Plurivalent Logics

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1 Introduction: Plurivalence

About 100 years ago, Nikolai Vasil'év published a number of ground-breaking papers in which he introduced what he termed *Imaginary Logic*. In the actual world, for any predicate, P something is either P or not-P—not both and not neither. But there could be imaginary worlds in which something could be both P and not-P.[1] In modern terms, one way in which one might describe Vasil'év's project is as countenancing the possibility that a statment, 'a is P', might have more than one truth value. Of course, Vasil'év was writing in the context of Aristotelian logic; and the developments in logic that were going on elsewhere in Europe at the time would soon make this framework obsolete. But with all the resources of modern formal logic at our disposal, we can ask whether there are now better ways of implementing Vasil'év's project. In this paper I will suggest one such way.[2]

I will describe a technique for generating a novel kind of logical semantics, and explore some of its consequences. Some particular cases of this technique are already known, as I shall point out in due course.[3] But as far as I know, no one has noted that there is a general and interesting construction to be had. It would be natural to call the semantics produced by the technique in question 'many-valued'; but that name is, of course, already taken. I shall

[1]For a detailed description of Imaginary Logic, see Priest (2000).

[2]What follows is published as 'Plurivalent Logic', *Australasian Journal of Logic*, 12 (2014), pp. 2-13. It is reprinted with their permission.

[3]In particular, it is deployed in Priest (1984), (2008b), (2010), (2014). These papers can be consulted for some of the philosophical considerations which might motivate such a construction. The point of the present paper is simply to spell out its technical details.

call them, instead, 'plurivalent'. In standard logical semantics, formulas take exactly one of a bunch of semantic values. I shall call such semantics 'univalent'. In a plurivalent semantics, by contrast, formulas may take one or more such values (maybe even less than one, but I set this possibility aside till the last part of this paper). The construction I shall describe can be applied to any univalent semantics to produce a corresponding plurivalent one. In this paper I will be concerned with the application of the technique to propositional many-valued (including two-valued) logics. Sometimes, as we shall see, going plurivalent does not change the consequence relation; sometimes, as we shall also see, it does. We will explore these possibilities in detail with respect to one small family of many-valued logics.

2 The Basic Construction

Let us start with a brief summary of a standard many-valued logic.[4]

Let L be a propositional language. A univalent semantics for the language is a structure $M = \langle V, D, \delta \rangle$. V is a non-empty set of truth values; D is a subset of V, the designated values. And for every n-place connective in the language, $\circ$, $\delta_\circ$ is the truth function for $\circ$; that is, it is a map from V^n to V. An interpretation is a pair $\langle M, \mu \rangle$, where M is such a structure, and μ is an evaluation function from the propositional parameters of the language to V. Given an interpretation, μ is extended to a map from all formulas to V recursively: $\mu(\circ(A_1, ..., A_n)) = \delta_\circ(\mu(A_1), ..., \mu(A_n))$. And an inference with conclusion A and premises Σ is valid iff in any interpretation in which all the premises are designated, so is the conclusion. I will write the consequence relation as $\models^M_u$ (The u is for univalent; and I may omit the M when this is clear from the context.)

Given a univalent interpretation, the corresponding plurivalent interpretation is the same, except that it replaces the evaluation function, μ, with a one-many evaluation relation, $\rhd$, between propositional parameters and V. That is, every propositional parameter relates to some number of values in V. The relation $\rhd$ is extended to a relation between all formulas and values in V pointwise. That is:

- $\circ(A_1, ..., A_n) \rhd v$ iff $\exists v_1, ...v_n(A_1 \rhd v_1, ..., A_n \rhd v_n$ and $v = \delta_\circ(v_1, ..., v_n))$

For the moment, we will assume that $\rhd$ satisfies the *Positivity Condition* (PC): for every propositional parameter, p:

- for some $v \in V$, $p \rhd v$

[4] See Priest (2008a), ch. 7.

Every parameter relates to a positive number of values. Clearly if every propositional parameter satisfies PC, every formula does.

Let us say that $\rhd$ *designates* A iff for some v such that $A \rhd v$, $v \in D$. Then the consequence relation is defined in the obvious way:

- $\Sigma \models_p^M A$ iff for all $\rhd$, if $\rhd$ designates every member of Σ, $\rhd$ designates A

(The p is for plurivalent, and I may drop the M when it is clear from the context.)[5]

Call the following the *Uniqueness Condition:*

- For every propositional parameter, p: there is exactly one $v \in V$ such that $p \rhd v$.

As is easy to see, if $\rhd$ satisfies the Uniqueness Condition, every formula relates to exactly one value. And in this case, it is simply a notational variant of a univalent interpretation. In other words, every univalent interpretation is a plurivalent interpretation. It follows that:

(1) $\Sigma \models_p^M A \Rightarrow \Sigma \models_u^M A$

The converse may or may not hold. If it does, we will say that the plurivalent semantics is *conservative* over the univalent semantics.

3 Many-Valued Logics and Homomorphisms

In later sections we will investigate the results of applying plurivalence to a family of many-valued logics. In order to do this, some standard results concerning homomorphisms will be useful.[6]

Given two univalent semantics for the same language, $N = \langle V, D, \delta \rangle$ and $N' = \langle V', D', \delta' \rangle$, a *homomorphism* from N to N' is a map, θ, from V to V' such that:

- $v \in D$ iff $\theta(v) \in D'$

and for any n-place connective, $\circ$:

- $\theta(\delta_\circ(v_1, ..., v_n)) = \delta'_\circ(\theta(v_1), ..., \theta(v_n))$

[5] An interesting alternative is to define '$\rhd$ *designates* A' as: for *all* v such that $A \rhd v$, $v \in D$. I defer discussion of this possibility to a brief appendix to the paper.

[6] These can be found in Czelakowski (2001), esp. Prop 0.3.3.

Given such a homomorphism, θ, and any evaluation for N, μ, the composition map $\mu' = \theta\mu$ is an evaluation for N', and the homomorphism ensures that for any A, $\mu'(A) = \theta\mu(A)$. So:

(2) $\Sigma \models_u^{N'} A \Rightarrow \Sigma \models_u^{N} A$.

For suppose that $\Sigma \not\models_u^{N} A$. Then for some μ, $\mu(A) \notin D$ but $\mu(B) \in D$, for every $B \in \Sigma$. Hence $\mu'(A) = \theta\mu(A) \notin D'$ and $\mu'(B) = \theta\mu(B) \in D'$. That is, $\Sigma \not\models_u^{N'} A$.

Moreover, if θ is onto, then the converse also holds. For suppose that $\Sigma \not\models_u^{N'} A$. Then for some μ', $\mu'(A) \notin D'$ but $\mu'(B) \in D'$, for every $B \in \Sigma$. By the Axiom of Choice, let θ^{-1} be a map from V' to V such that if $v' \in V'$, $\theta^{-1}(v')$ is one of the $v \in V$ such that $\theta(v) = v'$. Since θ is onto, this is a map from V' to V. Consider $\mu = \theta^{-1}\mu'$. This is an evaluation for N, and clearly $\theta\mu = \mu'$. Hence, $\Sigma \not\models_u^{N} A$. In other words, if there is a homomorphism from N onto N':

(3) $\Sigma \models_u^{N} A \Leftrightarrow \Sigma \models_u^{N'} A$

Now, given any univalent semantics $M = \langle V, D, \delta \rangle$, its corresponding plurivalent semantics can itself be seen as a univalent semantics, $\dot{M} = \left\langle \dot{V}, \dot{D}, \dot{\delta} \right\rangle$. $\dot{V}$ contains the non-empty subsets of V. (Hence, if M is an n-valued logic, $\dot{M}$ is a $(2^n - 1)$-valued logic.) $\dot{v} \in \dot{D}$ iff for some $v \in D$, $v \in \dot{v}$, and the truth functions, $\dot{\delta}$, are those induced by δ. That is, $v \in \dot{\delta}_\circ(X_1, ..., X_n)$ iff for some $v_1 \in X_1, ..., v_n \in X_n$, $v = \delta_\circ(v_1, ..., v_n)$. If $\rhd$ is a plurivalent evaluation for M, let us write $\rhd [A]$ for $\{v \in V : A \rhd v\}$. Every plurivalent evaluation on M corresponds to a univalent evaluation, $\dot{\mu}$, on $\dot{M}$, where $\rhd [p] = \dot{\mu}(p)$, and vice versa. And it is easy to check by induction that for all A, $\rhd [A] = \dot{\mu}(A)$. Hence:

(4) $\Sigma \models_p^{M} A \Leftrightarrow \Sigma \models_u^{\dot{M}} A$

A corollary of these results gives us a sufficient condition for a plurivalent semantics to be conservative. If there is a homomorphism from $\dot{M}$ to M then by (2) and (4) we have the converse of (1):

(5) $\Sigma \models_u^{M} A \Rightarrow \Sigma \models_u^{\dot{M}} A \Rightarrow \Sigma \models_p^{M} A$

One might note that the function which maps v to $\{v\}$ is a homomorphism from M to $\dot{M}$, though not onto. That is another way of inferring (1).

4 The *FDE* Family

We can now investigate applying the plurivalence construction to one family of univalent semantics. I will call this the *FDE family.* For this family, it is relatively easy to give a complete solution to the question of conservativity. In this section, we will lay out the details of the family.

All the semantics are substructures of a single 5-valued structure.[7] The language has the connectives $\wedge$, $\vee$, and $\neg$. $A \supset B$ may be defined in the usual way, as $\neg A \vee B$. In the semantics, $V = \{t, f, b, n, e\}$ (*true only*, *false only*, *both true and false*, *neither true nor false*, and *empty,* respectively). $D = \{t, b\}$. For the truth functions: any function gives an output e iff some input is e. For the other values: $\delta_\neg$ maps t to f, vice versa, and n and b to themselves. Conjunction and disjunction are the greatest lower bound and least upper bound of the familar diamond lattice:

$$\begin{tikzcd}
 & t & \\
b \arrow[ur] & & n \arrow[ul] \\
 & f \arrow[ul] \arrow[ur] &
\end{tikzcd}$$

The semantic structures we will be concerned with are those that contain the values t and f, plus some number (possibly zero) of the other values. Hence we can form a systematic taxonomy by citing those other values. I will delete set brackets for perspicuity. Most of the logics generated by these semantics are already known in one form or another.[8]

- $\emptyset$: classical logic, CL.
- e: Bochvar logic (also known as weak Kleene 3-valued logic), B_3.
- n: strong Kleene 3-valued logic, K_3.
- b: logic of paradox, LP.
- en: a logic not previously formulated (as far as I know).
- eb: the logic AL of Oller (1999).[9]
- bn: first degree entailment, FDE.

[7] Priest (2010) and (2012) call the logic this generates FDE_φ.
[8] See Priest (2008a), ch. 7. For B_3, see Haack (1996), pp. 169-70.
[9] Many thanks to Thomas Ferguson for drawing my attention to this.

- *bne*: FDE_φ.

The additions of b, n, and e, to $\emptyset$ have distinct effects. The addition of b (but not e or n) invalidates Explosion, $p \wedge \neg p \vdash q$. The addition of e (but not b or n) invalidates $\vee$-introduction, $p \vdash p \vee q$. The introduction of n invalidates Excluded Middle $p \vdash q \vee \neg q$, but so does the addition of e. The addition of n (but not b or e) invalidates a disjoined form of Excluded Middle, $p \vee q \vdash q \vee \neg q$. (One cannot make the premise designated by assigning p or q the value e. But assigning p the value t, and q the value n gives a countermodel.)

A first glance at the plurivalent versions of the above semantics reveals the following. Any plurivalent semantics which contains the values t and f (not just one in the FDE family) and on which negation works in the usual way, determines a paraconsistent consequence relation. For consider a $\rhd$ such that $\rhd [p] = \{t, f\}$ and $\rhd [q] = \{f\}$. This shows that $p, \neg p \nvDash^M_p q$. Hence if M is any semantics in the FDE family having an explosive consequence relation, its corresponding plurivalent semantics is going to deliver a strictly weaker consequence relation, and so is not conservative. In particular, the plurivalent consequence relation corresponding to $\emptyset$ is known to be that given by the semantics b.[10] By contrast, the plurivalent semantics given by b, *bn*, and *bne* are known to be conservative over the univalent semantics.[11] Why is going plurivalent sometimes conservative and sometimes not? In the next section I determine the consequence relations for the plurivalent semantics corresponding to each univalent semantics in the FDE family, in the process answering this question.

5 Plurivalence and the FDE Family

Let M be any many-valued semantics in the FDE family, and let M^b be the semantics obtained by adding b to its values, if necessary. We can define a map from the values of $\dot{M}$ to the values of M^b, and show that this is a homomorphism onto M^b. The definition is by cases:

- if $b \in X$ then $\theta(X) = b$
- else:
    - if $t \in X$ and $f \in X$ then $\theta(X) = b$

[10]See Priest (1984).

[11]For b this is proved by a model theoretic construction in Priest (1984). I generalise this in the next section. For *bn* and *bne*, the results are proved in Priest (2010) and (2012), respectively, by considering proof theories for these logics.

  - if $t \in X$ and $f \notin X$ then $\theta(X) = t$
  - if $t \notin X$ and $f \in X$ then $\theta(X) = f$

- else: if $n \in X$ then $\theta(X) = n$
- else: $\theta(X) = e$

Note that this definition makes sense for any semantics in our family.

θ is onto: $\theta(\{t\}) = t$, $\theta(\{f\}) = f$, $\theta(\{t, f\}) = b$, and if n or e is a value in M^b, $\theta(\{n\}) = n$, and $\theta(\{e\}) = e$. It is also clear that θ preserves designated values. In all our logics, $A \vee B$ can be defined as $\neg(\neg A \wedge \neg B)$. Hence to check that θ is a homomorphism, we need only check the cases for the truth functions for $\neg$ and $\wedge$.

For $\neg$, we need to show that $\theta(\delta_\neg(X)) = \delta_\neg(\theta(X))$. We check this by cases. If $b \in X$ then $\delta_\neg(\theta(X)) = \delta_\neg(b) = b$. And $\theta(\delta_\neg(X)) = \theta(Y)$, where $b \in Y$. So $\theta(\delta_\neg(X)) = b$. The other cases are similar, and left as exercises.

For $\wedge$, we need to show that $\theta(\delta_\wedge(X, Y)) = \delta_\wedge(\theta(X), \theta(Y))$. There are 36 cases to check, though since conjunction is commutative, this reduces the number to 21. Here are the first 6 cases. Suppose that $b \in X$:

1. $b \in Y$. Then $b \in \delta_\wedge(X, Y)$, so $\theta(\delta_\wedge(X, Y)) = b$. And $\delta_\wedge(\theta(X), \theta(Y)) = \delta_\wedge(b, b) = b$.
2. $b \notin Y$, $t \in Y$, $f \in Y$. Then $b \in \delta_\wedge(X, Y)$, so $\theta(\delta_\wedge(X, Y)) = b$. And $\delta_\wedge(\theta(X), \theta(Y)) = \delta_\wedge(b, b) = b$.
3. $b \notin Y$, $t \in Y$, $f \notin Y$. Then $b \in \delta_\wedge(X, Y)$, so $\theta(\delta_\wedge(X, Y)) = b$. And $\delta_\wedge(\theta(X), \theta(Y)) = \delta_\wedge(b, b) = b$.
4. $b \notin Y$, $t \notin Y$, $f \in Y$. Then $f \in \delta_\wedge(X, Y)$, but $b \notin \delta_\wedge(X, Y)$ and $t \notin \delta_\wedge(X, Y)$, so $\theta(\delta_\wedge(X, Y)) = f$. And $\delta_\wedge(\theta(X), \theta(Y)) = \delta_\wedge(b, f) = f$.
5. $b \notin Y$, $t \notin Y$, $f \notin Y$, $n \in Y$. Then $f \in \delta_\wedge(X, Y)$, but $b \notin \delta_\wedge(X, Y)$ and $t \notin \delta_\wedge(X, Y)$, so $\theta(\delta_\wedge(X, Y)) = f$. And $\delta_\wedge(\theta(X), \theta(Y)) = \delta_\wedge(b, n) = f$.
6. $b \notin Y$, $t \notin Y$, $f \notin Y$, $n \notin Y$. Then $\delta_\wedge(X, Y) = \delta_\wedge(\{e\}, \{e\}) = e$, so $\theta(\delta_\wedge(X, Y)) = e$. And $\delta_\wedge(\theta(X), \theta(Y)) = \delta_\wedge(e, e) = e$.

The other cases are left as exercises.

The existence of the homomorphism delivers us a complete characterisation of the plurivalent consequence relations in our family. For if M is any one of our semantics, then by (3) and (4), we have:

(6) $\Sigma \models_p^M A \Leftrightarrow \Sigma \models_u^{\dot{M}} A \Leftrightarrow \Sigma \models_u^{M^b} A$

The plurivalent consequence relation for M is just the consequence relation for the univalent semantics obtained by adding the value b. As a corollary, if the semantics contains b already, plurivalence is conservative. And if it does not, plurivalence is not conservative, since it turns an explosive consequence relation into a paraconsistent one.

6 General Plurivalence

In this section we will consider what happens when one drops the Positivity Condition. In plurivalent logics of this kind, a relation may relate a propositional parameter (and hence an arbitrary formula) to any number of truth values, including zero. I will call this kind of plurivalence *general plurivalence*. What I have so far called plurivalence I will now call *positive plurivalence*.

It is clear that, given a univalent semantics, the consequence relation delivered by its corresponding general plurivalent semantics can be no stronger than that delivered by its corresponding positive plurivalent semantics. For any positive plurivalent evaluation is a general plurivalent evaluation. Hence, if we subscript the general plurivalent consequence relation with a g, we have:

(7) $\Sigma \models_g^M A \Rightarrow \Sigma \models_p^M A \Rightarrow \Sigma \models_u^M A$

The general plurivalent semantics may or may not be conservative over the univalent one, and indeed, over the positive plurivalent sematics, as we will see in a moment.

Given a positive plurivalent semantics based on the structure M, we saw how to construct an equivalent univalent semantics, $\dot{M}$. Applying the same construction to the general plurivalent semantics based on M produces a corresponding univalent semantics, in exactly the same way. We may write this as $\ddot{M}$. The only difference between $\dot{M}$ and $\ddot{M}$ is that the empty set is a value of the latter, but not of the former. (Hence, if M is an n-valued semantics $\ddot{M}$ is a 2^n-valued semantics.) Exactly the same argument as before shows that:

(8) $\Sigma \models_g^M A \Leftrightarrow \Sigma \models_u^{\ddot{M}} A$

Now, take *any* univalent semantics, M (not just one in the FDE family) and augment it, if necessary, with a non-designated value e, such that an e input always gives an e output, and non-e inputs always give a non-e output. Call the result M^e. Then the consequence relation for the positive plurivalent semantics for M^e is the same as that for the general plurivalent semantics for M. That is:

(9) $\Sigma \models_p^{M^e} A \Leftrightarrow \Sigma \models_g^M A$

For the proof from left to right, suppose that $\Sigma \not\models_g^M A$. Let $\rhd$ be an evaluation which delivers a counter-model. Consider the evaluation, $\rhd^e$ which is the same, except that where $\rhd [p] = \emptyset$, $\rhd^e [p] = \{e\}$. This is a positive plurivalent evaluation for M^e. Moreover, it is not difficult to see that for any formula, A:

- if $\rhd [A] = \emptyset$ then $\rhd^e [A] = \{e\}$
- otherwise, $\rhd [A] = \rhd^e [A]$

The proof is by recursion on the formation of A. The details are straightforward. It follows that $\Sigma \not\models_p^{M^e} A$.

For the proof from left to right, suppose that $\Sigma \not\models_p^{M^e} A$. Let $\rhd^e$ be an evaluation which delivers a counter-model. Consider the evaluation, $\rhd$, which is the same as $\rhd^e$, except that it does not relate anything to e. This is a general plurivalent evaluation for M. Moreover, it is not difficult to see that for any value, v, distinct from e, and for any A:

- $A \rhd^e v$ iff $A \rhd v$

The proof is by recursion on the formation of A. The details are again straightforward. It follows that $\Sigma \not\models_g^M A$.

We see, then, that to obtain the general plurivalient consequence relation for a univalent semantics one just adds e to its values, and then takes the positive plurivalent consquence relation. In particular, if the semantics already contains the value e, positive and general plurivalentization produce the same consequence relation.

Thus, for logics in the FDE family, if the univalent semantics contains b and e, the general plurivalent semantics is conservative over it. Otherwise not. If the univalent semantics does not contain b, the consequence relation is explosive; the consequence relation for the general plurivalent logic is paraconsistent. If the univalent semantics does not contain e, its consequence validates $\vee$-introduction, which is invalid when e is present.[12]

[12] It is worth noting that Shramko and Wansing (2011), chs. 3 and 4, show how, given any univalent logic M, to construct a logic whose values are the values of $\ddot{M}$. Given their approach, it is natural to think of the empty set not simply as an *absence* of values, but as a positive value in its own right. This motivates different possible definitions for the truth functions in the logic, producing somewhat different results. In particular, with these definitions, the empty set does not generate a failure of $\vee$-introduction. In this context, it is worth noting that applying plurivalence to classical logic does *not* produce FDE.

7 Conclusion

In this paper we have seen that given any univalent semantics, we may produce the semantics which are positively and generally plurivalent with respect to it. For any univalent semantics, M, in the FDE family, the consequence relation for the corresponding positive plurivalent logic is the same as that delivered by the univalent semantics obtained by adding b to the values of M (if it is not already there); and the consequence relation delivered by corresponding general plurivalent semantics is the same as that for the univalent semantics obtained by adding b and e to the values of M (if they are not already there).

Of course, there are many more important things to be investigated. For a start, there should be a detailed investigation of the application of the plurivalent technique to logics other than the FDE family, such as, for example, continuum-valued logics. What are the logics generated in this way, and under what general conditions is plurivalence conservative over a univalent logic?

Moreover, the technique can be generalised. As we have seen, given any univalent semantics, M, going plurivalent produces an equivalent univalent logic with more values (identifying any value v with $\{v\}$ under this homomorphism)—in fact two, $\dot{M}$ and $\ddot{M}$. We can apply the construction to these, and then, if we wish, repeat the process. We can even "collect up the values", and consider the limit.[13] In the case of logics in the FDE family, this produces no further change in consequence relation, since after the first application, plurivalence is conservative, as we have seen.[14]

Moreover, as should be clear, the technique of plurivalence can be applied beyond the realm of propositional many-valued logics. We can apply it to any logic with a univalent world-semantics, by applying it to the truth values available at each world. We can apply it to any first-order logic with a univalent semantics. Also, we can apply it not only to truth but to denotation. In standard semantics, denotation is a function from constants into the domain. This can be replaced by a relation, allowing for multiple denotations.[15] I leave these matters for future investigation.[16]

[13] This is done for classical logic and positive plurivalence in Priest (1984).

[14] For the limit: anything valid in the limit logic is valid in each finite "approximation". Conversely, anything invalid in it is invalid in some approximation, since only finitely many values are employed in the counter-model.

[15] The construction is used, in effect, in Priest (1995), and (2005), ch. 8.

[16] Many thanks go to Lloyd Humberstone for very helpful comments on an earlier draft of this paper. Thanks, too, go to an anonymous referee for this journal.

8 Appendix

This appendix concerns a variation of the definition of designation for plurivalent logics. Specifically, we replace the definition of designation of Section 2 by: '$\rhd$ *designates* A' as: for *all* v such that $A \rhd v$, $v \in D$. This changes nothing till Section 5, except the definition of the designated values in $\dot{M}$, where the modification required is obvious. For Section 5, let M^n be the semantics obtained from M by adding the value n, if necessary. Then line (6) holds provided we replace M^b with M^n. The homomorphism that establishes this is defined as follows:

- if if $e \in X$ then $\theta(X) = e$
- else: if $n \in X$ then $\theta(X) = n$
- else: if $t \in X$ and $f \in X$ then $\theta(X) = n$
- else:
  - if $X = \{t, b\}$ then $\theta(X) = t$
  - if $X = \{t, f\}$ then $\theta(X) = f$
  - if $X = \{x\}$ then $\theta(X) = x$

One may check that this is a homomorphism, and that it is onto.

Things are quite different with respect to the general plurivalence of Section 6, however; and matters do not modify in such a straightforward way. In particular, the alignment between $\models_p^{M^e}$ and $\models_g^M$ disappears in both directions. Thus, for M take classical logic ($\emptyset$). Then, as one may check, $p \wedge q \models_p^e p$, $p \wedge q \not\models_g^\emptyset p$ (let $\rhd$ relate q to just f, and p to nothing); and $p \not\models_p^e p \vee q$, but $p \models_g^\emptyset p \vee q$ (for a counter-model, p must relate to just t; all the values of $p \vee q$ are then designated—even when q relates to nothing!). What one can say to characterise general plurivalence in this case is still an open question.

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