Offprinted from *MIND: A quarterly review of philosophy*
Vol LXXXVI No 341 January 1977

The Formalization of Ockham's Theory of Supposition

GRAHAM PRIEST AND STEPHEN READ

1. *Introduction*

The point of the paper is to establish that, contrary to the claims of many people,[1] the medieval theory of personal supposition can be formalized in standard modern logic. (There is, however, one important proviso.) We shall use this formalization (a) to establish that Ockham was mistaken in his analysis of the *suppositio* of the predicate in the O form, and to put him right; and (b) to correct the following claims that have recently been made about the theory: (i) that Ockham and the medieval logicians in general omitted some modes of *suppositio;* (ii) that Ockham lists too many modes; and (iii) that the theory is incapable of dealing with multiple quantification (i.e. the theory of relations).

2. *The Formalization*

We shall not give an exposition of Ockham's theory here, but will presuppose familiarity with it.[2]

Let W be the set of objects in the real world, and let $\{w_i : i \in I\}$ be an enumeration of W. Let L be a language like first-order logic except that it allows conjunctions ($\wedge$) and disjunctions ($\vee$) of infinite sets of sentences.[3] L has n-place predicates $P^n_j (j \in J, n \in \omega)$ (if $n = 1$ the superfix will be omitted) and an individual constant for each w_i (for simplicity we call these w_i too). Let $\mathcal{W}$ be a model for L with domain W and suitable interpretations for the P^n_j. We may think of $\mathcal{W}$ as 'the real world'.

Notation: If ϕ is any sentence of L, we write $\phi(\hat{A})$ to show a distinguished occurrence of the symbol A in ϕ, and $\phi(A/B)$ for the sentence of L obtained from ϕ on substituting the symbol B for that distinguished occurrence of A in ϕ. If t_1 and t_2 are terms of L, we write $t_1(t_2)$ for $t_1 = t_2$.

With each monadic predicate P_j of L we associate a subset W_j of I as follows: $W_j = \{i \in I : \mathcal{W} \models P_j(w_i)\}$. It follows that:

$$\mathcal{W} \models (\forall x)(P_j(x) \leftrightarrow \bigvee_{i \in W_j} x = w_i) \tag{2.0}$$

$$\mathcal{W} \models (\forall x)(x = w_i \rightarrow \phi(x)) \leftrightarrow \phi(x/w_i) \tag{2.1}$$

$$\mathcal{W} \models (\exists x)(x = w_i \; \& \; \phi(x)) \leftrightarrow \phi(x/w_i). \tag{2.2}$$

1 For example, G. Matthews 'Ockham's Supposition Theory and Modern Logic', *Philosophical Review*, lxxiii (1964), 91–99; D. P. Henry *Medieval Logic and Metaphysics*, III, section 1.

2 Many expositions can be found, e.g. J. Swiniarski 'A New Presentation of Ockham's Theory of Supposition with an evaluation of some contemporary criticisms', *Franciscan Studies*, xxx (1970), 181–217; M. Loux *Ockham's Theory of Terms* (part I of the *Summa Logicae* translated and introduced by M. J. Loux), pp. 23–46, 188–221.

3 For example, the language $L_{\kappa^+\omega}$, where κ is the cardinality of W. See J. Bell & A. Slomson *Models and Ultraproducts*, ch. 14.

Note that if W were finite, we could take ordinary first-order logic for L. However, Ockham asserts that 'all men who can exist are infinite in number'[1]; so since we wish to formalize expressions such as 'This man is an animal, and that man is an animal . . . and so on for all men', an infinitary language is required. This is why we have chosen a language which has infinite Boolean combinations among its well-formed expressions. No modern commentators seem to have noted this requirement.

Ockham defines each mode of personal supposition in terms of the *descensus* possible. (One can, as in the thirteenth century, attempt to define each mode independently of mobility, and present the *descensus* as a consequence.) Our formalization of Ockham's definitions is as follows: the supposition of P_j in ϕ is

(I) *determinate* iff

$$\text{(D.)} \quad W \models \phi(P_j) \leftrightarrow \bigvee_{i \in W_j} \phi(P_j/w_i)$$

(II) *confused and distributive* iff

$$\text{(C.D)} \quad W \models \phi(P_j) \leftrightarrow \bigwedge_{i \in W_j} \phi(P_j/w_i)$$

(III) *merely confused* iff neither equivalence (D.), (C.D.) is true in W, but

$$\text{(M.C.)} \quad W \models \phi(P_j) \leftrightarrow \phi(P_j/ \bigvee_{i \in W_j} w_i).$$

These definitions and the simplifications made possible by logical equivalence such as (2.1) and (2.2) allow us to descend from a sentence ϕ to an equivalent sentence with only discrete supposition (that is, in which no monadic predicates or quantifiers occur).

Examples: (i) 'Some P_1 is P_2', i.e. $(\exists x)(P_1(x) \ \&\ P_2(x))$. Both P_1 and P_2 have determinate supposition. Thus, descending under P_1, we have

$$\bigvee_{i \in W_1} (\exists x)(w_i(x) \ \&\ P_2(x)) \qquad \text{(by D.)}$$

and so

$$\bigvee_{i \in W_1} P_2(w_i). \qquad \text{(by 2.2)}$$

Descending under each disjunct, we obtain

$$\bigvee_{i \in W_1} \bigvee_{j \in W_2} w_i(w_j). \qquad \text{(by D.)}$$

If we descend in the reverse order we obtain

$$\bigvee_{j \in W_2} \bigvee_{i \in W_1} w_i(w_j) \qquad (2.3)$$

which is trivially equivalent. The symmetry of (2.3) in i and j accounts for simple conversion.

(ii) 'All P_1 is P_2', i.e. $(\forall x)(P_1(x) \rightarrow P_2(x))$.

1 *Commentary on Perihermeneias*; see P. Boehner 'The Realistic Conceptualism of William of Ockham', *Traditio*, iv (1946), 323–324.

If we descend firstly under P_1, which has distributive supposition, we find

$$\bigwedge_{i \in W_1} (\forall x)(w_i(x) \rightarrow P_2(x)) \qquad \text{(by C.D.)}$$

whence

$$\bigwedge_{i \in W_1} P_2(w_i). \qquad \text{(by 2.1)}$$

In each conjunct here, P_2 has determinate supposition. So, descending, we obtain

$$\bigwedge_{i \in W_1} \bigvee_{j \in W_2} w_j(w_i). \qquad \text{(by D.)}$$

If, on the other hand, we descend firstly under P_2 (which has merely confused supposition) we obtain

$$(\forall x)(P_1(x) \rightarrow \bigvee_{j \in W_2} w_j(x)), \qquad \text{(by M.C.)}$$

in which P_1 has distributive supposition. Thus we have

$$\bigwedge_{i \in W_1} (\forall x)(w_i(x) \rightarrow \bigvee_{j \in W_2} w_j(x)) \qquad \text{(by C.D.)}$$

and so

$$\bigwedge_{i \in W_1} \bigvee_{j \in W_2} w_j(w_i). \qquad \text{(by 2.2)}$$

Thus we obtain the same result in whatever order we descend. This is unsurprising; since (2.1), (2.2), (D.), (C.D.) and (M.C.) are equivalences, the results of descent, in whatever order, must be equivalent to each other (and since the results contain no quantifiers, this must be a Boolean equivalence).

3. *Consequences*

We now consider the four consequences of our formalization stated in section (1).

(a) Ockham considered the predicate of the O form (viz. P_2 in 'Some P_1 is not P_2') to have distributive *suppositio*. Although he does not state this explicitly in the *Summa Totius Logicae*, he does in the later *Tractatus Logicae Minor* and *Elementarium Logicae*.[1] A number of modern commentators have realized this to be a mistake.[2] For

$$(\exists x)(P_1(x) \ \& \ \neg P_2(x)) \qquad (3.0)$$

is not equivalent to

$$\bigwedge_{i \in W_2} (\exists x)(P_1(x) \ \& \ \neg w_i(x))$$

1 See the editions by E. Buytaert in *Franciscan Studies*, xxiv (1964), 68 and xxv (1965), 212 respectively.

2 For example J. Swiniarski op. cit. pp. 211–213, G. Matthews '*Suppositio* and Quantification in Ockham', *Noûs*, vii (1973), 18 ff. Loux fails to notice it—op. cit. p. 29.

(that is, (C.D.) does not apply). They variously claim however that there is no 'possible notion of supposition adequate to remedy the deficiency' (Swiniarski, p. 213), and that 'O propositions are unprovided for by supposition theory' (Matthews, p. 20). But (3.0) *is* equivalent to

$$(\exists x)(P_1(x) \ \& \ \neg \bigvee_{i \in W_2} w_i(x)).$$

It follows that the supposition of P_2 is merely confused. A subsequent descent under P_1, which has determinate supposition, leads to

$$\bigvee_{j \in W_1} (\exists x)(w_j(x) \ \& \ \neg \bigvee_{i \in W_2} w_i(x)) \qquad \text{(by D.)}$$

and so to

$$\bigwedge_{j \in W_1} \neg \bigvee_{i \in W_2} w_i(w_j) \qquad \text{(by 2.2)}$$

which is intuitively right. (Incidentally, on treating the predicate as 'not-P_2'—the negation is within the scope of the quantifier—Matthews' own table suggests by symmetry a descent to a conjunctive predicate. This is equivalent to the above by De Morgan's law. Cf. (b)(i).)

(b) (i) Some commentators[1] have claimed that there is a fourth mode of supposition ('conjunctive' or 'impurely confused') which Ockham omitted. This is wrong. A simple induction over sentence formation, using (2.0) as the basis, proves that the equivalence (M.C.) holds for any $\phi(A)$. Therefore, if the supposition of any general term is neither determinate nor distributive, it must be merely confused; there is no possibility of nor need for a fourth mode. Ockham had a complete theory of supposition.

(ii) Ockham's theory has also been criticized on the grounds that the notion of merely confused supposition is unnecessary.[2] Indeed Geach (ibid. p. 104) argues in effect that provided we always descend in the correct order the distinction between merely confused and determinate supposition is superfluous. Apart from the fact that such a 'rule of preference' would be an encumbrance to Ockham's theory (cf. Swiniarski, p. 210), this claim is mistaken. Consider the sentence 'Only pigs don't fly', that is, 'Everything that doesn't fly is a pig'. Writing this as $(\forall x)(\neg P_1(x) \rightarrow P_2(x))$, we see that both P_1 and P_2 have merely confused supposition. Furthermore, if we descend by (M.C.) under P_1, P_2 still has merely confused supposition, and *vice versa*. It follows that the notion of merely confused supposition is not redundant.

(iii) Finally, Dummett claims[3] that supposition theory is incapable of dealing with multiple quantification. This must mean that the descent to singulars cannot be performed on sentences containing relation words. That this is not true can be seen by considering the following notoriously ambiguous example:

'Every boy loves some girl'. (3.1)

1 Swiniarski, op. cit. p. 212; P. Geach *Reference and Generality*, pp. 71 ff., 134; Loux, op. cit. p. 45 n. 9.
2 For example, E. Moody *Truth and Consequence in Medieval Logic*, p. 46.
3 *Frege: Philosophy of Language*, ch. 2 (esp. pp. 19–20); cf. Loux, p. 45 n. 10.

If we take $P_1(x)$, $P_2(x)$ and $P_3^2(xy)$ for 'x is a boy', 'x is a girl' and 'x loves y' respectively, (3.1) can be rendered either as

$$(\forall x)(P_1(x) \rightarrow (\exists y)(P_2(y) \;\&\; P_3^2(xy))) \tag{3.2}$$

or as

$$(\exists y)(P_2(y) \;\&\; (\forall x)(P_1(x) \rightarrow P_3^2(xy))). \tag{3.3}$$

Descending firstly under P_1, (3.2) reduces via (C.D.), (2.1), (D.) and (2.2) to

$$\bigwedge_{i \in W_1} \bigvee_{j \in W_2} P_3^2(w_i w_j),$$

whereas (3.3) reduces to

$$\bigvee_{j \in W_2} \bigwedge_{i \in W_1} P_3^2(w_i w_j)$$

(descending firstly under P_2 via (D.), (2.2), (C.D.) and (2.1)). This illustrates the scope ambiguity perfectly.

We may conclude (1) that, contrary to the views of Matthews and Henry, Ockham's *descensus* theory of personal supposition can be formalized in standard modern logic, and (2) that, contrary to those of Swiniarski, Geach and Dummett, it is a workable, coherent theory.

UNIVERSITY OF ST ANDREWS
UNIVERSITY OF WESTERN AUSTRALIA

PRINTED IN GREAT BRITAIN AT THE UNIVERSITY PRESS ABERDEEN