**True, False, Both, Neither, or None of the Above?**

It’s 340 BCE, give or take a decade or two, and Aristotle (384-322 BCE) is writing a text that later philosophers would call his *Metaphysics*. In Book 4 of this, he takes a number of his predecessors in his sights and argues for the Principle of Excluded Middle (PEM) and the Principle of Non-Contradiction (PNC).

To understand what these are, it will help to have a little bit of symbolism. If *A* is any statement, ~*A* is what you get when you negate it. So if *A* is ‘the Sun is shining’, ~*A* is ‘it is not the case that the sun is shining’; or if *A* is ‘Brutus killed Caesar’, ~*A* is ‘it is not the case that Brutus killed Caesar’. We might write these more colloquially as ‘the sun is not shining’ and ‘Brutus did not kill Caesar’, respectively.

The PEM tells us that for *every* statement, *A*, one or other of *A* and ~*A* is true. Either the sun is shining or it isn’t; either Brutus killed Caesar or he didn’t. A statement is either true or false. There’s no third possibility.

And the PNC tells us that for *every* statement, *A*, *A* and ~*A* can’t both be true. The sun can’t be both shining and not shining; it can’t be the case that Brutus both did and didn’t kill Caesar. A statement can’t be both true and false. That would be a contradiction; and contradictions just can’t be true.

Aristotle took some of his predecessors to be in violation of these principles. Whether they actually were so, modern scholars might debate. But that he thought they were is not in doubt.

Aristotle’s considered view on the PEM is in some doubt, since in the rather notorious Chapter 9 of another of his texts, *De Interpretatione* (as the Latins called it), he appears to argue that contingent statements about the future are neither true nor false. Gazing out over the Aegean, there is the Athenian fleet and the Spartan fleet. Will they fight tomorrow or will they not? There is as yet no fact of the matter. The future is open. Tomorrow, one or other of these statements will be true; but at the moment neither is.

Aristotle’s attitude to the PNC was unequivocal, however. He calls this the firmest of all principles and says (rather oddly, given what he says about his predecessors) that no one can really believe a contradiction.

From a modern perspective, Aristotle’s arguments for the PEM and PNC don’t look terribly compelling. His arguments for the PNC, in particular, are tangled, opaque, and often just beside the point. For example, he argues in some places that it can’t be the case that *all* contradictions are true—which is quite compatible with *some* being true.

Be that as it may, Aristotle’s text established the PEM and PNC as orthodox in Western logic—so much so that there are hardly any other attempts to defend the principles thereafter. They are simply assumed. People who assert contradictions, in particular, are held to manifest the nadir of rationality.

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It’s about 100 years earlier, give or take a decade or two, and about 6,800 km to the East, give or take a few lost camels, and the Buddha (Siddhārtha Gautama, fl. c. 450 BCE) is fielding questions from a student, Vaccha. The dialogue goes something like this:

*Vaccha*: We know it’s possible to achieve enlightenment in this life (because you have done so), but what happens to an enlightened person after they die. Do they still exist?

*Buddha*: No, I don’t claim that.

*Vaccha*: Well then, do they not exist?

*Buddha*: No, I don’t claim that.

*Vaccha*: So do they *both* exist and not exist.

*Buddha*: No, I don’t claim that.

*Vaccha*: Okay. So do they *neither* exist not not exist.

*Buddha*: No, I don’t claim that.

The Buddha is being a bit evasive. Why, is a matter of some dispute. Some of the sūtras that report such dialogues go on to have the Buddha say ‘Look, I’m telling you how to make your life better, and you want to waste time on metaphysics?!’.

But ignore that, and look at the framing of Vaccha’s questions. He is assuming that there are four possibilities: that something can be true (and true only), false (and false only), both true and false, or neither true nor false. In Sanskrit, this is called the *catuṣkoṭi*, literally, *four points* or *four corners (koṭi* = point). And notice that the Buddha does not say ‘Don’t be silly Vaccha. Something can’t be both true and false, or neither true nor false’. Clearly, he accepts that these are both possibilities. Nor is this simply a fact about Buddhism; several texts from that period appear to countenance the third and/or fourth *koṭi*. At any rate, the origins of the *catuṣkoṭi* are lost in the mists of time, but that the Indian thinkers of the period may hold that the possibilities concerning truth are four, not two, is clear.

The acceptance of the *catuṣkoṭi* was not as entrenched in Indian philosophy as acceptance of the PEM and PNC are in Western philosophy. By about the 5th Century CE, the dominant school of Hindu logic was Nyāya, and the Nyāya thinkers accepted the PEM and PNC. Indeed, even Indian Buddhist logicians came to accept these principles. Buddhism goes into China around the turn of the Common Era, and by about the 5th Century we see the development of the distinctively Chinese forms of Buddhism, such as Chan (or Zen, as it is known in Japan). Arguably, the Chinese Buddhists are not at all sympathetic to the PEM and PNC; but that is another story.

Whatever the vicissitudes of the *catuṣkoṭi* in the East, it is clear that the itmakes very little sense to someone who views it through the imperialist eyes of Aristotle-inspired logic. That one might tolerate the thought that something could be true and false, in particular, is likely to make eyes roll, and cause Western philosophers to write off such thinking as mystical nonsense. Western exegetes of the texts deploying the *catuṣkoṭi* have therefore bent over backwards trying to torture them into the procrustean bed of the PEM and PNC to avoid such a charge.

At least until recently.

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It is now 1920, and the logician Jan Łukasiewicz is writing in Warsaw. Logic is going through its most revolutionary change since Aristotle, as mathematical techniques are being applied to the subject for the first time. The German and English logicians Gottlob Frege and Bertrand Russell have just invented a new mathematical logic, now called—somewhat misleadingly—*classical logic*. Radical development though this new logic is, it incorporates the orthodoxy of the PEM and PNC. Accordingly, in the new logic, statements have one of the two “truth values” (as logicians call them): *t* (true and true only) and *f* (false and false only).

Łukasiewicz ponders the idea that there might be more than two, and so invents the subject of many-valued logic—that is, logics where there are more than two truth values.

In particular, Łukasiewicz is persuaded by Aristotle’s argument that some contingent statements about the future are neither true nor false. So he adds a new truth value, *n* (neither true nor false), to Frege and Russell’s two. We needn’t enter into the details of how this logic works here, except to note one thing. If the statement *A* is neither true nor false, then neither *A* nor ~*A* is true. Hence, as one would expect, the PEM fails in the logic.

It is still the case that nothing can be both true and false, however; the PNC still holds in the logic. Fast forward to about 50 something years later. A group of logicians centred around the work of Alan Anderson and Nuel Belnap is working in Pittsburgh on a subject they call ‘relevance logic’*.*

The driving idea behind this is the very natural one (not respected by classical logic!), that if a conditional—something of the form *if A then B*—is true, there must be some connection between *A* and *B*. At the heart of their systems they discover a very simple four-valued logic, which has since become known as *First Degree Entailment* (FDE). (Don’t ask.) In FDE a statement can have the values *t*, *f*, or *n*; but it can now also have the value *b*—both true and false. Again, we don’t need go into the details of how the system works, other than to note that the PEM fails because of the presence of *n*; and now the PNC also fails as well because of the value *b*. If the statement *A* has the value *b* then it is both true and false. That is, both *A* and ~*A* hold. Thus, the value *b* is assigned to contradictions that are true.

The values of FDE are often depicted in the form of a diagram called the *diamond lattice* as follows:

INSERT FIG. 1 HERE

Strikingly, the four corners of the *catuṣkoṭi* appear before our very eyes!

Of course, the Western logicians we have met knew nothing of Buddhism or the *catuṣkoṭi*. Equally, it would be thoroughly anachronistic to attribute to Ancient Indian Buddhists anything like the techniques and ideas of modern logic. But that is not the point here. Nor is the point to show that Buddhist philosophy is correct. That is an entirely different matter. So what is the point? FDE and other many-valued logics can be produced and analyzed with all the mathematical rigor and precision of modern logic. Hence, it can no longer be maintained that the *catuṣkoṭi* and related ideas areincoherent mysticism.

Nor is this to say that FDE is the correct logic. That is also another matter. Whether classical logic, FDE, or some other logic is the correct logic—or even whether the claim that there is a unique correct logic makes sense—is currently the topic of much debate in contemporary philosophical logic. But that modern logic has undercut the ground from those who would say that the *caṭuskoṭi* is nonsense is clear. On the other side, exegetes of Buddhist ideas are not longer required to torture the *catuṣkoṭi* to make it fit into the Aristotelian framework. All they have to do is reach for some of the now standard techniques of modern (non-classical) logic.

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Now let’s wind the clock back to some time in the 1st or 2nd Century CE, and to somewhere in India. Nāgārjuna, the foundational philosopher of all Mahāyāna Buddhisms (of which there are many) is writing has canonical text, *Mūlamadhyamakakārikā* (Fundamental Verses of the Middle Way). The *catuṣkoṭi* plays a central role in Nāgārjuna’s analysis, but he gives it a twist. In all schools of Buddhism there is a distinction between the conventional reality we experience and the ultimate reality unvarnished by our conceptual constructions. Nāgārjuna suggests that ultimate reality is ineffable. (Recall that the Buddha refused to answer certain questions?) So there appears to be a fifth possibility beyond the four of the *catuṣkoṭi*: ineffability.

It is straightforward enough to add a fifth value to FDE. Let us call this *e* (for emptiness—emptiness (*śūnyata*) is often used as a name for ultimate reality in Mahāyāna philosophy. *e* is the value of statements which are ineffable—or better, statements which describe ineffable states of affairs; statements themselves can’t be ineffable! Our five values now look like this:

INSERT FIG 2 HERE

Again, we do not need to go into the details of this logic here. All we need note is that if a part of any statement is ineffable then, as one would expect, so is the whole statement. And as before, all of this is perfectly rigorous, given the mathematical techniques of many-valued logic.

But, wait a moment. Some things, we are supposing, are ineffable. And are we not talking about them? Indeed so; and so did the Buddhist philosophers in question. So you *can* talk about the ineffable. Of course, that’s a contradiction. But the third *koṭi* of the *catuṣkoṭi* tells us that there is nothing *per se*, impossible about this.

However, that something is both effable and ineffable is not just any old contradiction. For a start, it’s about the ineffable, and this itself might be enough to make some Western philosophers weak at the knees—though they should remember how often the ineffable has entered into the history of Western philosophy: Plato (*chora*), Kant (noumena), Wittgenstein (form, in the *Tractatus*), Heidegger (being)—not to mention God in orthodox Christian philosophy.

Setting that matter aside, it remains the case that this is a contradiction not about what happens to some remarkable people after they die, but about our own ability to talk about the world we live in and the knots into which this is tangled. That’s philosophically bold stuff. Buddhist philosophers were, of course, well aware of the matter, and it occasioned much discussion. Where this lead them is another matter.

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Back to Western philosophy one last time. In the Introduction to the *Tractatus* (1922) Wittgenstein says:

Thus, the aim of the book is to draw a limit to thought, or rather, not thought, but the expression of thought: for in being able to draw the limits to thought, we would have to find both sides of the limit thinkable (i.e., we would have to think what cannot be thought).

It will therefore be only in language that the limit can be drawn, and what lies on the other side will simply be nonsense.

But from the very beginning, commentators on the *Tractatus* noted that Wittgenstein *does* talk about what is on the other side of the effable. And what he says about this most certainly does not appear to be nonsense.

True to his Introduction, though, Wittgenstein does declare much of his own book meaningless. In retrospect, one might say that the PNC occasioned a failure of nerve on his part. Had he had the techniques of the *catuṣkoṭi* at his disposal, he might more simply just have accepted that this was a spectacular case of the third *koṭi*—and a profound one.