Speaking of the Ineffable, East and West

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1 Introduction: Enter the Ineffable

This talk[1] concerns a phenomenon that arises in several locations in philosophy, both East and West. A thinker comes to the conclusion that there is something ineffable. They do this on the basis of various arguments. But in the process of giving these arguments, they are, of course, talking about the ineffable thing. Clearly, they are facing contradiction. Many philosophers, when in this situation, will try to wriggle out of it. But usually, the supposed cure is worse than the disease. This is the phenomenon at issue.

If one finds oneself in this situation there is, it seems to me, a better option: simply to accept the contradiction. One *can* speak about some ineffable things. Of course, Western philosophy, steeped as it is in its near universal acceptance of the Principle of Non-Contradiction (at least until recently), hardly had the resources to make this move. But Indian philosophy has better resources. There is a principle of logic/metaphysics called the *catuṣkoṭi*, which recognises the possibility of accepting a contradiction as one in logical space. The point of this talk is to explain all this.

In the first part of the talk, to illustrate the phenomenon we are dealing with, we will look at three thinkers, or traditions, which manifest it. In the second part of the talk, we will look more carefully at the machinations of

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the catuṣkoṭi and its variations. In the final part of the talk, we will see that the possibility envisaged is my no means a flight of contemporary fantasy, but can be found in a very importrant Buddhist sūtra, the *Vimilakirti Sūtra*.

2   Some Key Thinkers

2.1   Wittgenstein

So to the first of our thinkers, the Wittgenstein of the *Tractatus*. In this, Wittgenstein describes a distinctive picture of the world, language, and the relationship between them. On the world side, states of affairs are composed of objects arranged with a certain form. On the language side, propositions are composed of names, arranged with a certain form. And a proposition expresses a state of affairs iff the names in the proposition refer the objects in the state of affairs, and the names in the proposition are arranged with the same form as the objects in the state of affairs. That is, as mathematicians say, the state of affairs and the proposition are *isomorphic*: have the same form.

What sort of thing is this form, though? One thing is clear: form is not just another object. If form were just an object, it could not bind the objects of a state of affairs into a unity: when we added it to the objects, we would just get a bigger bunch of objects. Form is that way that objects are *combined*. But now we have a problem. The only way one can say something is to make a proposition whose names refer to objects. If form is not an object, one can say nothing about it: it is ineffable.[2] But the *Tractatus* does say a lot about it. As Russell puts it in his introduction to the English translation of the *Tractatus*:[3]

> Everything ... which is involved in the very idea of the expressiveness of language must remain incapable of being expressed in language, and is, therefore, inexpressible in a perfectly precise sense. ... [One may have] some hesitation in accepting Mr Wittgenstein's position, in spite of the very powerful arguments which he brings to its support. What causes hesitation is the

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[2]Form is, in fact, just one of a number of structural notions of which the *Tractatus* makes use. The situation is the same for all of these. For a more general discussion, see Priest (2002), ch. 12.

[3]Pears and McGuinness (1961), p. xxi.

fact that, after all, Mr Wittgenstein manages to say a good deal about what cannot be said...

And of course, Wittgenstein realised that he was in this situation. He reaction to it is captured in the stunning final propositions of the *Tractatus*:[4]

> 6.54 My propositions are elucidatory in this way: he who understands me finally recognizes them as senseless, when he has climbed out through them, on them, over them. (He must so to speak throw away the ladder, after he has climbed up on it.) He must surmount these propositions; then he sees the world rightly.

> 7 Whereof one cannot speak, thereof one must be silent.

But the move, for all its chuztpah, is disastrous. Not only *do* we understand the propositions of the *Tractatus* (we teach what Wittgenstein said to our students), if Wittgenstein is right, and the statements of the *Tractatus* really are meaningless, they provide absolutely no ground to suppose that they are talking of the ineffable, and so need to be cured by meaninglessness.

This is our first example of out target phenomenon.

2.2 Heidegger

For the seecond, let us turn to Heidegger. At the beginning of *Sein und Zeit*, Heidegger announces the question which was to dominate his thinking for the rest of his life, the *Seinsfrage*: what is *being*? All objects are, but what, exactly, is it to be? Also, right at the start of *Sein und Zeit*, he observes, or avers that, whatever it is, it cannot be just another being. Why, he does not explain there, but it is a familiar enough thought to anyone who knows about Neo-Platonism: the One is not an object: it is what gives rise to all objects.

It follows that one cannot answer the question of being: to say what being is, one would have to say 'being is such and such', and in the process, treat it as an object—just what it is not. Heidegger came to accept this conclusion. One cannot say what being is; but art, poetry, and so on, can open people's eyes to being *showing* itself. Of course, if one cannot answer the question of being, one cannot ask it either. To say 'what is being?' treats it as an object too.[5] But Heidegger does just this.

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[4]Pears and McGuinness (1961), p. 74.

[5]For a general discussion of the matter, see Priest (2002), ch. 15.

Naturally, Heidegger realised that he faced a contradiction, and he wrestled with the situation. Thus, in one place he says:[6]

> If we painstakingly attend to the language in which we articulate what the principle of reason [*Satz vom Grund*] says as a principle of being, then it becomes clear we speak of being in an odd manner that is, in truth, inadmissible. We say: being and ground/reason [*Grund*] 'are' the same. Being 'is' the abyss [*Abgrund*]. When we say something 'is' and 'is such and so', then that something is, in such an utterance, represented as a being. Only a being 'is'; the 'is' itself—being—'is' not. The wall in front of you and behind me is. It immediately shows itself to us as something present. But where is its 'is'? Where should we seek the presencing of the wall? Probably these questions already run awry.

And in another, he tries to say what he cannot by the technique of writing under erasure:[7]

> ... a thoughtful glance ahead into the realm of *being* can only write it as ~~being~~ . The crossed lines at first only repel, especially the almost ineradicable habit of conceiving *being* as something standing by itself ... Nothingness would have to be written, and that means thought of, just like ~~being~~.

Though Heidegger's words do not have the bluntness of Wittgenstein's, what they are saying is effectively the same: that his discourse about being is meaningless.

And the problem is exactly the same as that for Wittgenstein. The discourse is not meaningless. We do understand it. Even in the above quotes, we have to understand it, in order to understand how we are supposed to be getting around what cannot be said. Heidegger's words belie his claims in a self-undercutting way.

This is the second example of our target phenomenon.

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[6]Lilly (1991), p. 51f.
[7]Kluback and Wilde (1959), p. 81.

2.3 Mahāyāna Buddhism

For the third example, we turn East. The discussion here will take a little longer. This is for two reasons. The first is that we are not dealing with a single thinker, but with a whole tradition. The second is that we will meet the idea that will take centre-stage in the second part of the talk: the catuṣkoṭi.[8]

Let us start by turning back the clock to the origins of Buddhism (c. 5th century BCE). The foundational ideas of this were spelled out by the historical Buddha, Siddhārtha Gautama (*Pāli*: Gotama. 'Buddha' is an honorific—like 'Christ'. It just means someone who is awakened, enlightened.) Only a small part of this will concern us here. The early sūtras record the sayings of the Buddha and his discussions with his followers. They often asked him questions, and in a couple of these, they ask the natural question: what's going to happen to an enlightened person after they die? The discussion goes like this:[9]

> 'How is it, Gotama? Does Gotama believe that the saint exists after death, and that this view alone is true, and every other false?'
>
> 'Nay, Vacca. I do not hold that the saint exists after death, and that this view alone is true, and every other false.'
>
> 'How is it, Gotama? Does Gotama believe that the saint does not exist after death, and that this view alone is true, and every other false?'
>
> 'Nay, Vacca. I do not hold that the saint does not exist after death, and that this view alone is true, and every other false.'

If one were in Aristotle's Lyceum, one would expect the matter to halt there. All the bases seem to have been covered. But the questions continue:

> 'How is it, Gotama? Does Gotama believe that the saint both exists and does not exist after death, and that this view alone is true, and every other false?'

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[8]For a longer discussion of the material in this section, see Priest (2014a).

[9]Radhakrishnan and Moore 1957, 289 f. I note that 'saint' is not a particularly good translation. It is a reference to someone who is enlightened.

'Nay, Vacca. I do not hold that the saint both exists and does not exist after death, and that this view alone is true, and every other false.'

'How is it, Gotama? Does Gotama believe that the saint neither exists nor does not exist after death, and that this view alone is true, and every other false?'

'Nay, Vacca. I do not hold that the saint neither exists nor does not exist after death, and that this view alone is true, and every other false.'

Four possibilities are countenanced: existence, non-existence, both, and neither. This is the catuṣkoṭi (Gk: *tetralemma*; Eng: four corners). And since this is the first time we are meeting it, let us pause over it for a moment.

According to the catuṣkoṭi, given any claim, there are four possibilities: that it is true (only), false (only), both true and false, neither true nor false. In Aristotle's logic there is a principle of excluded third: everything is true or false, and that's it: *tertium non datur*. The catuṣkoṭi is effectively a principle of excluded fifth: *quintum non datur*.

The historical origins of the catuṣkoṭi are lost in unrecorded history, but they clearly pre-date Buddhism. Note how the questioner just assumes this framework, and the Buddha accepts it.

Now to continue the story. The Buddha, you will note, refused to assent to any of the possibilities. Why? In some of the sūtras he goes on to say that it is a waste of time worrying about such things. 'Hey, I'm here telling you how you can really improve you life, and you want to do philosophy?'. But in some, there is a hint that there is something else going on. None of these four possibilitis 'fits the case'. That thought is never really fleshed out, and lies on the table for about the next 700 years. So let us move on to this.

Around the turn of the common era, a new sort of sūtra starts to emerge: the *Prajñāpāramitā Sūtras*, and these presage the development of a whole new kind of Buddhism: Mahāyāna. The philosopher who established the movement—perhaps the most influential philosopher in the history of Buddhism, other than Siddhārtha himself—was Nāgārjuna, who flourised around the 2nd or 3rd centuries CE. In his *Mūlamadhyamakakārikā* (MMK, Fundamental Verses of the Middle Way), he subjects the older metaphysical picture to a devastating attack.

The older form of Buddhism held that our familiar world is a conceptual construction, but it is constructed from *dhārmas*, atoms, things which exist

independently of our concepts and of each other. They are what they are intrinsically; they have *svabhāva*, self-being. Much of the MMK attacks this view: nothing has self-being. Everything is empty (*śūnya*) of svabhāva. Many of the arguments are by *reductio*. Nāgārjuna runs through all the possible cases on a position, and shows that each of them is unacceptable. The four possibilities are, in each case, the four cases of the catuṣkoṭi.

But in one of the later chapter of the MMK, we get the following:[10]

> 'Empty' should not be asserted.
>
> 'Non-empty' should not be asserted.
>
> Neither both nor neither should be asserted.
>
> These are used only nominally.
>
>
> How can the tetralemma of permanent and impermanent, etc.
>
> Be true of the peaceful?
>
> How can the tetralemma of finite, infinite, etc.
>
> Be true of the peaceful?

Nāgārjuna enumerates the four cases of the catuṣkoṭi, and says that none of them applies. And since these are the only things one *can* say, the implication would appear to be that something is ineffable. What? The peaceful. From the rest of the context, it is clear that this is exactly the state of the enlightened person after death.

But Nāgārjuna has bigger fish to fry. There is a standard distinction in Buddhism between two realities.[11] One is the conventional reality of the world we are familiar with; the other is what we do not normally see: ultimate reality. Conventional reality is a world of conceptual construction; ultimate reality is what is there once all such constructions have been removed.

Now, a few verses later in the chapter (XXII: 16) Nāgārjuna tells that the peaceful is not only the state of the enlightened person after death, but ultimate reality itself. And one can see why that should be ineffable. To describe it would be to deploy concepts; but ultimate reality is exactly what is left, after all concepts have been removed.

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[10]MMK XXII: 11, 12. Translation from Garfield (1995), p. 61 f.

[11]The word is *satya*, which can be translated as either truth or reality. 'Truth' in the more usual translation, but 'reality' is usually the better one.

At any rate, we are back with our familiar situation. We are being told that something is ineffable, at the same time as talking about it.

What does Nāgārjuna make of this contradictory situation? He does not comment on it. Maybe that is because the catuṣkoṭi explicitly allows that some contradictions may be true—that's the third koṭi. But later Buddhists, more influenced by schools of thought that endorsed the Principle of Non-Contradiction, were troubled by it.

One standard line adopted is that when one appears to be talking about the ultimate, one is, in fact, talking about a simulacrum. Here is the 15th century Tibetan Buddhist, Gorampa, explaining why one cannot talk about ultimate reality:[12]

> The scriptures which negate proliferations of the four extremes refer to ultimate truth but not to the conventional, because the ultimate is devoid of conceptual proliferations, and the conventional is endowed with them.

His response to the obvious contradiction present here is described by the Gorampa scholar Connie Kassor as follows:[13]

> In the *Synopsis*, Gorampa divides ultimate truth into two: the nominal ultimate (*don dam rnam grags pa*) and the ultimate truth (*don dam bden pa*). While the ultimate truth ... is free from conceptual proliferations, existing beyond the limits of thought, the nominal ultimate is simply a conceptual description of what the ultimate is like. Whenever ordinary persons talk about or conceptualize the ultimate, Gorampa argues that they are actually referring to the nominal ultimate. We cannot think or talk about the actual ultimate truth because it is beyond thoughts and language; any statement or thought about the ultimate is necessarily conceptual, and is, therefore, the nominal ultimate.

When we appear to be talking about the ultimate we are not really talking about it. We are talking about something else, the nominal ultimate, something we can describe. But this takes us out of the frying pan into the fire. If, when we are talking about the ultimate, we are talking about the nominal ultimate, then the claim that the ultimate is ineffable is just plain

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[12] *Synopsis of Madhyamaka*, 75. The translation is taken from Kassor (2013), p 401.
[13] Kassor (2013), p. 406.

false. For the nominal ultimate *is* effable. The move is just as undercutting as the responses of Wittgenstein and Heidegger to their predicaments.

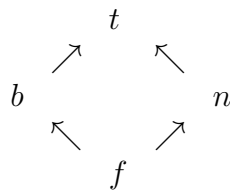This is the third and final example of our target phenomenon.

3 Making Sense of This

3.1 The *Catuṣkoṭi*

We have now seen how our target phenomenon arises in three quite different cases. Now, if one finds oneself in the situation of describing the ineffable, is there a better way to proceed than those discussed? I think there is, and the catuṣkoṭi shows the way. We can just accept the contradiction, appropriately shaped. Of course, to those schooled in the ways of the Principle of Non-Contradiction, this will appear incoherent. It is not. A language to formulate and express these ideas can be given a perfectly rigorous and precise semantics. In the subsections of this section we will see how.

We will be dealing with a standard propositional language, with a bunch of propositional parameters, and the connectives $\wedge$, $\vee$, $\neg$ (*and, or, not*). If one wishes, $A \supset B$ can be defined in the usual way, as $\neg A \vee B$.

We will take the construction of the semantics in three stages, starting with the vanilla flavoured catuṣkoṭi. There is, in fact, a very well know non-classical logic which can very naturally be seen as representing this idea, First Degree Entailment (FDE).[14]

The logic is a four-valued one, the values being t, f, b, n (true only, false only, both, and neither). These may be represented in the following Hasse diagram:

$$t$$
$$\nearrow \quad \nwarrow$$
$$b \qquad\qquad n$$
$$\nwarrow \quad \nearrow$$
$$f$$

The four corners of the catuṣkoṭi appear before our very eyes.

One of these values is assigned to each propositional parameter. A value is then assigned to each complex sentence in the standard recursive fashion. The connectives work as follows.

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[14]For more details on what follows, see Priest (2008), ch. 8.

Negation maps t to f, vice versa, n to itself, and b to itself. (So if something is neither true nor false, so is its negation; and if something is both true and false, so is its negation.) Conjunction is delivered by the greatest lower bound in the Hasse diagram. That is, the conjunction of two values is the greatest value that is less than or equal to both of them. (So the conjunction of b and f is f; and the conjunction of b and n is also f.) Disjunction is delivered dually: it is the least upper bound. That is, the disjunction of two values is the least value that is greater than or equal to both of them. (So the disjunction of b and f is b; and the disjunction of b and n is t.)

Finally, to define validity, $\models$, in a many-valued logic, we need a notion of *designated value*. These are the values that valid arguments preserve. In the present case, the designated values are t and b. (They are, after all, both kinds of truth.) So a valid argument is one such that whenever, under an assignation of values, all the premises are t or b, so is the conclusion.

The consequence relation delivered by these semantics deviates from the classical consequence relation in two notable ways. *Neither* of the following holds:
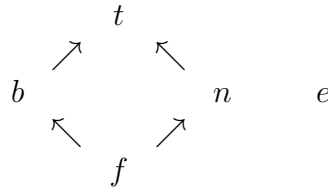
Explosion: $A \wedge \neg A \models B$ (because of the value b)

Implosion: $A \models B \vee \neg B$ (because of the value n)

Further details of the consequence relation can be found in the technical appendix below.

3.2   The Fifth Corner

The second stage of our construction brings ineffability into the picture. This is a fifth possibility, and it may be accommodated simple by adding a fifth value. Let us call this e (for emptiness). Our Hasse diagram now looks like this:

$$
\begin{array}{ccccc}
 & & t & & \\
 & \nearrow & & \nwarrow & \\
b & & & & n \qquad e \\
 & \nwarrow & & \nearrow & \\
 & & f & &
\end{array}
$$

How should our connectives behave with the addition of e? If A is effable, so is $\neg A$; if A and B are effable, so are $A \vee B$ and $A \wedge B$. But conversely, if

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A is ineffable so, presumably, is any more complex thing in which one might incorporate it. Hence, something takes the value e iff some component does.

The addition of e does nothing to change our designated values (e is not, in itself, a species of truth). So our definition of validity is as before. Call this logic FDE_e. The main difference between FDE and FDE_e is that $\vee$-introduction is no longer valid. It is not true that $A \models A \vee B$ because B may take the value e. Instead, we have a restricted version: $A, B^* \models A \vee B$, where B^* is anything containing all the parameters in B. Further details can be found in the technical appendix below.

One additional subtlety should be noted. Logicians normally take semantic values to be assigned to sentences. Sentences are not the *kind* of thing that can be ineffable; so we now have to think of our semantic bearers, not as sentences, but as propositions or states of affairs—something about which it makes sense to say that they are or are not the content of some sentence.

3.3 More than One Corner

In the third and final stage of our construction, we must incorporate the possibility that something may be both effable (and so have one of our four standard values) and ineffable. We can do this by allowing semantic bearers to take more than one value.

Standardly in logic, semantic bearers are assigned a unique value. That is, the assignation is a *function*. We now change this to a *relation*, and require that every parameter relate to *at least* one value of our five values.[15] How do our connectives work now? We simply operate point-wise. So if the set of values that A relates to is $\{f, e\}$, we obtain the set of values that $\neg A$ relates to simply be negating each of these, to obtain $\{t, e\}$. And if the sets of values A and B relate to are $\{b, e\}$ and $\{f, e\}$, we obtain the set of values $A \wedge B$ relates to by conjoining every possible combination: (i) b, f (ii) b, e (iii) e, f (iv) e, e; giving us $\{f, e, e, e\}$. That is, just $\{f, e\}$ since repetitions don't count.

All that remains is to define validity. Say that a semantic bearer is designated in a new sense, if it relates to a value that was designated in the old sense, t or b. Then an inference is valid in this logic iff it preserves designation in this new sense.

We may call the logic obtained in this way *plurivalent FDE_e*. A perhaps

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[15]Further details of the following construction can be found in Priest (2014b).

surprising fact is that an inference is valid in plurivalent FDE_e iff it is valid in ordinary FDE_e. Further details can be found in the technical appendix below.

This is the third and final stage of our construction, and with it, our goal has been achieved. We have a precise technical model which shows how something can be ineffable and have one of the values of effability too.[16]

4   Speech and Silence

So much for technical matters. But, it might well be thought, this is all a flight of contemporary logician's fantasy. Could one really hold that something is both effable and ineffable? The answer is *yes,* and we will see this by looking quickly at the *Vimilakīrti Sūtra*. The sūtra is an Indian Mahāyāna text, of uncertain date, but possibly about the 1st century CE. It's impact on the development of Indian Buddhism was limited, but it had an enormous impact on the development of Chinese Buddhism.

It is an unusual sūtra for a number of reasons reasons. First, its hero is neither the Buddha nor a monk, but a layman, Vimilakīrti. Next, a woman—a goddess, no less—plays an important role in it. (Women are notably absent from nearly all classical Buddhist texts.) Third, it has clear moments of humour.

A number of issues are addressed in the sūtra, but a central one is the transcendence of dualism, including the important dualism which is our central concern here: speech and silence. Common to all Mahāyāna is the thought that ultimate reality, whatever it is, must be free of all dualisms—whatever that means; indeed that is a matter addressed in the text.

So let us turn to the text. At one point in it, the goddess appears in the room, and causes petals to flutter down. These slide off enlightened people, but stick to people who are unenlightened. The petals stick to Śāriputra (a hero of a number of the pre-Mahāyāna sūtras), and he is not very happy about this. A conversation between him and the goddess ensues:[17]

> "Venerable sir," said the goddess, "how long has your attainment
> of emancipation been?"

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[16]In fact, it allows for more possibilities than this. Any (non-empty) collection of our five values is a possibility, though this need not concern us here.

[17]Watson (1997), p. 87.

Śāriputra was silent and did not answer.

The goddess said, "With your great wisdom, venerable sir, why do you remain silent?"

Śāriputra replied, "Emancipation cannot be spoken of in words. Therefore I do not know what I can say to you."

Śāriputra, appealing to the idea that enlightenment, the realisation of ultimate reality, is ineffable, takes the 5th Amendment. The goddess is not impressed (*ibid*):

The goddess said, "Words, writing, all are marks of emancipation. Why? Because emancipation is not internal, not external, and not in between. And words, likewise, are not internal, not external, and not in between. Therefore, Śāriputra, you can speak of emancipation without putting words aside. Why? Because all things that exist are marks of emancipation."

The reply is dark. The thought would appear to be that words are not something over and above ultimate reality, which can be—indeed, must be—prized off it. They are part of it, and so can be used to describe it. But whatever the exact meaning of the goddess' words, it is clear that she says that one can speak about ultimate reality.

If one left the text at this point, one would just think that the doctrine of ineffability had been dismissed. But this is not so. A chapter or so later, the topic of discussion turns explicitly to the question of what it means to transcend duality. Many bodhisattvas (people on the path to enlightenment) are brought into the discussion, and each takes it in turn to say what this means. For example, the bodhisattva Good Eye says:[18]

The unique in form and the formless constitute a dualism. But if one understands that the unique in form is in fact the formless, and then does not seize on the formless but sees all as equal, one may in this way enter the gate of nondualism.

The last bodhisattva to speak is the most important of them all. This is Mañjuśrī, the Bodhisattva of Wisdom—so he should know what he is talking about. He says:[19]

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[18]Watson (1997), p. 104.
[19]Watson (1997), p. 110.

> To my way of thinking, all dharmas are without words, without explanation, without purport, without cognition, removed from all questions and answers. In this way one may enter the gate of nondualism.

Then, Vimilakīrti, the real hero of the dialogue is asked what he thinks (*ibid*):

> Then Mañjuśrī said to Vimilakīrti, "Each of us has given an explanation. Now, sir, it is your turn to speak. How does the bodhisattva enter the gate of non-dualism?"

> At that time Vimilakīrti remained silent and did not speak a word.

> Mañjuśrī sighed and said, "Excellent, excellent! Not a word, not a syllable—this truly is to enter the gate of non-dualism."

Vimilakīrti remains silent. But unlike the silence of Śāriputra, this is praised. What is the difference?

Context. Mañjuśrī has just *said* that you cannot speak about ultimate reality (thereby contradicting himself). Vimilakīrti *shows* the same thing. The duality of speech/silence is transcended precisely by showing how both of these things are the same.

We see, then, that the Sūtra endorses speaking of the ineffable—indeed, insists on it as a way of overcoming duality.

5   Conclusion

We have been concerned, here, with a certain phenomenon. The phenomenon arises when a philosophical view claims that something is ineffable, but explains why that is so, in the process talking about the thing, so contradicting the claim that it is ineffable. We have seen, also, how some philosophers have tried—not very successfully—to wriggle out of the contradiction.

There appears to be a more robust way to face the issue: simply to accept the contradiction. One can speak of the ineffable—or at least, some of it. We have seen that the idea can be made quite precise and coherent with the use of the techniques of contemporary non-classical logic, and particularly those of many-valued and plurivalent logic. We have seen, also, how at least some people in one philosophical tradition took this idea on board.

I'm sure there's more to be said about all these matters... Maybe it's ineffable.

6   Technical Appendix

In this appendix, I specify the consequence relation of the three logics we have met, by giving a natural deduction system which is sound and complete with respect for each of them.[20]

The following is a (specially chosen) natural deduction rule system for classical logic. A double line indicates a two-way rule, and over-barring an assumption means that the rule discharges it.

$$\frac{A \; B}{A \wedge B} \qquad \frac{A \wedge B}{A} \qquad \frac{A \wedge B}{B}$$

$$\frac{A}{A \vee B} \qquad \frac{B}{A \vee B} \qquad \frac{A \vee B \quad \overset{\overline{A}}{\underset{C}{\vdots}} \quad \overset{\overline{B}}{\underset{C}{\vdots}}}{C}$$

$$\frac{A}{\neg\neg A} \qquad \frac{\neg(A \vee B)}{\neg A \vee \neg B} \qquad \frac{\neg(A \wedge B)}{\neg A \vee \neg B}$$

$$\frac{A}{B \vee \neg B} \qquad \frac{A \wedge \neg A}{B}$$

A rule system for FDE is obtained simply by dropping the last two rules (Implosion and Explosion).

A rule system for FDE_e is obtained by taking the rule system for FDE, dropping the two rules for disjunction introduction, replacing the first with:

$$\frac{A \; B^*}{A \vee B}$$

where B^* is any formula containing all the propositional parameters in B, and modifying the second in the same way.

The consequence relation for plurivalent FDE_e is extensionally the same as that for FDE_e, and so the same natural deduction system provides what is required.

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[20]Further details can be found in Priest (2010) and (2014b).

References

[1] Kassor, C. (2013), 'Is Gorampa's "Freedom from Conceptual Proliferations" Dialetheist? A Response to Garfield, Priest, and Tillemans', *Philosophy East and West* 63: 399-410.

[2] Garfield, J. (1995), *The Fundamental Principles of the Middle Way: Nāgārjuna's* Mūlamadhyamakakārikā, New York, NY: Oxford University Press.

[3] Kluback, W., and Wilde, J. (trs.) (1959), *The Question of Being*, Harrisonburg, VA: Vision.

[4] Lilly, R. (tr.) (1991), *The Principle of Reason*, Bloomington, IN: Indiana University Press.

[5] Pears, D. and McGuinness, B. (trs.) (1961), *Tractatus Logico-Philosophicus*, London: Routledge and Kegan Paul.

[6] Priest, G. (2002), *Beyond the Limits of Thought*, 2nd ed., Oxford: Oxford University Press.

[7] Priest, G. (2002), *Introduction to Non-Classical Logic*, 2nd ed., Cambridge: Cambridge University Press.

[8] Priest, G. (2010), 'The Logic of the Catuṣkoṭi', *Comparative Philosophy* 1: 32-54.

[9] Priest, G. (2014a), 'Speaking of the Ineffable...', ch. 7 of J. Lee and D. Berger (eds.), *Nothingness in Asian Philosophy*, London: Routledge.

[10] Priest, G. (2014b), 'Plurivalent Logic', *Australasian Journal of Logic* 11: article 1, http://ojs.victoria.ac.nz/ajl/article/view/1830.

[11] Radhakrishnan, S., and Moore, C. (eds.) (1957), *A Source Book in Indian Philosophy*, Princeton, NJ: Princeton University Press.

[12] Watson, B. (tr.) (1997), *The Vimalakīrti Sūtra,* New York, NY: Columbia University Press.