tests of correctness, other fallacies, etc.

. The principle does not, however, directly rule out joint assertion and denial: it says (as distinct from implies, given further logical connections) nothing about assertion or denial.

. The argument is hardly conclusive: courts would simply have to resort to more elaborate procedures to determine perjury and guilt than the inconclusive more classical procedure now taken by logicians to be used. A discussion of when and how particular contradictions are rejected can be found in Priest [198+a].

. To Arruda we owe the main exposition of Vasil'év's work.

. The mentalistic restrictions, or reductions, that Vasil'év tries to impose are inessential. Still the idealist shift is commonplace (and evidently helpful) in attempts to break free from the empirical.

. But it is only recently that proper investigation of their logical and other properties has begun. Historically much more effort has been devoted to dispelling, or suppressing, the idea of alternative, and especially superior, worlds.

. Consider, e.g. "Man is not not-man" and "Meinong is not not-Meinong" and their various sentential and predicate formal representations.

. This, syntactical LNC, is one of the forms Arruda also gives of LC, the other being effectively a modalisation of this.

. Vasil'év's argument on these points (discussed by Arruda [1977]) is decidedly suspect, and, it seems, somewhat confused. One reason is that he thinks that the meaning of negation is determined by real world considerations and by the way our sense perceptions in fact operate, and that in order to amend Aristotelian (equated with: real world) logic we have to change the meaning of negation. However what he is groping for is clear enough: a nonclassical negation rule.

. Or at least, like Meinong, an inconsistent-admitting predicate negation but a more classical (consistent) sentential negation.

. Less well-known relevant logics illustrate the point better.

. The difficulty in obtaining a workable positive/negative predicate distinction only compounds reservations properly felt about this way of trying to distinguish real and imaginary worlds. However the affirmative/negative distinction is not what matters, and may be dispensed with: what is important is the obtaining of incompatible facts.

. The paper, in Russian, is briefly reviewed by Church in [1939-40], and is discussed in more detail in Rescher, [1964], p.294 ff.

. But a kind of a dual thereof, nonsignificance treatments standing to dialethic ones somewhat as incompleteness stands to inconsistency.

. In Bochvar's three-valued logic of 1939, 'Bochvar proposes to construe (the third value) I as "undecidable" in the sense of "having some element of undecidability about it" (Rescher [1968], p.67). But Rescher goes on to say, what is puzzling, 'We are to think of I not as much as "intermediate" between truth and falsity but as paradoxical or even meaningless. We can think of such meaninglessness in terms of what is at issue in the classical semantical paradoxes ...' (p.67). For these are three very different interpretations of the third value - as undecidable, paradoxical, and meaningless - and characteristically go with different matrices. In fact the matrices ("truth tables") Bochvar presents are those that fit with the interpretation of the third value as meaningless, not as undecidable - or as paradoxical.

. Insofar as they are genuinely different from the nonsignificance interpretation. The point is argued in detail in Goddard and Routley [1973], chapter 5. Consider, e.g. the conjunction $A \wedge B$ of A and B where A is false and B is undecidable: then $A \wedge B$ is not undecidable in truth value, for however B would be decided $A \wedge B$ is false. The undecidability cannot be as to truth value but must indicate a semantic defect, i.e. the matter is one of significance after all.

. In the notation Rescher uses, the principle $p \wedge \neg p \Rightarrow q$ is a thesis. The truth connective T, with matrix

	T	F	N
T	T	F	F

coincides with Bochvar's external operator A.

. The matter is considered in more detail shortly when Jaŝkowski's work is studied.

. On both see White [1979]. Among the serious drawbacks are the unavailability of extensional axioms without inconsistency. These drawbacks are avoided in relevant sublogics of $L_{\aleph_0}$.

. This follows immediately from the first three axioms of Wajsberg's axiomatisation of $L_{\aleph_0}$, namely from (1) $A \supset . B \supset A$, (2) $A \supset B \supset . \; B \supset C \supset . \; A \supset C$, (3) $\sim A \supset \sim B \supset . \; B \supset A$: cf. Rescher [1968], p.39.

. Naturally Wittgenstein's change of positions did not occur in an historical vacuum. His shift to a later paraconsistent-leaning position was influenced by German idealism for instance, as occasional examples in his work reveal. As to Wittgenstein's acquaintance with idealism and its influence on his work, see Toulmin and Janik [1973].

. Though there is some doubt about this as the article by Goldstein in this volume shows.

. Qualification is necessary because Wittgenstein was also tempted to say, at this stage, that "contradictions" in pure calculi are not strictly contradictions, and should be differently represented, e.g. by a sign 'Z'. (The point is discussed in Appendix 2 of Routley and Plumwood [1983], which complements the present discussion of Wittgenstein's position.) Subsequently Wittgenstein was quite clear that calculi containing contradictions (nontrivially) are still calculi and may be perfectly good parts of mathematics (e.g. [1964], p.181).

Regrettably not enough of the transitional assumptions are expunged in later work; instead what tends to happen is that the assumptions or motivation for them are later obscured. A striking example concerns the matter of hidden contradictions which were to Wittgenstein something of an anathema. In 1930 Wittgenstein said roundly that 'it does not make sense to talk of hidden contradictions' (McGuiness [1979], p.174), the ground being an (indefensible) verification principle to the effect that where one doesn't have an effective method or criterion for something talk of it doesn't make sense. In later work the underlying verification principle ruling "hidden contradictions" meaningless has disappeared from sight (though it is still operative in the intuitionistic bias of the work in philosophy of mathematics), and the theme as to hidden contradictions amended to this: as long as they are hidden they don't matter, and when they come into the open they can do no harm ([1976], p.219, for instance). As will be seen, this stand is most unsatisfactory: for contradictions can certainly matter when still hidden, and can do much harm even when visible.

. One has simply lost one's way. Containing a contradiction in a harmful way is like not knowing one's way about ([1964], p.104). This comparison breaks down on elaboration. One may know one's way about an elementary inconsistent calculus quite well, and be lost in a consistent one, etc.

. This also helps explain how Wittgenstein can talk of a 'true contradiction' ([1964], pp.178-9) and give the impression that it is alright, in certain cases, to assert contradictions.

. Of course a nonsignificance approach would normally endeavour to separate logico-semantical paradoxes from other (noncompulsory) contradictions. There is little going for the view that everyday contradictions don't make sense, except a confusion of making sense with having content construed connexively.

. Nor does the notion of correctness, or objective determinacy, require radical modification in the way some of Wittgenstein's remarks suggest. The discussion in the text may however be criticised, along the lines indicated in Wright ([1980], pp.310-1), for making an illicit assumption Wittgenstein would reject, to the effect that there are some underlying facts of the matter which determine correctness (including correctness of application), whereas 'for Wittgenstein, there is no Olympian standpoint from which it may be discerned who is giving the right account of the matter' (p.311). The opposite view, that correctness can sometimes at least be discerned, is argued in EMJB chapter 11 and, for a range of logical principles, in RLR, chapter 2.

. Thus Wrigley [1980]; cf. also Wright [1980], p.310, discussed above. Note that although Wittgenstein's conventionalism is linked with construal of mathematics as a game, conventionalism as such does not entail that mathematics is a game.

. Goldstein [1981]. We are much indebted to Goldstein for several references.

. But of course questions about the conventionality of the practices remain.

. Naturally not all worlds, and certainly not all logics, are on a par. The view is perfectly compatible with there being a unique factual world, and even One True Logic.

. Wright's whole discussion of inconsistent systems, such as an arithmetic containing a contradiction, turns on the erroneous assumption that such systems are trivial (see e.g. p.306, top. p.308, middle). He assumes without warrant that intuitive arithmetic is based on a logic with spread principles.

). A similar writing-in of a consistency assumption, on this occasion into the notion of calculation, is attempted on p.307. This is illicit. Relevant and paraconsistent theories reveal quite straightforwardly that calculation is 'not ... frustrated in an inconsistent system...'.

Popper claims 'to have gone into this question...whether we can construct a system of logic in which contradictory statements do not entail every statement...and the answer is that such a system can be constructed. The system turns out, however, to be an extremely weak system. Very few of the ordinary rules of inference are left, not even the modus ponens...' (Popper [1940], see p. 321). However Popper's claim is not only innacurate but decidedly misleading. First, Popper's implicit claim that there is only one paraconsistent formal logic is false, as we shall see in the introduction to the next part of the book. Secondly, there are many quite strong paraconsistent systems in which modus ponens is valid, as we shall see there too. Thirdly, Popper's "negation" operator is not really a negation operator. It is a sort of dual to intuitionist "negation" for which $\vdash A, \neg A$ holds, but $A, \neg A \vdash A$ fails. It is therefore as Popper implicitly admits ([1940] p. 323 top) a subcontrary forming operator, not a contradiction operator. To this extent, it is like da Costa's "negation" operator (see the introduction to the next part of the book, section §2 II). Popper glimpsed the possibility of formal paraconsistent logic, but no more.

151. See RLR 12.8

152. A tradition running from the Jains (see above) through Nozick [1981], Introduction. The semantical analysis of discursive logics through possible worlds also enables an elegant representation of philosophical pluralism, a synthesis of different positions.

153. See the 1980 issue of Artificial Intelligence devoted to nonmonotonic logics, i.e. logics for which premiss augmentation fails.

154. There are several different ways discourse can be represented logically. A non-telescopic method simply represents participants within the theory, as in the work of Hamblin and of McKenzie, for details of which see McKenzie [1979].

155. For base modal logics weaker than S5 the relation can be generalised to allow m-time iteration of $\Diamond$, thus
$DL1^m$. A is a thesis of DL^m iff $\Diamond^m A$ is a thesis of base logic L,
where $\Diamond^m$ is a sequence of m occurrences of $\Diamond$, i.e. $\Diamond^m B = \Diamond(\Diamond^{m-1} B)$ and $\Diamond^1 B = \Diamond B$. Several results concerning such systems are summed up at the beginning of Kotas and da Costa [1977].

156. If Material Detachment were valid, the rule $\Diamond A, \Diamond(A \supset B) \twoheadrightarrow \Diamond B$ would be valid in S5 in which case $\Diamond(A \supset B) \mathbin{\&} \Diamond A \supset. \Diamond B$ would be a thesis of S5, which as Jaśkowski points out (p.150), it is not. To establish paraconsistency it is enough to find a falsifying assignment involved (in rejecting the nonthesis), which verifies $\Diamond A$ and $\Diamond{\sim}A$ and falsifies $\Diamond B$.

157. Namely $A \rightarrow_D' B =_{Df} \Diamond A \supset \Diamond B$. A problem with this is that it validates the principle $(A \rightarrow_D' B) \rightarrow_D'. {\sim}(A \rightarrow_D' B) \rightarrow_D' C$, which would violate the spirit of paraconsistency.

158. This claim is further defended in the Introduction to Part 2. Naturally discursive logics can be based on relevant logics. Then they will reject Conjunctive Spread, but they may unnecessarily toss out Adjunction with it. Reinstating Adjunction requires a somewhat different approach to discursive logics.

159. The departure from normality is rather like that of intuitionistic negation: just as discursive conjunction can be characterised in terms of & and $\Diamond$, so intuitionistic negation can be characterised, at least in some contexts, in terms of $\sim$ and $\Diamond$.

- See the end of his [1951]. We do not concede that he is successful, but his arguments are of surprising weight, especially given the usual consensus opposing $\Diamond A \,\&\, \Diamond B \rightarrow \Diamond(A \,\&\, B)$. And the arguments have an interesting nonadjunctive character.

- For such possibility functors do not pick out the worlds of creatures' discourse, assertions, beliefs, positions, etc., in the way much literature on intensionality has supposed. The point is shortly elaborated in the text; and for much more detail, see EMJB 8.12.

- The following proof works in fairly minimal modal logics: since $q \supset \Diamond q$, $\Diamond q \supset \Diamond\Diamond q$ and, by the same principle, $\Box\Diamond p \supset \Diamond p$, $\Box\Diamond p \,\&\, {\sim}\Diamond\Diamond q \;\supset.\; \Diamond p \,\&\, {\sim}q$, whence, by the principle again $\Box\Diamond p \,\&\, {\sim}\Diamond\Diamond q \;\supset.\; \Diamond(\Diamond p \,\&\, {\sim}q)$. The rest of the argument turns primarily upon distribution of $\Diamond$ over $\vee$ using the equivalence $\Diamond(A \vee B) \equiv. \Diamond A \vee \Diamond B$. Since, by sentential logic, and eliminating $\Box$, $\Diamond{\sim}\Diamond p \vee \Diamond\Diamond q \vee \Diamond(\Diamond p \,\&\, {\sim}q)$, $\Diamond[{\sim}\Diamond p \vee \Diamond q \vee (\Diamond p \,\&\, {\sim}q)]$, i.e. $\Diamond[\Diamond p \supset q \;\supset.\; \Diamond p \supset \Diamond q]$.

- For example, the powerful condition, $TRH_1 \,\&\, TRH_2 \;\supset.\; H_1 = H_2$, i.e. there is only one world accessible from the base world T, will guarantee it, without collapsing modal logics to extensional. Obtaining an exact modelling is more difficult but can be accomplished, e.g. within the framework of relevant modal logic.

- There are other less direct linkages, e.g. through the dialogue-conversation picture where one of those involved (perhaps the ideal participant, who keeps a record of all decidable theorems) is a computer.

- Asenjo [1954]. See also Asenjo [1966].

- Asenjo [1966] perpetuates some common errors. In particular, he confuses <u>ex falso quodlibet</u> with Reductio. A calculus of antinomies does not require, contrary to what Asenjo claims (p.104) either the rejection of Reductio or the rejection of Non-Contradiction.

- Though it is easy enough to obtain it, as in Goddard and Routley [1973], chapter 6.

- Asenjo and Tamburino [1975]. This paper is largely a condensation of Tamburino [1972] - largely, but not entirely, for the unexplained conditions on the comprehension axioms of the antinomic set theories given differ in perhaps significant ways.

- Thus da Costa's motivation compared directly with Anderson and Belnap's in arriving at the system E of entailment, where the objective was design of a system as strong as it could (reasonably) be within the classical confines, which satisfies the requirements of relevance and necessity. But Anderson and Belnap succeeded no more than da Costa did: see RLR3.

- Again there is the assumption throughout that they constitute <u>the</u> logic of inconsistent systems.

- These are the main sets of systems, not the only ones. Arruda and da Costa also (like Popper earlier, and Fitch [1952]) studied systems where the rule of Modus Ponens fails, namely the J systems, for which see Arruda and da Costa [1968], [1970] and [1974]. An overview of da Costa's main work on paraconsistent logics and theories is given in Arruda [1980]. On da Costa's systems, see sections 5 and 6.

2. Da Costa [1963a], our paraphrase. The main results of this note are recorded in da Costa [1974].

3. Da Costa was aware of this, since he refers to Jaśkowski when he introduces the hierarchy of systems based on C_1 as other possible solutions to the problem: there he says 'on the subject of this note see Jaśkowski ...' ([1948], p.3792).

4. R-W, and more generally relevant systems lacking Contradiction and Reductio, are examined later in the book.

5. For details of most of these systems see Arruda [1980].

6. The main reason is the inclusion of full positive logic, especially the inclusion Contraction principles, in the C systems. See especially the discussion of the Curry objection in the Introduction to Part II.

7. Arruda and da Costa, [1963], p.83. The idea of a logic which permits unrestricted abstraction (almost implicit in Meinong) goes back technically at least to Moh [1954] and ultimately to Frege. The idea, which is a good one, has since occurred to many others independently, including the authors.

8. Neither J systems nor P systems by <u>any</u> means exhaust the types of approach they here represent.

9. As to the first and second points see Routley and Loparic [1978], and for elaboration, RLR 14. As to the third point, see Arruda and da Costa [198+].

. Also Ackermann's rule δ of commutation (viz. $A \rightarrow . B \rightarrow C, B \rightarrow A \rightarrow C$) was removed, but <u>unlike</u> γ, δ is easily shown to be an admissible rule of E, by straightforward syntactical argument. The separation of E and Π' is (then) essentially through rule γ.

. The principle is challenged and rejected in RLR3.7.

. It is not at all difficult to reconcile: see R. Routley and V. Plumwood [1983]. Indeed a neat synthesis can be obtained.

. This is so although at the same time as <u>Entailment</u> was being drafted at Pittsburgh, Asenjo's and Tamburino's work on inconsistent theories was being produced (in a different department however). There was apparently no cross-fertilisation or noteworthy exchange between relevant and paraconsistent enterprises, though they were being undertaken in close proximity.

. See Anderson and Belnap [1975] p.461ff.

. For in dealing with Prior's family of semantical paradoxes, in [1961], Mackie worked with the assumption that assertions can be, as well as true ($\underline{T}$) and false ($\underline{F}$), either <u>paradoxical</u> (when $\underline{T}$ iff $\underline{F}$, i.e. given his two-valued logical frame, when both $\underline{T}$ and $\underline{F}$) or <u>vacuous</u> (when neither $\underline{T}$ nor $\underline{F}$). The Liar-paradoxical assertion and assertions which logically reduce to it are paradoxical, while their opposites, such as the Truth-teller (e.g. 'What I am now saying is true'), are vacuous or empty. But Mackie does not logically elaborate the theory indicated: he nowhere suggests what the theory immediately indicates, a 4-valued logic with values $\underline{T}$, $\underline{F}$, $\underline{P}$ or $\{\underline{T}, \underline{F}\}$, and $\underline{V}$ or $\{\ \}$, and would never have considered $\underline{P}$ a designated value. Although he contrasts "Logic", that is two-valued logic, with 'a wider sort of logic' needed 'to detect emptiness', a 'logic that pays attention to the method of deciding whether a given item is true or false (or obeyed or not obeyed, etc.) and sees whether this method is circular' (p.241), he did not conceive of this logic as a formal logic. (Nor was such a logic seriously considered in his seminars when these ideas were developed and thrashed out, seminars in which one of the authors participated.)

And although he formulaed 'some general principles that tell us when to expect emptiness and paradoxicalness' (p.241), he took 'these principles as indicating regions in which emptiness and paradoxes are to be expected, and are to be guarded against or understood as the occasion demands' (p.243). Any more formal guarantee against emptiness and paradox 'would need some kind of type-restriction ...' but for most purposes this would be too sweeping' (p.243). Thus he took paradoxes as items to be excluded or avoided as genuine statements, and the paradoxes as having solutions of a statement-incapability sort (an influential position at Oxford, which Mackie absorbed and helped import to Australasia). The wider logic 'has a bearing in the paradoxical cases too; for one way of expressing the solution of a paradox of the Liar type is to show that the paradoxical item is empty and therefore that the contradiction which would be generated if we took it as a genuine item does not matter' (p.241). Although rudimentary elements of a paraconsistent position appeared, then, in Mackie's otherwise thoroughgoing empiricism, crucial elements of paraconsistency - specifically the taking of paradoxical statements to hold somewhere (to be designated in some situations) - were lacking, and were bound to be excluded by themes of empiricism (for reasons explained in EMJB 9.10).

. As is evident in his recent (English written and derivative) material on paradoxes included in his [1973].

. On both Characterisation Postulates and the route therefrom to paraconsistency, see EMJB.

. On the I systems, see e.g. R. Routley [1972], and for a much fuller exposition RLR chapter 7.

. See V. Routley [1967], some of which was absorbed in RLR chapter 2.

. This was eventually published in shortened form as Routley and Routley [1972].

. See the semantics of entailment series, initial papers of which appeared in the Journal of Philosophical Logic 1972-3 on. Detailed citations are given in RLR.

. Among the series of papers developing this material were Routley and Meyer [1976], and R. Routley [1977] and [1979].

. Unfortunately the enormous costs of travelling between the countries has curtailed further visits.

. Apart from Petrov [1979], the main work from Bulgaria is indicated in papers included below.

. See her paper in this volume.

. This is in the tradition of "logic of paradoxes" and covers part of the same ground as that traversed in Priest [1979].

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CHAPTER 2: AN OUTLINE OF THE HISTORY OF (LOGICAL) DIALECTIC

In this essay we trace the historical development of the notion of dialectic. This is a thorny job. Dialectic has long meant a variety of different things. (This was already recognized in third century A.D.[1] but is even truer now). Moreover, as we shall see, it is a notion that has repeatedly tended to be generalised, often in unilluminating ways. For example, sometimes dialectic has been identified with logic,[2] sometimes with debate, and more recently it has been equated with scientific method and even with certain philosophical theories. However, dialectic _in stricto sensu_ has always been a much tighter subject. It comprises methods, characteristically of argument or analysis, and at its core lies the notion of contradiction. (That this core encapsulates the strict sense of dialectic is the view even of Soviet philosophy.[3]) Moreover, even though the notion of dialectic has evolved, it has remained throughout a method - one of the fundamental methods of philosophical argument and analysis of concepts or situations, with its central concern always remaining the use and function of contradictions therein. Thus the role of contradiction in this methodological context not only defines our subject but provides the historical backbone of our account.[4]

Contradiction in itself has become a slippery notion. However, we will understand it in the precise logicians' sense: a contradiction is a pair of contradictory statements, characteristically of the form A and ~A (i.e. it is not the case that A). The notion has certainly been generalised, especially in the last hundred and fifty years. However, we will discuss this later. Till then, 'a contradiction' means a pair of statements of the form A, ~A.

1. _The Origins of Dialectic in Western Philosophy._[5] No adequate account of dialectic can omit Heraclitus (c 500 B.C.), to whom the dialectical law of unity the interpenetration of opposites, is commonly ascribed. But it is obviously difficult to attribute views, with reasonable assurance, to a somewhat cryptic poet/philosopher, most of whose work has been lost. However, it is generally agreed

that one of Heraclitus' basic doctrines was that everything is in a continuous state of change.[6] It follows that everything that is, must cease to be. Thus everything changes into its opposite; life becomes death, hot becomes cold,[7] etc. On this basis it is argued that the world is a unity of opposites.[8] (This is later to become a principal point of C18-C19th German dialectic.) So far this allows the opposites to occur at different times. However, certain fragments strongly suggest that opposites can be realized at the same time too. For example:

> Things taken together are whole and not whole, something which is being brought together and brought apart, which is in tune and out of tune: out of all things can be made a unity, and out of a unity all things.[9]

Thus Heraclitus may have been the first (at least in the west) to assert that there are some true contradictions. He was certainly a source of important methodological principles included in later dialectic: the dynamic principle, the unity of opposites, and perhaps the holistic principles.

But it is not to Heraclitus, but to Zeno of Elea (c. 460 B.C.) that dialectic - really negative or destructive dialectic - is often traced. In fact, it is reported that Aristotle claimed that Zeno was the inventor of dialectic.[10] What Aristotle seems to have meant is that Zeno was the first person to use the method of argument we now call <u>reductio ad absurdum</u>. That is, Zeno attacked the views of his opponents, and especially their supposition of plurality, by trying to show that they lead to absurd or contradictory conclusions,[11] and accordingly had to be rejected. This, at any rate, is the orthodox construal of what Zeno was up to. (Hegel, as we shall see, had another.)

As is well known, Zeno was a student of Parmenides, who held that the world was one changeless unity. Since Zeno's arguments attempt to show that plurality and change involve contradiction it seems reasonable to suppose, as is orthodoxly assumed, that Zeno was arguing in favour of Parmenides' position. It may well be that in arguing against the possibility of motion, Zeno also took himself to be attacking Heraclitus' position. However, Heraclitus would perhaps not have been too worried by Zeno's arguments. Indeed he might well have welcomed them as extra support for the unity of opposites: change does require the (simultaneous) realization of opposite states. In any case, Zeno's paradoxes of motion have been important in the history of dialectic; in particular they have been widely accepted in the Hegelian and Marxist traditions as valid arguments showing that motion, and more generally change, involves contradictions essentially. It will therefore pay us to consider Zeno's arguments, or rather standard reconstructions of them, if briefly.[12] The two major arguments are (i) the Racecourse, and (ii) the Arrow. For the third argument, Achilles and the Tortoise, is a variant of the Racecourse, and the fourth, the Stadium, is commonly said to be somewhat obscure.

(i) The Racecourse. This argument runs as follows: Suppose that an object is in motion from A to B. Before the object arrives at B it must arrive at a point half-way between A and B. Before it reaches this point it must reach a point half-way between it and A. Before it reaches this point it must reach a point half-way .... Thus before it reaches B it must complete an infinite number of motions. But this is impossible since one can do only finitely many things in any finite time.

The argument is ingenious, but it is doubtful that it succeeds. The received opposing[13] view is that it is quite possible to do an infinite number of things in a finite time provided the acts are part of one continuous action. There is perhaps no sense in which this can be proved other than tendentiously (e.g. by an appropriate definition of continuity), and conceivably Zeno had some additional arguments against it. However, so long as sound arguments for the finitude assumption are lacking, the received move successfully defuses the paradox.

(ii) The Arrow. This argument goes as follows: Suppose an object is in motion from A to B. Consider any instant of the motion. At that instant, since it is only an instant, the object makes no progress on its journey. But the motion is composed of such instants. Hence, it makes no progress at all, i.e. it never moves.

Again, it is doubtful that the argument succeeds. It assumes that if the distance moved at any point is zero, the distance moved at any sum of points is zero. Plausible though this is, the principle is now rejected if the sum has the right size of infinity. Technically, that an uncountable set of points, each with measure zero, can have non-zero measure can be shown given a suitable definition of 'measure'.[14] The fact that this is surprising may be put down to the fact that our intuitions, drawn from finite cases, often break down where infinitudes are concerned.

Because crucial assumptions on which they are based can be readily rejected we are unable to endorse Zeno's arguments. However, in the present context that does not matter so much. The important point of this section is simply that originally dialectic was conceived as a method of argument showing that certain views entailed contradictions.[15]

2. The Development of Dialectic in Greek Philosophy. Thus conceived, dialectic was practised widely, amongst post-Zenonian philosophers. In particular, Euclides of Megara (c 400 B.C.) was fond of "attacking demonstrations not by the premises but by the conclusions", meaning, presumably, that Euclides attacked his opponents' positions by drawing consequences from them.[16] In fact, members of Euclides' school,

the Megarians, were called dialecticians, presumably because they practised dialectic.

At much the same time but in another place (around Athens), the Sophists were using a similar technique (sometimes called 'anti-logic') which consisted of drawing out the contradictions latent in popular or other beliefs.[17] The technique was a slight generalization of the Zenonian one, in that the dialectician did not have to produce a single continuous argument to a contradiction, but was allowed to argue by posing a series of yes/no questions leading the holder of the belief in question to admit that his views contained a contradiction.[18] Thus the dialectician actually showed that a set of beliefs, rather than just a single belief, were inconsistent. (However, in virtue of the logical equivalence between a finite set of beliefs and their conjunction, this modification is not very important.) It was through this slight generalisation also that the discussive component entered into the dialectical argument technique. (The word "dialectic" itself comes from διαλέγεσθαι meaning "discuss".) The same technique, now normally called "elenchus", was used by Socrates.[19] Socrates' 'mission' was to show people that they didn't really know what they claimed to know.[20] To this end the sophist-style dialectical argument was a very effective weapon.

Important steps in the development of dialectic occurred at the hands of Socrates' pupil, Plato. What, in the hands of Socrates and the Sophists was a weapon for showing negative results, Plato turned into a methodological approach for determining positive ones.[21] The point of the exercise was to delineate a Form, usually in the form of a real definition. To this end a hypothesis was put up, and this was examined for contradictions or other unacceptable consequences. (For example in the Theatetus (151 e), Theatetus defines knowledge to be perception from which Socrates draws conclusions which force its abandonment.) If these were found a new hypothesis was produced and the procedure repeated. If a hypothesis could be found which did not have unacceptable consequences, this was taken to be a correct definition (at least tentatively).[22] This process is illustrated in many of Plato's dialogues. Platonic dialectic was essentially a sequence of Zenonian dialectical arguments with a certain telos.[23]

There is another important feature of the Platonic dialectic, which though never perhaps stated explicitly by Plato is nonetheless visible in his dialogues. That is, that very often, when a hypothesis is refuted, a new one is not produced de novo but the old one is modified in such a way as to preserve its insights whilst trying to accommodate what has been learnt from the dialectic criticism. For example, in the Euthyphro, the first definition of "piety" mooted is that piety is that which the gods love. However, since the gods may disagree, it follows from this that an act may be both pious and impious. The definition of "piety" is then changed to "what all the gods love".

> The features of the Socratic [sic] dialectic are these: (1) it starts with a partial definition; which (2) on examination contradicts itself; (3) the further definition which reconciles the contradiction, though it negates the initial definition as such, yet contains modified and absorbed within it the grain of truth which the definition held.[24]

Obviously we are here not too far removed from what will be Hegel's notion of aufhebung.[25] At any rate this fact gives the dialectic a certain appearance of convergence.

> The method of hypothesis therefore seems to be a method of approximation, though there is no such description of it in the dialogues. We are continually making alterations in our whole set of opinions, according as contradictions are revealed among them by the powers of deduction. In this manner we render them more and more adequate as time goes on.[26]

Two further points should be made about Platonic dialectic. The first, and minor, is that it is possible that the Platonic dialectic is due to Socrates. Socrates is certainly shown as practising it in some of the dialogues. It is notoriously difficult to determine what the historical Socrates actually did. Our interpretation of the situation seems to us the most plausible. However, nothing much hinges on this for our purposes. The second, and more important, point is that Plato's conception of dialectic was not a uniform or stable one. In some of the later dialogues Plato applies the term "dialectic" to another method of finding correct definitions. This is the method of "division and collection",[27] essentially finding a definition by producing a taxonomic tree. While such a search procedure can be usefully combined with the dialectical procedures of earlier dialogues, the extension of the term to apply to these taxonomic methods on their own, does suggest that by this time Plato was taking the term "dialectic" as a general word of commendation[28] - a phenomenon not unknown in this century with certain Marxists.

3. Dialectic in Later Greek and Medieval Philosophy. Dialectic undergoes no very substantial development, but shows progressive degeneration, in Greek philosophy after Plato. Most of what is said rings the changes on either the Zenonian concept or the Platonic. Moreover both have a tendency to be diluted or else generalised in such a way as to render them near-trivial, and often banal.

Thus Aristotle generalised and weakened the sense of "dialectic": dialectic is, for him, the science of arguments from nonevident premisses.[29] More exactly, Aristotle takes dialectics to be a form of reasoning distinguished by the fact that its premisses are "probable", i.e. held but not proven. Thus "[dialectical reasoning] is distinguished from demonstrative reasoning ... by reference to the premisses from which it starts": dialectic for Aristotle means reasoning which takes men's convictions for its premisses; the premisses of the demonstrative syllogism are "true and primary".[30] Since one cannot prove an opinion by reasoning from it, the point, presumably, of dialectical reasoning (if it has a point - Aristotle says it may be merely mental gymnastics)[31] is to refute it. This suggests that Aristotle's dialectic is just a variant of Zeno's. The point is borne out by the fact that in the main place where Aristotle himself uses (as distinct from discusses) dialectical reasoning (namely Metaphysics Γ) the arguments are all variants of reductio.

However, according to Aristotle, dialectic can be also useful in determining the first principles of science (which can not, by definition, be demonstratively proved).[32] This would appear to be an echo of the Platonic dialectic. Thus: "'Reasoning' [Aristotle] says 'is dialectical if it reasons from opinions that are generally accepted ...' Topics 100^a 30. And as we read him we constantly observe Aristotle establishing his results by the gradual development of a more comprehensive and coherent theory through the criticism and modification of other men's conflicting doctrines. In respect of its method an Aristotelian treatise is a Platonic dialogue stripped of its dramatic [and discussive] form and reduced to more or less continuous lecture notes".[33]

Although the Stoic conception of dialectic was also continuous with the Platonic, and evidently (and acclaimedly) evolved from the Socratic dialectic, the notion of dialectic was again not so much enriched as markedly watered down. Dialectic was one of two branches of logic[34] and was conceived of very generally, to include reasoning or what we would not call logic, but was much more like the Platonic dialectic.

> .. in Stoicism, as Plato, dialectic is a science which has the real nature of things as its field of study.[35] Not that dialectic means the same procedure in both systems. For Plato the dialectician is someone who arrives by a process of question and answer at true definitions, and who discovers in this way what things are. The Stoics recognised question and answer as one of the methods dialectics uses but ... [not as] the only proper way of philosophy.[36]

In fact anything that concerned "knowledge of what is true, false or neither true nor false" or of "words, things and the relations which hold between them"[37] seems to have been thought part of dialectics. Thus dialectics becomes just a general name for semantics, epistemology and metaphysics.[38] Whilst it is understandable that it should have been generalized in this way it is clear that this empties the precise notion of dialectic of interest. Dialectic, a powerful method founded by Zeno, and reaching its zenith with Plato, ends in Greek philosophy impotent.

Medieval philosophy - though, it is now beginning to emerge, a highly creative period in general - accomplished little of value for the notion of dialectic. "Dialectic" was commonly used, as by the Stoics, as a blanket term for logic and semantics.[39] But even where the method of dialectic was that of "trying to discover the truth by discussion which would reveal the unacceptable consequences of various suggestions", the method was quite inadmissably restricted. For it "was not dialectic as the Greeks knew it", for the reason that the medieval schoolmen were constrained to "reach conclusions consistent with revelation".[40] Since Socrates' "willingness to question all accepted ideas and rely on reason so far as he could", to follow the method to whatever unpalatable places it led, was an important part of his dialectical method, medieval dialectic at its best was not Socratic dialectic.[41] It is not surprising, given the medieval educational curriculum, that the term "dialectic" was also transferred to refer to a certain form of discussion method or debate, especially in the universities, whose stylised form was obviously a somewhat debased descendant of Platonic dialectics.[42]

4. Kant and Fichte. The next major step in the development of dialectic was brought about by Kant. Dialectic had degenerated so much at the hands of Aristotle and the medievals that Kant thought the natural meaning of "dialectic" to be more or less synonymous with sophistry:

> "However various were the significations in which the ancients used 'dialectic' as the title for a science or art, we can safely include from their actual employment of it that with them it was never anything else than a logic of illusion. It was a sophistical art of giving to ignorance, and indeed to intentional sophistries, the appearance of truth, by the device of imitating the methodical thoroughness which logic prescribes, and of using its 'topic' to conceal the emptiness of its pretentions".[43]

Kant, rightly dissatisfied with this, decided to use "dialectic" for critical argumentation which aims at showing incorrectness.[44] (In this of course he was merely returning the word closer to its Zenonian meaning.) Specifically in the part of the *Critique* called the "Transcendental Dialectic", Kant aims to show that certain kinds of arguments, commonly used in metaphysics are incorrect. The most crucial part of this is the section called "The Antinomy of Pure Reason". In this section Kant produces four pairs of arguments, the antinomies, each pair for a contradictory conclusion. Although similar to *reductio ad absurdum* arguments, their interpretation, according to Kant, is much more sophisticated. According to Kant, neither of each antinomic pair is fallacious in any straightforward sense: they are not mere sophisms, but in some sense, a product of reason itself.[45] Kant is obviously skirting the paraconsistent position. However this is not his conclusion. What it is, is best explained by considering one of the antinomic pairs. Consider the first antinomic pair (which is fairly typical). It is as follows:

(i) The world has a beginning in time.

Proof. Suppose not. Then before any particular event an infinite number of events must have occurred. Thus a whole infinite sequence of events has occurred. But it is impossible that an infinite sequence be completed. Contradiction.

(ii) The world has no beginning in time.

Proof. Suppose not. Then there was a time when nothing existed. But nothing can come into existence out of nothing. Hence the world did not start to exist. Contradiction.[46]

Now first of all, let us make it clear that we think that both of these arguments are incorrect:- the first because an infinite sequence can often be completed, as Zeno's arguments reveal; the second because if the world had a beginning, it was coincident with that of space-time.[47] However, the defectiveness of the arguments is not to the point here. Both arguments employ the notion of a certain totality, of all events, or of all events before a certain time. Now although we might experience each event, we can not experience the totality. In Kantian jargon, each event is given to us by an intuition, whereas the totality is an object constructed in reason alone. Now Kant defuses the arguments by insisting that our concepts can legitimately apply only to objects of intuition,[48] i.e. experiences, and not to the likes of infinite totalities. To see what this means, consider argument (ii). It appeals to the principle "nothing comes from nothing", i.e. "every object has a material cause". This, according to Kant is true, but "everything" must be understood as everything given in experience. Thus the move to "the totality had a cause" is incorrect. In a similar way in (i) we can not apply the principle "If A_1 occurs before t and A_2 occurs before t and ... then the sequence A_1 A_2 ... occurs before t" where the sequence is an infinite one, not given to us in experience. Thus what Kant took this and the other antinomies to show is that certain a priori principles, whilst true enough, can be applied only to things experienced, i.e. that the quantifiers in the principles must be taken as restricted to experiences only.

The crux of Kant's position is that reason and its categories are dependent for content on experience. Indeed that is what Kant takes the antinomies to show.[49] However, it is difficult to find a direct argument for this assumption other than some very general empiricism. Once empiricism is openly rejected and reason, far from being parasitic on experience, is admitted to lead a life of its own, Kant's position on the antinomies collapses. Reason, by unimpeachable arguments, produces contradictions, which must therefore be true. This line of thought is of course precisely that pursued in post-Kantian German idealism, and especially by Hegel.

The lynch pin between Kant and Hegel is Fichte. In his Science of Knowledge Fichte starts from the position that philosophy should be a science, and that means, according to him, that it should all follow (in some sense) from some basic proposition.[50] Of course, this idea of science is perhaps naive, but that doesn't matter in the present context. What is the basic proposition of philosophy? Here Fichte turns to Kant. He criticises Kant's postulation of the thing-in-itself as totally spurious.[51] This leaves the other part of Kant's ontology, the transcendental ego. Thus Fichte's basic postulate: the existence of the ego thinking about (positing) itself. Here, however, the ego faces a problem. You can not think about, or have a conception of something unless it is in opposition to something else. (Compare Spinoza: omnis determinatio est negatio.) So at least thought Fichte. Hence the ego, to think itself, must posit something different, the non-ego, against which it can conceive itself.[52]

But, this realises a contradiction. This is precisely reason leading a life of its own (cf. the last paragraph on Kant). For the ego must make itself non-ego. It is therefore both ego and non-ego. As Fichte puts it:

> insofar as the not-self is posited in [the self] the self is not posited in the self[53]

but

> insofar as the not-self is to be posited in [the self], the self must be posited therein.[54]

Thus the self is both posited and not posited and the posited is both self and not-self. As Fichte puts it a few lines later, "self = not-self and not-self = self".[55] Moreover the contradictions do not stop here. For "the second principle [i.e. the positing of the non-ego] annuls itself, and it also does not annul itself".[56]

Fichte calls the positing of the ego and non-ego the "thesis" and "antithesis", respectively. The contradiction that these produce has to be "resolved" in a "synthesis". Exactly what this means is, to say the least, somewhat obscure. The ego and the non-ego cannot co-exist (unlimitedly). Each has to "limit" the other, i.e. each has to "abolish the reality" of the other,[57] "not wholly but in part only".[58] What this seems to mean is something like this. Suppose government X declares itself to have jurisdiction over a country (the thesis). Government Y then declares itself to have jurisdiction (the antithesis). The synthesis is obtained by partitioning the country into two halves, one under the jurisdiction of X and one under that of Y. Or, perhaps better, each accepts restricted and non-overlapping jurisdiction (as in the case of State and Federal governments). Exactly how this metaphor is to be cashed out in Fichte's case is a more difficult matter which we need not resolve here. However, we shall return to the subject of syntheses when we discuss Hegel. For the present we will simply note that in some sense the contradiction is resolved in the third phase of the dialectic. According to Fichte the new synthetic state will produce its negation, there will be a new synthesis and so on.[59] However, Fichte does not fill out the details much further. The stage is now set for the synthesis of the whole of dialectic so far: Hegel's.

5. Hegel. Hegel starts from the notion of the Fichtean ego. However, under the prompting of Schelling this has become something much grander, world spirit or Geist. Geist is a difficult notion and we do not need to go into it now.[60] However we may, as a first approximation think of it as bearing the same relation to the physical universe as a person's thought does to his or her body (provided we think of thought in Aristotelian, not Cartesian terms). For this reason we will translate "Geist" as "Thought". Thought has a certain essence and this is expressed in its telos, namely to come to think (understand) itself. To this end, it, like the Fichtean ego, posits its opposite, Nature (the material world). For reasons we have already discussed in Fichte, this realises a contradiction. Finally the contradiction is resolved. What this means is not that the contradiction disappears but that Thought comes to see that the contradictories are identical, that thought and Nature are not only different but are the same too. This is Hegel's notorious thesis of the identity of identity and difference.[61] To see this is at once to resolve the contradiction.[62] and to see what Thought is, thus realising the telos of thought.

We can hardly claim that this notion of a resolution of a contradiction is transparent. However, an analogy from paraconsistent theory will shed light on it. Consider a paradoxical sentence, say '$\{x:x \notin x\} \in \{x:x \notin x\}$'. Call this R. From R we can deduce the negation of R. (R posits its negation). But from $\sim$R we can deduce R. Thus though we normally assume that a proposition has a very different sense from its negation, in this particular case R and $\sim$R, since they entail each other, express, so as to say, the very same proposition. Thus the negation of R is exactly the same as R, i.e. the opposite of R is identical with itself. To see how a sentence can mean the same as its negation is to see how a contradiction is possible and hence for it to be understood (resolved). We might also think of the deduction

$$\begin{array}{c} R \\ \vdots \\ \sim R \\ \vdots \\ R \end{array}$$

in Hegelean terms. The deduction is a movement through negation, and the negation of negation. The final R returns us to our starting point but at a "higher level" since we now see that R and $\sim$R are identical.

The formal similarity between the structure of paradoxical deductions and the movement of Thought is remarkable. In not noticing it (perhaps historically excusable) Hegel undoubtedly missed a trick. The analogy may help to show how something can be at once identical and different to itself. Whether it actually says anything for Hegel's position (as opposed to just making it intelligible) is another matter, which we are content to leave open here.

Anyway this fundamental, we could say 'global', dialectic is not to be realised in a trice. It is to be realised after a long development. Thought goes through a whole series of stages to reach its telos. The progression here is not a temporal one but one of "logical" development. (However, as we shall see in a moment, it is connected with a temporal one.) Take any category of Thought (such as being, cause, infinity). This category contains a contradiction. (One might say that being part of the whole it reflects the global contradiction.) Here Hegel shows the influence of Kant. For Hegel, the Kantian antinomies are just the tip of an iceberg; extended, antinomic arguments show that all our categories are inconsistent, in the sense that any category must apply to things to which the contradictory category applies.[63] By analysis of the contradiction in a category we are led to posit another which "transcends" (aufhebt) the old. (Thus, for example, analysing being we determine that there are things which must both be and not be. We posit the category of becoming to apply to these. The contradiction is thought of as realised at the moment of change.) However this new category is itself contradictory and so the development progresses until we arrive at the most fundamental of categories, the Absolute Idea,[64] which is the category with which thought can think itself. (It applies to the "biggest contradiction of all".)

We could call this progression Hegel's logical dialectic, as opposed to the global dialectic. The logical dialectic is explained, appropriately enough, in his Logic. (Whether Hegel is always successful in showing that a certain category is inconsistent is yet another matter we will not detour to discuss. But it is extremely doubtful that he is.) There is also a third dialectic to be found in Hegel. Thought, it will be recalled, is embodied in Nature and in particular, in man and his institutions. Thus, the logical dialectic is mirrored by a parallel development in man and his consciousness. This is a historical development, and we could call it Hegel's historical dialectic. It is explained in his Phenomenology, Philosophy of History and several other books. The progression proceeds in the familiar dialectical way. The culture and the institutions of a certain epoch contain contradictions. They therefore pass away and are replaced by a new culture and institutions which resolve the contradictions and transcend (aufhebung) the old. But what do "contradiction" and "transcend" mean here? It is easy enough to see what inconsistent categories are - those that force there to be objects like Thought which have inconsistent properties, i.e. which force simple inconsistency. But what is it for an institution to be inconsistent?

Hegel catches a number of things under this rubric. However, the most important and fundamental seems to be something like this. An institution, like any human product, has a telos. If the realization of that telos requires the production of factors or situations, which inhibit or prevent the realization of that telos, we have a contradiction.[65] Hegel's most famous illustration of this kind of situation is the master/slave predicament.[66] To realize himself (the telos) a man requires the recognition of other men. To this end they are captured and enslaved. But this makes them less than men. Thus their recognition will not serve its required end. In nuce the telos requires us to possess the free. Obviously an impossible requirement.[67]

This notion of contradiction is obviously related to the proper notion, of a sentence of the form p & ~p. However it is obviously a slight displacement of the notion. Hence, the notion of transcendence must be slightly displaced too. The institution which transcends the old does so by minimally changing either the means or the end so that the contradiction is overcome. In this sense, the "contradiction" is defused and the old system is "preserved" whilst at the same time destroyed. But again we will not sidetrack to discuss Hegel's illustrations of this historical process, which range from convincing to unconvincing.[68]

By now it should be clear that Hegel's dialectic is a development of that of Kant and Fichte. A little reflection shows it to be a development of Greek dialectic too. Hegel's insistence that things are in a continuous process of development is of course Heraclitus' flux, and his insistence on the realization of contradictions is Heraclitus' principle of the unity of opposites.[69] Zeno's dialectic is cited with approval by Hegel. Hegel endorses Zeno's paradoxes. Zeno of course thought they showed motion to be impossible. Hegel, who took motion to be actual, thought they showed that motion, and change in general, realises contradictions.[70] Zeno's argument therefore played the same role as Kant's antinomies. As for Plato's dialectic, it is clear that Hegel's logical and historical dialectics are just Plato's dialectic transposed into different keys. Both are the attainment of a certain telos by the successive transcendence of contradictions.[71]

Before we leave Hegel, there is one final point worth emphasising. As we have seen, Hegel accepts the view that reality is inconsistent, that there are true contradictions. In fact he criticises Kant frequently for holding that contradictions are only "in thought". For Hegel, because of his idealism, the categories of thought are the structure of reality. Inconsistent categories are therefore inconsistent reality.[72] This has been too much for some commentators, especially Anglo-American ones, to stand. Many have assumed that contradictions cannot be true and then (mis)applied the principle of charity to suggest that when Hegel asserts that reality is inconsistent or even asserts a bald contradiction,[73] he cannot mean what he says. Not surprisingly, this results in gross distortion of Hegel's views. Hegel can be properly understood only from a paraconsistent point of view (which is not of course, to say that a paraconsistentist must agree with all of his zany metaphysics). When it is actually argued that Hegel could not have meant what he said, the arguments are usually bad. The most frequent one is the artless, "but a contradiction implies everything".[74] Not only is this false, but there is no reason to suppose that Hegel held it to be true, and reason to suppose that he did not. What is even worse, Popper, for one, assumes that Hegel does mean what he says about contradictions and so writes him off for just the artless reason.[75]

6. Marx and Marxists. Around the turn of this century, Hegel's views were influential in many places, especially Britain. What happened, however, was that British Hegeleans such as Green took up Hegel's idealism but left his dialectic.[76] Perhaps the major exception was Bradley who endorsed Hegel-type arguments to the end that space, time, causality and other categories are inconsistent, but who could not brook the view that reality is inconsistent, and who therefore consigned the whole lot to the realm of appearance.[77]

If we wish to see further developments in dialectic we must look not to the Hegelians, but to Marx and his successors. Marx transformed Hegel's dialectic by 'demystifying' it. What this means we will now see. Take first Hegel's global dialectic, that is, the movement of Thought, to non-Thought, with the corresponding synthesis. Marx, in effect, changes "Thought" to "Man". The dialectic of Man then goes as follows. Man has a certain essence (i.e. defining property), which is his labour. However, this alienates itself, i.e. comes to exist over and against Man. Alienated labour (objectified labour) is of course just capital (essentially, the labour theory of value) which exists in the form of private property. This is the fundamental opposition, between worker and capital. The synthesis is obtained in a communist society where private property disappears, Man labours for himself again and thus his essence is returned to him. In fact the goal of history is this synthesis: the production of Man as he ought to be.

The reinterpretation of Hegel is obviously substantial. However the important thing to note is that the alienated state is still a contradictory one, in the literal sense. For in the alienated state man loses his essence (loses his "species life"). Yet of course the essential properties are precisely those which, by definition, can not be lost. Contradiction. For example, in the Paris Manuscripts, Marx writes:

Estrangement (Alienation) is manifested in the fact that my means of life belong to someone else ... but also in the fact that everything is itself something different from itself - that my activity is something different from itself.[78]

Of course it is still my activity, otherwise it would not be different from itself. Hence we have a contradiction. In fact it is just Hegel's identity of identity and difference: my work is both identical with itself and different to itself.

The theory of alienation is discussed mainly in Marx's earlier writings. It does not appear a great deal in his later writings and the question of how much it is presupposed is a moot one which we need not discuss. The relevant dialectical material of the later works, especially Capital, derives not so much from Hegel's global dialectic but from his historical dialectic. In fact this dialectic is much the same as the Hegelian one except that economic factors become dominant. Thus, for example, the famous passage in the Preface to the Critique of Political Economy which cites the contradiction between the forces of production and the relation of production is exactly of this kind. The forces of capitalist production have a certain *telos*. These produce bourgeois relations of production. However, ultimately these prevent the realization of the *telos*. The system of capitalist production therefore ceases, and a new one is formed which preserves a certain amount of the old (particularly the forces of production) whilst getting rid of some of the rest (particularly the relations of production). As another example, consider the use of machinery in capitalist production.[79] The *telos* of capitalist production is the making of profit, which is a certain form of surplus value. To this end, when machinery is developed which is less expensive than a human worker, a manufacturer will employ it to realize more profit. But of course all capitalists will eventually do the same thing. Eventually therefore the manufactured article in question will contain less human labour, have less value, and hence less surplus value. Thus the amount of profit realized in the long run is less. The system of production in furtherance of its *telos*, produces factors which actually inhibit the realization of the *telos*. Thus Marx says:

> Hence there is an immanent contradiction in the application of machinery to the production of surplus-value, since, of the two factors of the surplus-value created by a given amount of capital, one, the rate of surplus-value (crudely, profit divided by wages) cannot be increased except by diminishing the other, the number of workers.[80]

There is then, a quite determinate sense of "contradiction" other than the literal sense to be found in Marx's writings. Moreover, Marx follows Hegel's apparent lead in broadening the sense of 'contradiction' to other cases. For example, in the Poverty of Philosophy he says:

> Meanwhile the antagonism between the proletariat and the bourgeois is a struggle of class against class, a struggle which carried to its highest

expression is a total revolution. Indeed, is it at all surprising that a society founded on the opposition of classes should culminate in brutal contradiction, the shock of body against body, as its final denouement?[81]

On another occasion Marx describes the fact that the means of production in capitalism are worked socially but owned privately as a contradiction:

.. this expropriation (of the private means of production) appears within the capitalist system in a contradictory form, as appropriation of social property by a few ...[82]

Clearly these two uses of "contradiction" have only loose connections with contradiction in the strict sense. One may put them down to a loose form of expression, or possibly just to polemics (something never completely absent in Marx's writing). However they and occurrences like them began a tendency (or at least accentuated a tendency already to be found in Hegel) to expand the sense of contradiction beyond legitimate logical levels.

This extension, and erosion, of the notion of contradiction has been taken further by other Marxists. For example, in Anti Duhring Engels uses the term "contradiction" both in the sense of logical contradiction[83] and in the other senses we have encountered.[84] But, he also uses it to describe the fact that man's potential knowledge is unlimited whilst his actual knowledge is limited.[85] This is accompanied, inevitably, by an even more dismaying stretching of the notion of negation.[86]

The notion of contradiction, and therewith negation, is stretched further again by Lenin who lists as examples of contradictions:[87]

In mathematics: + and -, and differential and integral.
In mechanics: action and reaction.
In Physics: positive and negative electricity.
In chemistry: the combination and disassociation of atoms.
In social science: the class struggle.

(However there are precedents for some of these in Hegel.[88]) Other Marxists use the term "contradiction" not only for any pair of contradictory categories, whether or not it can be shown that there is some one thing to which both apply, but even as regards any opposing forces or tendencies.[89]

The heterogeneity of all these things need hardly be emphasized. In this way the notion of contradiction has been emptied of much of its content. Consequently dialectic, the "science of contradictions", too has become generalized and diluted to the point, we think, where its essence has been lost. Part of the reason for this seems fairly obvious. The ideology of consistency has affected even dialecticians to the extent that they can no longer believe that Hegel and Marx meant what they actually said.[90]

But part of the reason is also that dialectic as a method again achieved honorific status (in certain circles). Accordingly, though there was some attempt to tighten up the method by formulation of some older principles as dialectical "laws", dialectic was extended to cover more and more favoured enterprises, notably science. Engels seems to have had a prominent role in this, by first simplifying Hegel - "according to Hegel dialectics is the self-development of the concept" - and then under the materialist transformation, which was alleged to put Hegel on his head, grossly changing the notion: "dialectics reduced itself to the science of general laws of motion, both of the external world and of human thought ..."[91] Lenin follows suit and equates the dialectical method with scientific method, sometimes localised to sociology:[92]

> .. what Marx and Engels called the dialectical method is nothing more or less than the scientific method in sociology. ... It all amounts to regarding social evolution as a natural-historical process of development.

This sort of generalization can be to some extent accommodated by amended accounts of scientific method and science in terms of a less corrupted notion of dialectic, though such a reverse procedure assumes (what is at best decidedly dubious) that "science always means the discovery of contradictions, inherent in all products of nature - and in society too".[93] And since Lenin the degeneration of dialectic from a comparatively tight and powerful method has, for the most part, continued within the wider Marxist tradition. (Just one important example is in Sartre [1976]).

7. Summary and Prospects. It will be clear that dialectic, as befits a theory of development, has developed markedly over two and a half thousand years of philosophy. We have isolated two major phases: Classical Greek philosophy and Modern German philosophy. Although they are very different, one is the development of the other, and there is an important parallel between the phases. Both started off concentrating precisely on contradiction within the setting of (perplexing) arguments. Both then developed into a theory of development in which contradiction plays the central role. Finally, both went into a period of decline when the specific essence of dialectic, literal contradiction, was forgotten, and consequently dialectic became a subject of high generality but little content. All this we have documented.

Of course the evolution of dialectic will continue and we think that we are at the start of a new phase of growth, during which symbolic logic will play a fundamental role. It will again start with a concentration on contradiction itself within the framework of argument procedures, especially convincing arguments which lead to contradiction. To an extent this has already happened with the matter of logical paradoxes and of paraconsistent logic. However it is also evident that a correct understanding of the history of dialectic is essential for further progress. To this end an analysis of the history of dialectic, and particularly Hegel's dialectic, using the techniques of modern logic is essential. From what has been said it is obvious that such an analysis will have to accommodate the notion of a true contradiction. Thus paraconsistent logic will be essential here too.[94] This analysis has already started, but remains in its earliest stages.[95] Where the whole modern study will take us, we can only speculate.

FOOTNOTES

1. By Alexander of Aphrodisias: see Long [1974], p.101.

2. See Kneale and Kneale [1962], p.7.

3. 'In its proper meaning, dialectic is the study of contradictions within the very essence of things.' Lenin quoted by Stalin [1973], p.305. Lenin goes on of course to say how much more than this core dialectic comprises.

4. This is the reason that we qualified 'Dialectic' with the adjective 'Logical' in the title of the paper. Of course, were dialectic to be synonymous with logic (which it was at one time) both 'logical dialectic' and 'dialectical logic' would be pleonasms. But dialectic and logic parted company certainly by the third century A.D., when the term 'logic' appeared in approximately its modern sense. See Kneale and Kneale [1962], p.7.

5. The practice of dialectic in Eastern thought is certainly older: see e.g. the previous chapter, part II.

6. See e.g. the fragment on p.381 of Kirk [1954].

7. Kirk [1954], p.134.

8. See the fragment in Kirk [1954], p.88.

9. Kirk [1954], p.168.

10. This is reported by both Sextus Empiricus (Adversus Mathematicos vii, 7) [1912-54] and Diogenes Laertius ([1951] viii, 57 and ix, 25). See Kneales [1962], p.7.

11. Kneales [1962], p.7.

12. The arguments are set out simply in Vlastos [1967].

13. Vlastos [1967].

4. The treatment of sets of measure zero in contemporary measure theory does, however, rely essentially on what amount to paradoxes of implication, and is accordingly open to serious objections from a paraconsistent stance: see the Appendix to Routley [1980]. There is sufficient evidence, moreover, that alternative measure theories can, like alternative logics, be devised. Zeno's summation principle can presumably be incorporated in the framework of such a theory. So there are presumably coherent theories in which some of Zeno's paradoxes are accepted. The question then becomes: which of the measure theories is true? For a further discussion of the arrow paradox, which concedes greater force to it, see Priest [198+a] and Peña [198+].

5. 'It seems then that the first precise meaning of the word "dialectic" was <u>reductio ad impossibile</u> in metaphysics.' Kneales [1962], p.9. Negative dialectics had a similar role in Nāgārjuna's thought.

6. Kneales [1962], p.8.

7. See Kerford [1967].

8. See the section "Socratic Method" in Ryle [1967].

9. Ryle [1967].

0. See Robinson [1953] p.13.

. The Kneales [1962], pp.9-10, claim that this is difficult to understand, and mysterious. However, as we shall see, it is quite straightforward. The Kneales run into trouble through presupposing a dubious positive/negative distinction, linking reductio arguments and refutations as negative invariably with negative results. But Zeno's procedure already indicates how results such as Parmenides' thesis that motion is impossible, a thesis of high generality, can be enforced by dialectical methods, e.g. supposing the opposite and deriving unacceptable conclusions.

. See Robinson [1953], p.107.

. Those who are familiar with Popper's account of science can not fail to notice a similarity here.

. Mure [1932], p.29.

. Those familiar with Lakatos' account of the growth of mathematics in [1976] will also notice a similarity with the way theorems are modified, on his account, in response to counter-examples.

. Robinson [1953], p.108.

. Kneales [1962], p.9. The Kneales go on to say, uncharitably and incorrectly, that:

> the only feature common to Plato's use of the term "dialectic" seems to be that it signifies a co-operative method of philosophical investigation, involving a search for definitions, and approved by Plato at the time of writing.

But the co-operative method remains specifically a <u>discussive</u> one, taking a definite question and answer form; it involves more than a search for definitions, but inquires into the nature of things; and it does not compromise merely what is arbitrarily approved by Plato. The method of division, e.g., retains a crucial feature of the earlier wider procedure, namely the rejection of unacceptable alternatives.

- Thus there is some basis, especially in Plato's later work, for Robinson's overstatement: "The fact is that the word 'dialectic' has a strong tendency in Plato to mean the 'the ideal method, whatever that may be'. In so far as it was thus an honorific title, Plato applied it at every stage of his life to whatever seemed to him at the moment the most hopeful procedure". Robinson [1953], p.70.

- Kneales [1962], p.10.

- Long [1974], p.122.

- Ross [1923], p.56.

- Ross [1923],pp.56-7.

- Mure [1932], p.217.

- The other being rhetoric. See Long [1974], p.121.

- In total contrast to Aristotle's account where dialectic is not a science, has no specific subject matter, and is chauvinistically defined in terms of human procedures and assumptions.

- Long [1974], p.122.

- Long [1974], p.122.

- Even more disconcertingly, the Stoics characterise dialectic, like logic, in several nonequivalent ways, so it appears:- Firstly, the Stoics define dialectic as the science of speaking well, and make speaking well consist in speaking things that are true and fitting (cited, though with reservations, in Long [1974], p. 102; the source is Diogenes Laertius, who however conflates this account with a narrowly semantical one: see [1951], p. 742). On this account, dialectic includes not only semantics broadly construed but also pragmatics and rhetoric and elements of epistemology. By contrast, however, the most widely attested Stoic definition of dialectics is as "the science of things true and false and neither true nor false", which is rather a semantical or metaphysical account. On yet a third, and more Platonic, Stoic account of dialectics, "of the two forms of inquiry which fall under the virtue (of dialectic), one considers what each thing that exists is [its real definition], and the other what it is called [its nominal definition]" (Diogenes Laertius [1951], 7.83). And, fourthly, it is also said that "Chrysippus agreed with Plato and Aristotle that the philosophical argument, formally conducted, is the only proper procedure for the demonstration of truth. ... called the expert in this a dialectician" (Long [1974], p. 113). In later Stoicism the notion degenerates entirely. Long ([1974], p.108) gives a "catalog of dialectical virtues" attributed to Chrysippus - Stoic dialectical virtues in his account, since Long goes on to call Stoic dialectic "Chrysippian dialectic". Long then suggests (p.117) and claims (p.120) that this is explicit in Epictetus: indeed (Long, [1974], p.123ff.) "dialectic may be regarded as a method of self-discovery" - because it "contributes to the understanding of man himself and of the rationality of the universe"!

39. See for example the discussion of Abelard's Dialectica, Kneales [1962], p.204ff.

40. The quotes are from Kneales [1962], p.203.

41. The fact that everything is open to dialectical assessment, and revision, does not of course imply that beliefs and assumptions cannot be adhered to, until shown faulty. It is no criticism of the Socratic technique that Socrates had (if he did) "firm belief in supernatural agencies which transcended reason and which it would be both foolish and dangerous to disregard": Zaehner [1974], p.13.

42. See Hall [1967] and Kneales [1962], pp.202-3, who describe the procedure as follows: all philosophy (and most else) was "studied by consideration of quaestiones. At the beginning of each quaestio authorities who oppose, or seem to oppose, each other are set in array, and then the teacher shows his mastery by producing distinctions of meaning that suffice to solve the problem and dispose of all difficulties" (Kneales [1962], p.202).

43. Kant [1950], A61 B86.

44. Kant [1950], A62 B86.

45. Kant [1950], A297 B354f; A339 B397.

46. Kant [1950], A426 B454ff.

47. This seems to be the consensus concerning the general theory of relativity. Alternatively one can argue that something can come into existence from nothing (space-time itself being an example of this) and trace the mistaken belief that this is impossible back to the Reference thesis. (See Routley [1980], Ch.2).

48. Kant [1950] A498 B526 ff.

49. A similar strategy, perhaps at a more sophisticated level, is deployed in the Buddhism of Nāgārjuna: see the previous chapter, part II.

50. Copleston [1963], p.55ff.

51. Copleston [1963] p.58.

52. Copleston [1963], p.65.

53. Fichte [1970], p.106. Quotation rearranged.

54. Fichte [1970], p.106.

55. Fichte [1970], p.107.

56. Fichte [1970],p.107.

57. Fichte [1970], p.108.

58. Fichte [1970], p.108.

59. Fichte [1970],p.113.

60. A full account can be found in, e.g., Taylor [1973].

61. See Taylor [1973], p.80.

62. We could say, to see the contradiction is not a contradiction.

63. Taylor [1973], p.228.

64. Taylor [1973], p.339.

65. Taylor [1973], p.131.

66. Taylor [1973], pp.153-7.

67. Hegel would seem to be open to an ad hominen argument here. If contradictory situations are realizable (which they certainly are since Thought is both identical to itself and not identical to itself) why can't you have a person who is both bound and free? The answer, of course, is that though Hegel is committed to the view that some contradictions are realizable, he is not committed to the bizarre view that all are. Which can be, and which cannot be, is an important question, but this is another matter.

However Hegel's discussion of the master-slave relationship does give grounds for saying that not only is the master bound (in some respects, at any rate) as well as free, but that the slave is free, as well as bound, For the slave turns out to be free in a way, since it is the slave who mediates between the master and the world and thus has a degree of control over the master. (This point is due to J. Norman.)

68. Details can be found in any book on Hegel. For example, Taylor [1973].

69. "There is no proposition of Heraclitus which I have not adopted in my Logic". Hegel [1955], Vol. 1, p.299.

70. Hegel interprets Zeno as holding his own position. Hegel [1955], pp.266-7.

71. Hegel is well aware of this. See the last section of Hall [1967].

72. Taylor [1973], pp.230 f, 342.

73. Which he often does. See for example his [1929], Vol. II, pp.66-7.

74. See, for example, the section "Dialectical Method" in Acton [1967].

75. Popper [1963]. Given his earlier logical work, Popper should have known better. He shows, p.321, that he is aware of the possibility of formal paraconsistent logic, but naively supposes that only one, extraordinarily weak, such logic is possible.

76. Giles [1967].

77. Acton [1967a].

78. Marx [1959], Vol. III, p.314.

79. Marx [1976] Ch. 15 §3.

80. Marx [1976], p.531. It is worth noting that a number of writers also see a dialectic analogous to Hegel's logical dialectic in Capital, for example in the

way that economic categories are deduced from the contradictions in the notion of a commodity. See Ilyenkov [1982] chapter 5.

81. Marx [1977], p.215.

82. Marx [1959], Ch. XXVIII, p.440.

83. Engels [1975], p.140 f. Some of which however are very dubious.

84. Engels [1975], p.326.

85. Engels [1975], p.140.

86. Engels [1975], p.156 ff.

87. V.I. Lenin [1972].

88. See for example, the section on Hegel's Philosophy of Nature in Acton [1967].

89. See, for example, Mao Tsetung [1968], p.32. For other examples, and further discussion of negation and contradiction, see Routley and Plumwood [198+].

90. On this, see for example Acton [1967b], p.392.

91. Engels [1941], p.44. Engel's "reduction" would make the Newtonian laws of motion, for example, dialectical laws along with such principles as the unity and opposites.

92. Lenin [1978], Part I. It is only fair to add however that much of Lenin's writing was polemical and not tightly theoretical. In such a context dilution of the notion of dialectic is (if not thereby exonerated) understandable, and of course sometimes advantageous.

93. Quoted in Cornforth [1965], p.292. However, note that in contrast to Popper [1963] and others, we are not denying that dialectic, and its laws duly amended, can play a significant role in accounting for scientific method and the growth of science. For a further discussion of the relation between science and dialectic, see Priest [198+].

94. It follows that the few attempted formalizations that have appeared which use classical logic are doomed to failure. Some of these can be found in Marconi [1977].

95. See, for example, Routley and Meyer [1975]; da Costa and Wolf [1980]; Priest [1981]; and Peña [198+].

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CHAPTER 3: SYSTEMS OF PARACONSISTENT LOGIC

1. Paraconsistency: Characterization and Motivation

Let $\models$ be a relation of logical consequence. $\models$ may be defined either semantically ($\Sigma \models A$ holds iff for some specified set of valuations, whenever all the formulas in Σ are true under an evaluation, so is A) or proof theoretically ($\Sigma \models A$ holds iff for some specified set of rules, there is a derivation of A, all of whose (undischarged) premisses are in Σ), or in some other way. $\models$ is explosive iff for all A and B, $\{A, \sim A\} \models B$. It is paraconsistent iff it is not explosive. A logic is paraconsistent iff its logical consequence relation is. If a logic is defined in terms of a set of theses it may have more than one associated consequence relation. For example, $\{A_1 \ldots A_n\} \models B$ iff $\vdash (A_1 \wedge \ldots \wedge A_n) \rightarrow B$ or $\vdash A_1 \rightarrow (\ldots \rightarrow (A_n \rightarrow B) \ldots)$ or $A_1, \ldots , A_n / B$ (the last representing the theorem-preserving or weak inferential connection). In this case all its associated consequence relations should be paraconsistent.

Let Σ be a set of statements. Σ is inconsistent iff, for some A, $\{A, \sim A\} \subseteq \Sigma$. Σ is trivial iff for all B, $B \in \Sigma$. The important fact about paraconsistent logics is that they provide the basis for inconsistent but non-trivial theories. In other words, there are sets of statements closed under logical consequence which are inconsistent but non-trivial. This fact is sometimes taken as an alternative definition of 'paraconsistent' and, given that logical consequence is transitive, it is equivalent to the original definition. The proof is this:- If Σ is an inconsistent but non-trivial theory then obviously the consequence relation is paraconsisent. Conversely, suppose that $\{A, \sim A\} \not\models B$. Let Σ be the transitive closure of $\{A, \sim A\}$ under logical consequence. Then Σ is inconsistent but $B \notin \Sigma$. Because of the equivalence we also call any inconsistent but non-trivial theory paraconsistent, and derivatively, any position whose deductive closure provides a paraconsistent theory.

Why should one be interested in paraconsistent logics? Among the many reasons are proof theoretic and semantic ones.

I) The proof theoretic reason is that there are interesting theories T which are inconsistent but non-trivial. Clearly the underlying logic of such theories must be paraconsistent - hence the need to study paraconsistent logics. Examples of inconsistent but non-trivial theories are easy to produce, and many will be given in what follows. A first example, that will recur again and again, is naive set theory, the theory of sets based on the full abstraction axiom scheme, $\exists y \forall x(x \in y \leftrightarrow A)$. This, together with extensionality, characterises the intuitive conception of set. The theory is inconsistent since it generates the set theoretic paradoxes (e.g. where R is the Russell set, defined as $\{x : \sim x \in x\}$, standard paradox arguments show that $R \in R$ and $\sim R \in R$). Yet it is non-trivial because there are many claims about sets which the intuitive notion rightly rejects (e.g. that $\{\Lambda\} \in \Lambda$, where Λ is the null set). A very similar, and likewise important example, is naive semantics, the truth theory based on the full T-scheme, $Tr\ulcorner A \urcorner \leftrightarrow A$. This characterises the intuitive conception of truth. It is inconsistent because it generates the semantic paradoxes (e.g., Liar paradoxes). Yet it is non-trivial since there are many claims concerning truth which the naive notion rightly rejects (e.g. that $Tr\ulcorner A \vee B \urcorner \leftrightarrow Tr\ulcorner A \urcorner \wedge Tr\ulcorner B \urcorner$).

Another group of examples of inconsistent but non-trivial theories derive from the history of science. Consider, for example, the Newton-Leibniz versions of the calculus. Let us concentrate on the Leibniz version. This was inconsistent since it required division by infinitesimals. Hence if α is any infinitesimal, $\alpha \neq 0$. Yet it also required that infinitesimals and their products be neglected in the final value of the derivative. Thus $\alpha = 0$. (As much was pointed out by Berkeley in his critique of the calculus.[1]) Despite this the calculus was certainly non-trivial. None of Newton, Leibniz, the Bernoullis, Euler, and so on, would have accepted that $\int_0^1 x dx = \Pi$. A very different but most interesting example of an inconsistent but non-trivial theory in the history of the natural sciences is the Bohr theory of the atom.[2] According to this an electron could orbit the nucleus of an atom without radiating energy. However, according to Maxwell's equations which formed an integral part of Bohr's account of the behaviour of the atom, an accelerating electron, such as an electron in orbit, must radiate energy. Despite this the Bohr theory of the atom was non-trivial. Someone who suggested to Bohr that it followed from his theory that electrons moved in squares would, rightly, have received a sharp answer. Many other examples of inconsistent but non-trivial theories from the history of science could be given.[3] Indeed it could be persuasively argued that the whole state of scientific knowledge at any time is such a theory.[4] However these two examples will suffice for present illustrative purposes.

A third group of examples of inconsistent but non-trivial theories are certain bodies of information which are theories only in a somewhat attenuated sense. What justifies their inclusion in the present setting is that inferences are made, and made commonly, from the information. Thus ideally they may be conceived of as deductively closed corpuses or theories. Many examples could be given here, and will be introduced subsequently.[5] Among the more interesting nonphilosophical examples are certain bodies of law, such as bills of rights and constitutions. The following is a convenient hypothetical example which, however, makes the point clearly. The constitution of a certain country contains the clauses (a) 'No person of the female sex shall have the right to vote', (b) 'All property holders shall have the right to vote'. We may also suppose that it is part of the common law that women may not legally be property holders. As enlightenment creeps over the country this part of common law is changed to allow women to hold property. Inevitably, eventually, a woman, call her Jan, turns up at a polling booth claiming the right to vote. A test case ensues. Patently the law is inconsistent. According to the law Jan both does and does not have the right to vote. Patently, also the law is not trivial. Someone who argued that her cat should be allowed to vote on the basis of (a) and (b) would not get very far. Actual historical examples of inconsistent legal situations are of course more complex and, therefore, more controversial. However two actual examples are the case of Riggs v Palmer and Lincoln's Proclamation of Emancipation. In the former the clear legal right of inheritance was contradicted by the legal principle that no one shall acquire property by crime. The benefactor had, in fact, murdered the deceased. In the second, the freeing of slaves, who were undoubtedly legal property, with no compensation, contradicted the Fifth Amendment, which says that property shall not be taken without just compensation.[6]

Other examples of inconsistent information from which inferences are drawn include: the data presented to a jury in a trial; the information fed into a computer; a person's set of beliefs.[7] In each of these cases the information may obviously be inconsistent. Moreover, inferences are obviously made from this information. Yet clearly people are <u>not</u> at liberty to conclude anything they like from the information. That there are inconsistent but non-trivial theories is thus well established.

II) The Semantical Reason. A second reason for being interested in paraconsistent logics is the fact that there are true contradictions, that is, there are statements A and ~A such that both are true. Because of this, some inferences of the form A, ~A/B must fail to be truth-preserving (let alone valid) since some statements (take one such for B) are not true. Thus, Logic is paraconsistent.

Examples of the alleged true contradictions are not difficult to provide. Under the influence of Zeno's paradoxes, Hegel thought that a moving object realized a contradiction: a body in motion was both at a certain place at a certain time and not at it.[8] However, the validity of Zeno's arguments is decidedly doubtful.[9] Hence such dialectic examples of true contradictions are perhaps not so plausible. Much more persuasive examples of true contradictions are provided by the logical paradoxes. These are examples of arguments in set theory and semantics which appear to be perfectly sound arguments issuing in contradictory conclusions. If this is indeed the case then clearly the contradictory conclusions are true. Those who wish to deny this conclusion must show that the paradoxical arguments are not really sound at all. This poses the problem of where to locate the unsoundness. It is some measure of the unworkability of the unsoundness position that there is still absolutely no consensus as to where to locate the unsoundness (as there is, for example, with Zeno's paradoxes) and this some 2,000 years after the initial discovery of a logical paradox.

But how is it possible for a contradiction to be true? Quite simply. For example consider the sentence

(c) This is a false sentence of English.

This has two components, a subject 'this' and a predicate 'is a false sentence of English'. Each of the components has certain semantic conditions. Thus, the semantic condition of 'this' is its referring to a certain object - in this case, (c) itself. The semantic condition of the predicate is that it applies truly to a certain class of objects, viz. those which are false English sentences. Now of course (c) is contradictory. In other words the semantic conditions of the components of (c) overdetermine its true value. They determine it to be both true and false. Of course, one can state dogmatically that this is incorrect, that we have got the semantic conditions of the components wrong or something of this kind. But this is to elevate consistency into an inviolable constraint on semantics; and why should we suppose it is? Semantic conditions were not laid down by God or even by some Hilbert who kibitzed them for consistency before unleashing them on the world. They have grown up in a piecemeal and haphazard way. It would, quite frankly, be amazing if they were consistent. Semantic conditions can be seen as determining a field of meaning. Overdetermining truth conditions produce singularities and other discontinuities in the field. But such are to be expected, and in no way interfere with the rest of the field. Of course this is only a metaphor, but it can help to break a mental set.

Once one gets past this mental block - past the consistency hang-up - there are other plausible examples of true contradictions. For example, in the hypothetical legal set up described in the previous section it seems that 'Jan has the legal right to vote' and 'Jan does not have the legal right to vote' are both true. A somewhat more controversial example[10] concerns the application of multicriterial terms. For instance, to determine whether a phrase, such as 'below 0°C', correctly applies to a certain situation, we may observe the behaviour of either a correctly functioning alcohol thermometer or a correctly functioning thermo-electric thermometer. These work on quite different principles, and there is no sense in which one is more basic to our determination of, or understanding of, temperature than the other. Certain behaviour of either of these instruments provides a sufficient condition for the correct applicability of the term 'below 0°C' or its negation, and both have equal claim to determine an operational meaning of the phrase. Normally the world is such that these two criteria hold or fail together. However in a novel situation they may well fall apart. In such a situation both the assertion that the phrase applies and the assertion that its negation does are true. By the symmetry of the situation neither claim can be truer than the other. Hence either both are true or both are false. To suppose that both are false would be to deny that they were criteria in the first place. Thus they must both be true. An historical example of where criteria fell apart in this way is the Michaelson-Morley experiment. Because of rigid rod measurements, 'The arms of the Michaelson-Morley interferometer are congruent' was true. Because of measurement in terms of time taken by light rays, 'The arms of the Michaelson-Morley interferometer are not congruent' was true.[11]

Clearly there is a relationship between the proof-theoretic and the semantic motivations for paraconsistency. If the semantic rationale is correct, then the proof theoretic one is too. For if S is the set of things true in some domain containing true contradictions then S is an inconsistent but non-trivial theory. However, it is possible to accept the proof theoretic motivation without accepting the stronger semantic one outlined. For one can hold that there are inconsistent but non-trivial theories which are interesting, have important applications, useful properties, and so forth, without accepting that they are true. Instrumentalists and formalists would, of course, have no problem in accepting such a theme, though they might well find difficulties in clearly distinguishing the stronger, dialethic position from the weaker, more pragmatic, position.[12] Whether either position is tenable on other grounds is another matter, which we will investigate in more detail as we proceed. Indeed, several of the issues raised above will be taken up in more philosophical detail in subsequent introductions. The discussion so far merely serves to indicate some of the motivation for paraconsistency.

§2. Approaches to Paraconsistent Logical Theory: Initial Systemic Taxonomy of Paraconsistent Logics; Zero Degree Formulas

Having shown that paraconsistent logic is well motivated, we need to specify a paraconsistent logical theory or theories. One thing is perfectly clear, classical two-valued logic is of no use: it is explosive; it is not paraconsistent. Nor for that matter are its extensions such as modal logic. Nor are intuitionist logic or its extensions, for they too are explosive, in virtue of their spread principles such as $A \rightarrow. \sim A \rightarrow B$ and $A \wedge \sim A \rightarrow B$. Many lesser known (but nonetheless significant) logics also fail to meet paraconsistency requirements, and are accordingly logically inadequate to accommodate a range of important philosophical and scientific theories and positions. Among them are various connectional, or broadly relevant logics (i.e. systems satisfying some variable-sharing principle linking antecedents and consequent), in particular connectional logics which validate Disjunctive Syllogism, $A \wedge (\sim A \vee B) \rightarrow B$, and retain some residual form of Rule Transitivity. Representative of this are conceptivist logics such as Parry systems and connexivist logics.[13] Furthermore, other logics that are technically paraconsistent, such as minimal logic, are not <u>interestingly paraconsistent</u> because, although they avoid the disaster of entirely trivialising inconsistent theories, they have the same effect for a whole syntactically determined class of statements. For example, minimal logic would have as holding, in any inconsistent theory at all, all statements of the form $\sim B$ in virtue of its spread principle, $A \rightarrow. \sim A \rightarrow \sim B$.

Beyond this negative data, much less is clear. What should a paraconsistent logical theory be like? There are three fairly well developed answers to this question. What follows will be largely an attempt to explain these three main approaches and to assess, in so far as possible, which is the most viable approach. While no claim is made that these are the only approaches, a well motivated approach fundamentally different from any of these is difficult to envisage, for the following reasons: any adequate logic - adequate that is for the basic relation of deduction - will contain an implication connective, $\rightarrow$, which conforms to Modus Ponens, i.e. $A, A \rightarrow B \ / \ B$.

Hence any adequate paraconsistent logic will have to break negative paradoxes of implication such as $\sim A \rightarrow . A \rightarrow B$, and there are only so many general strategies for doing that are compatible with paraconsistency. A first distinction is between approaches that do not break corresponding positive paradoxes, such as $A \rightarrow . B \rightarrow A$ and therefore are characteristically obliged to sacrifice parts of negation theory, in particular Contraposition, and on the other side, approaches that also defeat positive paradoxes. Approaches of the first type, the *positive-plus* approaches, can, like intuitionism, avail themselves of the full strength of Hilbert's positive logic (or extensions thereof), whereas approaches of the latter type, while they can retain negation theory intact, have to adopt a less extravagant positive logic, in effect either some type of modal system or else a relevant positive logic. The modal approach cannot be quite the usual one - though modal substitutivity conditions can be retained, justifying use of the term 'modal' [14] - because paraconsistency requirements would be violated by the following route through conjunction:-

1. $B \wedge \sim B \rightarrow B$, from Simplification, $A \wedge B \rightarrow A$, a modal thesis.
2. $A \wedge \sim A \leftrightarrow . B \wedge \sim B$, since $A \wedge \sim A$ and $B \wedge \sim B$ do not differ, e.g. in truth conditions, in any modal (i.e. complete possible) worlds, where $C \leftrightarrow D$ is $(C \rightarrow D) \wedge (D \rightarrow C)$.
3. $A \wedge \sim A \rightarrow B$, from 1 and 3 by (modal) substitutivity conditions.
4. $\{C, D\} \models C \wedge D$, i.e. Adjunction, a usual modal rule.
5. $\{A, \sim A\} \models B$, from 3 and 4.

Something has to give, and what has given, and had to give, in the modal approach is Adjunction, so yielding the *non-adjunctive* approach to paraconsistency. For to abandon equivalence 2, and acompanying substitutivity, would be to abandon a modal approach, for something in the order of a relevant one, while to reject 1 would be to opt for connexivism, which, since it blocks inference from inconsistency, is not at all congenial to paraconsistency (as we have already noted), and in any case also leads back to a *broadly relevant*, or connectional, approach.

The three main approaches are accordingly, the non-adjunctive approach, the positive-plus approach (of da Costa), and the (broadly) relevant approach. We will investigate these sorts of systems, as far as possible, via their appropriate semantics since these offer, in our view, the clearest understanding of the strengths and weaknesses of the approaches. In so far as paraconsistent logic differs from classical logic, it does so mainly at the propositional level. Hence our discussion will be primarily focussed on the zero order level. Quantifiers and other first order devices can be added in a fairly obvious and straightforward way to all the systems considered. However, at the zero order level it is useful and illuminating to separate the zero degree fragment from the rest. The zero degree fragment of these systems concerns the purely truth functional connectives, $\wedge$, $\vee$, $\sim$, and a number of the important theoretical disagreements between the approaches appear already at this stage. Higher degrees concern the (iterated) behaviour of implication, $\rightarrow$, the issues concerning which are best dealt with separately. Accordingly, we will start our discussion at the zero degree level and reserve the implicational issues until the next section.

I) Non-Adjunctive Systems

The non-adjunctive approach was pioneered by Jaśkowski[15] (see the first introduction to part 1). The line has been further developed formally by da Costa and a number of co-workers,[16] and had recently appeared again, in thinly disguised form, in the work of Rescher and Brandom.[17] Basically, the idea is as follows: A (piece of) discourse may be produced by a number of different participants. Each contributes to the discourse by producing information which is assumed self-consistent, but which may contradict the information of others. (Perhaps a paradigm example is that of the information presented to a jury at a trial; another example is that of data from different sources fed into a computer). The things that hold in the discourse (or are true in the discourse) are things which are put forward by some participant.[18] How is this approach to be formalized? We may suppose that each participant has a position. This is the story s/he is prepared to tell, the set of things s/he believes etc. and since this is self-consistent, this can be identified with the set of things true in a classical propositional evaluation, or possible world of standard modal logic. The discourse is just the sum of the participants' positions. Hence the things which hold in the discourse are just the things which hold in any one of the worlds which is a participant's position. Consequently let M be a possible world model of some modal logic. Let us say S5 for the sake of definiteness. (Different modal logics will give rise to different paraconsistent logics; we will comment where that difference is of any significance.) The definition of 'A holds at world w ($w \models A$)' is as usual. We will define 'A holds discursively in M ($M \models_d A$)' as follows:

$M \models_d$ A iff for some world w in M, $w \models A$. (α)

We can now define discursive logical validity and discursive logical consequence in the obvious way.

$\models_d$ A iff for all M, $M \models_d$ A.

$\Sigma \models_d$ A iff for all M either $\exists B \in \Sigma$ $M \not\models_d$ B or $M \models_d$ A.

It is evident that the things which are discursively logically valid are precisely the things which are S5 valid. In particular, a purely truth functional, zero degree formula A is discursively logically valid iff it is a two-valued tautology. By contrast the deducibility relation is anything but classical. For quite clearly $\{A, {\sim}A\} \not\models_d B$. A countermodel is easy to specify; it simply reflects the picture of discourse with contradictory inputs.

But although the motivation for discursive paraconsistent logic is clear and intelligible, and has good historical roots, there are grave doubts about its adequacy with respect to the basic motivation for paraconsistency. For a start discursive logic fails to be adjunctive. It is easily seen that $\{A \wedge B\} \models_d A$. However it is equally easy to see that $\{A, B\} \not\models_d A \wedge B$. This means that conjunction has decidedly non-standard behaviour. This by itself may not be a very heavy point. In any paraconsistent logic *something* must behave non-standardly (that is, non-classically). However, in this particular case it casts doubt upon whether conjunction *really is* conjunction in discursive logic. For conjunction just is that connective which has the truth (holding) conditions: $\ulcorner A \wedge B \urcorner$ is true (at a world) iff A is true and B is true (at that world). So something that fails adjunction is not then conjunction. Of course, there is no particular objection to having a non-standard operator '$\wedge$' with curious truth conditions, and hence strange meaning. But it is a serious criticism that '$\wedge$' has *no* recursive truth conditions, i.e. it is impossible to find a condition ψ such that

$$\mathcal{M} \models_d A \wedge B \text{ iff } \psi(\mathcal{M} \models_d A,\ \mathcal{M} \models_d B).$$ [19]

A more important point is however that there can be no objection to there being a genuine conjunction in the system. Perhaps, then, discursive logic just suffers from an omission? Suppose we add a genuine conjunction to the language with the semantic conditions

$$\mathcal{M} \models_d A + B \text{ iff } \mathcal{M} \models_d A \text{ and } \mathcal{M} \models_d B. \qquad (\beta)$$

The problem now is how to define the truth conditions of truth functions of sentences of the form A + B. There are two possibilities.

The first is that we can find some formula of the unaugmented modal language with two propositional parameters, with which + can be identified. Condition (α) then provides the truth conditions of formulas in a straightforward way. This approach has been adopted by da Costa, who defines discursive conjunction, using the possibility functor M, thus:

$$A \wedge_d B = MA \wedge B.$$ [20]

Discursive conjunction can quickly be seen to satisfy condition (β), at least in S5. Actually the lack of symmetry between A and B in the definition of $\wedge_d$ is displeasing and makes the definition appear to float in mid-air. It would be clearer to define

$$A + B \text{ as } MA \wedge MB \qquad (\gamma)$$

Condition (β) is still satisfied.

The problems with this approach to conjunction are two-fold. First, it is totally opaque why a modal functor such as M should poke its nose into the meaning of ordinary extensional conjunction. Granted that (β) fixes the extension of +, (γ) fixes the sense. This makes it quite clear that + is not ordinary conjunction, even though it has the right extension.

Secondly, this approach to conjunction totally destroys the normal relationships between conjunction, disjunction and negation. For example, none of the following holds:

$$\{\sim A\} \models_d \sim(A+B); \{\sim A+\sim B\} \models_d \sim(A \vee B); \{\sim A \vee \sim B\} \models_d \sim(A+B)$$

This is little more than the consequence of the fact that a modal functor has got embroiled in conjunction. It is worth saying again that some classical logical relations will have to go paraconsistently. However, the wholesale destruction of the relations normally taken to hold between conjunction, negation and disjunction clearly speaks against discursive conjunction. This is especially true when there are other options (such as the relevant one) which preserve virtually all these relations.

The other possibility is to refuse to identify A+B with any sentence functor of the unaugmented language, but to give the truth conditions of compounds of + sentences in the usual way, e.g.

$$\mathcal{M} \models_d \sim(A+B) \text{ iff } \mathcal{M} \not\models_d A+B$$

and similarly for conjunction and disjunction. This, at least, preserves all the classical relations between conjunction, disjunction and negation. However, it runs into other problems. In particular, it reinstates a very general form of non-paraconsistency. For it is now easy to see that

$$\{A+B, \sim(A+B)\} \models_d C$$

and as a special case

$\{A \to A, \sim(A \to A)\} \models_d C.$

Now, not only is it difficult to discern a connection between the premisses and the conclusion, but this is little better than the full, horrible, ex falso quodlibet. Should one participant in a discourse say 'It is raining and it is raining' and another say, 'No, that's not the case', the whole thing, quite counterintuitively, blows up. No one who takes paraconsistency seriously can accept this option.

The second objection to approaching paraconsistent logic discursively is more damaging than the first. It concerns the relation of logical consequence which is (as befits a paraconsistent logic!) both too strong and too weak.

First it is too strong. It is easily seen that $\{A\} \models_d B$ iff B is a classical two-valued consequence of A. This means that discursive logic is only half-heartedly paraconsistent. For everything does follow discursively from a conjoined contradiction: $\{A \wedge \sim A\} \models_d B$. What stops discursive logic from lapsing into non-paraconsistency is just the non-standard behaviour of conjunction. Because single premiss discursive validity coincides with classical validity, discursive logic is extremely badly suited to be the underlying logic of some of the most important inconsistent theories.[21] For example, classically $\{\exists y \forall x(x \in y \leftrightarrow x \notin x)\} \models \exists y(y \in y \wedge y \notin y)$, and $\exists y(y \in y \wedge y \notin y) \models B$. Hence if Σ is the set of instances of the abstraction scheme of set theory, $\Sigma \models_d B$. Thus discursive paraconsistent logic is totally unsuitable as the underlying logic of naive set theory. Similarly it is unsuitable as the underlying logic of naive semantics.

Rescher and Brandom try to avoid this difficulty[22] by suggesting that instances of the abstraction scheme which give rise to trouble be split into two halves. Thus the instance generating the Russell paradox becomes the pair comprising $\forall x(x \in R \to x \notin x)$ and $\forall x(x \notin x \to x \in R)$. Set theory is then split into essentially two distinct theories, one of which contains the first of these and the other of which contains the second. Each of these two theories then holds in different a possible world.

In fact this strategy has only the appearance of paraconsistency. In essence it is just a revisionist classical position. For paring an inconsistent theory down to various consistent subtheories is a game[23] that classical set theorists have been playing for eighty years. The classicist is quite happy with both the above fragments of set theory. Hence this line does not take the first motivation, for inconsistent theories,(as opposed to consistent fragments of inconsistent theories) seriously. All that Rescher and Brandom add to the classical position is the insistence that both fragments be true. However, the classicist will understand this as 'true in some possible world' and there will be no disagreement. Neither can the discursivist really object to the classicist understanding. For this is, in effect, what his understanding of paraconsistent truth amounts to as well.

The other side of this objection to discursive logical consequence is that it is too weak. To be exact, let Σ be a non-null set of zero degree formulas and let A be a first degree formula. Then, if $\Sigma \models_d A$ there is some $B \in \Sigma$ such that $\{B\} \models_d A$. To see this, suppose for reductio that there is no $B \in \Sigma$ such that $\{B\} \not\models_d A$. Then for every B we can find a model M_B such that, for some world w in M_B, B is true in w, whilst for no world w, A is true in w. Let M be the collection of all the worlds in every M_B. Then M is a countermodel to $\Sigma \models_d A$.

Hence there is no such thing as a valid multi-premiss discursive inference![24] This shows that as a logic for drawing inferences in real life situations, discursive logic is useless. (This too is important since one of the main motivations for paraconsistency was that useful conclusions should be drawn from actual inconsistent data, e.g. laws, judicial evidence, etc. Paraconsistent logic should, as Jaśkowski puts it, 'be rich enough to enable practical inference'.)[25] For no premisses can be combined to draw conclusions. Conceivably we might consider each of the participants in a discourse to be offering one long conjoined statement. However, by the very motivation, the contributions of each participant are not to be considered as conjoined. What follows in a discourse is all and only what follows from the contribution of any one participant. (The judge cannot infer from the statements of witness A that Jones was in the room and of witness B that no one else was in the room that Jones was the only person in the room!) This shows that discursive logic is not really acceptable even according to its own rationale, namely the drawing of reasonable inferences from inconsistent data from different sources. In fact both the other approaches to paraconsistency we will consider are better suited to this end.

We can sum up the foregoing discussion simply. Discursive logic may be either single premiss or multiple premiss. In the first case it is classical. In the second it is really no logic at all. In neither case it is suitable for the investigation of inconsistent theories. The main problem with the discursive approach is just that it does not take the second, dialetheic, motivation (that there are true contradictions) seriously. Contradictions may be "true" but this amounts to no more than "true in different worlds". Moreover each possible world is as consistent as any classicist could wish: the approach is much too modally based to accommodate inconsistency satisfactorily.[26] For all these sorts of reasons, the non-adjunctive modal approach to paraconsistency should be dismissed.

II) Positive-plus Systems: da Costa's Main Systems

The most detailed study of positive-plus system was initiated (as we saw in a previous introduction) by da Costa, who proposed a family of paraconsistent logics C_i, where $1 \leq i \leq \omega$.[27] The systems differ in points of detail but share the same basic semantical motivation. In fact the axiom systems came first and the semantics only later.[28] But the semantics are the most illuminating path to da Costa's approach, so we will concentrate on these, and in particular the semantics of system C_ω.

Unlike discursive logic, da Costa does take the idea that there are true contradictions seriously. da Costa formalizes this as follows. Given a propositional language, a da Costa evaluation is a function ν which maps every formula to 1 (true) or 0 (false) satisfying the conditions

(1) $\nu(A \wedge B)=1$ iff $\nu(A)=1$ and $\nu(B) = 1$

(2) $\nu(A \vee B)=1$ iff $\nu(A) =1$ or $\nu(B)=1$

(3) $\nu(\sim A) =1$ if $\nu(A)=0$

(4) $\nu(A) =1$ if $\nu(\sim\sim A)=1$

There are also conditions for $\supset$ too. We will consider these later. The above conditions can be shown to be characteristic for the zero degree part of C_ω. The conditions for $\wedge$, $\vee$ are normal ones and ensure that these really are conjunction and disjunction. The deviation from classical logic is only in the conditions for $\sim$. (3) ensures that at least one of A, $\sim$A is true (though both may be). And the rationale for (4) seems to be something like this: if it is not the case that not-A then since (by (3)) one of A and $\sim$A is true, A is true. Logical truth and consequence are defined in the usual way:

$\Sigma \models_C A$ iff for all evaluations ν, either $\nu(A)=1$ or for some $B \in \Sigma$ $\nu(B) \neq 1$.

$\models_C A$ iff for all evaluations ν, $\nu(A)=1$.

Quite clearly, neither $\{A, \sim A\} \models_C B$ nor $\{A \wedge \sim A\} \models_C B$.

The problems with da Costa's approach are perhaps not so obvious as those with the non-adjunctive systems. However, in the end they are equally telling.

The first objection is that condition (4) of the da Costa semantics is ill-motivated. (4) appears to follow from (3). (It does not, since otherwise it would be redundant.) The argument gets by by reading '$\nu(\sim\sim A)=1$' as 'It is not the case that $\sim A$' and then supposing that the latter means $\nu(\sim A)=0$. This is a fallacy of equivocation since the inference from $\nu(\sim\sim A)=1$ to $\nu(\sim A)=0$ is invalid even in da Costa's terms.

Without this argument, the motivation for condition (4) is totally obscure on this approach. If the truth values of A, $\sim A$, and $\sim\sim A$ are independent enough to let all be true, why shouldn't they be independent enough to let the first be false and the last two be true? Compare this with the next approach we deal with, where the connection in truth-value between a sentence and its negation falls, quite naturally, out of the motivating considerations. Of course condition (4) could be dropped from da Costa's semantics. However in that case, negation would have virtually none of the properties traditionally associated with negation. (It has few enough anyway.) This would strengthen our subsequent argument that da Costa's negation is not really negation at all.

The second objection to da Costa semantics is that they are non-recursive. Now whilst non-recursive semantics may be admirable for many technical purposes, there are good reasons for not being philosophically satisfied with them. The arguments are well known but the crucial point is something like this: since speakers of a language are able to understand sentences they have never heard before, the sense or meaning of a sentence must be determined by the senses of its components. In particular, then, an adequate semantics must specify recursively the meaning of a sentence in terms of the meanings of its components. Thus generally speaking the specification of semantic conditions must be recursive. Now da Costa semantics are certainly not recursive since the truth conditions of $\sim A$ are not determined by the truth conditions of A. (If $\nu(A)=1$, $\nu(\sim A)$ could be 1 or 0.) Thus these semantics have problems. This argument against da Costa semantics is not completely conclusive. It could be met by arguing that meaning is not completely determined by truth conditions, and that some other factor, let us call it <u>sense</u>, is involved. It can then be argued that whilst meaning conditions are recursive the truth conditions of a compound may depend upon the sense (rather than the truth value) of its components. In particular the truth value of $\sim A$ may be determined by the "sense factor-X" of A. This general approach to meaning is of course the one adopted in Montague semantics. We will not detour to examine the adequacy of this general approach to the theory of meaning. For it is enough to observe the following: First, even if this approach could be made to work (and it cannot in general[29]), da Costa semantics, as they stand, are radically incomplete.

Secondly, if this approach were to work it would show that $\sim$ is not our friendly neighbourhood extensional negation, but a radically intensional functor of some sort. Of course this point may be countered, but it is the first bit of evidence we will muster to show that da Costa negation is not really negation.

Let us turn from da Costa valuations to the set of zero degree logical truths in da Costa's approach. Since every classical evaluation is a da Costa evaluation then we have that if $\models_C A$, A is a classical two-valued tautology. The converse however, is not true. The most notable exception is the law of non-contradiction:

$$\sim(A \wedge \sim A) \qquad (\varepsilon)$$

The omission of this from a system of paraconsistent logic is not surprising. Nor is it a coincidence that it happens in da Costa's system; for he lays down as a condition of adequacy on a paraconsistent logic that (ε) not be valid.[30] The rationale for the omission of (ε) appears to be clear enough: some statements of the form $A \wedge \sim A$ <u>are</u> true. However, we should proceed with care. This does not settle the matter - even by da Costa's standards. For the fact that $A \wedge \sim A$ is true does not prevent $\sim(A \wedge \sim A)$ from being true too. In fact, insisting that the absence of (ε) be a condition of adequacy on a paraconsistent logic is far too strong. It is quite open for a paraconsistentist to adopt (ε), as the next approach we examine will show. Of course if we do adhere to (ε) then any contradiction $A \wedge \sim A$ (let us call this a <u>primary contradiction</u>) will generate another $(A \wedge \sim A) \wedge \sim(A \wedge \sim A)$ (let us call this a <u>secondary contradiction</u>). However, obviously there is no <u>a priori</u> bar to this for the paraconsistentist.

Is it best then to hold on to (ε) or to reject it? We do not wish to be too dogmatic about this. However, presumably any case against (ε) will hinge on the undesirability of secondary contradictions. Conceivably we might invoke the razor that contradictions should not be multiplied beyond necessity. However, even if this is correct (and is it?) it does not get us very far until we know what "necessity" is. We think the case in favour of (ε) much more plausible. Part of it goes like this. The law of non-contradiction has traditionally been seen as a central property, if not a defining characteristic, of negation. And this is true not only of traditional and classically oriented logicians such as Aristotle and Russell, but also of those who believed in true contradictions such as Hegel.[31] That an account of negation violates the law of non-contradiction therefore provides prima facie evidence that the account is wrong. This is the second piece of evidence that <u>da Costa negation is not negation</u>.

In fact, we can make the claim more precise. Traditionally A and B are sub-contraries if $A \vee B$ is a logical truth. A and B are contradictories if $A \vee B$ is a logical truth <u>and</u> $A \wedge B$ is logically false. It is the second condition which therefore distinguishes contradictories from sub-contraries. Now in da Costa's approach we have that $A \vee \sim A$ is a logical truth. But $A \wedge \sim A$ is not logically false. Thus A and $\sim A$ are sub-contraries, not contradictories. Consequently da Costa negation is not negation, since negation is a contradiction forming functor, not a sub-contrary forming functor.

Let us now turn our attention to the relation of logical consequence. Again it is easily seen that this is a sub-relation of classical two-valued logical consequence. However, the following <u>fail</u>, showing that it is a proper sub-relation.

$$\{\sim A\} \models_C \sim(A \wedge B) \qquad \{\sim(A \vee B)\} \models_C \sim A$$

$$\{A\} \models_C \sim\sim A \qquad \{\sim A \wedge \sim B\} \models_C \sim(A \vee B) \qquad (\kappa)$$

$$\{\sim A \vee \sim B\} \models_C \sim(A \wedge B)$$

Moreover as we shall be able to see later the following also fail.

$$\{A \supset B\} \models_C (\sim B \supset \sim A)$$

$$\{A \supset B,\ A \supset \sim B\} \models_C \sim A \qquad (\lambda)$$

This shows that da Costa negation has virtually none of the inferential properties traditionally associated with negation. (Compare this with negation in the next approach we consider, which has all the above properties.) This is a further piece of evidence suggesting that da Costa negation is not really negation. We have now mustered strong evidence to this effect and the case seems pretty conclusive. It is time to ask what da Costa negation <u>is</u>.

The key to this problem is provided by our discussion of the logical truths. We saw there that da Costa negation behaves like a sub-contrary forming operator, not a contradictory forming operator. Indeed, the truth conditions of negation (3) make this reading of $\sim$ almost mandatory. Hence we suggest that da Costa's negation is an operator which turns a formula into a sub-contrary. This not only explains the truth condition of $\sim$ and the behaviour of logical truths, but is also well confirmed for other reasons. First, if $\sim$ is a sub-contrary forming operator then we should expect all the inferential principles (κ), (λ)

to fail, which they do. Secondly, this fact explains why $\sim$ is not truth functional. For the truth value of a subcontrary of A is not determined by the truth value of A. Thus $\sim$ is not an extensional functor. All this fits the picture.

So, $\sim A$ is a sub-contrary of A, but which? For although the contradictory of a statement is unique, it may have many sub-contraries. Which is $\sim A$? It must be a sub-contrary which satisfies condition (4) of the semantics. However, this is by no means sufficient to determine the functor $\sim$ uniquely. If A and B are <u>any</u> sub-contraries then the functor which maps A to B and vice versa satisfies this condition. There are no other constraints on $\sim$ to determine which sub-contrary functor it is. Hence the answer to this question must be radically indeterminate.

Is the lack of a genuine negation operator in the C systems merely a matter of omission? The answer is a quick and simple 'No'. For if we were to add an operator, $-$, with the obvious conditions for negation,

$$\nu(-A)=1 \text{ iff } \nu(A)=0,$$

it is easy to see that non-paraconsistency would be reinstated. For then $\{A\wedge -A\}\models_C B$. Thus the C systems achieve their paraconsistency only at the cost of dispensing with negation.

So much for da Costa's general approach to zero degree formulas - points that rub off on to the more comprehensive positive-plus approach. Before we set such approaches aside, however, it is worth discussing the way da Costa strengthens system C_ω to produce the systems C_i, $1\leq i<\omega$. For the sake of definiteness we will fix our attention on C_1 (though all the points made apply equally to the others.)

It is clear that on a da Costa evaluation there are two kinds of statements: those that are "paradoxical", i.e. those such that $\nu(A)=\nu(\sim A)=1$ and those that are "classical", i.e. such that $\nu(A)\neq\nu(\sim A)$. Although classical logic does not hold for all sentences, it would be reasonable to suppose that it holds for sentences with classical values. (Actually in C_ω it does not.) Moreover, it is reasonable enough to suppose that this should in some sense be expressible in the language itself. In particular suppose we write 'A°' for 'A has a classical truth value', then the following is reasonable:

If B is a compound of $A_1 \ldots A_n$ and $\Gamma \models_C A_1^\circ \wedge \ldots \wedge A_n^\circ$ then

$\Gamma \models_C B$ iff B is a classical consequence of Γ (ξ).

Achieving this is precisely the motivational move from C_ω to C_1.

First, a "classicality" operator $\circ$ has to be produced. There are two different approaches possible here. The syntactic approach is to identify 'A is paradoxical' with '$A \wedge \sim A$' and hence define 'A has a classical value (A°)' as '$\sim(A \wedge \sim A)$'. The semantic approach is to give the truth conditions of A° directly as:

If $\nu(A) \neq \nu(\sim A)$, $\nu(A^\circ)=1$

If $\nu(A)=\nu(\sim A)=1$, $\nu(A^\circ)=0$.

da Costa wants to follow both these approaches. He defines A° as $\sim(A \wedge \sim A)$. From this, the first of the semantic conditions follows. For if $\nu(A) \neq \nu(\sim A)$, $\nu(A \wedge \sim A)=0$ and $\nu(A^\circ)=\nu(\sim(A \wedge \sim A))=1$. However, the second does not follow in C_ω semantics. Hence it has to be enforced with a new semantical postulate to this effect:

If $\nu(A \wedge \sim A)=1, \nu(\sim(A \wedge \sim A))=0$[32] ($\mu$)

The only other semantical postulates for C_1 ensure that all formulae compounded entirely from formulae with classical values have classical value, thus:

If $\nu(A^\circ)=\nu(B^\circ)=1$ then $\nu((A \wedge B)^\circ)=\nu((A \vee B)^\circ)=\nu((A \supset B)^\circ) = \nu((\sim A)^\circ)=1$.

These conditions ensure that (ξ) holds,[33] thus fulfilling the motivation.

There are a few points to make about extending C_ω with a "classicality" operator. The first is that it in no way affects our conclusions about the interpretation of da Costa negation. Even in C_1, negation is non-extensional, the law of non-contradiction still fails and so do all the principles of inference (κ) and (λ). Thus our conclusion that $\sim$ is a sub-contrary forming operator still stands. (Although, of course, the extra semantic constraints on $\sim$ add some further constraints on which sub-contrary of A, $\sim$A can be.) The second and more important point is that the addition of a classicality operator in this way leads to new trouble. It is easily checked that for any formula B, $\nu(B \wedge \sim B \wedge B^\circ) = 0$ for any ν. Thus

$\{B \wedge \sim B \wedge B^\circ\} \models_{C_1} A$ (χ)

Hence if we can ever prove a theorem of the form $B \wedge \sim B \wedge B^\circ$, things reduce to triviality. But it is easy to produce a theorem of this form in a semantically closed language (as da Costa has noted).[34] By the usual self-referential construction we can find a statement β such that $\beta \leftrightarrow (\sim\beta\wedge\beta^{\circ})$. (This sentence is false and has a classical truth value.) It is then easy to prove with reasoning valid in C_1 (in fact in C_ω) $\beta\wedge\sim\beta\wedge\beta^{\circ}$. A similar argument can be performed in naive set theory. Hence C_1 is entirely unsuited to formalizing two of the most important paraconsistent theories. Moreover, the trouble extends much more widely, to other paraconsistent theories, and to restricted forms of set theory.[35]

This is, of course, a major additional argument against the C_1 approach to paraconsistency. However, there is a more general lesson to be learnt here. Since the situation does not arise in the same way in C_ω, the problem lies with the classicality operator. We can locate the trouble more precisely. In the proof of $\beta\wedge\sim\beta\wedge\beta^{\circ}$, the only fact specifically about β° that is used is that $\sim(\beta^{\circ}) \rightarrow \beta\wedge\sim\beta$ and this is guaranteed by the syntactic definition of A°. What produces the special case of <u>ex falso quodlibet</u> (γ) is precisely the semantic conditions of B°. Thus we see that the semantic approach to a classicality operator, and the syntactic approach are incompatible. Plausible as both may seem, one has to give. Let us move on to the third approach to paraconsistency. This is the relevant one taken by both authors.[36]

III <u>The Relevant Approach</u>

Semantically there are several ways of proceeding. We will choose one that has seemed (especially to the more classically-inclined) particularly simple to grasp.[37] Like da Costa's approach, the relevant approach takes seriously the view that some statements are true <u>and</u> false. However, instead of insisting that every sentence take a unique truth value, it allows statements to have both.

Formally, let $V=\{\{1\}\{0\}\{1,0\}\}$. Here $\{1\}$ is (the classical) true and true only; $\{0\}$ is (the classical) false and false only; $\{1,0\}$ is (the paradoxical) true and false;

A valuation is a map ν from the set of zero degree formulas to V such that

1a) $1 \in \nu(\sim A)$ iff $0 \in \nu(A)$ b) $0 \in \nu(\sim A)$ iff $1 \in \nu(A)$

2a) $1 \in \nu(A\wedge B)$ iff $1 \in \nu(A)$ and $1 \in \nu(B)$ b) $0 \in \nu(A\wedge B)$ iff $0 \in \nu(A)$ or $0 \in \nu(B)$

3a) $1 \in \nu(A\vee B)$ iff $1 \in \nu(A)$ or $1 \in \nu(B)$ b) $0 \in \nu(A\vee B)$ iff $0 \in \nu(A)$ and $0 \in \nu(B)$

Logical truth and consequence are defined in the obvious way.

$\Sigma \models_R A$ iff for all evaluations ν either $1 \in \nu(A)$ or for some $B \in \Sigma$, $1 \notin \nu(B)$

$\models_R A$ iff for all evaluations ν, $1 \in \nu(A)$

It is easy to see that these truth conditions are paraconsistent, i.e. that $\{A, \sim A\} \not\models_R B$. Moreover, the truth conditions look very familiar. Indeed they are just the classical ones. Of course in the classical case the second one of each pair is redundant. However, this is no longer the case when we have grasped the paraconsistent insight that things may be both true and false.

Some of the more important features of the deducibility relation are as follows:

$\{A, B\} \models_R A \wedge B$

$\{A \wedge B\} \models_R A$

$\{A\} \models_R A \vee B$

If $\{A\} \models_R C$ and $\{B\} \models_R C$ then $\{A \vee B\} \models_R C$

If $\{A\} \models_R B$ and $\{B\} \models_R C$ then $\{A\} \models_R C$

$\{\sim A\} \models_R \sim(A \wedge B)$

$\{\sim A, \sim B\} \models_R \sim(A \vee B)$

$\{\sim(A \vee B)\} \models_R \sim A$

$\{A\} \models_R \sim\sim A$

$\{\sim\sim A\} \models_R A$

$\{\sim A \vee \sim B\} \models_R \sim(A \wedge B)$ (and all the other De Morgan principles).

Moreover it is straightforward to establish that $\models_R A$ iff A is a two-valued classical tautology.[38]

These properties make it easy to see that this approach avoids the problems of the two previous approaches. Unlike the non-adjunctive systems, it has an adequate conjunction and a decidedly non-trivial multi-premise deducibility relation. The properties of negation are neat and simple and no extra semantic postulates have to be added, as in da Costa's approach, to ensure bits of double negation. Moreover, there can be no doubt that the negation of this approach _is_ negation. The semantics are recursive and extensional. Thus ~ is not an intensional functor. Both the laws of excluded middle and non-contradiction hold and negation has all the deducibility relations one would expect.[39] Someone might try to make out that the negation of this system is not really negation. But in virtue of all the above points, they would have little ground to stand on. The negation of A is that statement which is true if A is false and false if A is true. But this is exactly what the relevant truth conditions say.

A pleasing feature of the semantics is that the set of zero degree logical truths is exactly the set of classical tautologies. This shows that this is a particularly stable set of formulas valid in both classical and inconsistent contexts. Moreover, it shows that in a sense relevant paraconsistent logic subsumes classical logic at its zero degree level.

Turning to the deducibility relation, it is easy to see that this is a sub-relation of the classical one. Indeed on pain of non-paraconsistency, this must be a proper sub-relation. Those running through the list of valid consequences given above, and not familiar with relevant logic, might wonder exactly what of classical logic is paraconsistently invalid. The answer is that it is the principle of the disjunctive syllogism.

$$\{A, \sim A \vee B\} \models B$$

and its cognates such as

$$\{A, \sim(A \wedge B)\} \models \sim B.$$

This is in fact the only major principle of classical inference that is rejected on the relevant paraconsistent approach. Despite this, its rejection has drawn some fire from various sources. A full discussion of the issue would involve a considerable detour.[40] However, a few points are worth making. First, as we pointed out right at the beginning, if paraconsistency is to be taken seriously, something of classical logic has to be rejected. It is therefore no argument against this approach _per se_ to point out that the disjunctive syllogism is rejected. Indeed the relevant approach holds the losses from classical logic to a minimum at the zero degree level. Both of the other approaches we have considered lose the disjunctive syllogism and much else besides. This is the only loss on the relevant paraconsistent position. Moreover, the loss of the disjunctive syllogism is not as great a blow as might be thought. First, many of the cases of disjunctive syllogism occurring in natural practice use an intensional 'or', $\underline{\vee}$. This can be defined simply

$$A \underline{\vee} B = \sim A \rightarrow B$$

The intensional disjunctive syllogism

$$\{A, \sim A \underline{\vee} B\} \models B$$

is certainly valid.[41] In fact it is little more than _modus ponens_.

The second, and more important reason is that although the disjunctive syllogism is generally invalid, it is usable in certain contexts. The point needs to be handled with some care as a later paper in this collection[42] shows. However, basically the point is this. The reason that the disjunctive syllogism fails is that the sentence A may be paradoxical. If A and ~A are true, then so are A and ~AVB, whatever B is. However, if this case is ruled out no more counterexamples to the disjunctive syllogism can be produced. Thus, provided we are not in a paradoxical situation (i.e. one where A is both true and false), the disjunctive syllogism can legitimately be used. Now it is easy to see that if the disjunctive syllogism is added to zero degree relevant paraconsistent logic, classical logic results. Hence what we see is that in non-paradoxical, consistent contexts (which are of course the only ones countenanced by classical logic anyway) classical logic is acceptable. Thus the general failure of the disjunctive syllogism is not a serious problem.

With the rejection of this - perhaps the major objection to relevant paraconsistent logic - we conclude that the relevant approach is the best one to paraconsistency, at least at the zero deree level.

One final point: if one admits truth-value gluts (i.e. statements that are both true and false), it might seem natural to accept truth-value gaps (i.e. statements that are neither). In fact all the approaches to paraconsistency we have discussed can be modified in fairly obvious ways to allow for this possibility. However, the matter of truth-value gaps is a separate issue, in no way entailed by the paraconsistent position. Accordingly, the issues raised by the modification of these logics to allow for truth-value gaps are not, strictly speaking, relevant to paraconsistency. It is for this reason that we can avoid opening this problem here.[43]

§3. Approaches to Paraconsistent Logical Theory: Implication

So far we have concentrated on features of the various approaches to paraconsistency at the zero degree level. However, all the approaches have distinctive implication operators. This is no accident. Implication is a central logical connective. Any adequate logic must give an account of its behaviour. The classical analysis of the implication operator $\rightarrow$ identifies $A \rightarrow B$ with $A \supset B$ (i.e. $\sim A \vee B$). This results in an equation of *modus ponens*, $A, A \rightarrow B/B$, with the disjunctive syllogism, which (as we saw at the end of the last section) fails in all the semantical approaches we considered. Yet *modus ponens* is the fundamental principle governing implication. No operator which fails to satisfy this can be implication. Hence each of the approaches must find a different, non-classical, account of implication.

I) Non-Adjunctive Systems, such as Jaśkowski's System.

Since the non-adjunctive approaches use possible world semantics, the natural implication operator in this context would certainly seem to be strict implication. Let us define $A \prec B$, as usual, to be $\Box(A \supset B)$. Then it is easily verified that $\{A, A \prec B\} \models_d B$. Observe that although strict implication suffers from paradoxes which _appear_ to make it unsuitable for paraconsistency (e.g. $(A \wedge \sim A \prec B)$), this is not the case _given_ that adjunction fails. Jaśkowski is well aware of this possible definition of implication,[44] though he opts for another possibility which we will discuss shortly. Nonetheless, we should ask whether strict implication is a satisfactory implication operator in the context of discursive logic.

The answer is that it is not. The first point is that although _Modus Ponens_ holds for strict implication if the underlying modal logic is S5, it fails for weaker logics. However, the two most important objections are ones which we will meet several times in this part; hence it is worth giving them names.

The first objection is the _irrelevancy objection_. The point here is that an implication should hold between A and B only in virtue of some common content between A and B. The truth-value of an implication should not depend simply upon the truth-value of one of its components, nor on the modal value of one of its components. Implication is _essentially relational_. This, though fairly banal, runs against classical (though not traditional) orthodoxy. It is a mark of the extent to which indoctrination of the classical view has been effective, that the irrelevancy of classical logic has not been seen as a defect in need of a remedy, and that vast amounts of argument[45] have been necessary to try to reopen people's eyes to the point and to reorient vision towards the True. However, given the enormous amount that has been written on relevance, it would be otiose for us to argue the case for it here again. Let us therefore merely endorse, or re-endorse, the arguments of Anderson, Belnap, Meyer, ourselves and many others, that implication is relevant. Now relevant logic and paraconsistent logic are not the same thing. It is possible to have irrelevant paraconsistent logics (as we are just about to see) and vice versa.[46] Hence relevance is not _de rigeur_ for a paraconsistentist. However, while we are in the process of reworking logic we might as well get implication right - in which case irrelevance is a failure of a paraconsistent logic.

So far so good. But what exactly _is_ the relevance requirement? Again this is a deep question and, since this is a book about paraconsistency, one that we can fortunately largely avoid. For present purposes all that is necessary is a test for irrelevance, and for this the Anderson and Belnap variable-sharing test will do nicely.[47] A sufficient condition for a (purely) propositional logic to be irrelevant is that it have a theorem (logical truth) of the form $A \rightarrow B$ where

A and B have no propositional variable in common. In such a case A and B have no common content. Having got this far it is now easily seen that strict implicaton is irrelevant, even in a discursive context. For $\models_d (A \wedge \sim A) \dashv B$, $\models_d B \dashv (A \vee \sim A)$, and all the other horrors of strict implication. Thus this approach to implication fails the relevancy objection.

The second objection is the Curry objection. There is an argument, due to Curry,[48] which shows that under certain conditions, naive set theory and semantics are trivial, that is, anything can be proved in them. The argument can be put in a number of different forms. Here is one of them.

Let β be the sentence 'If this sentence is true, A is' where A is arbitrary, i.e.

$$\beta = \ulcorner Tr\beta \rightarrow A \urcorner$$

By the truth scheme of naive semantics

$$Tr\beta \leftrightarrow (Tr\beta \rightarrow A) \qquad (1)$$

Hence by absorption $(C \rightarrow (C \rightarrow D))/C \rightarrow D$
from left to right

$$Tr\beta \rightarrow A \qquad (2)$$

So by (1), (2) and modus ponens

$$Tr\beta \qquad (3)$$

and by (2), (3) and modus ponens

A.

Thus if naive semantics is based on a logic which contains modus ponens and absorption, it is trivial. A similar result holds for naive set theory. Now one of the main motives for paraconsistent logic was the investigation of interesting inconsistent theories, of which naive set theory and semantics are perhaps the two most interesting. Thus any logic which contains both modus ponens and absorption is an unsuitable paraconsistent logic. In fact, since modus ponens is essential to any implication operator, it follows that a paraconsistent logic is objectionable if it contains absorption.

It is easily seen that absorption is true of strict implication, i.e. $\{ A ⥽ (A ⥽ B)\} \models_d A ⥽ B$. Hence this is not a suitable paraconsistent implication.

The third and final objection we will present against strict implication is similar to the second but a bit more parochial. For an additional reason, strict implication is quite unsuited for the role of the underlying implication of naive set theory and semantics. This is because $\{A \Leftrightarrow {\sim}A\} \models_d B$, where $\Leftrightarrow$ represents strict coimplication. An application of the abstraction axiom of naive set theory (or the truth scheme of naive semantics), with the implication operator being considered as strict implication, yields $\{ x|x\notin x\}\in\{x|x\notin x\} \Leftrightarrow {\sim}\{x|x\notin x\}\in\{x|x\notin x\}$, whence, again naive set theory and semantics are trivial.

As we said before, Jaskowski did not accept the obvious modal candidate, strict implication, as an account of implication. His candidate for this, called 'discursive implication' ($\supset_d$) is defined as follows:

$$A\supset_d B \text{ iff } MA\supset B.$$

If we recall that the things true at some possible world are the story or position of some participant in the discourse, we can understand Jaskowski's gloss of $A\supset_d B$ as 'if anyone states that A, then B'.[49] Leaving aside the question of the adequacy of this gloss, it is easy to check that discursive implication at least satisfies modus ponens:

$$\{A,\ A\supset_d B\} \models_d B.$$

However, discursive implication fares little better than strict implication. It is straightforward to establish, as Jaśkowski himself did,[50] the following fact:-

Let A be any formula which contains only the connective $\supset$, and let A_d be A, with every occurrence of '$\supset$' replaced by '$\supset_d$'. Then $\models_d A_d$ iff A is a two-valued tautology.

The proof is as follows. Suppose that A is a two-valued tautology. We need to show that for all M, A_d holds in M, which, by the completeness theorem for S5 is true iff MA_d is a theorem of S5. Now consider MA_d. It is easily checked that $\models_{S5} M(A\supset_d B) \Leftrightarrow (MA \supset MB)$. By repeated application of this strict equivalence we can drive all the 'M''s in MA_d inwards as far as possible, replacing all '$\supset_d$''s wi[illegible] '[illegible]''s. We then end up with a formula which is a substitution instance of

A, which is certainly provable in S5. Conversely, suppose that A is not a tautology. Let w be a classical world at which it fails and let M be the model which contains only that world. Then $M \models MB \leftrightarrow B$ and since A_d is obtained from A by the suitable insertion of M's, $M \not\models A_d$.

Thus the pure calculus of discursive implication is just the pure calculus of material implication. It is not true that the $\sim$, $\wedge$, $\vee$, $\supset_d$ fragment of discursive logic is identical with classical logic. (For example, it is easily checked that $\not\models_d A \supset_d (\sim A \supset_d B)$.) However, the initial result is damaging enough. For it shows, first, that discursive implication, like material implication, falls to the irrelevancy objection since for example $\models_d A \supset_d (B \supset_d B)$, and second, that discursive implication falls to the Curry objection since $(A \supset (A \supset B)) \supset (A \supset B)$ is a classical tautology.

Discursive implication does not fall to the other objection mooted against strict implication, but only because of a sleight of hand on Jaśkowski's part. Suppose we were to define discursive equivalence $\equiv_d$ in the obvious way, viz. $A \equiv_d B = (A \supset_d B) \wedge (B \supset_d A)$; then it is easy enough to check that $\{A \equiv_d \sim A\} \models_d B$, and so the objection would apply. Jaśkowski, presumably realizing this (but failing to give any reason) chose to define $A \equiv_d B$ as $(A \supset_d B) \wedge (B \supset_d MA)$. This avoids the problem. However, it produces a lopsided account of equivalence which is, intuitively, a symmetric operation. Moreover it results in the failure of the clearly desirable $(A \equiv_d B) \supset_d ((A \supset_d B) \wedge (B \supset_d A))$, though the converse implication holds.

It might be thought that it would be better to define $A \supset_d B$ as $MA \supset MB$. This receives the perfectly natural gloss 'if A holds discursively, B holds discursively'. So defined it would still satisfy modus ponens and now, moreover, discursive equivalence can be defined in the obvious way without disaster since

$$\{(MA \supset M{\sim}A) \wedge (M{\sim}A \supset MA)\} \not\models_d B.$$

However, this definition of $\supset_d$ would not have solved the other problems. For an argument exactly analogous to the previous one shows that, even as redefined, the pure $\supset_d$ fragment of the theory is the same as the pure material implication fragment of classical logic. Hence the account falls to both the irrelevancy and the Curry objections (see also footnote 187 of chapter 1).

Discursive implication whether defined in Jaśkowski's way or in our suggested way has some other undesirable features. In particular it fails several natural implication rules, e.g.

$\{A \supset_d B\} \not\models_d \sim B \supset \sim A$ $\{A \supset_d B,\ A \supset_d \sim B\} \not\models_d \sim A$

whilst satisfying such curios as

$\models_d (A \wedge \sim A) \supset_d B$ $\models_d A \supset_d (B \vee \sim B)$.

For all these reasons discursive implication is an inadequate account of implication. An obvious question to raise is whether there is any definition of discursive implication which would be satisfactory. Naturally the consequences of each definition have to be looked at separately. Yet it is easy to produce one objection to any definition.

Let $\phi(p,q)$ be any modal sentence with two propositional parameters p,q. Then $\phi(p,q)$ is not a suitable definition of implication. For either $\not\models_d \phi(p,p)$, i.e. implicational identity fails, or $\models_d \phi(p, p)$. In this case let A be any logically true sentence. Certainly $\models_d \phi(A,A)$. Now let q be any sentential parameter not in A. Then since $\models_{S5} A \leftrightarrow q \vee \sim q$, $\models_d \phi(A, q \vee \sim q)$. Thus the implication fails the relevancy objection.

Positive-plus systems similarly fail suitability requirements for paraconsistency, as we will next explain.

II) Positive-plus Systems, such as da Costa's Main Systems

Again we will start with the most accessible da Costa system, C_ω. Semantics for the full system C_ω, including its implication operator, due to Loparic,[51] take the following form: A semivaluation is any map ν from formulas to $\{0,1\}$ satisfying the conditions for a zero degree da Costa evaluation (see above) plus these conditions for $\supset$:

if $\nu(A \supset B)=0$ then $\nu(B)=0$; if $\nu(A \supset B)=1$ then $\nu(A)=0$ or $\nu(B)=1$.

A C_ω valuation is any semivaluation ν such that for any formula B of the form $A_1 \supset (A_2 \supset A_3 \ldots A_n) \ldots)$, where A_n is not of the form $C \supset D$, if $\nu(B)=0$ there is a semivaluation ν' such that $\nu'(A_i)=1$, for each i such that $1 \le i < n$, and $\nu'(A_n)=0$. Logical truth and consequence are now defined in the usual way.

These semantics for the full C_ω are not, on their own, particularly illuminating. Hence we will depart from our usual practice of analysing logics via their semantics and approach C_ω instead by its proof theory. The standard axioms for C_ω are as follows.[52]

1. $A\supset(B\supset A)$ 5. $A\supset(A\vee B)$ $[B\supset(A\vee B)]$

2. $(A\supset B)\supset((A\supset(B\supset C))\supset(A\supset C))$ 6. $(A\supset C)\supset((B\supset C)\supset((A\vee B)\supset C))$

3. $(A\wedge B)\supset A$ $[(A\wedge B)\supset B]$ 7. $A\vee{\sim}A$

4. $A\supset(B\supset(A\wedge B))$ 8. ${\sim\sim}A\supset A$

The only rule of inference is modus ponens for $\supset$.

Those who know their intuitionism will recognize that axioms 1 and 2 are axioms for the pure calculus of intuitionistic implication and axioms 1-6 are axioms for the positive intuitionist calculus. Thus C_ω contains both these theories. In fact the axioms suggest that the implicational fragment of C_ω is exactly the pure calculus of intuitionist implication and the positive part of C_ω is exactly the positive part of intuitionistic logic. Indeed Loparic's semantics can be used to show that this suggestion is correct. C_ω is a conservative extension of positive intuitionistic logic.[53]

Thus we see that C_ω is essentially positive intuitionist logic plus the "negation" operator - really a subcontrary operator - $\sim$. As is well known, neither 7 nor 8 is intuitionistically valid, though their "opposites" ${\sim}(A\wedge{\sim}A)$ and $A\supset{\sim\sim}A$ are. This shows a certain symmetry between the negation of C_ω and of intuitionist logic,[54] which fits in well with the discussion of C_ω negation in §2(II). For intuitionistic negation is plausibly seen as a contrary-forming operator (rather than a contradictory-forming one): $A\wedge{\sim}A$ is logically false and $A\vee{\sim}A$ is not logically true; and the connection between modal logic and intuitionist logic suggests that the intuitionist negation of A is to be understood as something like '${\sim}A$ is provable' or 'A will never be true'; both of which are contraries of A (at least as normally understood). The "opposite" of a contrary forming operator is a sub-contrary forming operator. And this is exactly what we argued the negation of C_ω to be.

Having got all this straight, we can now see quickly that the implication operator of C_ω is inadequate. For it, like strict implication, falls to both the irrelevance objection (since intuitionist logic contains irrelevancies such as $A\supset(B\supset B)$ and $C\supset(A\supset(A\vee B)))$ and the Curry objection (since it contains $(A\supset(A\supset B))\supset(A\supset B))$.

The transition from C_ω to C_1 (and the other C_1 systems) does not make matters any better; in fact it makes them worse. For if we add to the C_ω axioms those required for the C_1 classicality operator, viz.

$B^{\circ} \supset ((A \supset B) \supset ((A \supset \sim B) \supset \sim A))$

and $A^{\circ} \wedge B^{\circ} \supset ((A \wedge B)^{\circ} \wedge (A \vee B)^{\circ} \wedge (A \supset B)^{\circ} \wedge (\sim A)^{\circ}))$ then Peirce's law $((A \supset B) \supset A) \supset A)$ becomes provable and hence C_1 contains classical material implication.[55] In fact if we add the semantical postulate for the classicality operator to those for the Loparic semantics of C_{ω} we can then simplify the semantic condition for $\supset$ to the classical

$$\nu(A \supset B)=1 \text{ iff } \nu(A)=0 \text{ or } \nu(B)=1$$

and the difference between valuations and semivaluations vanishes. These semantics can then be used to show that the positive fragment of C_1 is exactly the positive fragment of classical two-valued logic. Thus C_1 is exactly classical positive logic plus da Costa "negation".

The fact that C_{ω} (C_1) contains conservatively the positive fragment of intuitionist (classical) logic, is no accident. For one of da Costa's motivating principles for the construction of the C systems is that they 'must contain the most part of the schematic rules of...[classical logic] which do not...[interfere with their paraconsistency, or make $\sim(A \wedge \sim A)$ provable]'.[56] Thus he is committed to a very strong implication operator. This is a mistake, not only because strong implication operators are irrelevant, but because this very fact forces on da Costa his inadequate treatment of negation. For example, the fact that the theory contains the paradoxical $A \supset (B \supset A)$ means that contraposition must fail. For that (together with the transitivity of implication) leads immediately to the paraconsistently unacceptable $A \supset (\sim A \supset \sim B)$. Similarly the fact that the theory contains the paradoxical $A \supset (B \vee \sim B)$ means that either contraposition or De Morgan's law must fail. For if they held, we would have both $\sim(\sim B \vee B) \supset \sim A$ and $B \wedge \sim B \supset \sim(\sim B \vee B)$, giving the paraconsistently unacceptable $(B \wedge \sim B) \supset \sim A$. The same point may be made about the failure of contraposition in discursive logic. Moreover, the fact that $\models_d (B \wedge \sim B) \supset_d A$ forces a discursive paraconsistentist to give up the law of adjunction $\{A,B\} \models_d A \vee B$. Thus although relevance is an issue separate from paraconsistency, a cavalier attitude to relevance causes infelicities, at least, in a paraconsistent logical theory. Neither material nor intuitionist nor strict nor discursive implication is a suitable account for a paraconsistentist.

III) The Relevant Approach

All this forces us back to the third paraconsistentist approach to implication: through a relevant implication. For our present purposes we again take a broadly relevant propositional logic to be one satisfying the Anderson and Belnap variable-sharing condition.[57] Clearly, any relevant logic will avoid the paraconsistently execrable <u>ex falso quodlibet</u> and therefore will be a <u>prima facie</u> candidate for a paraconsistent logic. However, there are many approaches to relevant logic. These may be usefully classified for present purposes as follows:

Accept Disjunctive Syllogism → Retain Transitivity → Connexive positions
Accept Disjunctive Syllogism → Retain Transitivity → Conceptivist (Parry) systems
Accept Disjunctive Syllogism → Reject Transitivity → Sieve positions

Reject Disjunctive Syllogism → Retain Absorption → Anderson-Belnap systems
Reject Disjunctive Syllogism → Reject Absorption → Depth relevant logics

Several of these approaches are not adequate for paraconsistency (and sometimes in their own terms). So much we have already seen in the case of connexivist and conceptivist systems, which allow the spread of inconsistency.

The third approach to relevant logic insists that a suitable logic should be obtained by imposing a condition of relevance or relatedness (as a sieve) on classical truth preservation.[58] Thus $A \rightarrow B$ is supposed to hold if A materially (or strictly) implies B and $R(A,B)$ holds where R is some suitable relation of relevance, usually taken to be some kind of meaning connection.[59] This approach we take to be fundamentally misguided, for a number of reasons. Here are some.

First, such approaches normally (and with superficial plausibility) take variable-sharing to be a sufficient criterion for relevance. If it is, then all of $(A \wedge B) \rightarrow B$, $A \rightarrow (A \vee B)$, $(A \wedge ({\sim}A \vee B)) \rightarrow B$ come out as relevantly valid. But these plus the transitivity of '$\rightarrow$' lead, by the usual Lewis argument to $(A \wedge {\sim}A) \rightarrow B$, which is clearly irrelevant. Thus, the transitivity of implication has to be given up.[60] This seems to be such a fundamental principle of implication, almost as fundamental as <u>modus ponens</u>, that it should be given up only under the most extreme of circumstances. Since there are other approaches which validate transitivity, these circumstances do not obtain.[61]

Secondly, although such approaches rather automatically avoid the irrelevance objection, they do not escape the Curry objection. For Absorption, $A \rightarrow (A \rightarrow B) \rightarrow . A \rightarrow B$ is a thesis of such systems along with <u>Modus Ponens</u>. For $A \rightarrow (A \rightarrow B) \rightarrow .$

$A \to B$ is a classical (or strict) thesis, and antecedent and consequent are related, i.e. $R(A \to (A \to B), A \to B)$, since $R(A \to B, A \to B)$ (because R is reflexive) and so relates the consequent of $A \to (A \to B)$ to $(A \to B)$. Thus such systems are quite unsuitable for major paraconsistent purposes.[62]

Thirdly the relevance relation R would seem to be, what it is usually taken to be, symmetrical. (If it is not, the nature of relevance becomes obscure.)[63] Now consider the clearly true: 'Today is Monday' implies 'Tomorrow is not Monday'. Let us write this as $A \to B$. Then since it is true, R(A, B) holds, and since R is symmetrical R(B,A) holds. Now suppose it is Sunday, then B is false. Thus the inference from B to A is materially truth preserving. Hence it is true that 'Tomorrow is not Monday' implies 'Today is Monday' - an obvious absurdity. A similar example can be made to work against those who wish to impose relevance on top of strict implication. '31 is an even number greater than 2' implies '31 is a composite number'. However '31 is a composite number' does not imply '31 is an even number greater than 2'. Thus relevance is not an extra condition to be tacked on, on top of truth preservation.[64] Rather relevance should be defined, as traditional logic has it, in terms of implication.[65]

A more enlightened approach to relevant logic is that of Anderson and Belnap, who start by trying to give an account of implication.[66] Their approach never, however, took due account of paraconsistency, and all their systems of relevant logic, namely E and T and R, fall to the Curry objection. All contain the offending rule $A \to (A \to B) / A \to B$.[67] Thus a suitable relevant paraconsistent logic can be found only in systems weaker than E, T and R, which have come to be known as 'depth relevant logics'.[68] Again there are a number of different approaches to these, and since our aim is not to give a survey of relevant logics, we will just outline one, which has a strong intuitive content.[69]

Let L be a language. Where A is a sentence of L, let [A] be the sense or objective content of A. Let $\leq$ be the relation of sense containment, i.e. $[A] \leq [B]$ iff the sense of A contains that of B (i.e. all the content of B is included in that of A). Clearly $\leq$ is a partial ordering. Moreover, assuming that the sense of a compound is a function of the senses of its parts, we can define the functions $\cup$, $\cap$ and $*$ thus: $[A] \cup [B] = [A \vee B]$; $[A] \cap [B] = [A \wedge B]$; $[A]^* = [\sim A]$. It can be convincingly argued that these operations turn the partial ordering of senses into a De Morgan lattice, i.e. a distributive lattice for which $\cup$ is the join, $\cap$ is the meet, and $*$ is an involution, i.e. a function such that $a^{**} = a$ and if $a \leq b$ then $b^* \leq a^*$. Thus, for example, the sense of $A \wedge B$ contains both the sense of A and that of B. Moreover anything that contains both senses also contains that of $A \wedge B$. Thus a De Morgan lattice can be seen as a lattice of senses.

Now an algebra of senses allows us to define entailment in a very natural way. For it is plausible to suppose, as many have done, that an entailment is true precisely if the sense of the antecedent contains that of the consequent. Thus $A \to B$ is true iff $[A] \leq [B]$. Formally, if T is the set of senses of true sentences, $[A \to B] \in T$ iff $[A] \leq [B]$. It is also reasonable to suppose that T is at least a prime filter on the lattice, i.e. that $a \cap b \in T$ iff $a \in T$ and $b \in T$; $a \cup b \in T$ iff $a \in T$ or $b \in T$; and if $a \in T$ and $a \Rightarrow b \in T$, $b \in T$ (where $[A] \Rightarrow [B]$ is $[A \to B]$).

Further details of the lattice and truth filter T are more negotiable, but it is already clear that these semantics show all the following to be logically true

$A \to A$, $A \to \sim\sim A$, $\sim\sim A \to A$, $A \wedge B \to A$, $A \to A \vee B$

and the following inferences to be truth preserving

$A \to B,\ B \to C / A \to C$ $\qquad$ $A \to B / \sim B \to \sim A$

$A \to B,\ A \to C / A \to B \wedge C$ $\qquad$ $A \to C,\ B \to C / A \vee B \to C$.

$A,\ A \to B / B$

which is what we would expect of an entailment operator $\to$. Hence it is clear that these details provide the basis of a semantics for entailment.

It may not be clear how these semantics relate to those for the zero degree case we discussed in §2(III). The connection is this:-[70] Suppose we define a map ν from zero degree formulas to $\{0,1\}$ as follows: $1 \in \nu(A)$ iff $A \in T$; and $0 \in \nu(A)$ iff $\sim A \in T$ Then ν is a zero degree valuation of the kind specified in §2(III). To be more precise the semantics as specified make ν a map to $V \cup \{\phi\}$, thus allowing for truth value gaps (see fn.43). The further condition: $a \in T$ or $a^* \in T$ makes ν a map to V. Thus these semantics subsume the zero degree semantics and extend them to higher degrees.

An implication based on these semantics is very satisfactory for paraconsistent purposes and suffers from none of the problems of the implications of the previous two approaches: it is relevant; the Curry-paradox generating $A \to (A \to B) / A \to B$ fails; negation has the right properties (contraposition, De Morgan, double negation), etc. Moreover, as we shall see subsequently, naive set theory and semantics based on this kind of relevant logic, though they may be inconsistent, are provably non-trivial.[71] Hence we conclude that this is the most suitable approach to implication for paraconsistent purposes, and that, more generally, the relevant approach to paraconsistency is the most satisfactory one.

NOTES

1. Berkeley [1734]. Further details of the story can be found in Boyer [1949].

2. See Lakatos [1970] §3(c2).

3. See e.g., Feyerabend [1978] IV.

4. See Priest [1980].

5. See the Introductions to parts 3 and 4 of the book.

6. For Riggs and Palmer, see 115 N.Y. 506, 22 N.E. 188 (1889) and Dworkin [1977] p.23. On the Proclamation of Emancipation, see Hook [1962] p.28. The section in which this point is made contains a discussion of several other inconsistencies in the American Bill of Rights.

7. For many examples of inconsistent sets of beliefs, see R. and V. Routley [1975]. For elaboration of the computer example, see N.D. Belnap Jr. [1977].

8. See Hegel, [1812] Vol. 1 Bk. 2 Ch.2 §C.

9. See the second introduction to part 1 of the book, where also further discussion of Hegel and Zeno, may be found.

10. This example is much further developed in the introduction to part 4, where two other plausible examples of true contradictions are given, e.g. those posed by certain impossible objects supplied by Meinong's theory of objects.

11. For further discussion see Priest [1980].

12. This is one of the many reasons for doubting the adequacy of such positions.

13. Details of these logics, and proofs that they fail to meet paraconsistency requirements, are given in chapter 2 of R. Routley et al. [1983]. See especially p.93 and p.101. Connexive logics are flawed paraconsistently because they admit the inference, $\{A, \sim A\} \models B$. The point, argued syntactically (p. 93), may be reargued semantically, as follows: Since in connexive logics A and ~A cancel one another, A and ~A are never designated together, and $A \wedge \sim A$ is always non-designated. Thus both $\{A, \sim A\} \models B$ and $\{A \wedge \sim A\} \models B$ hold (on designation-preserving accounts), and connexive logics are not paraconsistent. Quite apart from this, connexive logics would be pretty useless for genuine dialectic purposes. For, as with one of the leads Wittgenstein pursued (discussed in an introduction to part one of the book), contradictions stop things, so undercutting much legitimate reasoning. The argument that Parry logics and the like (e.g. Zinov'ev's system) fail to be paraconsistent, also given syntactically (p.101), may likewise be reworked semantically. For on the so far received semantics for these systems, A and ~A are never designated together, and $A \wedge \sim A$ is not designated.

14. See, for instance, the account given of modal logics in RLR, chapter 1, upon which the argument in the text depends.

15. Jaskowŝki [1948].

- See e.g., da Costa and Dubikajtis [1968], Kotas and da Costa [1978].

- See Rescher and Brandom [1980]. A more sophisticated form is to be found in Schotch and Jennings [198+]. Most of our criticisms apply to them too.

- The idea can be traced back at least to the Jains. It reappears in classical Greek thought: see the first introduction to part 1 of the book.

- Why non-recursive semantics are philosophically unsatisfactory is explained below. The truth of the claim is easily seen from the fact that we may have $M \models_d B$ and $M \models_d C$ whilst $M \models_d A \wedge B$ but $M \not\models_d A \wedge C$. Thus there is no such condition ψ in the standard (extensional) set-theoretic metalanguage of modal logic.

- See e.g. da Costa and Dubikajtis [1968].

- The charge is serious since this is one of the prime motivations of paraconsistency, and one moreover cited by Jaśkowski, [1948], p.143 ff.

- Rescher and Brandom [1980] ch.10. An analogous proposal, which drastically weakens set theory, was earlier investigated by Gilmore [1973] in his partial set theory.

- It remains an important game, which with paraconsistent theory can be tackled in a thoroughly systematic way for the first time. For the theory to be partitioned can now be formalised nontrivially.

- Jennings and Schotch's account avoids this problem.

- Jaśkowski [1948], p.145.

- A detailed critique of modal approaches to nonmodal matters (such as deducibility, entailment, and paraconsistency) is included in RLR, chapter 1, especially 1.6. This is not to imply that no intensional functors violate adjunction: some certainly do, but their proper treatment is not a modal one.

- He has written a number of papers on the subject. The best place to start is with da Costa [1974]. The C systens are not the only paraconsistent systems due to da Costa: he is also jointly responsible with Arruda for the basic P systems; see the first introduction to part one of the book. A rather different positive approach is that of Peña [198+]. See especially the appendix.

- See da Costa and Alves [1976], Loparić [1977].

- In fact it can be shown that under the very weak condition that no non-theorem has the same sense as any theorem, the slightly stronger da Costa system C_1 has no non-trivial recursive sense-semantics. For suppose it did. Let S be the set of senses (subsets of possible worlds or whatever) and let $\cong$ be the relation which holds between two formulas if they have the same sense. Then the set of formulas factored by $\cong$ would be a non-trivial quotient algebra for C_1. But there is no such algebra for C_1, as Mortensen [1980] shows.

- da Costa [1974] p.498.

- At least for formal logic. See Hegel [1830], note on §20, p.32.

. The da Costa formulation is the equivalent: if $\nu(A \supset B) = \nu(A \supset \neg B) = 1$ then $\nu(A) = 0$.

. See da Costa [1974], theorem 18.

. See da Costa [1974], p. 505.

. See, in particular, Arruda [1982]. See also Arruda and Batens [198+].

. See, e.g., Priest [1980a] and Routley [1979].

. This is the way of Priest [1979], though we formulate it in the manner of Dunn [1976]. For an alternative approach to the semantics of zero degree relevant logic, see Routley and Meyer [1972]. These and other approaches are elaborated and discussed in RLR 3.1 and 3.2.

. See Priest [1979] Theorem III.8.

. All the relations (κ) of §II hold. When we come to implication we will see that those of (λ) hold too.

. Further discussion is to be found in Priest [198+] and Routley [1978]. A detailed discussion may be found in RLR.

. See Anderson & Belnap [1975] §16.3.

. Priest [198+].

. As an example of how the semantics may be modified, consider those given in this section. We merely extend V to include the empty set (as in Dunn [1976]). All else remains the same. It is easy to check that the main effect of this is to ensure that $\not\models v \sim A$ and in fact that there are no theorems at all! The consequence relation is of course still non-trivial. A similar phenomenon occurs when the semantics are extended to allow for an implication operator. In this case although the logic now has theorems there are no purely extensional ones, i.e. ones containing only $\wedge$, $\vee$ and $\sim$. The holding or failing of purely extensional theorems is of little technical relevance to paraconsistency: it can be done either way. However, for reasons indicated in the text we think that a semantics which does not validate the laws of excluded middle and non-contradiction opens itself to the charge that it has not given a semantic account of negation. A discussion of truth value gaps in the context of Meinong's theory can be found in EMJB §1.2.

. See Jaśkowski [1948] p.147.

. For example see Anderson and Belnap [1975], Routley and others [1983], Routley and Norman [1983].

. For example, Ackermann's original Strengen Implikation in [1956] which uses the disjunctive syllogism in rule form.

. See Anderson and Belnap [1975] p.32f.

. See Curry [1942]. Different versions are given in Priest [1979] and Meyer, Routley and Dunn [1979].

. See Jaśkowski [1948] p.150.

. Jaśkowski [1948], Theorems 1, 3.

. See Loparić [1977].

, See da Costa [1974].

. See Loparić [1977] p.838.

, Giving some warrant to the label 'anti-intuitionistic' sometimes applied to logics like the C systems.

. See da Costa and Guillaume [1965].

. da Costa [1974] p.498.

, As before a purely propositional logic is (weakly) relevant iff there is no theorem of the form A→B, where A and B have no propositional variable in common.

. For examples of this approach see Epstein [1979], Copeland [1980], Epstein and Szcerba [1979].

, In fact various conditions of relevance (beginning with simple syntactical requirements such as variable overlap or inclusion) can be imposed on a wide variety of logics. The more general procedure enables not merely various nontransitive logics, but also certain Parry and depth relevant logics, to be represented as imposing a filter on classical or modal logics.

. Non-transitive theories of implication, especially a feature of the Cambridge tradition go back at least to Strode in the Middle Ages. Typically nontransitivity resulted by adding further requirements to material or strict implication, often epistemic requirements concerning ways of coming to know the truth of the implication. The initially unspecified relation R, of relational implication, is simply the latest, and in some respects the crudest, of these attempts to filter out the bad guys, but like the usual sieves fails abysmally in this task, admitting Disjunctive Syllogism and eliminating Transitivity. On the respective virtues of these principles, see RLR, chapter 2.) Another subtler way of instituting the filter, with a rather similar outcome, however, is that of Tennant [1982]. The resulting relevant system satisfies Disjunctive Syllogism, at the expense of faulting transitivity. Thus, the system is unsuitable for the representation of many inconsistent theories which are closed under logical consequence. Furthermore the system is open to the Curry objection.

. The case for transitivity and against its rejection is developed at length in RLR, especially in the initial parts of chapter 2.

. Some of the leading exponents of relatedness logics in fact seem oblivious to the fact that an important role for deductive - and also inductive - logic is reasoning from, or in the presence of, inconsistency. Otherwise, presumably, Woods and Walton would not have begun their text on fallacies with the following unclassified fallacy:

> Unlike deductive or inductive logic, the plausible model of argument allows us to deal with cases where we are confronted with contradictions ([1982] p.vii).

. A relation R for which symmetry is not required is considered by Epstein [1979]. But it remains uninterpreted, other than merely formally, whereas the symmetric relation can be interpreted in terms of shared topics or subject matter. Naturally there are nonsymmetric relations of interest here, e.g. the transitive relation of variable inclusion, important for some Parry logics. But these are not, or not merely, relevance relations. (For a discussion of Parry logics see RLR.)

. There is much else wrong with the tack-on idea, as for instance implemented in relational logic. For example, it validates Ackermann fallacies, such as $A \rightarrow. (A \rightarrow B) \rightarrow B$, which are also readily counterexampled as implicational and conditional principles. It also fouls up expected and legitimate substitution conditions, such as inter-replacement of co-entailments, and can interfere with substitution on variables.

. Thus, for example, A is (implicationally) relevant to B iff A is not independent of B, i.e. iff A is either a superimplicant, subimplicant, equivalent, contrary, subcontrary or contradictory of B, i.e. iff one of $A \rightarrow B$, $B \rightarrow A$, $A \rightarrow \sim B$, $\sim A \rightarrow B$ holds.

. See Anderson and Belnap [1975].

. Even a version of R without the absorption axioms has a version of the Curry paradox: see Slaney [198+]. Further arguments against the Anderson and Belnap systems and in favour of depth relevant logics can be found in Priest and Routley [1982].

. For the term see Brady [1982].

. This is given in Priest [1980a]. It is similar to the range semantics of Routley and Routley [1972] with the restriction to first degree wff lifted, something in effect carried out - and done explicitly for the dual content semantics - in Routley [1978], where further details may be found. Moreover these semantics are readily embedded in more familiar semantics for relevant logics; see e.g. RLR, chapter 3.

. See Priest [1980a] Appendix.

. See Brady [198+].

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CHAPTER 4: APPLICATIONS OF PARACONSISTENT LOGIC

1. Introduction: the variety and types of applications.

The most important application of paraconsistent logics is their application to possibly inconsistent theories. However one needs to interepret "theories" here fairly liberally, as any body of doctrine, statements, axioms etc. which can be thought of as inferentially closed. The theories can be historical, current, embryoic or merely entertained. Of course the formalization of such theories often requires much wider logical apparatus than the mere first order deductive logic discussed in the introduction to part two of the book. This may include probability, inductive logic, the logic of various modalities and other intentional notions such as belief, and so on. Such things, or at least some of them, have been considered by logicians. But, by and large, the logical theories produced have been tuned to classical or at least intuitionist logic. This is singularly inappropriate since as often as not, the material to which the logical apparatus is applied is inconsistent, as we shall see. Accordingly the ideas of paraconsistency need to be applied to the logical theories of modality, probability, etc. themselves to produce adequate logical machinery. In this essay we will consider first some interesting inconsistent theories, some of them in some detail, and then move on to consider the remodelling of various logical theories. It should be stressed that the studies of many applications mentioned are in their infancy, and we can often do no more than make suggestions for the directions of future research.

2. Historical and extant inconsistent theories.

There is a wide variety of inconsistent but non-trivial theories, some of them important. And some of them are true. Some of these important true theories, such as naive set theory, have been alluded to already in previous introductory sections. Of course paraconsistentists are not committed to the view that all contradictory theories are true - or even, if their position is weak enough, that _any_ are true. Thus in many cases the formal reconstruction or paraconsistent representation of an inconsistent theory may be of mainly or merely a matter of historical interest. Formalisations of Bohr's theory of the atom, Spinoza's _Ethics_ and Aristotle's theory of motion, inconsistencies and all, would be enterprises of this kind. These theories are no longer alive, or active. However an important philosophical consequence of paraconsistency is that it allows the refloating of certain historical theories which had been pronounced dead prematurely. Meinong's theory of objects (which we will discuss in the introduction to the next part) is a theory of just this kind (it went wrong because of imported features of the prevailing logical paradigm). For some theories, such as the early theory of infinitesimals, it may not be clear which of these classes, the alive or the dead, they are in. Fortunately this is an issue we do not need to try to settle. We will start by giving a general overview of the range of inconsistent theories and then consider a few of the more interesting ones in detail.

Inconsistent theories are to be found in almost every discipline, but especially in

α) Philosophy and theology.

Among philosophical theories of this type we might mention the theories of Heraclitus, Hume, Hegel, Frege, Meinong and Wittgenstein, and some dialectical theories of change. We will not examine these theories here, since most of them are discussed elsewhere in the introductory chapters.[1] Some of these theories have parts which are of live interest for the development and elaboration of new theories.

Within this general category of philosophical theories we might also put certain theologies. In fact, most sophisticated theologies are inconsistent. Some, such as Christianity, run into inconsistency over issues such as the Trinity, the substantiality of God and the humanity of Christ. Other religions such as varieties of Buddhism, especially Zen, seem to court contradictions of the "mind is no-mind" type. In fact any religion which posits the existence of an all powerful God will run into the standard paradoxes of omnipotence (e.g. that He can invent a problem that He cannot solve, produce an immovable object, etc.); and in a similar way the assumption of an omniscient God is open to paradoxes of omniscience.[2]

In virtue of this sort of problem, some theologies, both medieval and modern, have bitten the bullet and allowed God to be an inconsistent object, though this solution may create more theological problems than it solves (e.g. how such an object can then exist, or be worthy of worship; how God can interact with the world at all). Undoubtedly the philosophically most sophisticated of these inconsistent theologies is Hegel's, where God is identified with the Absolute, with Self-Positing Spirit. Indeed inconsistency is part of the very essence of Hegel's Absolute.[3]

It is not merely specific philosophical or scientific theories that are inconsistent and call for applications of paraconsistent logic: the more general theory of theories is also of this sort.[4]

β) Natural and social sciences.

As we have already begun to see, the sciences too have produced their share of inconsistent theories. For example: Bohr's theory of the atom, some versions of the Everett-Wheeler interpretation of quantum mechanics, and some other parts of quantum mechanics which involve causal anomalies or the Dirac δ function.[5] Some of these theories are certainly still of live interest. Travelling a bit further back into history it is quite arguable that Copernicus's joint theory of astronomy and dynamics was inconsistent, as were versions of the phlogiston theory, some theories of light, very likely Aristotle's theory of motion, and certainly earlier theories of motion which admitted Zeno's arguments.

The social sciences too have their share of contradictory theories. In particular, Freudian metapsychology is inconsistent. More generally any sociology or economics based on Marx's theory of alienation is inconsistent.[6] Similarly inconsistent is any theory incorporating conditions such as those unearthed by Arrow, for a general social or environmental theory.[7]

γ) Logic and mathematics.

A further class of theories, especially rich in contradiction, is that belonging to logic and mathematics. In this area we should cite, yet again, semantics, the theory of attributes (and of propositions), set theory, and the early theory of infinitesimals. It is almost certain that many other branches of mathematics - perhaps most - were inconsistent in their early versions. Recently research into the history of the growth of mathematics (as in Lakatos [1978]) seems to confirm this theme. However, the rewriting, and consequent transformation, of classical mathematics which has dominated this century, from Principia Mathematica to Bourbaki, has made the true historical situation more difficult to ascertain. But anyone who thinks that mathematics has always been done à la Bourbaki is guilty of serious historical anachronism.

3. A more detailed look at some of these theories.

Obviously we cannot (for want both of space and of research time) discuss each of the above theories in detail. However, to give just a flavour of the investigation of inconsistent theories we will look at a few of them in more detail. In particular we will look at semantics, set theory, the infinitesimal calculus and some bits of quantum theory. We intend our discussion to be as neutral as we can make it with respect to the underlying paraconsistent logic we employ. But we will assume for the sake of definiteness that we are basing our theory on a suitable quantificational extension of a relevant logic such as that discussed in the introduction to part two (§3 III). This is certainly the most versatile paraconsistent logic as well as the most philosophically adequate (as we argued in the introduction to part 2). If another sort of paraconsistent logic is not suitable, we will mention this explicitly.

α) Naive semantics.

Semantics is the theory of satisfaction, truth, denotation and other relationships between language and the world. No classical theory can adequately express its own semantics, on pain of inconsistency. However, an inconsistent theory obviously can be allowed to express its own semantics. And this is precisely what naive semantics does. Naive semantics is the theory of truth, satisfaction, denotation, definition, etc., which is capable of giving a semantics for itself.

This theory must be paraconsistently based because of the semantic paradoxes, and cannot be based on any paraconsistent logic which contains the absorption principle $A \to (A \to B) \,/\, A \to B$. This is because such a principle would trivialise the theory (see again the introduction to part 2 §3 III). There remains however a good deal of scope for formalising the theory in different ways: with one satisfaction predicate or many; with infinite sequences, finite sequences, and no sequences; as a many sorted theory or as a single sorted theory. We have chosen a way that seems particularly simple.

For every $n \geqslant 0$ the theory has an $n+1$ place predicate Sat_n such that $Sat_n\, y\, x_1 \ldots x_n$ is thought of as "$x_1 \ldots x_n$ satisfy y". In this context y is thought of as a formula with n free variables. We may suppose that if y has the wrong number of variables or indeed is not a formula at all, $Sat_n\, y\, x_1 \ldots x_n$ is false. The only other non-logical symbol is a functor $\ulcorner\ \urcorner$ satisfying the following formlulation clause:

if φ is any formula or term, $\ulcorner\varphi\urcorner$ is a closed term.

$\ulcorner\varphi\urcorner$ is thought of as the name of φ.

The theory has the family of axiom schemes:

$$Sat_n\ulcorner\varphi\urcorner x_1 \ldots x_n \leftrightarrow \varphi(v_{i_1}/x_1 \ldots v_{i_n}/x_n) \qquad \text{(SS)}$$

where $v_{i_1} \ldots v_{i_n}$ are the free variables of φ in increasing order in some standard enumeration, and v/x denotes the substitution of 'x' for 'v'.

The satisfaction scheme represents the naively correct and analytic principle of satisfaction which generalises claims of the kind

John and Mary satisfy 'x loves y' iff John loves Mary.

We could, at the cost of certain complications, simplify (in one sense) the axiomatisation thus: if we restrict (SS) to the cases where φ is atomic, but add recursive clauses such as

$$Sat_n\ulcorner\phi \vee \varphi\urcorner x_1 \ldots x_n \leftrightarrow Sat_n\ulcorner\phi\urcorner\ x_1 \ldots x_n \wedge Sat_n\ulcorner\varphi\urcorner x_1 \ldots x_n$$

the more general scheme can be proved. We will leave this as a non-trivial exercise.

The theory gives satisfaction conditions for all the formulas in the language including those which contain the satisfaction relation. It therefore formulates its own semantics. Of course it is inconsistent. For if we let $n = 2$ and take φ to be $\sim Sat_2\ x\ x$ we get

$$Sat_2\ulcorner \sim Sat_2\ x\ x\urcorner y \leftrightarrow \sim Sat_2\ y\ y$$

Now for y take $\ulcorner\sim Sat_2\ x\ x\urcorner$ to derive what is, in effect, the heterological paradox.

Other semantic notions are simply accommodated. In particular, the satisfaction scheme for $n = 0$ is

$$Sat_0\ulcorner\varphi\urcorner \leftrightarrow \varphi.$$

Hence 'Sat_0' is the truth predicate for the language. As for denotation Δ, if t is any closed term of the language, we simply take $\Delta\ulcorner t\urcorner x$ as $Sat_1\ulcorner y = t\urcorner x$. The satisfaction scheme for $n = 1$ then gives

$$\Delta\ulcorner t\urcorner\ x \leftrightarrow x = t$$

The paradoxes of truth and denotation characteristically depend on machinery that is not available in the very limited theory we have sketched. However, if the axioms of arithmetic were added, giving the theory of semantically closed arithmetic, the liar paradox, Berry's paradox, and so on, would be forthcoming in the usual way. The triviality of the theory specified has not yet been investigated (except indirectly through its representation in set theory). (On Berry's paradox, see Priest [1983].)

Unlike naive set theory, naive semantics has not been much developed; investigation of its theorems has not been carried very far. Much waits to be done. However, the theory affords a clear axiomatisation of the intuitive semantical notions, which are built into natural language.

β) Naivė property theory, set theory and category theory.

Naive semantics encodes intuitive (and correct) views about truth, satisfaction, etc. Naive property theory and set theory do the same for these notions. Neither theory can be formalized nontrivially without paraconsistent logic; neither can be formalized using a paraconsistent logic which admits absorption without trivialisation occurring.

Let us take the theory of properties first. This uses a variable binding, term forming operator λ such that $\lambda x\varphi$ is thought of as the property expressed by the open sentence φ as a condition on x. The only additional predicate required is η, 'has the property'.

The axiom scheme for properties is the obvious abstraction principle

$$y \; \eta \; \lambda x\varphi \leftrightarrow \varphi(x/y) \qquad \text{(AP)}$$

A slight generalisation of the theory is provided by allowing for n-place properties. For each $n \geqslant 0$ we now have an operator λ_n which binds n free variables, and an n + 1 place predicate η_n (for which we will write only its last argument to the right) which satisfies

$$y_1 \ldots y_n \; \eta_n \; \lambda_n x_1 \ldots x_n \; \varphi \leftrightarrow \varphi(x_1/y_1 \ldots x_n/y_n)$$

When n = 0, this theory is just the theory of propositions, with $\lambda_0\varphi$ expressing the proposition that φ and η_0 being, in effect, the truth predicate for propositions. However, for simplicity, we will restrict our further discussion to the fairly representative 1-place case.

Again, the theory is patently inconsistent since we have

$$y \; \eta \; \lambda x(\sim x \; \eta \; x) \leftrightarrow \sim y \eta y$$

Taking $\lambda x(\sim x \, \eta \, x)$ for y gives the expected contradiction, the impredicativity paradox. The non-triviality of this theory follows from that of naive set theory which we will discuss shortly.

Provided the language we are using contains modal functors, we can express the familiar identity condition for properties, namely that two properties are the same iff they are necessarily coinstantiated, i.e.

$y = x \leftrightarrow L\forall z\ (z\ \eta\ y \leftrightarrow z\ \eta\ x)$.

At this point we should perhaps warn that the precise formulation of the axioms for = itself, is a sensitive business. The simple substitutivity principle

$x = y \rightarrow (\varphi(z/x) \leftrightarrow \varphi(z/y))$

leads to curiosities such as $x = y \rightarrow (\varphi \rightarrow \varphi)$, and to irrelevance. Moreover there are good reasons for supposing it to be invalid if the language in question contains more highly intensional functors concerning belief, etc.[8]

The formulation of naive set theory is now a simple business. For the only real difference between sets and properties as usually conceived, lies in their identity conditions. Thus if we write 'ϵ' (is a member of the set) for 'η' and '$\{z|\varphi\}$' (the set of objects z which φ) for '$\lambda z\varphi$', the abstraction scheme for properties AP, becomes the abstraction scheme for sets AS:

$$y \in \{x|\varphi\} \leftrightarrow \varphi(x/y). \qquad \text{(AS)}$$

The identity condition for sets amounts to the familiar extensional one:

$x = y \leftrightarrow \forall z(z\epsilon y \leftrightarrow z\epsilon y)$

Naive set theory is the one inconsistent theory that has had its theorems investigated, at least to a certain extent. In particular, virtually the whole apparatus of basic set theory, Boolean operations, ordered pairs, functions, power sets, etc., can be developed in much the same way as normal, though some changes are necessary. For example, if we define the null set Λ in the usual way as $\{x/x \neq x\}$ then we can no longer prove that $\Lambda \subseteq x$ since this trades on the paradox of material implication $A \rightarrow (\sim A \rightarrow B)$. However, if we define Λ as $\{x|\forall yx\epsilon y\}$ this and the other usual properties of Λ are forthcoming. That there are infinite sets is also provable in a simple way. For example, let V be the universal set, defined as $\{x|\exists yx\epsilon y\}$. Then V is mapped into a proper subset of itself by the map $x \mapsto \{x\}$. Hence V is infinite. Thus naive set theory appears to provide for the set theory required in all normal mathematics.[9] The extent to which classical set theory itself, including the theory of transfinite ordinals and cardinals, can be developed or represented is still an open problem, as is the problem of what interesting structure inconsistent sets such as $\{x|x \notin x\}$ have and yield.[10]

Let us return now to the abstraction scheme AS itself. If we formulate the abstraction scheme without set abstracts, it is the usual:

$\exists z\forall y(y\epsilon z \leftrightarrow \varphi)$

Where φ is arbitrary except that z may not occur in it.

The qualification is required in consistent set theories since if it is violated inconsistencies are soon forthcoming. However this is no reason for keeping it in naive set theory, and the condition can be dropped. If we do this we can prove the existence of some more interesting sets. For example, consider the set defined thus:

$x \in f \leftrightarrow \exists uv(u \in y \wedge v \in u \wedge x = \langle uv \rangle) \wedge \forall uv_1 v_2(\langle uv_1 \rangle \in f \wedge \langle uv_2 \rangle \in f \rightarrow v_1 = v_2)$

It is not difficult to check that f is a function and that for all $u \in y$, $f(u) \in u$. Thus f is a choice function on y, and we have proved the axiom of choice[11]. In fact we can take y to be V. Hence we have the global axiom of choice. Obviously this raises the important question of whether the continuum hypothesis or generalized continuum hypothesis can be settled by naive set theory. The answer to this is as yet unknown. However it is known that naive set theory is non-trivial even without the restriction on z in its formulation.[12] We will discuss the signficance of this in the introduction to the next part.[13]

Before we leave the topic of set theory we should mention the situation with category theory. If we take ZF set theory and define the notion of category in the standard way, a category has to be a set. Thus we are precluded from considering such categories as the category of all sets. Alternatively if we allow categories to be proper classes, we are not able to consider the category of all proper classes or even all groups, since some of these are proper classes. These are well known difficulties. Standard solutions to them, such as the Gröthendeick hierarchy are not very successful.[14] However, if category theory is developed in naive set theory, we can define such categories as the category of all groups and be sure that all groups are in it. We can define the category of all sets, and since this is a set it will be a member of itself. Similarly we can consider the category of all categories. This not only frees the category theorist, whose hands are chained by ZF, but also introduces exciting new possibilities within naive inconsistent category theory. But to what extent these further non-well-founded categories exhibit interesting category theoretic properties remains to be seen.

In this last part we have been concerned with inconsistencies involving very large objects, such as certain infinite sets and categories. We turn next to inconsistencies involving the very small: infinitesimals and microphysical objects.[15]

γ) The infinitesimal calculus.

The third theory we should mention is the theory of infinitesimals. It is often suggested that the reworking by Robinson of the infinitesimal calculus in terms of non-standard analysis shows that the theory was not really inconsistent. But however elegant and useful the non-standard analysis theory of "infinitesimals" is, it is a gross anachronism to suggest that it is the original theory.[16] For infinitesimals had to be genuine inconsistent objects: in the calculation of a derivative, at different points, it had to be assumed that an infinitesimal was both zero and non-zero. Thus the theory is highly suitable for a paraconsistent formalization. Exactly how this is to be done is, however, a subject which requires a good deal more research. For the present we consider only the following suggestion for an absolutely naive infinitesimal theory, and some of its features.

First, the theory is based on the second-order theory of reals, which, we may suppose, is formulated to allow for specification of functions by λ-abstraction. Division is to be taken as a primitive symbol satisfying the condition

1) $x \neq 0 \quad x.y/x = y$.

The theory has one additional function symbol 'd', 'an infinitesimal part of' satisfying the two extra axioms

2) $dx = 0$.

3) $dx \neq 0$.

The derivative Df, of a function, f, can be defined in the usual way:

$$Df = \lambda x\left(\frac{f(x + dx) - f(x)}{dx}\right)$$

Thus the derivative of f at x is the ratio of the change in fx produced by an infinitesimal change in x. The calculation of a derivative can now proceed in the absolutely obvious way. For example, let f be $\lambda y\ y^2$. Then

$$Df = \lambda x\left(\frac{(\lambda y\ y^2\ (x + dx) - \lambda y\ y^2\ (x))}{dx}\right)$$

$$= \lambda x\left(\frac{(x + dx)^2 - x^2)}{dx}\right) = \lambda x\left(\frac{2xdx + dx^2}{dx}\right)$$

$$= \lambda x\left(\frac{dx(2x + dx)}{dx}\right)$$

But by 3), 1) and the properties of λ,

$$Df = \lambda x\ (2x + dx).$$

And by 2) and the properties of λ,

$$Df = \lambda x\ 2x.$$

To prove various further properties of derivatives, extra axioms are required, such as $dfx = f(x + dx)$. However this will suffice to indicate something of the general shape of the theory. Perhaps the nicest thing about the theory is the way it allows d to be what it was originally thought to be, namely, an infinitesimal forming functor - where an infinitesimal is now thought of as an infinitely small inconsistent object.

Regrettably, a nasty thing about the theory is that it is trivial, or at least so close to trivial as to make no real difference (as observed by Dunn). For 0 = 1 can be proved, and thus applied to prove every equation. Since $0 = 0 + 0$, by 2), $dx = dx + dx$. Hence using 3) and 1), $\frac{dx}{dx} = \frac{dx}{dx} + \frac{dx}{dx}$. Therefore $1 = 1 + 1$, whence $0 = 1$, and disaster. A less naive, genuinely paraconsistent theory, which should be a conservative extension of arithmetic, will have to proceed more circumspectly.

There are various possibilities to be explored. One proposal is that arithmetical operations on infinitesimals be limited. Another, suggested by the practice of the pristine theory, is that axiom 2) is contextually qualified, e.g. it only applies in certain d-contexts. The idea here is that while dx is not strictly zero, it is so close to zero that suitably placed d-terms elsewhere can absorb it.

δ) Quantum mechanics.

There are many parts of quantum theory which suggest paraconsistent formalization, because on the face of it they yield contradictions. Areas of especial sensitivity as regards consistency are those concerning the collapse of wave packets upon measurement, and in particular the matter of the exact determination of operators such as those of position and momentum. We will look briefly at some of these areas, beginning with the formalization of the Dirac delta function.

Very commonly, quantum mechanics is formulated in terms of Hilbert spaces. Thus the state description of a system is a member of the Hilbert space, H, which is the set of total functions from the reals $\mathbb{R}$ to the complex plane $\mathbb{C}$, with suitable operations defined. There is no insuperable problem in axiomatizing such a theory and we will leave it as an exercise for the diligent reader.[17] The important point at present is that it will imply that

$\forall\psi \in H, \forall x \in \mathbb{R}, \exists y \in \mathbb{C}, \psi(x) = y$ (α).

Now to solve many problems it is necessary to invoke the Dirac δ-functions and to suppose them to be elements of the Hilbert space. The δ-functions are characterised by the axioms

i) $\delta_x \in H \wedge \delta_x = \delta_x^*$

ii) $\forall y \neq x\ \delta_x(y) = 0$

iii) $\int_{-\infty}^{+\infty} \delta_x(y)dy = 1.$

If we add these to the axioms for the Hilbert space we can quickly derive an inconsistency. For since $\delta_x \in H$, $\forall y \in \mathbb{R}, \exists z \in C, \delta_x(y) = z$. Thus $\exists z \in C\ \delta_x(x) = z$. But since $\delta_x = \delta_x^*$, z is real. Hence $\int_{-\infty}^{+\infty} \delta_x^*(y)dy = 0$. Contradiction.

Thus the theory as it stands requires paraconsistent formalisation. That the theory is in deep classical trouble was observed by von Neumann.

> The method of Dirac ... in no way satisfies the requirements of mathematical rigour - not even if these are reduced in a natural and proper fashion to the extent common elsewhere in theoretical physics. For example, the method adheres to the fiction that each self-adjoint operator can be put in diagonal form. In the case of those operators for which this

is not actually the case, this requires the introduction of "improper" functions with self-contradicting properties.[18]

Of course a paraconsistent reformulation is only one way of surmounting the problem, and we are certainly not claiming that it is the best. The theory of the Dirac δ-function can for instance be formalised using the theory of distributions, though the adequacy of this formalisation is another question. All we are claiming is that this is one not unreasonable formalisation and one, moreover whose consequences it might well be fruitful to investigate.

More immediate than the problems of the δ-function or general wave packet reduction are, what underlie these problems, the causal anomalies of quantum mechanics. Perhaps the simplest example is provided by the famous two-slit experiment:- Suppose we fire a beam of light through a screen with two slits, A and B, in it. Having passed through the slits the light hits a screen. We wish to make sense of this in particular terms. If one slit is open a certain characteristic pattern of light is observed on the screen. If the other slit is open a similar pattern is observed. It would seem that if both slits are open the pattern obtained should be the simple superposition of these two patterns; but it is not. Consider first the proof that it should be, before the weak points in it are assessed. The intensity of light at a certain point x on the screen is determined by the probability of a photon hitting it. Let us write r for 'a photon hits x', a for 'a photon passes through A' and b for 'a photon passes through B'. We are interested in $Pr(r/a \vee b)$. This can be calculated as follows:

$Pr(r/a \vee b) = Pr(r \wedge (a \vee b)) / Pr(a \vee b).$

(*) $= Pr((r \wedge a) \vee (r \wedge b)) / Pr(a \vee b).$

(**) $= Pr(r \wedge a) / Pr(a \vee b) + Pr(r \wedge b) / Pr(a \vee b)$, since $\sim((r \wedge b) \wedge (r \wedge a))$. But

(***) $\sim(a \wedge b)$. Hence $Pr(a \vee b) = Pr\ a + Pr\ b$,

and by symmetry $Pr\ a = Pr\ b$.

Thus $Pr(r/(a \vee b) = Pr(r \wedge a)/2\ Pr\ a + Pr\ (r \wedge b)/2\ Pr\ b$

$= \frac{1}{2} (Pr\ (r/a) + Pr\ (r/b)).$

Orthodox quantum logic tries to block the proof of stage (*) by rejecting distribution. While this does what is required in the given two-slit example, there is good reason to doubt that the strategy succeeds in the larger quantum mechanical context in which it has eventually to be set. For, firstly, the damaging superposition proof can be adapted to work with what orthodox quantum logic appears to allow; and secondly, when combined with arithmetic, which is essential to any larger venture, orthodox quantum logic permits the proof of distribution.[19]

Paraconsistent strategies are different. Leading paraconsistent options are to reject steps (**) and/or (***). According to the stronger, dialethic option, even though $\sim(a \wedge b)$ is true, $a \wedge b$ is not thereby ruled out. Hence both those steps of the argument fail.[20] What this means in qualitative terms is that the particle which obviously cannot pass through both slits, actually does so. Far fetched as this may seem, once the idea that some inconsistencies are true is taken seriously, who is to say that some inconsistencies are not realized at the micro-level? (That micro-particles are not also waves?) It would be very strange; but we already know that strange things happen in this domain. In fact, given that the other steps of the argument are acceptable, the experimental evidence shows that sometimes a particle must (as a wave) pass through both slits,[21] even though this is impossible!

Once we have our eyes attuned to the possibility of particles doing the impossible, several other phenomena in quantum physics spring to mind, for example, the penetration of a potential barrier by a particle with (classically) insufficent energy. However, we need not pursue this issue further.

On the face of it the quantum-theoretical account of measurement is also inconsistent. For 'the result of a measurement is a superposition of vectors, each representing the quantity being observed as having one of its possible values': yet 'in practice we only observe one value', not many.[22] The predicament of Schrodinger's cat provides a celebrated example of the problem: the wave function for the system has 'a form in which the living cat and the dead cat are mixed in equal proportions' (De Witt, p.31), but only one cat, a living or else a dead, is observed. There are several well known attempts - none particularly convincing - to resolve the matter, to consistencize in a coherent way; in particular, the Copenhagen collapse of the wave-packet, the hidden variable interpretation, Wigner's conscious interference proposal (see De Witt, p.32). A different attempt - very much in the tradition of Jainist pluralism and discursive logic - is the EWG (Everett-Wheeler/Graham) interpretation, according to which all the possible values are realised in different worlds, to a distinguished one of which we observers are confined. The semantical framework of discursive logic is ready-made for the interpretation. But despite its paraconsistent connections the EWG theory is not really a paraconsistent one at all. For the branching tree of alternative worlds fits in an evolving Hilbert space (that of the nested superposition) which conforms to classical logic.[23]

4. Paraconsistency and wider logical notions.

Let us now consider the application of paraconsistency to the theories of logic themselves. What an adequate paraconsistent logic does, at bottom, is to provide canons of good reasoning that can be used in all situations - including the many that misbehave classically.[24] However, it is but a limited basis for this. For it will account at best for deductive reasoning concerning a very restrictive class of logical notions. Beyond that there are other types of reasoning, such as inductive methods, and - not unrelated - there are many other notions we use in our reasoning, such as probability, various modalities, and so on; and each of these notions and types of reasoning must be properly tuned to the paraconsistent. We will make a beginning on showing how some of the adjusting of notions is to be accomplished in subsequent subsections. But first let us consider a little more generally the matter of the construction of adequate logics.

α) Reason, inference, fallacies, and the inconsistent.

As should by now be very clear, reason and inference do not break down in inconsistent situations (whatever the friends of consistency in logic and artificial intelligence may say). If one finds an inconsistency in one's reasoning one certainly does not invoke ex falso quodlibet and conclude that one ought to accept everything; nor does one grind to a complete halt. Of course it is common, once one finds a contradiction, to take evasive action, to modify one's views until they are consistent. But common enough though this is, it is by no means rationally obligatory. The rational thing to do may well be to accept the contradiction, or at least to see what emerges from it. We will discuss this further in the introduction to the next part. The important point for now is just that a theory of reason certainly must be paraconsistent. The attempt of much recent literature to provide an account of rational human reason based on classical logic and probability theory - from which human inferential practice frequently and perversely deviates - is misguided. The classical theory is no ideal, but is itself defective.

Reasoning (whether artifical or natural, human or otherwise) is of several types: deductive, inductive, analogical, dialectical.[25] A logic for each one of these kinds of reasoning should tell us which principles of inference to accept. Moreover, each kind of reasoning will have associated fallacies, inferences that it is the business of logic to reject. Formal systems such as those of paraconsistent sentential logic codify only a small part of reasoning practice, namely some accepted principles of deductive reasoning.

Such systems can be expanded however, in the fashion of Lukasiewicz, to encompass rejected rules of deductive reasoning too, through rules of rejection. In this way they can reflect and codify classes of fallacies. But even this enterprise has not yet been undertaken for intensional logics, though it would have interesting features. For example, central principles of classical logic would be rejected as fallacious in paraconsistent logic. In particular such obvious fallacies of relevance as the Lewis paradoxes would be rejected: i.e., $\dashv p \wedge {\sim}p \rightarrow q$ $\dashv q \rightarrow p \vee {\sim}p$. Given the expected linkage between assertions and rejections, e.g.

(*) $\vdash A \rightarrow B \rightarrow (\vdash A \rightarrow \vdash B)$

(**) $\vdash A \to B \to (\dashv B \to \dashv A)$,
several other rejections would follow, perhaps most controversially, $\dashv p \wedge (\sim p \vee q) \to q$. For since, $\vdash p \wedge \sim p \to p \wedge (\sim p \vee q)$, $\vdash (p \wedge (\sim p \vee q) \to q) \to ((p \wedge \sim p) \to q)$,
whence, using (**), instances of the disjunctive syllogism are rejected. (In a similar way, given the set abstraction scheme, it can be shown that instances of $A \wedge (A \to B) \to B$ are rejected in depth relevant logics.)

To provide an adequate norm for reasoning, the reach of relevant/paraconsistent logic has to be expanded beyond the state to which it has currently advanced. Even when this has been accomplished, there remains the question of the relationship of norm to practice. Analytically, the norm is a guide to correct practice and correct practice must bear some suitable relation to practice (though teaching and brainwashing can seriously affect the relation). Anyway this question is a little premature at the moment since there has been little unbiased testing of the way people (and other creatures) actually reason, especially in inconsistent situations. It is enough for the present purposes, however, that clear cases of irrelevance, such as the Lewis paradoxes exhibited, are widely recognised in preanalytic thought as fallacious or mistaken, and that reasoning continues, in non-classical fashion, in incomplete, inconsistent and paradoxical situations. For this indicates that the normative theory of reasoning adopted should be a relevant/paraconsistent one.

β) Extending paraconsistent logic by intensional functors: modal and tense operators.

With this firmly in mind let us revert to the question of the tuning of notions used extensively in reasoning to the paraconsistent, beginning with, what is relatively straightforward, modal and tense notions.

Tense and modal operators can be added to a paraconsistent logic in standard syntactical and semantical ways. Consider, for example, logical modality from a semantical slant. Let K be a class of evaluations suitable for a paraconsistent logic. (These might be da Costa evaluations for C_1, a set of Routley/Meyer worlds, a set of truth filters on a De Morgan lattice, etc.; see the introduction to part 2.) Let R be some binary relation, considered as relative possibility, with domain K. Then in the usual way, we can define:

1) $L\varphi$ holds at $w \in K$ iff for all $v \in K$ such that wRv, φ holds at v.

2) $M\varphi$ holds at $w \in K$ iff for some $v \in K$ such what wRv, φ holds at v.

What modal principles hold for the logic so defined depend, of course, not only on relation R but on the underlying paraconsistent logic. However, for most paraconsistent logics it is easy enough to produce paraconsistent versions of such systems as T, S4, S5, etc. Completeness proofs are usually forthcoming from a fusion of the completeness proofs for standard modal logics and those of extant paraconsistent logics.[26]

Such paraconsistent bases allow for the elaboration of essentially novel modal logics. Little has as yet been done in this area, but we might mention as an example systems formed by adding Nietzsche's Theme:

$M\varphi$ - everything is possible.[27]

Any normal modal logic which contains Nietzsche's Theme is inconsistent, but if the underlying logic is paraconsistent, there is no reason why it should be trivial. In fact, all we need to realise Nietzsche's Theme in a modal model structure is that the structure contain the trivial evaluation/world (where everything is true). Of course, though this is sufficient it is not necessary.

The addition of tense operators to a paraconsistent logic is also fairly straightforward. The truth conditions for G (it will always be the case that) and F (it will be the case that) are given by 1) and 2) with 'G' replacing 'L' and 'F' replacing 'M'. The truth condition of H (it has always been the case that) and P (it was the case that) are given in the same way except that R is replaced by its converse $\check{R}$. R is now thought of as temporal order.[28]

Paraconsistent tense logic is important in connection with the dialectical theory of change. It is sometimes said that the postulation that time is real is necessary to restore the contradictions produced by change. What is meant by this is that if a thing changes it is both P and not P. The apparent contradiction is resolved when we admit that it is P at one time and not P at another. (So Idealists such as Bradley who denied the reality of time were therefore forced to admit that change involves contradiction. Because of this they relegated change to the realm of appearances, which, unlike reality, could be inconsistent.) Even _if_ the postulation of time is necessary, it is certainly not sufficient. For there may be instants of time at which a contradiction holds. This is precisely what those who accepted a dialectical account of change believed. Something which is moving from here to somewhere else is, at one and the same time, both here and not here.[29] And more generally if something is changing from P to not P at a certain instant it is both P and not P. Let us call this view _Zeno's principle_ since it was he who first argued it. Then within a paraconsistent tense logic this view results in principles such as

$$(A \wedge F\sim A) \rightarrow F((A \wedge \sim A) \wedge F\sim A \wedge PA) \qquad (Z)$$

and its past/future dual. The completeness of (Z) with respect to Zeno's principle is obviously a question which depends upon many details concerning the underlying logic, so we will not pursue it here. However it is clear that to make much sense of Zeno's principle a paraconsistent logic is required. For classically we can prove

$\sim F(A \wedge \sim A)$ and hence $\sim(A \wedge F\sim A)$ i.e. there is no change. Thus (Z) plus classical logic produces Parmenides' position, that there is no change. Hence paraconsistent tense logic is an important logical theory for investigation of the dialectical account of change. But principle (Z) and its further logical consequences remain to be investigated.

γ) Moral dilemmas: deontic logic.

Another philosophically significant extension of paraconsistent logic is that to deontic logic. The deontic operators 'O' (it is obligatory that) and P (it is permissible/permitted that) can be added in much the same way as the modal operators of the previous section were. We can now think of the relation R as one of "moral accessibility" (i.e. xRy means that y is obtained from x by the performance of some morally permissible acts) or alternatively as affording access to ideal worlds, (i.e. y is ideal as seen from x). The truth conditions for 'O' and 'P' are obtained simply by replacing 'L' with 'O' and 'M' with 'P' in (1) and (2) of the previous section. As usual a range of deontic logics is obtained by allowing R to have various properties. Details of sound and complete axiom systems depend, of course, also on the underlying paraconsistent logic, as does the range of deontic systems encompassed.

Paraconsistent deontic logics are particularly important; for they rectify the gross distortions of the concepts of obligation that obtain in classically based deontic logics. For example, it is notoriously the case that we may incur inconsistent obligations. Examples of moral dilemmas abound in the literature. Not all of these are cases of inconsistent obligations. Some moral dilemmas are just situations where it is difficult to decide what the right thing to do is. Others are cases where there is no inconsistency since it is clear that one prima facie obligation is overridden by a more important one. For example the obligation produced by my promising to arrive home at a certain time disappears if I delay to save someone's life. However there are cases where we do end up with genuinely inconsistent obligations.[30] I promise to go to dinner with x the next time he is in town. I promise to go to dinner with y, and x announces that he will be in town during that time (and only that time). Let us write X for 'I go to dinner with x' and Y for 'I go to dinner with y'. Thus we have both OX and OY. But assuming that I can not go to dinner with one if I go to dinner with the other (maybe because they will be in different parts of town), we have $X \rightarrow \sim Y$. And since OX, we obtain $O\sim Y$. (If this inference be doubted, just consider the case where I promise x I will do something and, forgetfully, I promise y I will not do it.) Hence we have $OY \wedge O\sim Y$, which is equivalent to $O(Y \wedge \sim Y)$.[31]

This last formula describes a realisable state of affairs. Yet it can not be true in any world of standard deontic logic. However, worse is to come. For classically $\vdash Y \wedge \sim Y \rightarrow A$. Hence, distributing O, $\vdash O(Y \wedge \sim Y) \rightarrow OA$. Thus according to classical deontic theory if I incur inconsistent obligations I ought to do everything. This is ridiculous. In the situation described it is obviously not permissible, let alone obligatory, to go and shoot one of the friends or some innocent bystander. However, even worse is to come. Assuming (as for usual deontic logics) that every world has some accessible extension, we have $\vdash \sim(OY \wedge O\sim Y)$, and thus $\vdash (OY \wedge O\sim Y) \rightarrow B$. In other words, by making inconsistent promises I bring it about that Paris is the capital of China. In fact I make the world trivial!

None of these ridiculous consequences follow from a paraconsistent deontic logic of the relevant sort: there are worlds in which $O(A \wedge \sim A)$ is true and (therefore) $(OA \wedge O\sim A) \rightarrow OB$ (and a fortiori $(OA \wedge O\sim A) \rightarrow B$)) fails.

One important consequence of this approach is that the Kantian dictum, Ought implies Can, i.e. 'everything that is obligatory is possible' (or conversely 'if something is impossible it is not obligatory'), needs to be rejected. On the semantics given it is of course true that $OA \rightarrow MA$ (at least when the deontic R is a subrelation of the altheic R, and $\forall x \exists y xRy$ for the deontic R). But presumably if $O(A \wedge \sim A)$ is true at w, all the worlds accessible to w will be "impossible" worlds. But the Kantian moral maxim presumably means that everything that is obligatory is true in some possible world. So let C be the set of consistent (classical) worlds or evaluations. If we characterise a connective M' thus:

M'A holds at w iff, for some x in C such that wRx, A holds at x,

then the intended Kantian maxim is expressed by $OA \rightarrow M'A$. But this obviously fails in general.

We have taken our examples of inconsistent obligation from the area of morals. But the sphere of obligation is obviously much wider, and it would be easy to produce inconsistent obligations by considering political situations, legal systems, contracts, constitutions, games and so on.[32] Thus the application of paraconsistent deontic logic has broad scope.

Virtually all the points we have made concerning inconsistent obligations can be repeated with respect to inconsistent orders (which may be produced both intentionally or by accident). Hence our discussion applies mutatis mutandis to imperatival logics. Satisfactory imperatival logics will be paraconsistent.

δ) Belief systems: doxastic logic.

Thought is more comprehensive than reasoning. While reasoning is included in thought, thought also involves assumption and (as the term is commonly used) belief.

Reasoning proceeds in accord with principles; thought may involve the adoption of the principles, reflection upon them, and much else. Reflection alone, before assumption and beliefs are brought in, may include more than reasoning: it may include such things as sorting or assembling and comparison of things so sorted or assembled. The operations included in reflection beyond those of reasoning have been little investigated in modern logic, though they were included, at a time when psychology was in a much more primitive state, in traditional logic and are sometimes said to have an important place in Hegel's logic, and so in dialectical logic. With belief, and to a lesser extent assumption, the situation is somewhat better: elements of doxastic logic have been furnished, though mostly on an inadequate classical basis, by direct analogy with weaker modal logics.

A key feature of belief, as of many psychological functors, in contrast to reason, is that it is not deductively closed. A creature may perfectly well believe A but not believe B though B is deducible from A. Thus the following theses of some doxastic logics should be rejected:

$\dashv (A \rightarrow B) \rightarrow (xBelA \rightarrow xBelB)$

$\dashv ((A \rightarrow B) \wedge xBelA) \rightarrow xBelB.$

Other principles that need to be rejected are various consistency postulates, in particular

$\dashv xBel\ (A \wedge {\sim}A)$

$\dashv xBel{\sim}A \rightarrow {\sim}xBelA.$

It is clear that an agent may well believe a contradiction either wittingly or unwittingly. Moreover (as we will argue in the introduction to the next part), the agent may rationally believe a contradiction. The rejection of consistency postulates would cause serious problems if we were to try to base a doxastic logic on classical possible-world theory. However, the more general worlds-theory of paraconsistent logics enables the rejection of these principles without any trouble.

In contrast to standard modal logics doxastic logic is very weak. (Indeed this is little more than a corollary of belief not being closed under entailment.) For this reason it is characterized as much by the principles it rejects as those it accepts. But one might well wonder whether one should accept any principle that involves belief essentially. In fact, one should. An example is the conjunction principle

$\vdash xBel(A \wedge B) \leftrightarrow .\ xBelA \wedge xBelB.$

While simplification, $xBel(A \wedge B) \rightarrow .\ xBelA \wedge xBelB$, is not in much dispute, its converse, $xBelA \wedge xBelB \rightarrow xBel(A \wedge B)$, is moot. But it can be persuasively argued that it correctly characterises belief.[33] This converse principle is important. For amongst those who duly concede that our beliefs may well be inconsistent, it is common to propose a non-adjunctive paraconsistent logic,[34] on the grounds that though one may believe A and believe ~A one will not believe $A \wedge {\sim}A$. Obviously if the conjunction principle for belief is correct, this defense - one of the main defenses - of non-adjunctive systems, fails.

The logic of rational belief, while certainly closed at least under more elementary logical operations (such as adjunction no doubt), is, like the logic of belief, not encumbered by consistency postulates. Hence it too is not satisfactorily based on classical logic. Nor therefore, since probability assignments are so intricately tied to rational belief assessments, is probability theory - which leads to another major newer application (though one with older roots).

ε) Probability and inductive reasoning.

The standard approaches to probability theory are squarely based on classical logic. However, they can alternatively, and easily, be based on a paraconsistent logic and, as we shall see, doing so produces a number of advantages. There are many different approaches to classical probability theory. One of the easiest to adapt to paraconsistency is Carnap's. Let C be a class of paraconsistent worlds/evaluations suitable for some paraconsistent logic. Let m be a normal measure function defined on C. In particular then $m(C) = 1$. The probability of a formula A, Pr(A), may now be defined as $m(\{x \in C \mid A \text{ holds at } x\})$. It is easy to check the following:

i) $0 \le Pr(A) \le 1$

ii) if A entails B in the logic (i.e. if every valuation at which A holds, B holds), $Pr(A) \le Pr(B)$

iii) $Pr(A \vee B) = Pr(A) + Pr(B) - Pr(A \wedge B)$

(assuming that conjunction and disjunction behave in the normal way).

iv) if A is a logical truth (i.e. holds in all evaluations), $Pr(A) = 1$.

In general, all the principles of probability theory that do not concern negation will carry over straightforwardly into paraconsistent probability theory. Typically, where paraconsistent probability theory diverges from the classical theory is in the vicinity of negation. In particular, it will not in general be true that $Pr(A) + Pr({\sim}A) = 1$. If $(A \vee {\sim}A)$ is a logical truth it will certainly be the case that

$$1 = Pr(A) + Pr({\sim}A) - Pr(A \wedge {\sim}A),$$

by iii) and iv), but of course $Pr(A \wedge {\sim}A) \neq 1$ in general.

An especial advantage of this approach is the following. The standard definition of conditional probability Pr(A/B) is $Pr(A \wedge B)/Pr(B)$, which of course makes sense only if $Pr(B) \neq 0$. It follows from what we have already said therefore that $Pr(A/B \wedge {\sim}B)$ may be well defined. Thus we can have sensible evaluations of the probabilities of statements relative to inconsistent data. Preanalytically, this is something we do all the time. For example we estimate what is happening in various countries (with degrees of probability) given the inconsistent newspaper reports we read. Yet according to the classical theory this is impossible. More generally, it is a feature of many paraconsistent logics (once again, the relevant and positive-plus systems, but not the non-adjunctive ones) that any formula holds under some evaluation, or more correctly, in some non-trivial class of evaluations. Hence if we choose our measure with care, we can insure that $Pr(A) \neq 0$ for all A. This means that conditional probability is always defined - a very pleasing feature. It might be said that if paraconsistently $Pr(A) \neq 0$, this shows that a paraconsistentist is crazy enough to countenance anything. However one could put it in a slightly less biased way thus: there is nothing that a paraconsistentist will dogmatically and with a closed mind rule out a priori.

The non-classical behaviour of negation means that some parts of classical probability have to be reworked slightly. For example consider Bayes' theorem. In the usual way we can show that $Pr(h/e) = Pr(e/h).Pr(h)/Pr(e)$. Now suppose we have two hypotheses h and h_1 (the application to an arbitrary finite number is routine) and h and h_1 are exhaustive and exclusive in sense that $h_1 \vee h$ and $\sim(h_1 \wedge h)$, are logical truths. Then

$$\begin{aligned} Pr(e) &= Pr(e \wedge (h \vee h_1)) = Pr((e \wedge h) \vee (e \wedge h_1)) \\ &= Pr(e \wedge h) + Pr(e \wedge h_1) - Pr(e \wedge h \wedge e \wedge h_1) \\ &= Pr(h)\ Pr(e/h) + Pr(h_1) Pr(e/h_1) - Pr(h \wedge h_1)\ Pr(e/h \wedge h_1) \end{aligned}$$

Thus $$Pr(h/e) = \frac{Pr(e/h).\ Pr(h)}{Pr(h)\ Pr(e/h) + Pr(h_1)\ Pr(e/h_1) - Pr(h_1 \wedge h)\ Pr(e/h_1 \wedge h)}$$

This is the paraconsistent two-hypotheses case of Bayes' theorem. In the classical case the last summand in the denominator can be dropped since $Pr(h_1 \wedge h) = 0$. However $Pr\sim(h_1 \wedge h) = 1$ is no longer a guarantee of this.[35]

Probability plays a role in many other logical theories, and often a paraconsistent probability theory has a distinct advantage over a classical one. For example in confirmation theory it allows for the high probability/confirmation of contradictory hypotheses. For $Pr(e/h)$ and $Pr(\sim e/h)$ may both be $> \frac{1}{2}$. (Indeed $Pr(p/p \wedge \sim p) = Pr(\sim p/p \wedge \sim p) = 1$.) This has obvious connections with the grue paradox, where contrary hypotheses are both confirmed by the evidence.

As another example, consider the theory of rational acceptance. It is frequently mooted, and very plausibly so, that a claim should be rationally accepted just if it has a high enough probability. In the obvious notation:

(1) $Pr(A) \geqslant 1-\epsilon$ iff $Rat(A)$ $(\epsilon \leqslant \frac{1}{2})$

Since, as we have seen, we may well have $Pr(A \wedge \sim A) \geqslant 1-\epsilon$, $Rat(A \wedge \sim A)$, i.e. there are some contradictions that are rationally acceptable.

A standard problem with (1) is illustrated by the lottery paradox. Consider a fair lottery with n tickets. Let us write A_n for 'Ticket n wins'. Then obviously if we choose n large enough we have $Pr(\sim A_i) \geq 1-\epsilon$, $1 \leq i \leq n$, whilst $Pr(A_1 \vee \ldots \vee A_n) = 1$. Thus the set of rationally accepted beliefs includes $\{A_1 \vee \ldots \vee A_n, \sim A_1, \ldots, \sim A_n\}$ which is obviously inconsistent. This will not bother a paraconsistentist. Similar remarks apply as regards the paradox of the preface.[36]

It is often suggested in connection with rational acceptability, that the set of things rationally accepted should be deductively closed. This is obviously a serious problem for any classical logician who accepts the conclusion of a paradox since this move would trivialise rational belief. Plainly it is not a similar problem for a paraconsistentist. Despite this, we think that the suggestion is incorrect. Indeed, it is easily proved that this suggestion is incompatible with (1). For it is easy enough to produce situations where logical consequence is probability <u>decreasing</u>. (Just consider $\{A, \sim A\} \vdash A \wedge \sim A$.) Hence if we accept both (1) and the deductive closure of the rationally accepted, we could prove that there is an A such that $Pr(A) \geq 1-\epsilon$ and $Pr(A) < 1-\epsilon$. This is a contradiction there is no sufficient reason for accepting.

ζ) <u>Information content and data processing</u>.

Just as standard probability theory is based on classical logic but may be reworked paraconsistently to give a more satisfactory theory, so also can other classically-based theories; for example, the same is true of classical accounts of information content. Again, there are many possible approaches to content, but one that is most easily generalized to paraconsistent logic is due to Carnap and Bar-Hillel.

Let C be a class of worlds (evaluations) suitable for some paraconsistent logic. Then the information content of A, Con(A), is just $C-\{x \in C : A \text{ holds at } x\}$. Assuming that conjunction and disjunction behave normally then usual results about the contents of conjunctions and disjunctions are forthcoming, e.g.

i) A entails B in the logic iff $Con(B) \subseteq Con(A)$.

ii) $Con(A \vee B) = Con(A) \cap Con(B)$

iii) $Con(A \wedge B) = Con(A) \cup Con(B)$

However, as is to be expected, results concerning formulas containing negation differ. In particular we may have $Con(A \wedge \sim A) \neq 0$. A contradiction may therefore have determinate, non-trivial content, and different contradictions different content. Thus, claims that contradictions have no, or trivial, content can be sustained only by insisting that the valuations or worlds, over which content is defined, are consistent. Any justification of this is liable to beg the original question.

If a numerical measure of content, c, is required we can take a suitable measure function, m, defined on C and take

$c(A) = m\ Con(A)$.

Standard results about the numerical content of conjunctions and disjunction are then forthcoming. Relative content can be defined etc [37].

The question of information naturally suggests a further application of paraconsistent logic: data processing. We wish to store data in a computer and be able to retrieve it. However we usually want to do more than that: we want our computer to be able to make inferences from its data and to give us the conclusions. Not only is this a question of efficiency (it is quicker to program 'John has brown hair and no one else has' than 'John has brown hair; Fred has not got brown hair; Steve has not got brown hair, ...') but we want to be able to determine the consequences of our data when it is too large to handle humanly. Now notoriously, data collected from various sources is liable to be inconsistent. Conceivably, one might want to test the data before feeding it into the computer but even if an inconsistency is found (and of course there is no decision procedure in general for inconsistency) we are faced with the problem of how to consistentise it without throwing out too much data. In many ways it is much more sensible to let the machine have it all. But now it is obvious that the computer must be programmed with a paraconsistent logic. One that resulted in the computer answering 'yes' to every question including 'Is there a God?' when fed the speeches of virtually any politician would be useless. The question of how best to construct such a practical implementation of a paraconsistent logic in the computing field is an important one - one that will again depend, of course, on the logic in question. It is very tempting to think that a the logic should be a relevant one.[38]

η) Vagueness

Finally, it is worth noting the role of paraconsistent logic as the underlying logic for a language with vague predicates. It is frequently suggested that what characterises a vague predicate is that in a certain range of application objects satisfy neither the predicate nor its negation.[39] However, what intuition says is that the predicate and its negation are just as true of the borderline object as they are false. Hence an alternative treatment, consonant with this intuition, is that the object satisfies both the predicate and its negation, and hence that the situation is paraconsistent.[40] Moreover, there are reasons why this approach may be preferable, at least in particular cases and perhaps in general. First, consider a colour transition from red to blue through magenta. At the borderline area between red and blue, it seems much more plausible to suppose the colour to be both red and blue, than neither red nor blue. An argument that the paraconsistent approach is better in general, is that truth-value-gap approaches characteristically produce a failure of the law of excluded middle at the borderline area. Yet as Dummett and others have pointed out[41], this is not so plausible. In a borderline case between orange and red, we would be inclined to say that the colour is either orange or red, and it follows from this that it is either orange or not orange.

Actually, no standard three-valued (or by parity of reasoning, finite-valued) approach to vagueness is satisfactory, be it inconsistent or incomplete. This is because the area between definiteness and vagueness is itself vague. This has suggested that a continuum (or at least a dense sequence) of semantic values be used. As with the three-valued case, this can result in either incompleteness or inconsistency, depending on the truth conditions of negation and the set of designated values.[42] As with the three-valued case, there are reasons for supposing the inconsistent variant to be preferable.

5. Conclusion.

We have now given an overview of some of the applications of paraconsistent logic. The view has concentrated on breadth rather than depth. (Even so it can hardly claim to be comprehensive). What will be clear is that little more than a start - if that - has been made on most of the topics we have introduced. For a subject, the serious study of whose theory is little more than 20 years old, and which got away to a slow beginning, this is hardly surprising. However, it will also be clear that the investigation of these areas promises to be a fascinating task for paraconsistent theorists. The only thing that would really surprise us about future work in these areas would be its failure to produce surprises.

FOOTNOTES

1. See especially the Introduction to part IV.

2. See e.g. Routley, [1981].

3. See section 5 of chapter 2 of this book. For a further discussion of contradictory theology, see section 3 of chapter 1 of this book, and also Peña [1981].

4. The semantics for relevant logic of Routley and Meyer [1973] furnishes a fairly general framework for the static elaboration of the theory of theories. The

static development is taken much further in Meyer's unpublished work on the theory of theories, [198+].

5. See section 3 below.

6. See the section on Marx in 'Outline of the History of Logical Dialectics', part one of this book.

7. For fuller explanation and proof see Routley [1980].

8. However, we will not discuss these issues here. Details can be found in Routley [1977] §7, and in EMJB.

9. Further details can be found in Routley [1977] §8.

10. For initial investigations of the latter issues, in somewhat stronger paraconsistent settings, see Arruda and Batens [1983].

11. See Routley [1977] §8.

12. See Brady [198+].

13. See §3γiii.

14. See Fraenkel Bar-Hillel and Levy [1973] pp.143ff.

15. Symmetry of treatment would perhaps suggest a section on relativity theory, indicating parts of the theory ripe or fit for paraconsistent reformulation and investigation. Certainly relativity theory has generated its share of paradoxes.

16. 'At the time [circa 1740] ... mathematicians still felt that the calculus must be interpreted in terms of what is intuitively reasonable, rather than of that which is logically consistent', C.B. Boyer, [1949], p.232.

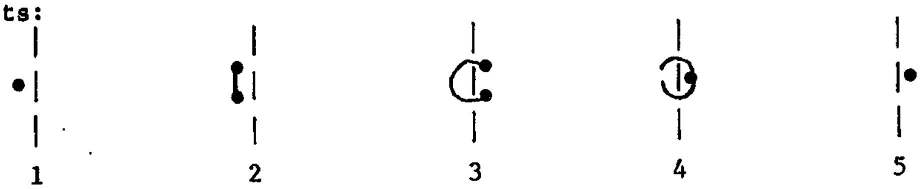
17. Elementary quantum theory is axiomatised in this way by J. von Neumann, [1953]. As was customary with mathematical axiomatisations, the precise logical base is not specified.

18. J. von Neumann, [1953]. von Neumann's point has been adumbrated by P. Feyerabend [1978], p.157, and then set down in Mortensen [1982], which we have made use of.

19. For the second, see J.M. Dunn, [198+]. On the first see P. Gibbins [1981].

20. Strictly this calls for a paraconsistent probability theory, an issue taken up in §4ε below. For elaboration of a nondialethic paraconsistent approach to quantum theory see Routley [1977], where a relevant position is advanced.

21. We can indicate - in a way - how it can happen, through the following sequence of snapshots:



Nothing stops visualising the impossible or diagramatic representation of the impossible: see EMJB, and also Canadian Education Progam [1982].

. De Witt [1970] p.30.

. Rescher and Brandom say that 'the systematisation of quantum-physics by the Everett-Wheeler approach invites (though it does not irrevocably demand) a logical apparatus that is inconsistency-tolerant', [1980] p.60 (italics added). But they do not say how. Moreover on the basis of what they do say the possible world theories of Democritus and David Lewis would equally invite "inconsistency-tolerant apparatus" - which they decidedly do not. (Accounting for the 'maverick worlds' of De Witt, p.34, would require a larger apparatus: the semantics for relevant logic would suffice.)

. This theme is much elaborated at the beginning of Routley [1977].

. We are certainly not claiming that these types are either exclusive or exhaustive.

. For details in the case of the most difficult of these, relevant logics, see RLR, chapters 8 and 9, where multiply intensional relevant logics are much more fully treated.

. Relevant logics of this kind are investigated in RLR Ch 8, and the philosophical issues involved are discussed in R. Routley [1983]. Nietzsche's nihilistic theme also has a notorious deontic analogue, Dostoievsky's Axiom: Everything is permissible (also investigated in the above sources).

. Further details can be found in RLR Ch 8 and in Priest [198+].

. Thus, e.g., Hegel [1812] Vol.II Bk.2 Ch.2§3.

. Many examples of such moral dilemmas are given in Routley and Plumwood [198+].

. In non-adjunctive paraconsistent logics this principle is rejected (in our view erroneously). In such theories there are moral dilemmas of the form $OA \wedge O{\sim}A$ but not of the form $O(A \wedge {\sim}A)$.

. As we discuss, in the legal case, in the introductions to parts two and four of the book.

. Arguments to this end are assembled in R. and V. Routley [1975]. The logic of belief there outlined is improved upon and elaborated in EMJB 8.11.

. See e.g. Lewis [1982], Rescher and Brandom [1980], Schotch and Jennings [198+], Ellis [1979].

. The development of the rest of paraconsistent probability theory lies beyond the scope of this introduction. For a fuller discussion of the developed theory and discussion of some other aspects of relevant probability theory, see Routley [1977].

. On this "paradox" see D. Makinson [1964-65]. The "lottery paradox" is further considered in R. and V. Routley, [1975].

. Details can be found in Routley [1977].

38. As to why, and for interesting suggestions as to how the elaboration should go see Belnap [1977].

39. See e.g. Haack [1974] p.109 ff.

40. This approach to vagueness is taken in Pinter [1980].

41. Haack op.cit p.114.

42. A version of the inconsistent variant is found in Peña [198+].

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CHAPTER 5: THE PHILOSOPHICAL SIGNIFICANCE AND INEVITABILITY OF PARACONSISTENCY

1. Introduction

Paraconsistency strikes at the root of principles which are fundamental to, and entrenched in, much philosophy. It is therefore bound to be philosophically problematic and to have important philosophical ramifications. In this introduction we will try to chart and analyse some of these issues. By its nature, this will require us to deal with a number of separate and not otherwise connected issues. However, we will start by looking at some important points raised by the arguments for paraconsistency in the introduction to Part 2 (§1). We will then go on to investigate some of the philosophical consequences of paraconsistency.

2. Reasons for Paraconsistency

In the introduction to Part 2 (§1) we gave two reasons for studying paraconsistency. The first, and weaker, was that some theories are inconsistent but non-trivial; the second was the truth of certain contradictions. Both of these claims are bound to be hotly contested, especially the second; for this reason we now consider them in greater depth.

I) There are natural inconsistent but non-trivial theories.

No one would deny that we can construct purely formal, uninterpreted calculi which have as theorems formulas of the form 'A' and '~A', but which are not trivial. If paraconsistency is to have real interest, it must be possible to do more than this: we must be able to find real life, philosophical or scientific examples of inconsistent but non-trivial theories. A number of these were suggested in the introduction to Part 2 (§1I). Let us look at these more closely.

i) Inconsistent bodies of law and the like

One example of an inconsistent corpus from which non-trivial conclusions are drawn concerns certain bodies of law. Now it is not difficult to find bodies of law which are *prima facie* inconsistent. But it will undoubtedly be claimed by some - the "friends of consistency" we will encounter again and again - that the inconsistencies are only *prima facie*, that when properly understood the law is consistent. The obvious move is to suppose that one or more of the laws which produce inconsistency contain implicit exceptive clauses which prevent them from being applied in the contradiction-producing case. For example, it is often pointed out that laws can be ranked in increasing order of strength through common law, statute law, constitutional law, etc. This may suggest that if a lower ranking law contradicts a higher ranking law, it *ipso facto* ceases to be applicable. Another way in which we may try to make good the idea that a law has implicit exception clauses is this. The preamble of the bill which passes a piece of legislation may make the intentions of the legislators quite clear. It may then be said that although a particular case falls under the act as literally worded, it was never really meant to apply to this kind of case. The intentions of the legislators therefore provide implicit exception clauses. (This is a somewhat problematic point however, since a judge will often uphold the letter of the law, even when it is clear that doing so goes against the intentions of the legislators.)

Moves of the above kind can sometimes be reasonable. However, someone who denies the occurrence of inconsistent bodies of law must do more than claim that this or that manoeuvre is sometimes performable. He must claim that it always is. And put like this, it seems most unlikely. A case may easily arise where both of the contradiction-producing laws have equal rank, where the intentions of the legislators are lost in the mists of time, are moot, or are even downright inconsistent, where there is no precedent for waiving one law rather than the other, etc. In short there is no objective situation which can be used to underpin the claim that one law has implicit exceptive clauses or takes precedence over another. In such circumstances to insist that nonetheless one law has implicit exceptions is mere whimsy; there may well be much more to the law than what is literally written in a parliamentary bill, but to suppose that something can be a fact of law when it is grounded in no aspect of the legal process is baseless obscurantism.

Of course, when a contradiction of the kind we have pointed to becomes important, there are procedures for resolving it. The matter goes to court where a judge makes a decision. Since there are, *ex hypothesi*, no legal reasons for deciding one way or another, the judge will decide on extra-legal (socio-political) grounds. However, the important point here is that the judge is not trying to find out what the (consistent) law is, but is himself *making* law. What the judge decides just is the law and that is that (until and unless the legislature decides to change it or a higher court, if any, amends it). In this situation there is no way in which a judge can be wrong, i.e. make a ruling which is incorrect.[1] Thus the judge, by making new law, is changing the corpus of the law. His action provides the basis for the law, henceforth, to be considered to have an exceptive clause, and hence after his ruling the corpus of law may no longer have this inconsistency. But this does not change the fact that before the ruling the corpus was genuinely and not just *prima facie* inconsistent. Thus there can be genuinely inconsistent bodies of law.

What holds of law applies also, given appropriate adjustments, to like bodies of (partly prescriptive) doctrine, such as those applied by morality or religion. Again there are evidently or demonstrably inconsistent bodies of doctrine whose inconsistency cannot be satisfactorily explained away. Important examples are offered by irresoluble moral dilemmas.[2]

ii) Inconsistent theories in philosophy and the history of ideas

A major reason for taking the paraconsistent enterprise seriously is that inconsistent but putatively nontrivial theories abound in intellectual endeavours. Indeed much of our intellectual history is comprised of such theories. This is particularly true of our philosophical heritage,[3] where it is not entirely implausible to advance the following (classically preposterous) thesis:

TH1. Any sufficiently complex and interesting philosophy will be inconsistent.

There are several ways of arriving at, and supporting, this large thesis. One is by direct argument from the character of such philosophies, another weaker but persuasive argument is by induction from the inconsistent nature of major philosophies. The themes used in the induction are of much independent interest, namely

TH2[M]. All [most of] the major philosophical positions, from the history of philosophy, are inconsistent.

TH3. No philosopher has succeeded in avoiding inconsistencies of a fundamental kind, those encountered in achieving the complex aims involved in working out a fairly comprehensive philosophical position.[4] In each case the themes concern major, complex or comprehensive theories. (Without doubt there are, or can be designed, miniature philosophical theories which are consistent, e.g. simple nominalist theories or theories pegged to a consistent segment of the cumulative hierarchy of sets.)

The latter themes give the appearance of being much more factual than the initial theme, which also makes predictions about future, and indeed merely possible, philosophies. But the appearance is something of an illusion: a variety of less factual and overtly normative considerations enter into attempts to show that the positions of given philosophers are inconsistent. For this reason, establishing TH2 and TH3, and even the weaker TH2M, is far from straightforward and cannot be achieved with any high degree of certainty. Open to the friends of consistency are always too many escape routes from (apparent) inconsistency, such as those that "interpretation" of a philosophy can supply.

Fortunately, then, a much weaker thesis will serve very well for paraconsistent purposes, namely,

TH4. Some major philosophical positions, which are not trivial, are inconsistent.

Rather surprisingly, given the dominance of philosophical positions (often ideologies) which entail that all inconsistent theories are trivial, TH4 is a thesis to which many philosophers will assent at once. Indeed, they will frequently go further, with a little or no prompting, and propound theses like TH1-TH3. Yet practically no-one believes that major philosophical positions which are inconsistent are trivial, or thereby trivialised. Accordingly, the situation cannot be accounted for under the usual (classical-type) methodology of philosophical theories. In short, thesis TH4 leads to the further thesis

TH5. The theory of philosophical theories must be paraconsistent;

no other type of theory is adequate to cope with the data; in particular, no classical account of philosophy will do.

The main detailed work which follows will concentrate on establishing the thesis TH4, that some major philosophical theories are inconsistent but nontrivial, that underpins TH5. Naturally, derivation of TH5 involves further assumptions, such as that philosophical theories are theories; that is to say, are at least deductively closed.[5] That this holds can be argued as follows: x's philosophical theory is given by what x is committed to philosophically. But if x is committed to A, and B is deducible from A, then x is committed to B, whether or not he asserts it. Thus philosophical theories, as encapsulating philosophical commitments, are closed under deducibility.

Similar points serve to distance a philosopher's theory from what the philosopher asserts. The separation is familiar from discussion of criteria for ontological commitment. There are analogous, but even more complex, problems in determining criteria for philosophical commitment. However, assertion, without (later) amendment or withdrawal, is normally sufficient for commitment; this (qualified) sufficient condition is crucial for the exegesis of positions from philosophical texts, and will be applied in what follows.

By no means all inconsistent theories are trivial. But recent philosophical theories which are inconsistent and also incorporate classical or intuitionist logic - or, more accurately, the theories of logical consequence these logics supply - are trivial, and accordingly are worthless unless repaired. Such theories, while they are relevant to theses TH3 and perhaps TH2, are not germane to TH4, and so will receive only brief presentation and exemplification. Such theories trivialise because, of course, they supply spread laws such as $A \& \sim A \vdash B$. Examples are theories of Frege, Russell, the early Wittgenstein[6] and Quine. Main inconsistencies detected in these theories are not, however, tied essentially to the underlying (classical) logic, so they have a wider interest.

Frege's theory, and likewise a transitional position of the early Russell, is inconsistent because of the logical paradoxes (to say nothing of the concept horse and the like). It is not merely that Frege's theory succumbed to the Russell paradox, but that his suggested amendment to avoid that paradox left him open to the derivation of further paradoxes.[7] Many philosophical theories seem indeed to succumb to logical or semantical paradoxes. Cantor's theory, if that is accounted philosophical, is one; Aristotle's theory with its (apparent) acceptance of the Liar at face value as both true and false[8] is another. Whether these theories are trivial or not turns on the question of what their underlying logics look like. More generally, paradoxes of one sort or another are a prime source of inconsistency not just in logical theories but in philosophical theories.[9]

Dealing with Russell introduces a complication already hinted at which is very important, both as regards determining what a philosopher's theory is, and as regards enabling a philosopher to escape from inconsistency; namely, the change or adjustment of theories over time. Russell, to take a more extreme example, did not develop a single philosophical theory: rather, he went through a sequence of somewhat different theories with significant common elements. We have already observed the phenomenon of theory change on a smaller scale (in terms of a number of changes) with Wittgenstein. Frequently, of course, inconsistency in a previous position is a major reason for moving on to a new one; and often this would take the form of inconsistency with - sometimes presented as inability to account for - data the position was supposed to comprehend. A somewhat Hegelian account of philosophical motion, or "progress", must be a proper part of any adequate theory of philosophical theory dynamics, especially of the theories of one individual or school.[10]

Inconsistency was certainly a moving force in Russell's development. He discarded several of the earlier positions he had held because, in large measure, of inconsistencies, e.g. "naive" logicism, the "naive" theory of denotation. He was halted entirely in his attempt in 1913 to work out a theory of intentionality by inconsistency[11], and forced thereby into extensional reduction in the form of logical atomism. This theory not only proved, rather quickly, inconsistent with much rather evident data, but was internally inconsistent. For example, Russell's logical atomism was inconsistent as to the existence of facts. On the one hand, the world consists, according to the theory, entirely of simples (including relations). On the other, inconsistent, hand, sentences are true only by virtue of appropriate facts, which are not however conveyed by any listing of simples - so that facts also demand, and obtain, basic ontological status:

> .. facts, which are the sorts of things you express by a sentence, ... just as much as particular chairs and tables, are part of the real world.[12]

But also,

> .. facts ... are not properly entities at all in the same sense in which their constituents are. That is shown by the fact that you cannot name them. You can only deny, or assert, or consider them. But you cannot name them because they are not there to be named...[13]

Russell's later work contains many lesser inconsistencies. One example (developed elsewhere[14]) concerns negation, where Russell both sponsors a classical theory and also elaborates an account which leads to a nonclassical model of negation.

Russell's multiple relation theory of belief leads to inconsistency: logical forms both are and are not constituents of judgement complexes.[15] Wittgenstein's first criticism of the theory also pointed to a connected inconsistency, that

'Socrates and Mortality are of the same type ... This directly contradicts Russell's claim ... that universals and particulars are of different logical types' (Griffin, p.173). As Griffin goes on to remark, this 'is a new version of an old problem: namely the problem in The Principles of Mathematics of the verbal noun which occurs as a logical subject, and which Russell wanted to be the same as the verb which occurs as relating relation' (pp.173-4).

Discovery of contradiction in his earlier position was a major source of motion for Russell from one position to a later. Thus he abandoned 'the relational theory of space together with Bradlerian monism' because of contradictions deriving therefrom.[16]

There was a further major inconsistency in Russell's 1913 research program, brought out by Wittgenstein's second criticism of Russell's theory of judgement which was (see Tractactus 5-5422) that 'the correct explanation of the form of the proposition 'A makes the judgement p' must show that it is impossible for a judgement to be a piece of nonsense (Russell's theory doesn't satisfy this requirement)'.[17] Griffin explains in detail the tensions in Russell's 1913 program: for a summary see p.180, from which it follows that the combination of theories Russell proposed to combine is inconsistent.

Most larger philosophies contain a good many minor inconsistencies: such appears certainly to be the case with the theories of Russell, Wittgenstein and Quine. A "minor" example from Quine concerns the proposition "God exists", which is rendered true by the theory of descriptions Quine adopts, but false by Quine's overall atheistic physicalism which leaves no existential place for God.[18] Such an example is minor because it can be avoided by a relatively minor change in the underlying theory of descriptions (as it is, Quine advances mutually inconsistent theories of descriptions). The fact that this example is "minor" and things comparatively easily rectified does not imply that it does not have a devastating impact on the theory as given, since classically any contradiction is a catastrophe.[19]

To obtain examples of inconsistent philosophical theories which are not trivial without change of logical base, it is not necessary to proceed backwards in time to periods before the ascendency of classical logic - especially since nonclassical theories are now being developed, as by isolated philosophers who were never caught in the classical trap, and as in this book - but it is advantageous to do so, especially if major philosophies are to provide examples. There are several well marked ways in which a philosophical theory can end in inconsistency, whether unintentionally or not; for example, by inclusion of sufficient self-referential apparatus for the formulation of paradoxes, by other paradoxes of a variety of types, through involvement in an infinite regress, by self-refutation of one sort or another, and so on. But as there is so far no very worthwhile classification of these ways to inconsistency (a thoroughly effective classification is not to be expected however), the preliminary grouping that follows is rather rough and ready. So also the selection of further philosophers we now present as having inconsistent theories is not at all systematic, nor is the particular selection of inconsistencies from the philosophers cited systematically determined. We set down what we fell over that was solid enough to trip us up. More exactly, we list examples of inconsistencies we managed to recall or came across or that others pointed out, where we thought we could make the claims good enough to stand up on their own.

Important sources of inconsistency in recent philosophy closely tied up with both features of mathematical objects and the logical paradoxes are (insufficiently qualified) characterisation postulates, that is, postulates which assign objects characteristics which serve to identify and distinguish them, such as that the object characterised as f is indeed f. The most famous modern examples are the inconsistencies concerning the round square and the existent King of France, that Russell located in the theory of objects of the Meinong school at Graz.[20]

More generally, comprehensive theories of abstract objects are especially liable to upsets of one sort or another, arising from characterisation postulates (inevitable for such objects) and ending in inconsistency. The stock example is Plato's theory of forms, which is revealed as inconsistent, for instance, by the problem of self-prediction and through the Third Man Argument.[21]

Another connected source of inconsistency to which philosophical theories, including recent theories, are especially prone derives from self-refutation. Inconsistency frequently emerges in this sort of way:- According to the theory t,
a) All philosophical [metaphysical, etc.] theories are of kind k. But
b) t itself is a philosophical [metaphysical, etc.] theory, at least according to its own lights,
c) t is not of kind k.
One famous example concerns logical positivism, which can be represented as a metaphysical theory to the effect that all metaphysical theories are nonsense. One of the damaging inconsistencies in Wittgenstein's early work is of this type, and issues in the proposed throwing away of the ladder or theory by which one has ascended.[22] Wittgenstein's later theory is in a similar predicament, with its theme to the effect that all philosophical theories are mistaken, or else that there are none.[23] Much the same applies to Collingwood's absolutist theory of metaphysical presupposition, where kind k amounts to involving metaphysical presuppositions. Because of their metaphysical presuppositions no philosophical theory can be taken as true or rejected as false; but this depends on the theory of metaphysical

presuppositions itself being taken as true.[24] As regards the history of philosophy, Collingwood's theory leads to derivative inconsistency, such as that some historical theories (e.g. Collingwood's) are both criticisable and not.

The trouble with Collingwood's theory resembles, in its self-refuting aspects, the trouble with Protagoras's 'man is the measure of all things' doctrine of more than 2000 years before (at least as that doctrine is interpreted by Plato). To engage properly in discourse at all, as he does, Protagoras has to assert something to be the case; yet he renounces the claim to be able to assert true statements, in his teaching that no one can inform one of anything.[25] In this way Protagoras's position leads to inconsistency (in fact of the general form already given above). Other positions of a thoroughgoing relativistic or sceptical cast do also.[26]

Inconsistencies over knowledge or belief, common among philosophers, sometimes arise, as in scepticism, out of self-refuting theses, sometimes not. It is not clear which mechanism is operative in Lao Tse, whom Bose has accused of inconsistency over knowledge. According to Lao Tse, it is both wise (sensible) to know the laws (of nature) and also not wise to know anything.[27] The situation looks a bit like the Socratic "paradox", that Socrates' wisdom consists in his knowledge that he knows nothing. Socrates is also inconsistent. For $K_s(p)(p \rightarrow \sim K_s p)$. But if this proposition, q, is true then it follows, since $(p)(p \rightarrow \sim K_s p)$ (by $K_s r \rightarrow r$), that $\sim K_s q$, i.e. $\sim K_s K_s (p)(p \rightarrow \sim K_s p)$. But in the case of such conscious knowledge, it seems certainly true that $K_s r \rightarrow K_s K_s r$ (i.e. an S4 principle holds[28]). So $\sim K_s(p)(p \rightarrow \sim K_s p)$, establishing inconsistency as regards q.

Bose goes on to charge that 'Locke and Rousseau are "bogged in inconsistencies"', but he unfortunately supplies no details. According to Passmore, however, who does give details, the inconsistencies in Locke are quite blatant. One instance is this: 'An idea, let us say, is whatever lies before the mind, and yet we can have before our mind the ideas' capacity to represent what is not an idea'.[29] Locke is, so Passmore contends, not inconsistent in an uninteresting way, for instance out of carelessness, or because he has not entirely overcome an older position he is working away from. Such inconsistency is not deep and is easily rectified, for instance, by minor adjustments to the theory or the restatement of certain themes or claims, so removing inconsistencies while remaining faithful to the original.[30] No, Locke is inconsistent because he has to be, to establish what he wants to establish: that is why he is inconsistent about ideas and about our power over our beliefs, and why that inconsistency is deep in his theory and not easily excised. Thus

> there can be no doubt that Locke would have liked consistently to maintain two theses, the first that rational human beings will regulate their degree of assurance in a proposition so that it accords with the evidence.... The

second, that human beings are so constituted as naturally to do this, were created rational, that they go wrong only where some of the evidence is not before them. But he finds it impossible to reconcile this second with his experience of the actual irrationality of human beings...[31].

Consistency of his belief with the evident data is never satisfactorily achieved, with such results also that there is not a uniform picture of belief in Locke but competing inconsistent theories, modelled on knowledge and, differently, on desire. The account of belief as an important surrogate of knowledge[32] is one source of a serious inconsistency in Locke's ethics of belief:- On the one hand, he maintains, on several occasions, that 'to believe this or that to be true is not within the scope of the will', is not a voluntary matter. On the other hand, he also maintains, and is theoretically committed to maintaining, that belief ought to be, can be, and is often a voluntary action.[33]

Inconsistency was not confined to seventeenth and eighteenth century empiricists, such as Locke and Hume. It much afflicted the rationalist alternative as well. Spinoza's Ethics,[34] in particular, appears to be riddled with inconsistencies, many of which seem sustainable, for example, the following, concerning God and the notion of love. On the one hand God loves himself (an immediate consequence of proposition 35, book 5), whence, by the definition of love (p.130: love is pleasure accompanied by the idea of an external cause, Definition of Emotion 6: see p.172), God has emotions, and God is affected with the emotion of pleasure. On the other hand, 'God is free from passions, nor is He affected with any emotion of pleasure of pain' (proposition 17, book 5). Derivatively then, God, an already perfect being, sustains both increases and also, worse, decreases in perfection.

Another deep inconsistency arises from Spinoza's strong determinism. Spinoza is committed, in the Ethics, to these theses:

(1) Δp (everything is non-contingent)

(2) $\Box p \supset p$ (whatever is necessary is true)

(3) $\Box p \supset Pp$ (what is necessary is permissible)

(4) $(\exists p)(p \,\&\, {\sim}Pp)$ (nonpermissible acts occur)

But (1) and (2) imply

(5) $p \equiv \Box p$,

and (3) and (5) imply

(6) $p \supset Pp$;

And (4) and (6) are inconsistent. This simple argument[35] to inconsistency presents a difficulty more generally for philosophers committed to a determinism with some teeth in it: for

(5') all (and only) what happens is determined, and

(3') what is determined is not reprehensible (forbidden).

Therefore nothing that happens is reprehensible. But, by independent evidence, some things that happen are reprehensible.

It has so often been charged that Descartes is inconsistent that a great deal of controversy surrounds the question. Until this is straightened out, if it can be, Descartes is even less guilelessly included as a philosopher with an inconsistent position than the other philosophers we have indicted. Among the more important grounds on which Descartes has been accounted inconsistent are the following nontrivial issues:- Firstly, clear and distinct perceptions need to be guaranteed by something beyond themselves, e.g. God; but clear and distinct perceptions do not need to be so guaranteed, because they are self-guaranteeing in virtue of their clearness and distinctness. Secondly, one both can trust one's senses and cannot trust one's senses, as Descartes' argument on scepticism reveals. Descartes finds reasons for rejecting sense evidence as a ground for truth claims. But his reasons require[36] an appeal to sense evidence, e.g. that he has sometimes been deceived by his senses, something determinable only by relying on, what he rejects, subsequent sense evidence. So he both relies upon and rejects sense evidence. It does not matter, for our purposes, that the arguments contain large gaps; more important is that the conclusions reached and maintained in the dialectic are inconsistent. Thirdly, the theory of mind-body interconnections was entangled in difficulties and inconsistencies. [37]Consider the nature of a relation relating mental and physical phenomena, of which there are many: it must be included either in the material or in the mental, yet in view of its relata it cannot belong to either area. Or consider the problems concerning rational purposive and voluntary behaviour. On the one hand these things can be distinguished from their opposites, yet on the other, given the theory, they cannot be so distinguished.[38]

Reporting examples as controversial as the Cartesian philosophy will make things much easier for the friends of consistency. These friends will, without doubt, claim to be able to remove many, or even all,[39] of the examples of (unintentional) inconsistency in major philosophers that we have assembled; and certainly they will be able to throw up enough dust to obscure the issue. So it is as well that we have several examples of admitted or intentional inconsistency to complete our case.[40]

Our main examples of inconsistent philosophical theories that were recognised as such have already been given. They include tentatively Heracleitus, fairly certainly various Indian schools such as the Jains, and quite definitely Hegel and Marx and Sartre. [41] For example, Hegel's theory is non-trivial because there are definite propositions - many from the history of philosophy - that Hegel rejected and which (on relevance grounds, for example) appear not to be entailed by his theory. And Hegel's theory is certainly inconsistent. A body in motion, for example, is both at, and also simultaneously not at, a given position. The basic categories, such as Being, are both self-identical and not. And so on.

Hume is another philosopher who is inconsistent in an extremely interesting way. Not only are some of Hume's inconsistencies integrally embedded in his philosophical system, but further, Hume is one of the rare major philosophers - apart from Hegel and his dialectical successors - who explicitly acknowledges and discusses inconsistencies in his philosophical system.

The commentator on Hume is faced with inconsistencies on considerable scale, as Passmore, Selby-Bigge and other commentators have said.[42] In a certain sense the whole of Passmore's Hume's Intentions is, as remarked by its author, about inconsistencies in Hume.[43] A great deal of scholarly argument is readily viewed as an attempt either to point to inconsistencies in major philosophies or to protect them against that charge. That is particularly so with Hume and Descartes. But what is astonishing about Hume is his confession of irremedial inconsistency in his system:

> I had entertain'd some hopes, that however deficient our theory of the intellectual world might be, it wou'd be free from those contradictions, and absurdities, which seem to attend every explication, that human reason can give of the material world. But upon a more strict review of the section concerning personal identity, I find myself involv'd in such a labyrinth, that, I must confess, I neither know how to correct my former opinions, nor how to render them consistent. If this be not a good general reason for scepticism, 'tis at least a sufficient one (if I were not already abundantly supplied) for me to entertain a diffidence and modesty in all my decisions. (Treatise, p.633)

Hume was however no dialectician:[44] he was very uneasy about contradictions, and would have removed them if he could, if he knew how. But his attitude to inconsistency in his own system, very definitely is not classical. Classically, Hume's system, if inconsistent, trivialises, which is not a good reason for modesty, though it is then not an unreasonable ground for scepticism. But with the rejection of a classical framework, inconsistency on its own is not a good general reason for scepticism: this much dialetheic theory shows. For if inconsistencies are, like other statements, restricted in their consequence cone, if they do not lead everywhere, then they may not lead to sceptical conclusions, such as that we do not know this or that we thought we did know. Generally, arguments to scepticism involve further assumptions than merely isolated inconsistent premisses. Further premisses supplied in Hume's system do give grounds for scepticism concerning personal identity, and given the underlying classical nature of his system, that scepticism may well get more widely distributed. The argument to limited scepticism takes the following lines. If, for some x, x knows facts about personal identity then these facts about personal identity are true, and hence, since facts about the world, consistent. But there is, given Hume's theory no consistent account of the facts concerning personal identity, hence, contraposing, x lacks knowledge. A different argument runs from inconsistency to what Hume pleads, as, 'the privilege of a sceptic', unintelligibility, that the matter is beyond (his) understanding. But this involves the assumption, rightly criticised in detail by Reid[45] that what is impossible is beyond understanding or conception.

A residual problem, however, with Hume's claim to unavoidable inconsistency is that the principles he 'cannot render consistent' though it is not in his 'power to renounce either of them' appear, at least on the face of it, not to be inconsistent at all. The principles are:

> that all our distinct perceptions are distinct existences, and that the mind never perceives any real connexion among distinct existences. Did our perceptions either inhere in something simple and individual, or did the mind perceive some real connexion among them, there would be no difficulty in the case (*Treatise*, p.636).

However, it is not a difficult feat to combine these principles with others of Hume's theses, to obtain an explicit and 'serious inconsistency in Hume's views'.[46]

iii) Inconsistent theories in science and the history of science.

Inconsistency of theories is by no means confined to more spacious philosophical edifices; other examples we appealed to in the introduction to Part 2 (§1I) were inconsistent theories in the history of science, for example Bohr's theory of the atom and the (early) infinitesimal calculus. There are many other examples, including (as Galileo showed[47]) the Aristotelian theory of motion.[48]

Now as in the legal and philosophical cases, the friends of consistency have to argue that the contradictions are only *prima facie*. The most plausible way of doing this is to suppose that, in practice, the theories were guarded by *ad hoc* auxiliary assumptions, or even slightly modified in an *ad hoc* fashion in such a way as to avoid overt contradiction. Again, it must be insisted that this was *always* done. However, history does not bear this out.

For example, when Bohr enunciated his theory of the atom he was quite clear that this contradicted Maxwell's equations. He *may* have had it in mind that Maxwell's equations would eventually have to be modified because of this. However, he provides no suggestions, even *ad hoc* ones, of how this is to be done, and goes ahead and uses Maxwell's equations whenever he feels like it. There is a blatant inconsistency which he ignores. Maybe others later tried to patch up the inconsistency. Maybe Bohr himself, at a later date, thought the correspondence principle might be used to solve the problem. However, there is no avoiding the fact that Bohr's theory of the atom as presented in 1913 was inconsistent.[49]

Similar points apply as regards the infinitesimal calculus. This was inconsistent and widely recognised as such. In this case various attempts were made to rework the theory in a consistent way. (For example, Berkeley had a theory of the "cancellation of errors"). However, the attempts did not meet with a great deal of success.[50] Moreover these attempts confirm the fact that the theory and certain of its parts, e.g. the Newtonian theory of fluxions, were inconsistent.[51] If they were not, attempted consistentizations would hardly have been necessary.

Thus, there are genuinely inconsistent theories in the history of science. Moreover, even if our claims specifically about Bohr's and Newton's theories were not correct, the friends of consistency could not claim a priori that prima facie inconsistent theories always had ad hoc consistentizations. That would, after all, beg the question against us. What would be required, rather, would be a detailed historical analysis of many and varied cases of such inconsistent theories. And such an analysis, which we are content to leave to suitably informed historians (those acquainted, among other things, with paraconsistency), would, we conjecture, issue in our favour.

iv) The matter of non-triviality

So far we have argued that in many domains of intellectual endeavour inconsistent theories abound. To complete the paraconsistent picture we need to argue that many of them are non-trivial. Of course this is not always true. As we have seen, philosophers such as Frege and Russell who produced inconsistent theories and who explicitly endorsed classical logic had the misfortune to produce trivial theories. However, in most of the other cases we have discussed we wish to claim that it is at least reasonable to suppose that the theories concerned are nontrivial. But it is not easy to argue this. None of the theories we have discussed is a formal theory. (Indeed one aim of our enterprise was to find inconsistent informal theories.) Hence a rigorous non-triviality proof is out of the question, since a very exact formulation is required before a start can be made upon applying modern techniques for establishing non-triviality of a theory. Conceivably some of the theories in question could be formalized,[52] thus posing the possibility of a rigorous non-triviality proof. However, then the question of the adequacy of the formalization would typically pose similar problems. How then are we to proceed? There are (at least) two possibilities.

The first is to pay particular attention to the sort of logic underlying the theory. If it fails to contain the principle ex falso quodlibet then the fact that the theory contains contradictions does not ipso facto produce a presupposition of triviality. This is perhaps good enough for our purposes. If the theories in question are theories of science or law where there is no selfconscious reflection on the logic being used, then this approach may be only marginally fruitful. For the underlying logic is exactly natural logic - the logic of ordinary discourse. Whether or not this is paraconsistent is precisely what we are arguing about. However, clearly, we think it is. With philosophical theories we are somewhat better off. for often philosophers explicitly or implicitly endorse certain logical systems. Perhaps the best example for our purposes is again Hegel. Despite the fact (or perhaps in virtue of the fact) that he wrote two enormous books on logic, we and many other people would not care to say exactly what his logical theory was. But this much is clear:- Hegel's philosophy is explicitly inconsistent. But no man in his right mind would have an explicitly inconsistent philosophy and a non-paraconsistent logic. Hence Hegel's logic was paraconsistent. (Perhaps the weakest part of this reasoning is the suppressed premiss.)

We can also mount an argument from the form of the underlying logic for the nontriviality of the main traditional theories, for example, empiricist and rationalist philosophies of the 17th and 18th centuries. Their received logical theory would have been some form of Aristotelian logic. Now despite the fact that the law of Non-contradiction is a keystone of Aristotle's logic, Aristotle's syllogistic (and indeed the larger traditional theory) is paraconsistent! Specifically the inference

$$\begin{array}{ll} & \text{S is P} \\ & \underline{\text{S is not P}} \\ \therefore & \text{S is Q} \end{array}$$

is not a valid syllogistic figure, and the inference

$$\begin{array}{l} \text{S is P} \\ \underline{\text{S is not P}} \\ \text{R is Q} \end{array}$$

is a fallacy of four terms.

The second approach is somewhat stronger and allows us to produce a legitimate presupposition of nontriviality. Although there is no general decision procedure for deciding whether something follows in a theory (even in the formal case) we may have well founded and reliable intuitions about this. Neither is this an appeal to some modish irrationalism. The intuitions in questions are to be obtained only by a lot of hard rational work, such as intelligent and informed guessing, and deductive and non-deductive reasoning. It is necessary to be thoroughly familiar with the theory; to know what typical proofs are like; to have tried to prove certain things unsuccessfully and understood why attempted proofs or arguments break down; it is necessary to know what sort of interpretations or partial interpretations the theory has, to know major heuristics for proving things in the theory and so on. It is exactly this kind of experience, for example, on which most contemporary logicians would base the judgement that ZF and Peano Arithmetic are consistent. Now in many of the cases countenanced such intuitions are available. For example, experienced lawyers know what sort of cases can be made out and what cases are hopeless on the basis of well understood laws. Yet no lawyer would claim that a body of inconsistent law (such as the one described in the introduction to part 2(§1II)) would allow him to be able to make out a good case for anything at all. Similarly scientists working on an inconsistent theory, such as Bohr's or the infinitesimal calculus, would oviously reject the idea that their theories could be used to prove everything. With philosophical theories which are somewhat more fluid, it may be difficult to build up the kind of intuition that is necessary. Yet we think that any Spinoza scholar would firmly reject the idea that from Spinoza's principles follows Cartesian Dualism, and any Locke scholar reject the claim that from Locke's theory follows the mind-brain identity theory.

Let us say again that these intuitions (roughly, intuitive theories) are by no means conclusive. However, they serve at the very least to establish a legitimate presupposition. We may even push the case further. Consider any of the inconsistent theories or situations we have so far mentioned, and suppose that we were able, with some form of reasoning to show that from the principles of the theory followed something absolutely antithetical to the spirit of the theory. Clearly someone who accepts the theory may give it up, or they may try to reformulate the theory in order to avoid this. However, they may simply reject the reasoning involved. For example, consider the following thought experiment. Suppose we had argued with Hune from the admitted inconsistency of his position, using <u>ex falso quodlibet</u>, to the conclusion that we can be certain that the sun will rise tomorrow. Would he have accepted this? Of course not. He <u>would</u>, quite rightly have rejected this form of inference. Obviously what is accepted and what rejected by a theory depends upon the underlying logic of the theory. The thought experiment illustrates that the converse may also be true. If we are certain that a theory (or a person holding it) strongly accepts A but rejects B, then we have good evidence to suppose that the person would reject the inferences from A to B. Moreover the fact that something is rejected means <u>ipso facto</u> that the theory, whatever its underlying logic is to be, is non-trivial. Since in all the cases we have considered we can be sure that the judges, scientists, philosophers, etc., in question would strongly reject certain things, we have good reason to believe the theories in question to be non-trivial.

When we go on to try to produce a theoretical account of natural logic, the logic of ordinary discourse, we run into the same type of phenomenon. We have an independent fix on neither the logic nor the theories, such as semantics and set theory, embedded in the practice. Hence we must determine the best theory of these jointly. The situation is not unlike that in accounts of radical translation, where we have no independent fix on either speakers' beliefs or their meanings but must fix the two simultaneously. In the present case we have firm dispositions to accept, e.g., the T-scheme and equally firm disposition to reject other beliefs about truth. We can not have a theory which endorses both these intuitions and the triviality-producing absorption principle: $A \to (A \to B)/(A \to B)$. Clearly we can reject the T-scheme and accept absorption. However, a simpler and far more plausible course of action is to accept the T-scheme and reject absorption (as we in effect argue at many places in the book.) This is particularly so since once we reject the mistaken identification of implication (or $\to$) with material implication and its mates, it is not at all clear that our intuitions for accepting absorption are very strong.

With segments of naive semantics and naive set-theory however, where the intended formal principles can be uncontroversially articulated, the matter of non-triviality can be more straightforwardly resolved. For we can choose, in well-motivated ways, underlying logics in terms of which non-triviality can be proved (see below).

v) Naive set theory

Another of the diverse examples we gave in the introduction to Part 2 (§1I) of an interesting inconsistent theory was naive set theory. This is the theory of sets produced and developed in the late nineteenth century mainly by Dedekind, Cantor, and Frege. The inconsistency of this theory is incontrovertible. The claim that the theory came complete with fail-safe devices, of the kind considered in previous sections, to prevent the deduction of naked contradictions has no plausibility. This is because, among other things, some people (such as Burali-Forti and Russell) went ahead and deduced contradictions.

The defence action by the friends of consistency has therefore had to take another direction in this case. The main move has been to suggest that whilst, for example, Cantor's theory was inconsistent it was not essentially so. That is, the theory of sets is at bottom a perfectly consistent theory. However, the early set theorists blurred a couple of fundamental points producing inessential contradictions. The immediate and crucial question is, therefore, what this reasonable and consistent theory of sets is. (Different answers to this will locate different sites of confusion in early set theorists.) If this cannot be answered, this line of resistance collapses. So what is the reasonable, consistent core of the naive theory? It is a matter of history that for a long time there was no clear answer to this question. Rival answers provided by Russell, Zermelo, von Neumann, Quine, et al, vied for place. However, it is now fairly clear that a consensus has emerged among mathematicians (though not perhaps among philosophers). The fundamental notion here is that of the cumulative hierarchy, i.e. the hierarchy defined by recursion on the ordinals thus:

$$V_0 = \phi$$

$$V_\alpha = P(V_\alpha), \text{ i.e. the power set of } V_\alpha$$

$$V_\lambda = \bigcup_{\beta<\lambda} V_\beta, \text{ for limit } \lambda$$

The surprising fact is that virtually every consistent set theory proposed this century can be seen as characterizing an initial segment of the cumulative hierarchy (in the sense that the segment is a fairly natural model for the theory).[53] Thus ZF set theory characteristics V_θ where θ is the first inaccessible ordinal, while the proper classes of Bernay's set theory can be taken to be just the members of $V_{\theta+1}$. Finite type theory based on the natural numbers is essentially $V_{\omega+\omega}$ and so on. The only proposed set theory which cannot be fitted into this picture is Quine's system NF (and its class-extension ML). It is indicative of the hegemony that the cumulative hierarchy has now achieved, that Quine's systems are, by and large, regarded as little more than curiosities.

Let us ask then whether the cumulative hierarchy is the consistent and reasonable core of naive set theory? (That it is an interesting and important structure is not in dispute.) The answer to this must be a fairly definite 'No'. One reason is that the notion of set produced by the cumulative hierarchy is very different from that produced by the naive theory. For a start the naive notion is clear and precise, whilst that of the cumulative hierarchy is, as we shall explain, inherently vague. The naive notion of set is that of the extension of an arbitrary predicate, a notion that can at once be spelt out in a pair of axioms, comprehension and extensionality. This is as tight an account as can be expected for any fundamental notion. It was thought to be problematical only because it was assumed (under the ideology of consistency) that 'arbitrary' could not mean arbitrary. However, it does. By contrast the notion of set given by the cumulative hierarchy is only as clear as the notions used in defining it. To begin with, the notion of an arbitrary ordinal is a highly problematic one. (The notion of an arbitrary sub-set also poses problems for the consistentist but we will pass this over.) Specifically, the construction of sets presupposes a prior construction of ordinals. However, this raises all sorts of problems about "how far" the construction can be continued, about sizes of infinities, etc. Indeed it is just these kinds of problems that the theory of sets was supposed to solve. We do not deny that once one has a notion of set one can noncircularly produce a notion of ordinal and use this in turn to define a special collection of sets, the cumulative hierarchy. But to suppose that one can use the notion of an ordinal to produce a non-question begging definition of 'set' is moonshine.

There is a second and stronger argument that the cumulative hierarchy is not the essential core of the naive theory. This is that there are quite essential features of the naive theory which cannot be handled by the cumulative hierarchy. Specifically, the cumulative hierarchy cannot handle such intuitively acceptable sets as the universal set, and such intuitively acceptable set theoretic operations as complementation. Sub-collections of the hierarchy which have members of arbitrarily high rank are just not sets acceptable at all. The hierarchist may dig his heels in and insist that there really are no such sub-collections, but this only illustrates our main point. The notion of such a sub-collection is a quite legitimate naive one. If it is not a legitimate one for the hierarchist, then the hierarchy does not grasp the essentials of the naive notion. It might be thought that the notion of a proper class would help here. It does not. One might conceive of a collection with members of arbitarily high rank as a proper class. But if one does this, there is no reason for not supposing that these proper classes can be members of hyper-proper classes. And what this shows is that we are still going "up" the cumulative hierarchy. So contrary to our original supposition, our "universe" is not <u>the</u> universe at all but merely a proper initial segment of the cumulative hierarchy. If we really did have the <u>whole</u> universe to start with, the notion of a proper class would take us nowhere.

The standard reply to the argument we have just advanced is that the sorts of set theoretic constructions we have been referring to are not part of the essence of set theory at all but part of the peripheral confusion. This however is an illusion encouraged by the fact that during the period 1920-1960 it seemed that the hierarchy did provide a sound basis for all mathematics - at least all mathematics for which set theory was just part of the auxiliary machinery. However, the illusion has now been shattered by category theory. Category theorists wish to refer to the category of <u>all</u> groups, sets, etc., or even the category of all categories. This cannot be done on the hierarchical view of set. Of course, there have been some attempts to get round the problem. Most of them work on some variant of the strategy of supposing that "all groups" means all groups of rank less than β for some nice ordinal β. However such ploys, which there is no need to disciuss in detail,[54] are just a subterfuge. The unpalatable fact (unpalatable to category theorists who accept the hierarchical view of sets, that is) is that one just cannot perform the constructions with global categories that category theorists need to make. There is no such thing as the category of <u>all</u> sets, etc., and that is that.

Thus the adequacy of the hierarchy view for all mathematics can no longer be maintained. Naive set theory has an essential power (essential for real mathematics, that is), which goes beyond that of the hierarchy; the hierarchy is not the consistent essence of naive set theory. Such a stable consistent essence has not been obtained. Nor is it likely to be: there are no doubt various consistent cutdowns of the inconsistent naive theory, but all will sacrifice significant features of the original whole; all will stop the expression of what can be expressed.

vi) Naive semantics

The final example of an interesting inconsistent theory that we gave in the introduction to Part 2(§1I) was that of naive semantics. Semantics, as usually understood, is the theory which concerns notions such as truth, satisfaction, denotation, etc. and naive semantics is the theory of such notions embedded in natural language discourse on such matters. Unlike naive set theory it has never been much elaborated, with distinctive theorems, results, etc. Nonetheless, *prima facie*, truth, denotation and satisfaction would seem to be characterized by the following axioms (already given in the introduction to Part 3):

$$Tr\ulcorner\phi\urcorner \leftrightarrow \phi' \qquad (1)$$

$$Den\ulcorner t\urcorner\, x \leftrightarrow t = x \quad (2)$$

$$Sat\ulcorner\phi\urcorner \leftrightarrow \phi'_s \qquad (3)$$

respectively, where ϕ is an arbitrary formula (closed in (1)), ϕ' a suitable translation, t is an arbitrary term, s a sequence, $\ulcorner\urcorner$ is the naming functor, and ϕ'_s is ϕ' with every free occurrence of the i'th variable replaced by 's(i)'. It is exactly this kind of insight which is enshrined in Tarski's calling (1) a *condition of adequacy* on any definition of truth.[55]

Each of (1)-(3) leads, as is well-known and as we have indicated, to its respective paradoxes. Both hard approaches, which fly in the face of the data, and softer approaches, which try to take some account of the continuing successful operation of natural languages, have been tried. Hard-liners, such as Tarski[56] and many logical positivists, grant that (1)-(3) do characterize our naive notions and have accordingly concluded that they are incoherent. But this effectively concedes what we are arguing for, that our naive notions are inconsistent. That they are incoherent is an unwarranted classical extrapolation. The friends of consistency who try to take a softer line have been forced by the paradoxes to maintain that, despite appearances, (1)-(3) do not characterize our naive semantic notions. It should be said straightaway that no argument has ever been produced for this claim *other than* that (1)-(3) lead to inconsistencies - which is no kind of argument at all against a paraconsistentist. Moreover, the sorts of thing that have been proposed to replace (1)-(3) are usually not only ugly but of dubious efficacy: they are either strong enough to produce some form of logical paradox or far too weak. But the detailed argument here comes down to the matter of proposed solutions to the semantical paradoxes, something we shall have to take up in detail below (in §II ,ii,a). What we will do now is to adduce one further argument as to why the T-scheme (1), in particular, cannot be jettisoned in semantics.

Perhaps the most fundamental insight in semantics in the last 100 years is that the meaning of a sentence is (given by) its truth conditions, or better, that to give the meaning of a sentence is to give its truth conditions. Though the insight is Frege's, he did not take it very far. However, it has without doubt provided the keystone of almost all the formal semantic theories this century. This means that the T-scheme must be an essential part of any semantics. For this, after all, is the scheme which states the truth conditions of ϕ. Admittedly, the T-scheme may not always occur in its pristine form. It does in Davidson's original theory. But it has to be context-sensitivised for indexicals. Moreover it has to be world-relativised as well in Montague or Routley/Meyer semantics, and it has to be constructivised for Dummett. Yet it is there playing its central role. In fact, without it we would be hard-pressed to know what a formal semantical theory looked like. Thus the friends of consistency, in urging us to junk the T-scheme, are in effect doing nothing less than urging us to junk semantics.

As always in philosophy, there are comebacks. The friends of consistency could claim - a claim yet to be made good - that formal semantics could be worked out with a limited T-scheme, which, like that of the hard-line levels-of-language people, coincides with the T-scheme for restricted languages and fragments of natural languages. And of course there is little problem in preserving the general truth scheme if we are concerned with the semantics of only certain fragments of natural language - in particular those which do not involve semantical notions themselves. This is what semanticists have by and large busied themselves with. However, once we try to map out the semantics of a whole natural language (including its own semantical discourse) - the doing of which has always been the pretension of semanticists despite their fragmented vision - the problem can no longer be shelved. Let us say it again: give up the general T-scheme and there must be some sentences whose truth conditions, and therefore meaning, cannot be given. Goodbye universal semantics.

With this powerful case for paraconsistency - from the semantical analysis of philosophically unavoidable inconsistent theories, such as those supplied by natural languages which quite properly include their own semantical terms - we move, or rather the dialectic moves us, from the questions of inconsistent theories to the question immediately raised by the apparent, or possible, truth of some of these theories: the matter of true contradictions.

II) The truth of some contradictions

The second reason we gave for paraconsistency in the introduction to Part 2 was the truth of certain contradictions (see §1II). This claim is likely to seem even more contentious. So let us examine it in more detail. The examples of true contradictions we gave concerned i) multicriterial terms and ii) the logical paradoxes. We shall consider each of these in turn. There are other examples that have been canvassed, but we prefer not to hang our main case on them.(We will discuss them later, in §3β).

i) Multicriterial terms

The situation *in abstracto* is this:- We have some term t, and two criteria C_1 and C_2 which are empirically determinable. That C_1 holds is logically sufficient for applying t. That C_1 fails is logically sufficient for applying ⌜not-t⌝. Similarly with C_2. C_1 and C_2 are not, however, synonymous. Thus a situation may arise where C_1 holds but C_2 fails, making both t and ⌜not-t⌝ true of some object or situation.[57] Now the friends of consistency must deny the possibility of the sort of situation described. On what grounds is this to be done? There are several.

The first is to deny that there are any criteria in the sense we require, namely such that they are logically sufficient for the application of a term. It might be argued that criteria are connected to the applicability of terms via empirical "correspondence rules" such as 'when C_1 occurs, x is t'. Thus when the situation described arises what we have is, in fact, a falsification of either the correspondence rule for C_1 or that for C_2. However, it is not possible to maintain this line. It is a fact of life that there are criteria of the kind described, where there is an analytic connection between the satisfaction of a criterion and the correct applicability of a term. Thus, for example, having two legs is logically quite sufficient for the correct applicability of 'biped'. Having male genitalia (or perhaps having a certain chromosomic composition) is logically sufficient for the correct applicability of 'male'. Measuring six inches by a ruler is sufficient for the correct applicability of 'six inches long', etc.

The second possibility is to admit that there may be an analytic connection between criterion and applicability of term, but to argue that if the situation we have described occurs, what this shows is just that, despite appearances, either C_1 or C_2 has in fact failed. This kind of approach might be backed by an appeal to the general fallibility of observation, etc. Now it has to be conceded that in the sort of situation described, the move of doubting that either C_1 or C_2 obtained is a possible one. However, it seems to us that we would never take the clash per se to show that either C_1 or C_2 failed. We might well investigate the holding of C_1 and C_2 to try to find independent evidence of their failure. However, there is no reason why, in general, this should be forthcoming. And, if it is not, we would find it very unreasonable to insist, nonetheless, that one or other has failed. For all the arguments about theory-ladenness notwithstanding, to deny that, for example, an animal has male genitalia, for no concrete reason and when the evidence is right before one's eyes, takes a lot of balls. In fact, if we tested C_1 and C_2 independently and found them both to hold, we would not insist that one of them failed; what we would do is move on to another tack, which is the third way one might try to argue that the situation described never really occurs.

The third way is this. We might argue that the fact that we have two criteria analytically connected with the applicability of a certain term, one of which is realizable, indeed, realized, whilst the other is not, shows that the term is in fact ambiguous, representing two quite distinct concepts. Let us call these t_1, corresponding to C_1, and t_2, corresponding to C_2. Thus in the case described, t_1 is correctly applicable (but not $\ulcorner$not-$t_1\urcorner$) and $\ulcorner$not-$t_2\urcorner$ is correctly applicable (but not t_2). Hence the contradiction is only apparent. This insistence that there be a (1-1) correlation between concepts and criteria of applicability is, in fact, a well-known position. It was argued by operationalists such as Bridgeman. Equally its shortcomings are well-known.[58] The essential point is that as a matter of fact the senses of terms are not individuated in this way, via criteria of application. This can be seen by the fact that if we did try to pursue science whilst enforcing this kind of individuation, the complex network of theoretical interconnections of science would break down irreparably. Thus we have, in outline, a transcendental argument against this position.

Notwithstanding any of the above, in the kind of situation described, concepts do have a tendency to split. As a response to the crisis provoked by the falling apart of the criteria, the old concept will frequently divide into two, one corresponding to each of the criteria.[59] Be that as it may, this does not show that there was no true contradiction. Ex post the contradiction is resolved into two non-contradictory statements. But this does not affect the fact that ex ante the contradiction stood: otherwise there would have been no conceptual change.

This exhausts the relevant possibilities: there is no way of avoiding the conclusion that true contradictions are produced by multi-criterial terms (even if in the ongoing dialectic these contradictions are duly resolved or removed, and replaced by others, and so on).

ii) Logical and semantical paradoxes

The logical paradoxes are (as we stressed in the introduction to Part 2 (§1II)) prima facie sound arguments with contradictory conclusions. Anyone who wishes to deny the truth of the conclusions must deny the soundness of the paradoxical arguments - of every single paradoxical argument, that is. But they must also do more than this. They must locate, specifically, the place where the argument fails (and be prepared to accept all the consequences thereof). However, just proposing a precise location of the unsoundness is not sufficient. That, after all, is too easy: merely list every principle used in a paradoxical argument, select one at random and deny it. Someone who wishes to reject the paraconsistent position must not only locate the source of the unsoundness but must explain, in a non-question-begging and coherent way, what is wrong with it. In fact, even more than this is required. An explanation of why the incorrect principle was found plausible in the first place is also required. Otherwise, the paraconsistent position, that all the principles are correct, still outstrips its rival in explanatory power. These are tall orders, which have never been met, as even many of those who would unreflectingly dismiss paraconsistency effectively acknowledge:

> No one, for example, who has thought at all seriously about the paradoxes will feel at ease with the supposition that they must contain one or more specific errors which, if presented to us, we should be readily capable of recognising as such and excising from our conception of admissible argument and definition. We have learned of a variety of strategies which seem to keep us out of trouble; but none of them has the simple intuitive appeal originally possessed by the 'naive' assumptions concerning class existence and predication, and what constitutes an admissible range of quantification, which featured in, for example, Frege's foundational theory. It is difficult to defend a notion of 'error' in this context for which the criterion is not precisely the potential to generate paradox; and this criterion, naturally, fails to discriminate in point of preferability between the alternative, seemingly successful strategies for avoiding paradox.[60]

These sorts of difficulties, for the friends of consistency, are basically the same whether the paradoxes are set theoretic or semantical ones. Indeed there is, we should argue, no essential difference between these types of paradox, a feature dialethic resolution reflects. Nonparaconsistent approaches have however generally been formed into a bogus distinction of types, and in order to deal critically with these approaches it is convenient to follow this artificial separation. The contemporary extensional bias in logic has moreover led to the widely assumed reduction of all logical paradoxes to set-theoretical paradoxes, with which we shall start.

a) Paradoxes in set theory.

The almost universally accepted villain of the set theoretic paradoxes is the abstraction axiom: the principle that every condition defines a set which is its extension. This, it is claimed, cannot be true in every case. Moreover, there is also a fairly general mathematical consensus as to which of its instances are acceptable: essentially those that are true in the cumulative hierarchy (see §2 , I v above). This answer is not completely definite. It suffers from the vagueness we noted before concerning the "height" of the hierarchy. (For example there is no way to use the conception of set to determine whether the condition 'the rank of x is less than the first inaccessible' defines a set.) However, the answer is perhaps just passable, and enough to get to first base. Unfortunately for its supporters it is put out before it gets to second.

For a start no one has ever explained what is wrong with the instances of the abstraction axiom which fail in the cumulative hierarchy. (It cannot even be claimed that they necessarily lead to contradiction when added to ZF.) If it could be shown that the cumulative hierarchy was the essential core of the theory of sets, then this would go some way towards answering this challenge; however, as we have seen, this cannot be done. Moreover, even if it could, the solution would still not be adequate. For why we should ever have thought that every condition defines a set remains a complete mystery. It doesn't even look a plausible claim. How could such a mistake be made? No: the genuine conception of set is that given by the unrestricted abstraction scheme, according to which a set is the extension of an arbitrary property or condition. The cumulative hierarchy is exactly what it appears to be with a little historical perspective: a consistent substructure of the inconsistent universe of sets, masquerading as the whole thing.[61]

b) In semantics

The matter of the semantical paradoxes is more complex, though prima facie it should be simpler. There is no consensus whatever amongst logicians concerning the location of unsoundness in the semantic paradoxes. "Solutions" are two-a-penny, if not cheaper. This should mean that the consistent position doesn't even get to first base. If the opposition cannot even field a team, we win by default. However, in practice this just makes life difficult for the paraconsistentist. For instead of having one major rival to argue against, there is a variety of competing "solutions" to counter - some plausible, some wildly implausible, some endorsed, some merely mooted, some ancient, some modern, and about all they have in comon is the ideology of consistency (plus the fact that they are wrong). Moreover, they all appear Hydra-headed: show that one distinction doesn't work and a dozen more appear in its place; show that an account runs counter to a well-supported philosophical theory and a dozen patched-up versions appear to replace it. In virtue of this, it would be understandable if we turned our back on the debacle and waited until at most one (and preferably none) survive the struggle for existence. However, something needs to be said if we are not to be accused of funking the issue. It would clearly be an impossible undertaking to criticize in detail all the rival theories that have been proposed - or even just the major ones - in an introduction, only part of which is devoted to the topic. It is fortunate, then, that it is unnecessary to do so here, since general arguments can be found that (largely) postpone the need for such detailed criticism.

First observe that given the derivation of a paradox, all the moves in it are at least plausible. This suggests that moves of these kinds are normally correct, though - if the consistency hypothesis is correct - they fail on certain occasions. On which occasions? One striking feature of the paradoxical derivations, often remarked, is that they all use sentences that are in some sense, self reflexive (or at least "ungrounded"). Exactly how to characterize this reflexivity is itself a difficult problem, but let us pass it by. Let us call this class of sentences R (for reflexive). We might suggest that an occurrence of a member of R in a normally valid principle of reasoning may suffice to invalidate it. Assuming that members of R are fairly rare, this at least suggests an answer to how we come to be under the illusion that the principles are universally valid.

However, the crux of the matter now is this. What exactly is it about members of R that may give them the ability to invalidate normal logical principles? To answer this question we need to isolate a certain class of sentences D (for defective) for which normal reasoning (including applicatons of the T-scheme, etc.) can be seen to fail. But what are these? This is where the picture starts to fragment. Many different answers have been suggested. Without attempting to be comprehensive, we think it is fair to say that the mainstream answers to this question are of two types:

α) the members of D are not well-formed. They may be well-formed sentences of English (or prima facie well-formed) but they are not well-formed sentences of a logically correct language (or of the deep structure of English);

β) the members of D are neither true nor false, lack a truth-value, fail to make a statement/proposition, or in some way fail to relate in an appropriate way to true/false.[62]

Once D has been isolated, the paradox is "solved" by insisting that the member of R used in the argument is in D.

We are now in a position to formulate some general criticisms. But before doing so we should point out that many of the solutions face internal difficulties concerning D even before R comes into the picture. For example, someone who espouses strategy (β) needs to give a detailed account of truth-bearers and why certain sentences may fail to express (be) them. Many problems lurk here, as a substantial literature attests. Similarly someone who espouses strategy (α) needs to show that the grammar at work is really the grammar of English (or at least the grammar of rational discourse). Notoriously difficult problems lurk here.

Our first criticism of the sort of solutions we have outlined is that they are almost all, without exception, ad hoc. It is rarely argued that the member of R is in D, except on the circular basis of the paradoxes themselves. It is just assumed. Even when a general criterion for being a member of D is formulated, it usually turns out to include a clause whose sole rationale is to capture members of R. Because of this, such solutions beg the question against paraconsistency. For some paraconsistentists may well conclude that sentences of the class D invalidate standard principles. However, they will just deny that the member of R in question is a member of D. For example, they may well be pushed into (erroneously) admitting that sentences which don't make statements cannot be logically manipulated.[63] However, they can still claim that the liar sentence, for instance, does make a statement, a paradoxical one.

Our second criticism is that it is usually not at all clear that the proposed solutions really do avoid the problem. For whilst simple forms of paradox may be avoided in this way, more complex forms are just around the corner. Specifically, paradoxes of the "extended" variety characteristically arise to trouble proposed solutions. Let us illustrate this with the extended liar paradox. The liar paradox is avoided by insisting that 'This sentence is false' is in class D. Thus the paradoxical derivation is stymied. But now consider 'This sentence is false or is in D'. The supposition that it is either true or false leads to the usual contradiction. Moreover, the escape clause that the sentence is in D now leads to a contradiction also. In this situation, the apologist can make one of two moves. He can isolate a new class of sentences, D', for which standard principles fail, and insist that the extended liar sentence is in it. This move will not help. Firstly, repeated, it leads to an infinite regress. The regress may not be vicious but it doesn't get us anywhere. For if (D_α) is the, possibly transfinite, sequence of sets generated by this process, we are still faced with the absolutely extended liar paradox:

This sentence is false or there is a β such that it is in D_β.

Secondly, it becomes more and more difficult to find new classes D', D", etc., which have any semblance of plausibility. So the whole thing bogs down very quickly. The other move the apologist can make is to insist that although the extended liar sentence is, in fact, in D, this cannot be truly said. This evidently opens the apologist up to an ad hominem argument of a devastating kind.

It also illustrates a third argument against "solutions" to the liar paradox. The purported solutions seem, without exception, to run the solver into the ineffable. The solver will be lead to the position that there are certain things which are the case which cannot be said or, more prosaically, the solutions will lead the solver to assert that certain things which can obviously be said, cannot be said. Ironically, one of these things often turns out to be the very solution itself.[64] This exclusion of sayable things as unsayable does not always occur in a uniform manner, but happen it invariably does. Moreover there are deep theoretical reasons why this should be so. The root of the problem is that English has an expressive power which is, in a sense, over-rich. It permits the saying of things whose semantic conditions determine that a contradiction is true (see the introduction to Part §2, III.) What all the solutions amount to in the end are proposals to limit this expressive power. However, this obviously means that there will be things which are the case, which can be expresed in English but which cannot be expressed in the self-imposed idiolect of the solver.[65] What this shows is that the original problem has not been solved but merely avoided. For the original proposal was to give an account of our semantic concepts and to show how, despite appearances, they do not, in fact, lead to paradox. In other words what is required is a semantic analysis of English, or at least those parts of it which themselves concern semantic notions, which shows their structure to be consistent. But what now transpires is that the semantic account offered in the course of "solving the paradoxes" is of notions which are expressively decidedly weaker than those embedded in English. Hence they are not those notions. Thus the semantic analyses are not those of our original concepts and do not therefore show them to be consistent.

These difficulties have suggested an heroic last stand to some friends of consistency. They (finally) concede that the semantic notions of English <u>are</u> inconsistent but urge that, in the cause of"science" (or whatever), they be ditched for ones that are. However this will not do. For, first, it concedes anyway, what we have been arguing, that the semantic conditions of English <u>do</u> determine certain contradictions to be true. Secondly, it recommends a move whose only benefit is the production of consistency. But once we are persuaded that the spread law, <u>ex falso quodlibet</u>, of classical and intuitionist logic is wrong, there is no objective benefit here (though it may, subjectively, make the friends feel better). Thirdly, the move occasions serious losses since it involves significant impoverishment of our expressive power (and so also of our logical powers). The adoption of some positivistic new-speak has therefore nothing to recommend it.

This completes our swift overview of the case against those who would "solve" the logico-semantical paradoxes. For the most part our arguments have to be understood rather as argument-schemes. For, strictly speaking, each proposed solution has to be taken in its own rights, its weaknesses exposed and explored.

Then the argument schemes can be instantiated to produce more concrete arguments against any such solution. No doubt many of the finer details will have to be hammered out, but we have no doubt that this can be done. Moreover some concrete positions will have prima facie replies to some of the arguments and these too will have to be handled on their individual (de)merits. If induction is a good guide, the replies will pose more problems than they solve. For this has been the whole character of the enterprise of "solving" the paradoxes. Indeed, the enterprise can be seen as a research program, starting in all seriousness at the beginning of this century. Its problematic is that the paradoxes indicate a flaw in some logico-semantic principle and the aim has been to find it. The strategies (α) and (β) given above are the main heuristics that have been used to find a solution. Each has been elaborated (in many ways), its weaknesses exposed, and its auxiliary belt of protective hypotheses multiplied. However, it is characteristic of the debate that rather than making bold progress towards a solution, it has bogged down in trying to solve no problems other than those spuriously created by the research program itself. It is, in Lakatos' terms, a degenerating problemshift.[66] By contrast, the newer paraconsistency program, which does not try to locate a fault in the paradoxical reasoning, is definitely advancing, solving interesting technical and philosophical problems. Eventually, therefore, we would expect to see the program of solving the paradoxes begin to wither away. It will come to be seen as a plausible idea that never worked properly.

3. Ramifications and consequences of paraconsistency, and further reasons for paraconsistency.

We have already argued that paraconsistency, and especially dialethism, has a major impact on logic and semantics, an impact which has many important philosophical consequences. The effect is equally devastating on the third of the conventional divisions of semiotics, pragmatics, and spreads out from semiotics to virtually all other reaches of philosophy. Because of the spread concerned we shall have to be very selective; we choose to concentrate, apart from pragmatics, on some facets of metaphysics and the philosophy of mathematics. We certainly make no claim that these are the only philosophical areas of consequence for paraconsistency: they are not.

α) Pragmatics

There are issues that come under the rubric of pragmatics - though this standard description is not an entirely happy one - questions concerning assertion, acceptance, and the use of argument to change beliefs rationally, that we can hardly avoid tackling. For there are various arguments to the effect that the ways assertion and belief function rule out paraconsistency. We will try to show that this is not so. In doing so we will show that certain standard conceptions of how they do function are incorrect and suggest better accounts. Thus paraconsistency has philosophical consequences in this area too.

i) Assertion: the question of content

Consider first someone who asserts a contradiction, say, to take the worst case, an explicit contradiction, such as A_o & $\sim A_o$. Obviously dialethicians are among such people. They are quickly confronted by several arguments to the effect that contradictions are not rationally assertable. If these were correct then strong paraconsistency would not be rationally espousable. Fortunately for paraconsistency, then, the arguments - though they do raise serious issues, for instance as to when it is reasonable to stop avoiding contradiction by resort to consistencizing strategems and rationally to accept an inconsistent theory - are generally rather feeble.

According to the first such argument, contradictions are not only untrue, but manifestly so. Hence it is irrational to assert them, since we should not assert (sincerely, of course) a blatant untruth. This argument simply begs the question: dialetheists hold that some contradictions are true. In reply to this, it might be argued that even on our grounds since the contradictions concerned are not only true but false, they still ought not to be asserted, since a rational man eschews falsehood. However, this again begs the question. If truth and falsity were exclusive, then the eschewal of falsity would follow from the aiming at truth. However once one sees that truth and falsity are inextricably intermingled, like a constant boiling mixture, the rational man must face the fact that the primary aim of complete truth-achievement can only be satisfied by accepting some falsehoods: the alternative of rejecting dialetheias would leave his grasp of truth partial and his knowledge incomplete. The second argument that contradictions are not rationally assessible is only marginally better. To see what it is, notice that someone who asserts A_o & $\sim A_o$ is denying an instance of the law of non-contradiction (LNC). It is often suggested that it is impossible to rationally deny the LNC, since even to do so presupposes it.[67] Now even granting that it is impossible to deny the LNC without presupposing it (which we find no good reason to believe), this objection need not worry a paraconsistentist. For what it shows is only that the contradiction-utterer is committed to certain secondary contradictions.[68] And to suppose that this is objectionable is again just to beg the question against paraconsistency. This kind of objection doesn't really take paraconsistency seriously.

The same points apply to semantical arguments sometimes used to back up the first and second arguments, for example by showing, most simple-mindedly by appeal to classical truth-tables, that contradictions can't ever be true. To appeal to such (classical) considerations is to assume several questions at issue. In any case, such semantical "verifications" can be met by semantical refutations, rival semantics which admit some contradictions as true.[69]

The third argument against the possibility of rationally asserting a contradiction is to the effect that contradictions have no sense or content. There is quite literally, therefore, nothing to assert and a fortiori assert rationally.[70] But in that event contradictions should have no sensible consequences, for the consequence relation would otherwise enable assertions with content to be got out of those with none, something to be got out of nothing. Yet contradictions do have consequences, A & ~A entailing A, for example. So what is the case, if any, for supposing that contradictions have no sense or content? It must depend, in the end, on some definition of sense or content which implies that result. Neither of the usual accounts of content - the semantic account in terms of the class of worlds a statement excludes, and the consequence account in terms of the statements the statement entails - have the requisite results; indeed classically and intuitionistically they, mistakenly, assign contradictions total content, and paraconsistenly (e.g. relevantly) they assign nonnull content.[71] Only in certain connexive logics, where contradictions entail nothing, do contradictions have zero content under the consequence account. But such a special logical base should be rejected as logically and paraconsistently inadequate, as we have seen.[72]

There is a better prospect of success, in bringing contradictions out as senseless (if that really is the objective), with the two accounts of sense which are dual to the accounts of content considered. For these accounts do render contradictions classically senseless. Under the first, which is often taken to be a Tractarian account of sense,[73] the sense of an assertion is something like the non-trivial set of worlds (or evaluations) at which it is true (i.e. its range). Then if contradictions were true at no worlds (under no evaluations), the conclusion would follow. However, there are evaluations which make contradictions true at some worlds.[74] Certainly if one restricts the totality of evaluations to classical ones,[75] (e.g. those for which $\nu(A) = \{0\}$ or $\nu(A) = \{1\}$ for all A), then there are no worlds at which contradictions are true. But to insist that one must do this when it is obviously unnecessary is, again, just to beg the question against paraconsistency. Under the alternative dual account of sense, the sense of an assertion is given by the complement of the set of assertions it entails. Should then we suppose that contradictions entail everything, they again come out with zero sense. However, the defining characteristic of paraconsistent logic is precisely the rejection of the counterintuitive view that contradictions entail everything. Hence this line of argument will not work against paraconsistency.

A similar set of points apply, with even more force, against specially rigged accounts of sense - or for that matter of rationality - which serve to bring out the antiparaconsistent conclusion. Rival, and more natural, accounts can be given, as we have just explained, which undermine the antiparaconsistent conclusion and show that it is far from obligatory. So why should it, and its grounds be accepted? The further advancement of the antiparaconsistent case resorts at this stage of the dialectic, and has to resort, to special pleading.

The objections to the rational assertibility of contradictions we have considered so far do little more than beg simple questions against paraconsistency. However, there is an apparently deeper theoretical argument to the effect that if we were to allow people to assert contradictions then no assertion would have any content.[76] The argument is this: for an assertion to have content it must exclude certain possibilities, otherwise it carries no information. Now when a paraconsistentist asserts A he thereby excludes nothing. Certainly he does not exclude ~A: that may be realized too. Hence his assertion has no content. The argument is very plausible. Indeed we can even strengthen it. For that the truth of A does not logically exclude the truth of anything else can be proved in the semantics of most paraconsistent logics. Any set of formulas has an interpretation in which the formulas are all true. However, the argument fails, and does so at the first step. There is no reason to suppose that for a sentence to have determinate and non-trivial content it must exclude anything. Consider '2+2=4' and 'Perth is in Australia'. If paraconsistency is right, neither of these assertions logically excludes its negation, or anything else. Yet each has a different determinate but non-trivial content. This is so because each carries information the other does not include. So the second implies that Perth is somewhere, that Perth is in either Australia or Indonesia etc. whilst the first does not. (Only the sentence 'everything is true' has total content: its content is determinate but trivial.) This notion of content can be captured by taking the content of an assertion to be the set of sentences it implies, or what comes to the same thing, its place in the De Morgan lattice of propositions.[77] At any rate it shows that logical exclusion is unnecessary for content.

As we have seen, paraconsistent theory can supply its own accounts of content, both semantical and consequential. (In fact the definitions take exactly the same form as the classical definitions but are based on different theories of worlds and of consequence respectively.) In this way paraconsistency shows that accounts of how assertions convey information in terms of what they classically exclude are misguided. Of course, if we accept such accounts of content we cannot use content-exclusion as a way of defining the sense, or content, of negation. But then there are plenty of other ways of doing this, for example, through a semantical account.[78]

A final thrust against paraconsistency may be based not on the notion of content, or what is asserted, or assertion, but on the notion of rationality. For it is widely assumed that contradictions, whatever they may assert (whether everything, something or nothing) cannot be rationally accepted or maintained. While it may be widely assumed, the assumption itself is usually not rationally based. Insofar as arguments are offered, they typically depend on a characterisation of rationality that simply makes consistency a necessary condition of rationality. But why do that? Imposition of such a powerful condition requires legitimation - otherwise it can simply be rejected (e.g. as stipulative) - by arguments to the effect that contradictions ought not to be accepted (where the ought is one of rationality, to be expanded semantically through a rule: In no ideally rational world...). What arguments there are, mostly deriving from Aristotle, implicitly invoke the principle that ought implies can, and try to show that contradictions cannot be accepted, by virtue of their logical character, e.g. they never hold anywhere, they lack content, are senseless, etc. We have already refuted these arguments: we have argued that contradictions do not have such radically defective logical character, that they are capable of being assumed, accepted and believed, and sometimes are, and that the arguments to the contrary, emanating from Aristotle, are one and all fallacious.[79] Furthermore, we have argued, in some detail, that in certain cases contradictions ought to be accepted, because there is no really rational way of avoiding them, and because they are true.

ii) Criticism and the change of belief

There is another way of trying to infiltrate traditional and classical rationality assumptions, through the question of the rational change of belief. It is often suggested that if true contradictions are admitted, then the process, indeed the possibility, of rationally forcing someone to change their beliefs by criticism is made impossible.[80] The central point is that if one criticizes a theory held by a dialetheist, there is nothing in the domain of logic to stop him accepting both his own view and the criticisms of the critic, even though they constitute a contradiction. Hence, it is claimed, his view cannot be rationally criticized.

The premiss of this argument is correct. The conclusion is a blatant non-sequitur. It assumes that just because some contradictions are true, any contradiction may be rationally accepted. This is almost as crazy as the view that just becaue some assertions are true, any assertion may be rationally accepted, and involves the same two fallacious slides: from some to all and from true to rationally acceptable as true. But, it will be objected, we need a criterion, a decision-procedure for telling which contradictions are acceptable and which are not. There is reason to suppose that the stronger demand for a decision-procedure cannot be met. There is no decision-procedure for determining when a contradiction is true. Moreover to demand one is unreasonable: we know there is no decision-procedure for truth, even in very simple cases. There is no decision-procedure for true sentences of the form ⌜Rab⌝, ⌜Fa⌝, ⌜$p \vee q$⌝, etc. Why should sentences of the form ⌜$p \vee \sim p$⌝ be any different?

How then are we to determine whether a given contradiction in a given context is rationally acceptable? A preliminary answer is that, at this stage, we need to consider each sort of case on its merits, and even then there may be no immediate clear-cut answer. There is perhaps no answer to the question 'when is an arbitrary contradiction rationally acceptable?' which is neither pretty vacuous, nor false. For there is, after all, no answer so far to the question 'When is an arbitrary assertion rationally acceptable?' which is neither vacuous nor false. But even though both a criterion and the need for one are in doubt, partial sufficient conditions are not. We have argued that certain contradictions are true. The arguments - whether cogent or not - are laid out (in §2II) above, for the reader to decide. The important point here is that these arguments are rational considerations driving us towards the acceptance of certain contradictions as true - and can be recognised by any rational agent of sufficient competence as such. Thus the sort of things which drive us to accepting a contradiction as true are exactly the same as those which drive us towards accepting other propositions as true. But if paraconsistency is correct, can we build up a rational case for every contradiction? Of course not. Consider the contradiction "This object is both an elephant and not an elephant", where the object in question is a telephone.[81] What can be said for this? We can certainly argue rationally for one limb of the conjunction. Elephants are grey, organic, have trunks, etc., whereas this object, the telephone is white, inorganic, has no trunk, etc. If anyone can put forward a serious rational case for the other conjunct (without, of course, using ex falso quodlibet or other sophistry), we will give up not only paraconsistency but much other rational activity.

In this way paraconsistency shows that rational criticism is not based on assumptions of consistency. A view is effectively criticized if it can be shown to lead to something that is rationally rejectable - be it a contradiction or not.[82] The insistence upon the total unacceptability of any contradiction (or at least for a decision procedure for the total unacceptability of some) is the last refuge of the "Euclidean" desire for certitude or conclusiveness,[83] which, once common in the oceans of epistemology, now lives on like some coelacanth in the stagnant waters of (classical) logic. It used to be insisted that rational procedures, especially those of science, provided certainty either in proof (for inductivists) or in refutation (for naive falsificationists). However it has become increasingly clear, particularly through the work of people such as Lakatos, that this is something of an illusion. The job of killing a theory is frequently a long business: there is mostly no "instant rationality", no experiment which is guaranteed to work.[84] Nonetheless a sufficient weight of evidence can eventually succeed. Paraconsistency takes us some steps further towards showing that there is no argument of any kind which is guaranteed to work. Thus it may well be that a person <u>can</u> rationally hang on to an inconsistent theory, including an explicit contradiction to which it leads, at least for a time. Perhaps, however, as other evidence and arguments build up, as this consequence of the theory, or others, are found to be too damaging, this may no longer remain rationally possible. Perhaps not. Paraconsistency thus helps dispose of the last vestiges of "instant rationality".

Similar points to those that meet the charge that paraconsistency renders impossible the rational process of forcing someone to change their beliefs by criticism, also serve to meet the allegation that paraconsistency itself is uncriticisable;[85] for the allegation is really a special case of the more general charge. While it may be more difficult to criticize a paraconsistent, since (as we have seen) one cannot automatically expect him to concede defeat if his position turns out to be inconsistent, still it is very far from impossible. For example, if an analysis of the way assertion and rational change of belief function were to show that these things would not be possible if paraconsistency were correct, we would have a powerful transcendental argument against paraconsistency. However, a correct analysis of these subjects does not show this, so we have argued. More generally, there are <u>many</u> ways in which paraconsistent arguments and logical procedures (including certainly our own) are open to criticism, and to reappraisal in the light of criticism, as criticisms of one paraconsistent position by or from another,[86] and resulting amendments of positions, reveals.

iii) <u>Consistency and the metatheory</u>

The thrust of the argument of this section so far is to the effect that, <u>pragmatically, contradictions are not so very different from other kinds of assertions</u>: they have a determinate sense, may be true, may be rationally believed, and may be rationally rejected. Are there, however, any special classes of contradictions which do function in a "classical" fashion? Nothing we have said so far commits us to a position on this, though we incline to the view that the answer is 'No'. However we wish briefly to discuss one class, which, it has been suggested, has this special status.

Specifically, it has been suggested that no contradictions of the form ⌜s is true and s is not true⌝ should be true. Now since the truth predicate is the main predicate of a semantic metalanguage, what is really at issue here is the consistency of the metalanguage. (If atomic formulas containing the truth predicate behave consistently, so will all usual compounds thereof.) Actually even to express it this way is somewhat obnoxious: for to talk of 'metalanguage' and 'metatheory' is already to start buying into the Tarski hierarchy which we do not want to do. So let us rephrase the issue: should our own semantic theory be consistent? It will be evident from all we have said on semantics (in §§2Ivi) and 2II ii) b)) that we think the answer is 'No'. This, after all, is the lesson of the liar paradox, 'This statement is not true'. This statement is both true and not true. We envisage no stratification to avoid this. Natural language (or a formal language which models it) should be catholic enough to formulate its own semantics within itself.

However, there are some arguments to the effect that there must be an (ultimate) metatheory which is consistent. We can now dispose of these fairly quickly. Such arguments are found in Batens[87] who, like Rescher and Brandom, espouses inconsistency but only at the "object level". One of his arguments (pp.227, 231) is to the effect that if we cannot locate a domain somewhere which we can guarantee to be consistent, the possibility of rationally criticizing a theory and rejecting it is impossible. We dealt with this objection in the last section. The other argument (p.227) is less clear. It is to the effect that 'one may describe an inconsistent domain [with a paraconsistent theory] but that something may be called a description only if its metatheory is consistent'. To be honest we find the argument for this not very clear, but the following quotation a few lines later suggests that the problem is that if the metatheory cannot rule out, absolutely, certain things being in the theory, the theory has no content: 'I cannot see how one could disagree with [Popper's] basic insight that only those theories are informative which 'forbid' something'. If we are right the problem is just like the one concerning content and exclusion we have dealt with above. But that response can be supplemented:- The mere fact that $A \in T$ and $A \notin T$ does not imply that $B \in T$ for arbitrary B. (Unless of course we reason classically.) Then the fact that the metatheory is inconsistent does not imply that T is trivial i.e. it rules out nothing.

β) Metaphysics

In §2II above we allowed for the possibility that there might be true contradictions other than those cited. It is now time to discuss some of them. Designing inconsistent theories has not been the prerogative solely of mathematicians and scientists. Philosophers too, as we have seen, have proposed their considerable share of inconsistent theories, sometimes intentionally, sometimes not. Obviously, should any of these theories be correct, then they would provide further examples of true contradictions. But almost invariably such theories have been rejected by philosophers working in a predominantly empiricist climate, just because they were inconsistent. An important immediate effect, then, of paraconsistency is to give these theories a new lease of life. While a large number of these might be cited, (cf.§21 ii) two in particular stand out: Meinong's theory of objects, and dialectics, which we discuss in turn:-

i) Meinong's Theory of Objects

Meinong's fundamental thesis is that every singular term of language signifies (something). The significata are objects, only some of which exist. The arguments for this position are numerous.[88] A main one, however, is that it enables a simple and uniform account of the semantic function of individual terms in sentences in which they appear. Classically the semantic function of an individual term is spelt out in terms of the object which is its denotation, and the properties which it has. However, traditionally this account has been thought to work only when the term signifies an existent object. For it is supposed that the notion of a non-existent object is a highly problematic one. Thus the classical account is drastically limited in application, since much common language appears to be about non-existent objects, e.g. fictional objects, objects of belief and other intentional objects, abstract objects such as numbers, properties, functions etc. Thus in giving an account of the semantics of this kind of language, about what does not exist, some other manoeuvres have had to be made. One is to insist that this sort of term does denote an existent, though non-actual object. This is "Platonism", which is particularly common in the standard semantical accounts of mathematical theories. Another manoeuvre, more common with respect to fictional and intentional objects, is to try to paraphrase away the apparent reference to objects. This is reductionism. All of these moves encounter severe problems, though this is not the place to go into them.[89] The Meinongian solution is to maintain essentially the classical account of the semantic function of names but to ditch the assumption that what is signified must exist. Thus, in effect, a denotational account of names applies, quite simply, across the board, and all the problems disappear.

What has all this to do with paraconsistency? There are, as might be expected, numerous objections to Meinong's solution. Perhaps the most common is that it is impossible to make sense of, or form a theory with, objects which do not, at least in some sense, exist. This is a hoary old problem and we do not intend to buy into it now.[90] The objection we are concerned with is this:- At least some non-existent objects must have properties. The semantic account of the function of names depends on this. Moreover we must be able to determine their properties, at least in some cases; otherwise we would never be able to distinguish them and tell whether sentences about them are true or false. How we are often able to determine the properties of existent objects is clear enough in outline, since they causally interact with us via our sense organs. However, this sort of answer obviously fails in general for non-existent objects. But if we can find no way of attributing properties to, or determining the properties of, non-existent objects, then they become very like Kantian dingen an sich and just as useless. How then are we to do this? Meinongians have a number of answers, but one of central importance concerns the characterization postulate. Consider a descriptive term 'a...', 'the...'. Let us use 'τ' for a general description operator. Then the term $\tau x\phi$ has just these properties, ϕ, by which it is characterized. This is the characterization postulate:

$$\phi(x/\tau x\phi)$$

It is analytic by virtue of the senses of the terms involved, and hence known a priori. Now application of the characterization postulate soon leads to contradictions, in the case of certain impossible objects. Consider, for example, an object which is strongly contradictory in having both F and not having F, $\tau x(Fx \& \sim Fx)$ say. Then $F(\tau x(Fx\&\sim Fx))\&\sim F(\tau x(Fx\&\sim Fx))$. Hence, should we reject all contradictions, the characterization postulate must also be rejected - an outcome with important (negative) implications for the whole Meinongian enterprise. Indeed, an instance of just this argument was used by Russell in his damaging critique of Meinong's theory of objects, one of Russell's underlying assumptions being that contradictions are, one and all, outlawed.[91] However, once the possibility of true contradictions is conceded what is there against supposing that impossible objects such as $\tau x(Fx \& \sim Fx)$ yield some of them? It may just be a fact of life that some Meinongian objects are inconsistent objects, in the sense of having contradictory properties as supplied by the characterisation postulate. The objection, on consistency grounds, to the characterisation postulate is thus foiled. As is now becoming better known, Meinong immediately responded to Russell's critique along these lines, emphasising that of course impossible objects have impossible properties and violate LNC, which holds at best for actual and possible objects.[92]

To be honest, the situation is not quite as simple as we have so far suggested; namely, go dialethic, and Meinong's theory of objects can be unproblematically rehabilitated. For even a dialethician cannot accept the characterization postulate in full generality, since using an unrestricted form, one can prove absolutely anything. It is enough to observe that $F((\tau x(Fx\&p))\&p$ is an instance of the unrestricted postulate. Accordingly, either the postulate is already implicitly restricted, e.g. in a natural way, or else some restrictions have to be imposed on it. This opens another avenue of escape for the Meinongian, who can try to formulate restrictions on the postulate which rule out any applications which lead to inconsistency. However, this makes life a lot harder and messier.[93]

ii) Dialectic

As we have seen in an introduction to Part I, dialectic is not so much a single theory as a cluster of ideas and themes to be found in a number of different thinkers, starting, in the modern period perhaps, with Fichte, going through

Schelling, Hegel, Marx, Engels into contemporary thinkers such as Lenin, Sartre and Mao-Tsetung. It would be foolish to suppose that there is a uniform account of dialectics to be found in all these people. What these people share often seems, at least to those unsympathetic to dialectic, to be little more than a form of words, whose meanings differ radically. However, let us start with these words. A central component of dialectics, as construed in the modern period, is that of contradiction.[94] The main things asserted about contradictions are:

D1) There are real contradictions: some situations realize contradictions. (This is one form of the law of the unity of opposites.)

D2) Change is brought about by the resolution of contradictions: in a dynamical system the state S' succeeding a state S is produced by resolving some of the contradictions in S. (S' is the negation of S.)

We grant that different dialecticians have understood the notion of contradiction in different ways.[95] Thus, for example, a contradiction can be a self-contradictory proposition, incompatible concepts, a conception of a situation different from the reality of that situation, a process which moves towards an end which is self-defeating, inverse operations, opposing forces, opposing interests, conflicting tendencies, and so on. Different senses of contradiction will of course give rise to different senses for claims D1) and D2) above. It would be rash to try to find much more than a family resemblance between the various notions of contradiction listed above, and we will not try. Moreover, some of these notions of contradiction have little connection with the way that a logician understands the term, which is primarily the first on the list. For example, the notion of opposing tendencies has comparatively little to do with this. We concentrate on the logical notion, not because we regard the other notions as incorrect or uninteresting, but because if dialethic and dialectic are mutually relevant, this will be the locus. Thus - let us emphasize again - we are not claiming that what we go on to say about contradictions is a correct analysis of all dialectic - far from it. What we do say is that some major dialecticians have often deployed this sense of contradiction,[96] and that this is the focal sense from which the others derive and draw their strength. For these reasons dialethism is a valuable technical aid, in fact absolutely essential, in understanding logically, what is going on in dialectic. By contrast, attempts based on received logics (classical or intuitionist or traditional) to explain what is going on are bound to be rather abject failures, and to either write dialectic off, as Popper does as 'a loose and woolly way of speaking' 'without the slightest foundation',[97] or else turn dialectic into something very different from what it has been historically.[98] However, our aim here is not one of historical exegesis but one of trying to establish the mutual relevance of dialectic and dialethic, and in this way to help to logically rehabilitate dialectic. To this end we consider contradictions in knowledge and contradictions in the natural world. These are not the only places of relevance, but they suffice to establish our main point.

a) Contradictions in knowledge and the corpus of science

According to the law of the unity of opposites, when applied to knowledge, the historical state of knowledge at a certain time is liable to contain contradictory propositions. That this is indeed so is fairly easily seen. First, we have already argued that certain theories within the corpus of knowledge may be internally inconsistent.[99] However, contradictions arise for other reasons as well. As Popper has emphasized, a well-corroborated experimental result may well contradict a well-corroborated theory. The corroboration of both would normally suffice to put them both in the corpus of knowledge.[100] Popper, of course, insists that the theory should be jettisoned in this context. However, as others such as Lakatos have argued against Popper, "falsification" and rejection are not historically contemporary events in most cases.[101] The contradiction in the state of knowledge persists, seeking a resolution. The third reason for contradictions in the state of knowledge is, as Lakatos has again emphasized, that at certain times it may contain competing research programs, whose "hard cores" will certainly contradict one another.[102] Thus D1 is vindicated in this context.

Having seen this, D2 is fairly trivial. These contradictions provide an important motor for knowledge. For the fact that they are included generally provides a good reason for changing the corpus of knowledge in such a way as to resolve them.[103] Here 'resolve' does not mean simply 'eliminate', but 'transcend', in the sense of finding satisfactory explanations for the corrorboration of both parts of the contradiction. Thus are the insights of Popper concerning the struggle between theory and experimental falsification, and of Lakatos concerning the struggle of rival research programs to supersede each other, built into a dialectical account of knowledge and the growth of science.[104]

It has been objected that our admission that some contradictions are true, undercuts this account of the dialectics of knowledge - and, more sweepingly, that D1 undercuts D2 - for the following reason. Knowledge may well develop by the resolution of contradictions. However, the reason that it does so is because a contradiction in knowledge is unsatisfactory, and it is this because it must be false. Once one denies this then there is no reason why the contradiction should not be allowed to stay and knowledge remain static.

The point is simple. It suggests that there is an incompatibility between progress through contradiction-resolution and belief by the historical actors in true contradictions (not, nota bene, true contradictions as such). But this is confusion. For even assuming that the aim of the actors which produces the change in knowledge is the elimination of falsehood, it does not follow that they will rest content with a contradiction. Neither dialetheists nor dialecticians believe that all contradictions are true: they are as likely not to be true as anything else, indeed, more likely. Given a contradiction in the corpus of knowledge, and given the belief that it, like most contradictions to be encountered, is false, strong motivation to eliminate it from the corpus remains. But what of those contradictions which are true and seen to be such? Will these be resolved? Not necessarily. They may well stay. There is no impetus for a dialethician to exorcise the logical paradoxes, for instance, from the corpus of knowledge. But neither is it any part of dialectics that all contradictions are change-producing. In fact, there is a standard distinction drawn in dialectic between antagonistic (i.e. change-producing) contradictions and non-antagonistic ones.[105]

But all this is to assume that the aim of the actors is simply elimination of falsehood. And this of course, is too simple. For falsehoods are not simply eliminated but transcended. As knowledge expands we produce deeper and deeper explanations.[106] The true is explained and its limitations shown, the false is shown to be false and its corroboration explained. Thus a false contradiction may be jettisoned not for itself, but as a result of increasing explanatory depth. Moreover, even true contradictions may be resolved in this way. For deeper theories may well produce meaning-change which resolve contradictions by meaning-fission.[107]

b) Contradictions in the natural world

Let us begin again with the law of the unity of opposites as encapsulated in D1. If the world is the Tractarian world of the totality of that which is the case, then it contains contradictions. The logical paradoxes are examples of these. But are there true contradictions concerning the natural world, as opposed to the analytic part of the world? We have already answered this question in the affirmative, too, through the analysis of multicriterial terms.[108] This, however, by no means exhausts the possibilities. A paraconsistent Marxist may well argue for true contradictions (in our sense) concerning society, and a dialetheic quantum mechanicist (at present a science fictional object) would argue for true contradictions at the subatomic level.[109]

This is still to neglect the much remarked connection between contradiction and change. For when dialecticians such as Hegel and Engels have emphasized the presence of contradictions in the natural world, they have done so in connection with change. For example, Engels is quite prepared to concede that a true description of the world, as it is at any one instant, a static account, may be consistent. However, once we correctly consider it as a dynamical system, in a state of flux, then a true description of what is going on must contain a contradiction.[110] Thus contradictions arise when a system is moving from one state to another and are resolved on terminating the motion. This is D2. It must be confessed that dialecticians have not, by and large, been prepared to <u>argue</u> the issue. Usually they have been content to cite the authority of Zeno, and his paradoxes of motion. Certainly some of Zeno's arguments (for example, the arrow) can be represented as arguments for the claim that in a state of change something both is and is not the case. But we have already discussed Zeno's arguments[111] and have not endorsed them. Let us, however, see what we can make of the situation without them.

Suppose a system is in a state S_0, and at time t_0 it changes to state S_1. What state is it in at t_0? <u>A priori</u> there are three possibilities: it is in one or other of S_0, S_1 but not both; it is in neither S_0 nor S_1; it is in both S_0 and S_1. Maybe on different occasions and with different sorts of change, all three possibilities are realized. But, in particular, if S_0 is p's being true and S_1 is ~p's being true, and a change of the third type occurs, then a contradiction is realized at t_0: both p and ~p are true. Of course this possibility is ruled out classically: the very least one can say for paraconsistency is that it opens it up.[112] To determine whether or not this possibility is actualised we need to seek further arguments. Such arguments can be found. For example, one concerns Leibniz' limit principle "whatever holds up to the limit holds at the limit". This has a good deal of plausibility where physical processes are concerned. And if it is correct it implies that both p and ~p are true at t_0 since t_0 is a limit of the intervals of time both before and after it. In this case, such change does involve contradictions, and D1 and D2 are both further confirmed as regards contradictions in the natural world.[113]

γ) <u>The Philosophy of Mathematics</u>

Not only can mostly defunct metaphysical theories be rehabilitated through paraconsistency; many programs in the philosophy of mathematics can also be reactivated. For largely historical reasons the philosophy of mathematics has been intimately tied this century to the investigation of logic. And virtually all the philosophy so done has been predicated on the unquestioned assumption that logic, <u>that</u> logic, is either classical or intuitionistic. Thus paraconsistency is bound to have consequences for the philosophy of mathematics, some of which we now examine.[114] The most devastating effect of paraconsistency is to undo many of the negative results that have emerged over the last fifty years, especially those arising from or concerning Godel's theorem, logicism and Hilbert's program, which we discuss in turn.

i) <u>Gödel's first incompleteness theorem</u>

Gödel's theorem can be stated in the following form: any (ω-) consistent theory which is strong enough to represent all recursive functions is incomplete. With this profound result we need not presently quarrel. Its proof requires only rather minimal assumptions concerning the underlying logic of the theory.[115] Of course, Gödel - and everyone else - again assumed that the underlying logic of the theory must be classical, or else intuitionistic, but at least in the case of Gödel's first theorem this is unnecessary. No, what we do take issue with are the bloated claims which are often made and which are supposed to follow from this result. The more modest of these[116] are usually to the effect that any axiomatic mathematics or arithmetic is incomplete.[117] Put another way, this is the claim that the set of true mathematical (or arithmetical) sentences is not recursively enumerable. Now as will be quite clear, this follows from Gödel's theorem if and only if the set of mathematical (arithmetical) truths is consistent. However in virtue of what we have said concerning the set theoretic paradoxes (in §2IIii)a), the set of true mathematical assertions obviously is not consistent.

But what of the set of true arithmetic assertions? It could be that Peano Arithmetic, for example, is inconsistent. It is not impossible, despite the consistency proofs, but it seems unlikely since the sort of conditions which seem necessary for producing paradoxes do not arise in Peano Arithmetic.[118] However, this does not show that the set of true statements about numbers is not recursively enumerable. It shows only that the set of such statements expressible in the language of Peano Arithmetic is not. For it is of course quite possible for a recursively enumerable set to have non-recursively enumerable subsets. But it is part of current dogma that anything mathematical that can be said about numbers can be said in the language of Peano Arithmetic. This is simply false.

Consider, for example, the theory elsewhere[119] called "semantically closed arithmetic". This can be thought of as Peano Arithmetic extended by new n+1-place predicates Sat_n (for all $n \geq 1$) and axioms:

$$Sat_n(\ulcorner\phi\urcorner x_1 \ldots x_n) \leftrightarrow \phi(v_1/x_1 \ldots v_n/x_n),$$

where ϕ is any formula with n free variables $v_1 \ldots v_n$ and $\ulcorner\phi\urcorner$ is its Gödel code. This theory will be able to express facts about numbers that are not expressible within the language of Peano Arithmetic, e.g. $Sat(\ulcorner x=1+1\urcorner 2)$. Presumably semantically closed Peano Arithmetic will not be a conservative extension of Peano Arithmetic.[120] If so, Peano Arithmetic is not only theorem incomplete but expressively incomplete, and, we suspect, the concurrence of these things may not be accidental. We have conjectured that semantically closed arithmetic, or at least some natural axiomatic extension of it, is (extensionally) complete; but the solution to this problem is, as yet, unknown.

Anyway these few observations are sufficient to sink, at least for the time being, most of the important purported philosophical consequences of Gödel's theorem (some more of which we will mention in the next two sections).

ii) Logicism

A main aim of logicism, the foundational program proposed by Frege and Russell around the turn of the century, was to show that all mathematical truths are logical truths.[121] The attempt to do so fell into, acording to recent extensional reconstructions of the program, two parts: a) showing that set theory was a branch of logic; b) showing that mathematics is reducible to set theory. Both a) and b) have run into a heap of problems, problems that are largely removed once we turn paraconsistent.

a) The set theory that Frege worked with was essentially an elaboration of naive set theory. His theory was, like the naive theory, found to be inconsistent, and so a consistent reformulation had to be sought.[122] The troubles for logicism begin right here. The main problem here is that set theory, as now reformulated, does not look much like logic at all. Frege's set theory could plausibly be seen as a theory about notions which are very general, subject-neutral and well within bounds of traditional logical concern, namely properties, concepts, and their extensions. Moreover its principles appeared (indeed we would claim are) analytic, a matter of logic (in a fairly tight sense). Thus Frege's set theory appeared, in all relevant respects, a part of logic. However, the same does not apply to the mangled forms that have passed for set theory in the twentieth century. The now-received theory of sets appears to be a subject whose concern is a quite specific domain of abstract objects, slightly more general than, though essentially no different from, other specifically mathematical objects such as groups, categories, etc. Thus, received set theory is often accounted a branch of mathematics and not logic at all. Moreover the axioms involved can hardly claim the self-evident analyticity of Frege's axioms.

By turning paraconsistent (by adopting a suitable underlying paraconsistent logic) we can however revert to naive set theory, and so, as with Frege's original theory, avoid all these objections, since the theory has, like Frege's, an evidently logical cast. This is not all. Much criticism has been directed at the inclusion of axioms of choice and infinity within the logicist reduction base, on the grounds, once again, that these axions are not logical in character and not (analytical) theses of logic. These problems too disappear. For both the axioms of infinity and choice, which have purely logical formulations, are provable in absolutely naive set theory.[123]

b) The question of whether mathematics is reducible to set theory is a little more open. That classical mathematics is largely reducible to, say, ZF set theory with classical underlying logic is now widely acknowledged - though there are some arguments to the effect that it is not completely so reducible, which we will consider in a moment. However, it is not at all clear yet whether mathematics, or even number theory, is reducible to naive set theory with a paraconsistent logical base. For although the purely set theoretic principles are a great deal stronger, the logic is correspondingly weaker. Thus a lot of the moves in the standard reduction cannot be made. This does not, of course, mean that reduction is impossible; it may well be possible in a different way. However to determine whether this is possible will require a great deal of work which has not yet been done. What little work has been done on the relevant/paraconsistent formalization of mathematics (e.g. by Meyer and by Brady), shows that very often it is possible to find relevantly/paraconsistently acceptable versions of theorems/proofs that traditionally are formulated/done assuming objectionable classical principles. However, work in this area is only just beginning.

There are arguments that such work would be a waste of time, at least as regards reinstating a logicist program. Let us consider then the standard theoretical arguments against the total reducibility of mathematics to set theory, and ask whether these hold against paraconsistent naive set theory. There are three such objections to be parried.

The first objection is that the embedding of category theory in set theory is impossible because of the problem of "large" categories. While this is a very real problem for standard, putatively consistent, set theories, (as we urged when we examined this issue in §2 I, v) above) it is sufficiently clear that naive set theory provides adequate conceptual apparatus for category theory.[124] This therefore ceases to be an objection to reducibility.

Secondly, there is the major objection from Gödel's theorem. Gödel's theorem appears to show that the set of mathematical truths is not recursively enumerable, and hence not capturable by any axiomatic system, naive or otherwise. However, as we saw in the previous section, paraconsistently there is no reason to believe this to be so. Hence this objection to logicism also lapses.

The third objection is related to the second. Gödel's theorem purports to show that any axiomatic set theory is theorem-wise incomplete. This objection is to the effect that any set theory is expressively incomplete in the sense that there are mathematical objects which cannot be defined in it.[125] Basically it is as follows:- Let # be a Gödel coding of formulas. Define a function F from the natural numbers to the ordinals thus:

$$F(\#\phi) = \begin{cases} \alpha & \text{if } \phi \text{ is a formula of one free variable} \\ & \text{and } \alpha \text{ is the least ordinal that satisfies it;} \\ 0 & \text{otherwise.} \end{cases}$$

The supposition that F is representative by a formula of set theory leads to the expected derivation of a contradiction. In fact the derivation is just a version of König's paradox.

All the objection shows, however, is that F is not definable in a consistent set theory. It obviously does not show that F is not definable in naive set theory, which we know to be inconsistent anyway. Moreover we know how to define satisfaction set theoretically. Thus we may suppose that naive set theory can define its own satisfaction predicate. And given it can, then F is definable within naive set theory by essentially the above definition. Furthermore, in the event that - because of the weakness of the underlying logic - naive set theory could not define its own satisfaction relation and prove it to have all the right properties, all this would show is that set theory as such does not exhaust all logical notions. For satisfaction clearly is a logical notion. Hence we could add it to naive set theory (with appropriate axioms, etc.) and in the extended theory F will be definable, just as above. This extension of the theory in no way undercuts the logicist claim since the extended theory is just as much logic as the original set theory. Thus this objection to logicism lapses, as did all the others. These points, taken together, meet the main objections to logicism.

iii) Hilbert's program

Hilbert's program was philosophically motivated, but technically what it came down to were a) axiomatizing mathematics or, at least, various of its parts, and b) proving consistency by finitary means.[126] The notion of being finitary was always vague to a certain extent. However, finitary methods had to be constructive in some sense, and certainly much less than full classical methods of proof. Hilbert's program ran into trouble at both stages because of Gödel's theorems.

a) The attempt to axiomatize even elementary arithmetic was given up because of Gödel's theorem, which was thought to show that this was impossible. However, as we have already seen, paraconsistency undermines this impossibility argument.[127]

b) The attempt to prove the consistency of axiomatic arithmetic by finitary means was abandoned because of Gödel's second incompleteness theorem, which seemed to show that the sentence which canonically asserts the consistency of a theory is not provable within the theory itself. It follows that the consistency of any reasonably strong theory of arithmetic is not provable by finitary means. Now this all lapses once we abandon classical logic. For the proof hinges on the fact that the underlying logic of the theory is classical. And without this the proof of Gödel's second theorem fails. This is most easily illustrated through the system R# of relevant arithmetic, that is, a system of arithmetic comprising suitable versions of the Peano postulates but based on a relevant logic R. The system R# is fairly strong: it can represent all recursive functions. Yet this system has a simple consistency proof, finitary by any standards, which is representable within the system itself.[128] Thus the whole question of "finitary" consistency proofs for various interesting mathematical theories is reopened.

Naturally once we move to inconsistent mathematical theories, of which, as we have seen, there are a number, the question of a consistency proof lapses. However, the subject does not lose its interest. For the role that is played by consistency classically is played paraconsistently by non-triviality. The important question becomes whether in certain mathematical theories everything can be proved. But the subject assumes new dimensions as well: for questions concerning degrees of inconsistency (and, correspondingly, extent of consistent subtheories) are raised. More specifically the idea is to chacterize what inconsistencies are provable. For example, in naive set theory are there provable any inconsistencies concerning small (e.g. finite) sets? The question of what mathematical methods are necessary to establish these results is also of interest. Thus this kind of investigation, largely initiated by Hilbert, still has much interest. Indeed, questions of degrees of inconsistency are, we suspect, deep and will come to play a very important role in the subject. However this kind of investigation is very much in its infancy. Only a start has been made: Brady has now proved that naive set theory is non-trivial.[129] But even the question of whether the obviously undesirable $\phi \neq \phi$ is provable is open. In these various ways, paraconsistency opens up main parts of Hilbert's program and associated areas that had been classically closed off, and reawakens interest in them.

4. Conclusion: the ideology of consistency

There are, we have argued, no insuperable philosophical problems in supposing that there are true contradictions and, moreover, there are substantial benefits attached to doing so. What mainly prevents the acceptance of this view is the ideology of consistency: the deep-seated and irrational view that the world is consistent.

It is worth inquiring why the view is so deep-seated. A superficial answer is to the effect that the belief in consistency is an unwarranted induction from common experience which, for the most part, is consistent. However, this does not get to the root of the problem. For belief in consistency has not been universal. The belief was rejected by many pre-Socratics and by most nineteenth century German philosophers. Why then should it be so dominant now? Part of the answer lies in the present dominance of Anglo-American analytical philosophy which is squarely in the empiricist tradition. It does not take much perception to observe the lines running between Locke, Hume, Mill, Russell, Carnap, etc. Underlying empiricism has always been an atomistic metaphysics[130] whether it is Hume's "world" of independent experiences, or Russell's logical atomism. Now atomism has always been ill-accommodating to contradictions. For each atom is quite independent of all others. It moves within its own "logical space" and can have no relationships with other atoms. Hence it cannot come into conflict with them and produce a contradiction. Thus, for example, Wittgenstein in the Tractatus, who simply echoes Hume, in slightly different terminology:

5.134 One elementary proposition cannot be deduced from another.

5.135 There is no possible way of making an inference from the existence of one situation to the existence of another, entirely different situation.

5.136 There is no causal nexus to justify such an inference.

These separability assumptions ensure consistency of elementary components, from which all else is built up. The dominant metaphor in nineteenth century German philosophy is of course very different. Instead of the collection of atoms, it is the organic whole, that is, a whole the parts of which are internally related to each other. The possibility of essential conflict, and so internal contradictions, is thus to be anticipated.

Placing the dominant philosophical paradigm in its empiricist atomist tradition indicates part of the answer to the question of why the tenor of mainstream Anglo-American philosophy is antagonistic to contradictions. Another piece of the jigsaw is this: atomism has always played, through individualism, a political role as well as a metaphysical one in empiricism. For society also is conceived of in terms of a collection of autonomous individuals or political atoms (contrast again the organicism of Marx and Hegel), and this picture has formed the basis of virtually all bourgeois political and economic theories, and certainly of the dominant politico-economic paradigm. Thus insofar as a consistency hypothesis and repugnance of contradiction are part of a general empiricist/atomist/bourgeois perspective, our allusion to the ideology of consistency is far less fanciful than it may at first have seemed.

Anyway these are deep issues and we shall not pursue them further now. In fact we have done little more than scratch the surface of the philosophy of paraconsistency. Some of the issues are taken further in this section, but we hope to have at least shown that the philosophical ramifications of paraconsistency are wide-ranging and deep. It is impossible to tell where exactly they will lead eventually, but such is the case with any radical new theory.

FOOTNOTES

. Strictly speaking there could be, if the decision could be overturned by a higher court. However for the court of highest jurisdiction, the claim is correct.

. On such dilemmas and their place in paraconsistent deontic logic see Routley and Plumwood, this volume. The theme that the natural logic of legal language is paraconsistent and not classical was first suggested by Quesada; see further his paper, this volume.

. Many nonphilosophical theories are in a similar "predicament", as we shall see.

. This theme is adapted from a note from J. Passmore.

. The point could be rendered analytic by appropriate distinction between _practice_ and _theory_ and tightening of the latter notion.

. There is substantial evidence however, assembled by Goldstein, that Wittgenstein adopted a theory of content in the _Tractatus_ which implies the rejection of classical spread laws. What this seems to show however is not, as Goldstein suggests, that the _Tractatus_ is based on a nonclassical logic, which lacks such principles as Contraposition - but a further inconsistency in the _Tractatus_ between the underlying classical logical theory and the theory of content grafted onto it.

. See e.g. W.V. Quine [1955]. But Wittgenstein wanted to insist that Frege's theory, though inconsistent, was not trivial, i.e. there were implicit restrictions on what rules could be applied where: see the first introduction to part 1.

. See the introduction to part 1 above. This is only one of the many apparent inconsistencies in Aristotle's philosophy. Another is as regards the extent to which the principle of noncontradiction (LNC) applies to appearances as well as to substances: see Łukasiewicz, [1971] p.502).

. Inconsistency of philosophers is almost to be expected. But logicians, who are supposed to be especially skilled at seeing the consequences of their assumptions, have a considerable record of inconsistency where they try to design more comprehensive systems. A list is impressive and includes such logicians as Frege, Church, Quine, Lewis, ...

. Inconsistencies, especially in the form of anomalies, are only part of the story; philosophical fashions as influenced or even controlled by underlying socio-economic conditions, are another crucial part of the fuller story. Movement to avoid inconsistency does not always represent progress (we follow the honorific use). Thus Russell's movement from logical paradoxes to the ramified theory of types, Mally's dismantling of the theory of objects in the face of perceived inconsistencies in favour of a much more complicated construction, the epicycling of degenerating philosophical theories such as Plato's later philosophy and the current extensional Davidsonian program, do not strike us as cases of philosophical progress.

. See Griffin [1980].

. From Marsh [1956] p.183.

. Marsh [1956] p.270: the passage quoted concludes: 'the knowing of facts is a different sort of thing from the knowing of simples'. P. Simpson, to whom we owe this example of inconsistency in Russell, points out that a rescue attempt might perhaps be mounted by arguing that facts only "exist" derivatively (or at a different level) in the way that complexes such as particular chairs and tables do. But that supposes that facts are, what they are not in the theory, certain - somehow independently determined - assemblages of simples.

. R. Routley and V. Plumwood [1982].

. See Griffin [1980], p.155.

. See N. Griffin, [1982] Appendix.

. See further Griffin [1980], pp.176-77.

. In [1951]: for details see EMJB 1.13.

. For other examples of lesser inconsistencies in Quine, see Routley [1982] and especially Gochet [1984], which also presents deeper tensions in Quine's overall position. One more serious difficulty arises from the attempt to combine physicalism (and individualism) consistently with the orthodox kind of set-theoretic methodology physicalists typically adopt. For an unsuccessful attempt to resolve part of this problem see Smart [1978]. And Quine's very recent changes to his philosophy, which, in abolishing individuals, upsets much of his mature work, can be viewed as an attempt - among other things - to remove this inconsistency.

A related inconsistency in Quine's mature theory concerns the status of classical mathematics, which emerges as both true and false; a similar inconsistency infects Smart's work (see EMJB, p. 620). Armstrong's naturalistic theory is also bogged down in connected inconsistency concerning mathematics (see again EMJB, p. 750).

. See, e.g. Russell [1905]. It is by no means clear that Russell's objections apply against Meinong himself (see EMJB). Inconsistent by virtue of such characterisation postulates is not only the Graz theory but also, for instance, Castañeda's 1974 theory (see EMJB p.880 ff). Castañeda's theory is also rendered inconsistent, and thereby trivial given its classical basis, by virtue of inconsistencies in guise-theory; see Clark [1981] and also [1978].

. The basic argument is in Plato's Parmenides, §132; the resulting inconsistency is widely discussed in the literature. The more extensive inconsistencies in Plato's theory of forms - some of them integral to the theory - are well-documented in Griffin and Johnson [1983], where it is also argued that leading modern attempts to consistencize the theory fail. To set the matter in the present context, see EMJB, p.639 ff.

. Other inconsistencies in Wittgenstein's theories have already been recorded, earlier on, and these draw upon only a small portion of his work. (Some of the other respects in which the work is pragmatically self-refuting, and so inconsistent when the further assumptions are applied, are so trite as to scarcely bear recording; e.g. whatever can be said can be said clearly.) Whether the more significant inconsistencies trivialise the later theory is, however, unclear, but it would seem not since classical logic is supposed to be restricted in application in inconsistent situations.

. A similar problem may be found lurking in Tao, which is a teaching that denigrates learning.

. The evidence for these claims, which are strictly in the nature of promissory notes, is to be found in Collingwood [1939], pp.58-75, and [1940], p.21ff. Collingwood's theory is on a direct collision course with this enterprise.

. This is a highly condensed version of Passmore's explanation of the historical charge of self refutation against Protagoras: see Passmore [1961], pp.64-70.

. See Passmore [1961] chapter 4, for examples in the case of scepticism and for further discussion of self-refuting theories which often issue in inconsistency.

. Bose [1967], p.15. But there is some doubt as to whether Lao Tse is genuinely committed to the latter proposition, though other sources seem to supply it also.

. See Hintikka [1962].

. Passmore, personal communication, 1982.

. The inconsistency in Reid over the admission of ideas <u>may</u> be of this shallower type. For it seems, at first sight anyway, that he has no real need to make the following concession to Locke from which the trouble starts. Referring to what he calls 'the appearance of colour', Reid says

> Mr Locke calls it <u>an idea</u> and it may be called so with the greatest propriety ... . It is a kind of thought, and can only be the act of a percipient or thinking being (<u>Inquiry</u> VI. iv; <u>Works</u> i.137).

But this concession is inconsistent with Reid's rejection of ideas in his critique of the Theory of Ideas. More generally, as S. Grave to whom we owe these points concerning Reid remarks, Reid's account of perception, by all senses except touch, is inconsistent with part of his attack on the Theory of Ideas. A way of avoiding such inconsistency emerges from the treatment of ideas and sense data in EMJB, but it is very doubtful Reid could, or would, like this way.

. Passmore [1978]; see p.207.

. Passmore [1978], p.187.

. On all the points, and for the quote from Locke, see Passmore [1978], pp.185-9. Passmore proceeds (in accord with the prevailing consistency assumptions operating in the history of thought) to try to reinterpret Locke to remove the contradiction: 'faced with so absolute a contradiction ... we have no option but to look again at our interpretation' (p.189). But as Passmore is well aware (e.g. p.190 middle), his various proposed ways out run into logical conflict with other parts of Locke's philosophy, and Locke would not have found them at all palatable.

. All page references are to Spinoza [1675]. As always, there are ways out of such inconsistencies, by distinctions the theory does make; e.g. by a distinction between intellectual love, which God has, and nonintellectual love, which God does not have. But this conflicts with the definition of <u>love</u>. Strictly, since <u>love</u> is defined through <u>pleasure</u>, and <u>pleasure</u> (human chauvinistically) for humans only (pp.128-30), none but humans can have love; neither animals nor God can. (There are inconsistencies also in the theory and treatment of animals in

Spinoza.) C.B. Daniels, to whom we owe this example, suggests there would be another way out if time has a beginning (see proofs of propositions 33 and 34, book 5).

- The argument is taken from Routley [1968].

- As several passages from the Discourse on Method, e.g. p.101, and Meditations e.g. pp.145-6, show. (Page references are to Haldane and Ross [1911-1912].) These references and this inconsistency in Cartesian scepticism we owe to J. Kleinig. Compare also Passmore [1961] on the self-refuting character of absolute scepticism.

- A good feel for the problem is given by Ryle [1949], p.12ff.

- See Ryle [1949], pp.20-1.

- They may however be happy to see some of the philosophical competition removed.

- Of course there are many other examples of unintentional inconsistency we could have developed, given sufficient time and energy. In an obvious sense, then, our case is, inevitably, incomplete. There are many philosophers we might have looked at (more closely) but haven't. For example, Mill is inconsistent in his account of causation (in A System of Logic) as to whether exhibited constant conjunction is sufficient or hypothetical conjunctions are also required. In Leibnitz, apart from the matter of the infinitesimal calculus, there are the inconsistencies that helped lead Russell to propound the double philosophy theory (in [1900]). And then there is Kant ...

- The theories may also be inconsistent in respects other than those recognised. For example, a main theme in certain kinds of Buddhism is that nothing is self-contained, that everything is attached to other things. Yet the objective recommended is to obtain release (Nirvana) (e.g. from pain and troubles) by detachment, by severing connections (e.g. important attachments to place and people): that is the ideal personal situation of self-containment, which is impossible. Yet Nirvana is attained.

  Deliberate inconsistencies in Hegel and Marx have already been documented, e.g. in chapter 2, but those in Sartre have not. Contradiction figures essentially in Sartre's accounts of anguish, said to arise from a paradoxical feature of existence, that I am the self that I will be but I am not the self I will be; and in his analysis of self-deception or bad faith, which require the 'forming of contradictory concepts which unite in themselves both an idea and the negation of that idea'. More generally, describing human existence adequately requires use of contradictions: 'we have to deal with human reality as a being which is what is not and which is not what it is'; and the same applies to true descriptions of persons (e.g. of the pederast). For a fuller elaboration of all these points, and for references, see Tormey [1982].

- Passmore [1952], p.1, and Selby-Bigge as referred to in Passmore.

- For further examples of (unacknowledged) inconsistencies in Hume see this work of Passmore. Yet other examples (we owe to F. White) derive from a systemic inconsistency, concerning external objects, God, and such theoretical objects as energy and force, to the effect both that these objects do not exist and that they do exist but we cannot know that they do. As regards the negative ontological claim, Hume says much to back up a theme of the Abstract of the Treatise that we have no idea at all of force or energy, and these words are altogether insignificant, or they can mean nothing but that determination of thought, acquired by habit, to pass from the cause to its usual effect.

Yet in the Enquiry Concerning Human Nature (IV, I, 29) Hume's case seems to be a negative epistemological one, simply that ultimate causal powers in things cannot be known to us. Two further inconsistencies in Hume (pointed out to us by D. Stove) concern induction and caused existence. As regards induction, Hume both argues that induction is fallacious (in sections 4-6 of the Enquiry) and also accepts induction as not fallacious in his argument against miracles (in section 10 of the same work). As to coming into existence, he both accepts and elsewhere rejects the proposition that something might begin to exist without a cause: for details see Stove [1975].

Though there is surprisingly dialectical-looking material at places in the Treatise, e.g. p.205, [1978].

The point is discussed in detail in EMJB, chapter 12.

Garrett [1981], see p.337. The inconsistency is spelt out on p.350ff.

Galileo Galilei [1914].

For further examples see, e.g. Feyerabend [1975] p.258, [1978] §iv.

Further discussion of the Bohr theory can be found in Lakatos [1970] §3(C2).

See Boyer [1949] Ch.6.

"[circa 1720] mathematicians still felt that the calculus must be interpreted in terms of what is intuitively reasonable, rather than of what is logically consistent". Boyer [1949], p.232.

This is a real possibility with theories such as parts of the infinitesimal calculus and quantum mechanics. With some philosophical theories, it is quite another thing. It is extremely difficult, if not impossible (for the more ordinary philosopher at least), to obtain a commanding view of a philosophical position like Hegel's, a view in terms of which one could begin formulating the theory in a suitably exact way.

See Fraenkel, Bar Hillel and Levy [1973], pp.321-331.

Details can be found in Fraenkel, Bar Hillel and Levy [1973], p.143ff.

See Tarski [1936], pp.187-8.

Tarski [1936], pp.164-5.

A slight generalization of the situation is presented in Pinter [1980] where the criteria C_1, C_2 are replaced by sets of criteria. However this change makes no essential difference. For concrete examples of this sort of situation see the introduction to Part 2, §1II.

See Hempel [1966], Ch.7, Papineau [1979], pp.8-10.

See Priest [1980] for a further discussion of this point, and Maund [1981] for further examples of splitting.

- Wright [1980], p.297.

- How could people make the mistake that it was the whole thing? That people will do (wildly) irrational things if it is demanded by an ideology is well-documented. In this case, the ideology of consistency demanded that an ersatz for the universe of sets be produced and the cumulative hierarchy was, if not exactly an ideal candidate, at least a lowest agreed common denominator which, as it turned out, captured, in a neat synthesis, several apparently rival proposals.

- The two main positions outlined are, of course, rather bloodless abstractions. Concrete proposals are always more complex and variegated; they need to be much more specific about the scope and basis of D, for example. Still some abstraction is necessary to make the discussion manageable.

- There are many counterexamples, including imperatival, erotetic and significance logics.

- For this sort of criticism of the theory of types, see Fitch [1952], Appendix C, and also Black [1944].

- See Priest [1983] for a further discussion of these issues.

- See Lakatos [1968] pp.128-9.

- See e.g. Stahl [1975], p.45. The argument goes back to Aristotle: see J. Lukasiewicz [1971]. Łukasiewicz devastates Aristotle's argument. For further detailed criticism of LNC as a law of thought and as impossible rationally to deny, see R. and V. Routley [1975].

- See the Introduction to Part two of the book, §2II.

- See the semantics of various of the paraconsistent logics discussed in the Introduction to Part II.

- An argument of this sort is presented fairly dogmatically by Rescher and Brandom [1980] pp.24-25:

  > We have little choice but to regard the...[contradiction] as self-destructive, as simply self-annihilating...a blatant contradiction [is unintelligible]....,

  and a similar nonclassical cancellation view is an important strand in Wittgenstein's later thought (see the Introduction to Part I). There are certainly other choices, and furthermore better choices, as is argued in Routley and Plumwood [1982].

- These content measures, classical and relevant especially, are investigated in detail in Routley [1977], and one of them is discussed in the introduction to part three, §4ζ.

- In the Introduction to Part II. A much more detailed critique of connexive logics may be found in RLR, chapter 2.

- The account of Wittgenstein's <u>Tractatus</u> is different, in that it brings out tautologies as well as contradictions as lacking sense. Exactly what non-contrived theory of sense underlies the <u>Tractatus</u> (if any) is still a matter for debate.

74. We have explained exactly what these are in the Introduction to Part II.

75. This is essentially what Rescher and Brandom do [1980]. Naturally the argument does not terminate where we have left it. The dialectic next shifts back to the questions of what counts as a world and what as an evaluation or interpretation.

76. This argument is hinted at by Rescher and Brandom [1980], and is to be found, in effect, as an argument for the existence of a "strong negation" in Batens [1980], pp.226-7. It is stated more explicitly in Lear [1980], p.112.

77. See Priest [1980a].

78. For example, as given in the Introduction to Part II, §2 .

79. The arguments that contradictions are not logically defective as objects of acceptance or belief, that they have content, etc., are given as indicated earlier in this subsection. The arguments that contradictions can and are believed by rational creatures are assembled in R. and V. Routley [1975], where too the arguments deriving from Aristotle are undone. The demonstration that Aristotle's arguments are fallacious is given by Łukasiewicz [1971]. A further (overlapping) case against the mainstream traditional assumption that rationality entails consistency is presented in R. Routley, EMJB, and also in Rescher and Brandom [1980]. But though Rescher and Brandom wax eloquent against the traditional assumption, they do not really notice that similar arguments tell against the modification of the traditional assumption that they adopt, that rationality entails no (truth-table) contradictions.

80. Lewis [1982] suggests this. Batens [1980] pp.230-1 also asserts as much: he says that paraconsistency makes all theories "unfalsifiable". And Popper [1963] states it explicitly p.317.

81. If this looks too like a category mistake select some other object, e.g. a goose, a shadow, etc.

82. This matter is explored further in Priest [198+].

83. See Lakatos [1962].

84. Men and women of practice have of course always known this.

85. See Lewis [1982].

86. As throughout this work.

87. In his [1980]. Baten's discussion by no means exhausts the arguments already abroad. For much more on the issue as to whether a consistent metatheory is required, see RLR chapter 3, and Priest [1983].

88. See EMJB, especially chapter 1.

89. They are discussed in EMJB, especially chapter 4.

90. Again a full discussion can be found in EMJB.

91. Russell's critique appears in a series of papers in Mind around 1905: see especially [1905].

92. These points were made by Meinong in several places, e.g. his [1907].

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