The Boundary Stones of Thought. Ian Rumfitt. Oxford: Clarendon Press, 2015. xiv + 345 pp. Cloth £35.

Starting with his doctoral dissertation in 1907, the Dutch mathematician L. E. J. Brouwer launched a principled attack on certain aspects of the mathematics of his day, and in particular, its treatment of infinities. By 1930, and with the work of Heyting and others, this had become an attack on the recently developed logic of Frege and Russell—classical logic, CL—which had been formulated precisely to do justice to the reasoning in that very mathematics. The intuitionist challenge was taken seriously by those interested in the philosophy of mathematics, but made little impact on philosophy in general.

Matters changed substantially in the 1970s. Drawing on the philosophy of language—and especially some themes in the later Wittgenstein—rather than the philosophy of mathematics, Dummett launched a sustained attack on realism in general, arguing that CL, especially in the form of the Principle of Excluded Middle ($A \vee \neg A$, PEM), harboured unacceptable realist assumptions, and that intuitionist logic, IL, was the correct logic. The debate that ensued dominated much of British philosophy for the next generation. It is fair, I think, to say that much of the debate has now subsided. There have also been numerous other attacks on the correctness of CL since the 1960s—by those who favour logics with truth value gaps, fuzzy logic, relevant logic, paraconsistent logic, sub-structural logics. But Dummett's anti-realist challenge to CL remains one of the most influential and best articulated.

In this book, Ian Rumfitt, an erstwhile Dummett student, faces afresh this challenge—other challenges being quietly set aside. He analyses the arguments; many of them, he finds, are wanting as normally given, but may be articulated in a way less easy to fault. In the process of assessing these arguments, he develops many new interesting views and techniques. In the end, though, he wishes to come down on the side of CL, but the journey was worth the making. The book is careful, insightful, and imaginative. I, for one, learned much from it.

After an introduction setting in context what is to come, the book has two parts. If we are going to be looking at critiques of logic, we should have some idea of what logic is. Part one of the book addresses this issue (for deductive logic—non-deductive logic is not on the agenda). Chapter two tells us that an inference is valid if there is no possibility in which the premises are true but the conclusion is not. Chapter three then tells us that when an inference is valid, it is so in virtue of the meanings of the logical particles, and so preserves necessity, in all of its forms. There is much one might wish to contest here, but the picture is familiar enough.

The second part of the book has five main chapters, each analysing an anti-realist attack on CL. Chapter four examines an argument which Rumfitt extracts from an early paper of Dummett, 'Truth'. The argument is rejected, but in the process, we are introduced to a number of the topological ideas that will play a core role in the rest of the book. In chapter five, Dummett's well known "manifestation" argument for intuitionism is explained and rejected. However, a stronger argument, based on some thoughts of McDowell, is articulated. What, if anything, grounds the Principle of Bivalence (Every statement is either true or false, PB) if the meanings of the logical particles themselves do not do so? This exposes the philosophical nerve of what is to follow.

Chapter six faces the challenge to CL from quantum mechanics, which one may naturally take as an anti-realist theory of physics. Quantum theory, quantum logic, and advocates of quantum logic such as Putnam, are discussed. The failure of distributivity in quantum logic ($A \wedge (B \vee C) \models (A \wedge B) \vee (A \wedge C)$) is used to foreground the behaviour of disjunction, and the virtues of its treatment in Beth semantics for intuitionism, as compared to Kripke semantics. Chapter seven returns us to the roots of intuitionism for a discussion of the infinite. Brouwer's arguments to the effect that the arithmetic of natural numbers delivers violations of the PEM are discussed and rejected. In the discussion, Rumfitt argues that what is really at issue between CL and IL (and so what is behind McDowell's challenge) is the question of whether statements must have "backs"; that is, whether, for every A, there is a B such that A is $\neg B$.

Chapter eight takes us into the realm of vagueness. A common thought is that in the transition region of a sorites progression between objects, x, such that Px, and objects, z, such that $\neg Pz$, there are objects, y, such that neither Py nor $\neg Py$ holds. Drawing on some ideas of Sainsbury, Rumfitt argues that the meaning of a colour term may be best understood in terms of the extent to which its bearers approach a paradigm of that colour, and parlays this into an appropriate semantics. How these semantics solve one version of the colour sorites is also demonstrated.

Chapter seven dealt with arithmetic, but left the rest of classical mathematics hanging. Chapter nine deals with this rest, in the shape of set theory. The worry here, as articulated in the work of Tait, concerns statements about absolute infinities if one rejects platonism about mathematical objects. Rumfitt wrestles with the issues, and concludes that classical set theory can be justified for a fragment (though a strong fragment) of ZF.

A brief conclusion to the book pulls some of its threads together, and explains why one should resist the move from the PEM to the PB.

In the process of all this, Rumfitt produces a number of different topologically-based semantics for CL and IL. These, he says, are helpful in 'adjudicating the debate' between IL and CL (p. 13). They certainly show the technical coherence of having the PEM without the PB. However, it was not always clear to me that the different semantics for each of these logics were philosophically compatible. I, for one, would have liked to see a less fragmented picture. But set that aside. Rumfitt's book is rich in ideas, arguments, and technical constructions—much more than can be discussed here. In the rest of this review I will focus on its core strategy.

This is to argue that CL is the correct default assumption. There may be challenges to it, as articulated in chapters four through nine. But these may (for the most part) be seen off. So the default stands. Both stages of the argument may be contested.

Part of Rumfitt's argument for the default is a quite general one to the effect that CL should be taken as correct until and unless it can be shown otherwise. He has two arguments for this. The first is its strength (p. 14): CL is Post-complete so, in a sense, as powerful as possible. It is not clear why strength should be a desideratum. After all, many of the standard critiques of CL are that it is *too* strong. But in any case, there are logics which are incomparable with CL. In particular, they contain inferences which are not valid in CL, and so are at least partially stronger. Connexive logics are like this. The second argument is that 'with only one class of exceptions, classically valid arguments conform to our intuitive sense of deductions whose conclusions follow from their premises' (p. 15). The exceptions concern conditionals. Relevant logicians will hardly be impressed by this qualification. Even setting this aside, one needs to ask who is the *us* in *our*? Anyone who has taught a first course in logic will probably agree that the students when they start are not "one of us". Of course, their intuitions can be wrought into shape by the end of Logic 100. But students are normally exposed only to sanitized examples. Examples which tend to draw non-classical responses, such as those concerning paradoxes, vagueness, and so on, are rarely given air. Indeed, once one remembers the origins of CL, that it should be taken as correct by default seems most implausible. Why on earth should one suppose that a canon of inference developed to account for mathematical reasoning in the late 19th century—even assuming it to be correct for this—applies to the rich variety of other sorts of discourse?

The second reason for the default assumption addresses explicitly the claim that every statement has a "back". Here, Rumfitt appeals to the medieval principle *eadem est scientia oppositorum* (p. 196):

> in order to attain a clear conception of what it is for A to be the case, one needs to attain a conception of what it is for A not to be the case. But then, having attained that conception, it is very hard, at least *prima facie*, to see how an assertion of A could fail to be equivalent to a denial of $\ulcorner \neg A \urcorner$.

Now, first, the principle, so understood, is highly disputable. For any A, it would seem, we have a grasp of what it is for $A \wedge \neg A$ to be false. Many have told me that they have no grasp of what it could be for it to be true. But setting this aside, why suppose that the possibilities ruled out by A are exactly the ones ruled in by $\neg A$? It is very hard to see otherwise, we are told. For whom? This takes us back to the question of who *we* are. Not intuitionists, for sure.

Turn now to Rumfitt's discussion of the challenges to the default position. These contain the meat of the book, and some of its most interesting discussion. However, Rumfitt makes some surprising concessions.

After discussing and dismissing intuitionist arithmetic in general as a cogent challenge, he turns to Smooth Infinitesimal Analysis: a theory of infinitesimals which demands intuitionist logic. Everything collapses if the PEM holds. Rumfitt calls this a 'consolation prize' for the intuitionist (p. 210). However, unless the theory is to be written off as incoherent (a line to which Rumfitt does not seem inclined), it is hard to see this as anything other than a demonstration that claims to the universal applicability of CL must be given up.

Next, the chapter on vagueness argues very persuasively that a semantics of the kind Rumfitt articulates is appropriate for colour terms, and other vague terms which may be thought to get their sense from nearness to paradigms. Thus, for colours, we have a linear spectrum with clearly distinguishable bands. Rumfitt is clear, however, that there might be other kinds of vague terms (p. 255). Indeed there would seem to be such. Merely consider 'right wing' and 'left wing' as political terms. The topology of political positions has no such clearly defined structure, and is something of a hotch-potch. (Recall that Hitler was a National *Socialist*.) Unless the semantics can be shown to apply to all vague terms, the suspicion remains that PEM will fail for some of them, so that CL is not universally correct.

Third, the final main chapter of the book turns to the question of how one might justify the use of classical logic in standard set theory if one eschews the platonist assumption of a determinate totality of all sets. The strategy here is different from those pursued before. Shorn of its technicalities, the thought is that if we interpret $A \vee B$ and $\exists x A$ as the intuitionist $\neg(\neg A \wedge \neg B)$ and $\neg \forall x \neg A$, respectively, then classical reasoning is intuitionistically kosher, and so can be allowed to stand. It seems to me to be a rather back-handed justification of classical logic if we are told that it works if we reinterpret the meanings of its words to be something else. It is of no consolation to a theist to be told that their views are incorrect as intended, but fine if 'God' is interpreted to mean 'the fundamental laws of nature'. But even setting this aside, there is, as Rumfitt notes, still a rub: not all the axioms of ZF are acceptable given this understanding. Crucially, the Power Set Axiom fails. All that can be justified are the axioms of the much weaker $KP\omega$ (Kripke-Platek set theory plus the Axiom of Infinity). Rumfitt therefore has to concede that classical reasoning is not, *in toto*, justified in set theory. (Section 9.7 gestures at a way in which one might try to justify classical reasoning that goes beyond intuitionist $KP\omega$, in terms of the applications of mathematics. I'm not sure that I really understand the details, but I find it hard to see how it can work when much mathematics has no application external to mathematics.)

Rumfitt opens his concluding chapter saying (p. 302) 'A theme of this book has been to vindicate classical logic without appealing to the Principle of Bivalence'. An intuitionist can, of course appeal to CL in certain circumstances: when reasoning about finite, or at least decidable, situations. What Rumfitt's book shows is that, for someone motivated by intuitionist concerns, the class of situations for which they may appeal to CL goes much beyond this. That's a substantial gain of ground; but not the whole nine yards.

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