Revising Logic

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1 What's at Issue?

Much ink has been spilled over the last few decades in disputes between advocates of "classical logic"—that is, the logic invented by Frege and Russell, and polished by Hilbert and others—and advocates of non-classical logics—such as intuitionist and paraconsistent logics. One move that is commonly made in such debates is that logic *cannot* be revised. When the move is made, it is typically by defenders of classical logic. Possession, for them, is ten tenths of the law.

The point of this paper is not to enter into substantive debates about which logic is correct—though relevant methodological issues will transpire in due course. The point is to examine the question of whether logic can be revised.[1] (And let me make it clear at the start that I am talking about deductive logic. I think that matters concerning non-deductive logic are much the same, but that is an issue for another occasion.) Three questions, then, will concern us:

- Can logic be revised?
- If so, can this be done rationally?

[1]Thanks go to Hartry Field for many enjoyable and illuminating discussions on the matter. We taught a course on the topic together in New York in the Fall of 2012. Many of my views were clarified in the process.

- If so, how is this done?

Unfortunately, debates about the answers to these questions are often vitiated by a failure to observe that the word 'logic' is ambiguous. Only confusion results from running the senses of the word together. Once the appropriate disambiguations are made, some of the answers to our questions are obvious; some are not. It pays, for a start, to be clear about which are which.

We may distinguish between at least three senses of the word, which I will call:

- *Logica docens*
- *Logica utens*
- *Logica ens*

What each of these is will require further discussion and clarification. But as a first cut, we may characterise these as follows.

Logica docens (the logic that is taught) is what logicians claim about logic. It is what one finds in logic texts used for teaching. *Logica utens* (the logic which is used) is how people actually reason. The first two phrases are familiar from medieval logic. The third, *logica ens* (logic itself) is not. (I have had to make the phrase up.) This is what *is* actually valid: what really follows from what.

Of course, there are important connections between these senses of 'logic', as we will see in due course. But the three are distinct, both intensionally and extensionally, as again we will see.

I will proceed by discussing each of these senses of 'logic', and asking each of our three target questions about them. We have, then a nine-part investigation.

2 Logica Docens

2.1 Can it be Revised?

Let us start with *logica docens*. The discussion of this will form the longest part of the essay, since it informs the discussion with respect to the other two parts.

The question of whether the *logica docens* can be revised is, however, the easiest to deal with. It *can* be revised because it *has* been revised.

The history of logic in the West has three great periods.[2] The first was in Ancient Greece, when logic was founded by Aristotle, the Megarians, and the Stoics. The second was in the new universities of Medieval Europe, such as Oxford and Paris, where Ockham, Scotus, and Buridam flourised. The third starts in the late 19th century, with the rise of mathematical logic, and shows no signs yet of ending. Between these three periods were periods of, at best, mainly maintaining what was known, and at worst forgetting it. Much of Greek logic was forgotten in Europe, but fortunately preserved by the great Arabic scholars such as Al Farabi and Ibn Rushd. Most of medieval logic was simply wiped out by the rise of the Enlightenment, and the consequent obliteration of Scholasticism. It is only in the 20th century that we have started to rediscover what was lost in this period.

At any rate, one needs only a passing acquaintance with logic texts in the history of Western logic to see that the *logica docens* was quite different in the various periods. The differences between the contents of Aristotle's *Analytics*, Paul of Venice's *Logica Magna*, the Port Royale *Logic, or the Art of Thinking*, Kant's Jäsche *Logik*, and Hilbert and Ackermann's *Principle's of Mathematial Logic* would strike even the most casual observer.

It is sometimes suggested that, periods of oblivion aside, the development of logic was cumulative. That is: something once accepted, was never rejected. Like the corresponding view in science, this is just plan false. Let me give a couple of examples.

One of the syllogisms that was, according to Aristotle, valid, was given the name *Darapti* by the Medievals, and is as follows:

$$\frac{\begin{array}{c}\text{All } A\text{s are } B\text{s}\\ \text{All } A\text{s are } C\text{s}\end{array}}{\text{Some } B\text{s are } C\text{s}}$$

As anyone who has taken first course on modern first-order logic will know, this inference is now taken to be invalid.[3]

For another example: Classical logic is not paraconsistent; that is, the following inference (Explosion) is valid for all A and B: $A, \neg A \vdash B$. It is

[2] The history of logic in the East has its own story to tell, but that will not be our concern here.

[3] For further discussion of the matter, see Priest (2006), 10.8.

frequently assumed that this has always been taken to be valid. It has not. Aristotle was quite clear that, in syllogisms, contradictions may or may not entail a conclusion. Thus, consider the syllogism:

$$\frac{\begin{array}{c}\text{No } A\text{s are } B\text{s} \\ \text{Some } B\text{s are } A\text{s}\end{array}}{\text{All } A\text{s are } A\text{s}}$$

This is not a valid syllogism, though the premises are contradictories. There are usually three distinct terms in a syllogism. The above has only two. But Aristotle is also quite explicit that two terms of a syllogism may be the same.

So when did Explosion enter the history of Western logic? Matters are conjectural, but the best bet is that it entered with the ideas of the 12th century Paris logicians called the *Parvipontinians*, whose members included Adam of Balsaha and William of Soissans, who may well have developed the argument to Explosion using extensional connectives and the Disjunctive Syllogism. After that, the validity of Explosion was debated. But it certainly did not become entrenched in Western logic till the rise of classical logic.[4]

2.2 Can it be Revised Rationally?

Logica docens, then, has been revised, and not in a cumulative fashion. The next question is whether revision can be rational.

Arguably, not all the changes in the history of logic were rational (or perhaps better: occurred for reasons that were internal to the subject). Thus, logic fell into oblivion in the early Middle Ages in Western Christendom because the institutions for the transmission of philosophical texts collapsed. And later Medieval logic was written off on the coat-tails of the rejection of Scholasticism during the Enlightenment.[5]

However, many changes that did arise were the result of novel ideas, reason, argument, debate. These are the things of which rational change are made.

This should be pretty obvious with respect to the only change that most logicians are now familiar with: the rise of mathematical logic. In the mid 19th century, text book logic ("traditional logic") was a highly degenerate

[4] For references and further discussion on all these matters, see Priest (2007), sec. 2.

[5] Actually, my knowledge of the history of these periods is pretty sketchy; but I think that these claims are essentially correct.

form of Medieval logic: essentially, Aristotelian syllogistic with a few medieval accretions, such as “immediate inferences” like *modus ponens*. But this was a period in which high standards of rigour in mathematics were developing. Mathematicians such as Weierstrass and Dedekind were setting the theory of numbers on a firm footing. And when it came to examining the reasoning required in the process, notably by Frege, it became clear that traditional logic did not seem to be up to the job. Hence Frege invented a logic that did much better: classical logic. The extra power of this logic made it much preferable rationally; and within 50 years it had replaced traditional logic as the received *logica docens*.

I will come back to this in the next section. For the present, let us move on to our third question.

2.3 Logic as Theory

So, what, exactly, is it in virtue of which one *logica docens* is rationally preferable to another, and so may replace it? To answer this question, we need to draw some new distinctions.

Let us start with geometry. There are many *pure* geometries: Euclidean geometry, elliplical geometry, hyperbolic geometry, and so on. And as pieces of pure mathematics, all are equally good. They all have axiom systems, model theories, each specifies a perfectly fine class of mathematical structures. Rivalry between them can arise only when they are applied in some way. Then we may dispute which is the correct gometry for a particular application, such as mensurating the surface of the earth. Each applied geometry becomes, in effect, a theory of the way in which the subject of the application behaves.

Geometry had what one might call a canonical application: the spatiotemporal structure of the physical consmos. Indeed this application was coevel with the rise of Eucliden geometry. It was only the rise of non-Euclidean geometries, which brought home the conceptual distinction between a pure and an applied geometry. And nowadays the standard scientific view is that Euclidean geometry is not the correct geometry for the canonical application.

So much, I think, is relatively unconsestable. But exactly the same picture holds with respect to logic. There are many pure logics: classical logic, intuitionist logic, various paraconsistent logics, and so on. And as pieces of pure mathematics, all are equally good. They all have systems of proof, model theories, algebraicisations. Each is a perfectly good mathematical structure. But pure logics are applied for many purposes: to simplify electrical circuits (classical propositional logic), to parse grammatical structures (the Lambeck calculus), and it is only when different logics are taken to be applied for a particular domain that the question of which is right arises. Just as with geometries, each applied logic provides, in effect, a theory about how the domain of application behaves.

And just as with geometries, pure logics have a canonical application: (deductive) reasoning. A logic with its canonical application delivers an account of ordinary reasoning. One should note that ordinary reasoning, even in science and mathematics, is not carried out in a formal language, but in the vernacular; no doubt the vernacular augmented by many technical terms, but the vernacular none the less. (No one reasons *à la Principia Mathematica.*) And so applied, different pure logics may give different verdicts concerning an inference. If it is not the case that it is not the case that there is an infinitude of numbers, does it follow that there is an infinitude of numbers? Classical logic says yes; intuitionist logic says no.[6]

In other words, a pure logic with its canonical application is a theory of the validity of ordinary arguments: what follows (deductively) from what. How to frame such a theory is not at all obvious. Many approaches have been proposed and explored. One approach is to take validity to be constituted modally, by necessary truth preservation (suitably understood). Another is to define validity in terms of probabilistic constraints on rational belief. Perhaps the most common approach at present is to take a valid inference to be one which obtains in virtue of the meanings of (at least some of) the words employed in it. This strategy has itself two ways in which it can be implemented. One takes these meanings to be spelled out in terms of truth conditions, giving us a model-theoretic account of validity; the other takes these meaning to be spelled out in inferential terms, giving us a proof-theoretic account of validity.

It is clear that a theory of validity is no small undertaking. It requires an account of many other notions, such as negation and quantification. Moreover, depending on the theory in question, it will require an articulation of other important notions, such as truth, meaning, probability. No wonder it is hard to come up with plausible such theories!

At any rate, it is crucial to distinguish between logic as a theory (*logic docens*, with its canonical application), and what it is a theory of (*logica ens*).

[6]Further on the above, see Priest (2006), chs. 10, 12.

In the same way we must clearly distinguish between dynamics as a theory (e.g., Newtonian dynamics) and dynamics as what this is a theory of (e.g., the dynamics of the earth). This is enough to dispose of the Quinean charge (still all too frequently heard): change of logic means change of subject.[7] If one changes one's theory of dynamics, one can still be reasoning about the same thing: the way the earth moves.

2.4 What is the Mechanism of Rational Revision?

With this substantial prolegomenon over, we can now address the question of the mechanism of rational change of *logica docens*. As we have seen, a pure logic with its canonical appication is essentially a theory of validity and its multitude of cognate notions. How do we determine which theory is better? By the standard criteria of rational theory choice.

Given any theory, in science, metaphysics, ethics, logic, or anything else, we choose the theory which best meets those criteria which determine a good theory. Principle amongst these is adequacy to the data for which the theory is meant to account. In the present case, these are those particular inferences that strike us as correct or incorrect. This does not mean that a theory which is good in other respects cannot overturn aberrant data. As is well recognised in the philosophy of science, all things are fallible: both theory and data.

Adequacy to the data is only one criterion, however. Others that are frequently invoked are: simplicity, non-(ad hocness), unifying power, fruitfulness. What exactly these criteria are, and why they should be respected, are important questions, which we do not need to go into here. One should note, however, that whatever they are, they are not all guaranteed to come down on the same side of the issue. Thus (the standard story goes), Copernican and Ptolemaic astronomy were about equal in terms of adequacy to the data; the Copernican system was simpler (since it eschewed the equant); but the Ptolemaic system cohered with the accepted (Aristotelian) dymanics. (The Copernican system could handle the motion of the earth only in an ad hoc fashion.) In the end, the theory most rational to accept, if there is one, is the one that comes out best on balance. How to understand this is not, of course, obvious. But we do not need to pursue details here.[8]

I observe that this procedure does not prejudice the question of logical

[7] Quine (1970), p. 81.

[8] Matters are spelled out in detail on Priest (2006), ch. 8, and especially, Priest (201+).

monism *vs* logical pluralism. If there is "one true logic" one's best appraisal of what this is is determined in the way I have indicated. If there are different logics for different topics, each of these is determined in the same way. Whether one single logic is better than many, is a "meta-issue", and is itself to be determined by similar considerations of rational theory-choice.

Let me finish this discussion by returning, by way of illustration, to the replacement of traditional logic by mathematical logic in the early years of the 20th century. In the 19th century, much new data had turned up: specifically the microscope had been turned on mathematial reasoning, showing all sorts of inferences that did not fit into traditional logic. Mathematial logic was much more adequate to this data. This is not to say that enterprising logicians could not try to stretch traditional logic to account for these inferences. But mathematical logic scored high on many of the other theoretical criteria: simplicity, unifying power, and so on. It was clearly the much better theory.

A word of warning: it would be wrong to infer that classical logic did not have its problems. It had its own ad hoc hypotheses (to deal with the material conditional, for example). It had areas where it seemed to perform badly (for example, in dealing with vague language). And why should one expect a logic that arose from the analysis of mathematical reasoning to be applicable to *all* areas of reasoning? It was just these things which left the door open for the development of non-classical logics. That, however, is also a topic for another occasion.[9] We have seen, at least in outline, what the mechanism of rational change for a *logica docens* is.

3 Logica Utens

3.1 What is This?

So much for the discussion of *logica docens*. Let us now turn to the next disambiguation. Before we address our three questions, however, there is an important preliminary issue to be addressed. What exactly is *logica utens*?

I said that it is the way that people actually reason. This may make it sound like a matter of descriptive cognitive psychology; but it is not this, for the simple reason that we know that people often reason invalidly. Set

[9] Some discussion can be found in Priest (1989).

aside slips due to tiredness, inebriation, or whatever. We know that people *actually* reason wrongly in systematic ways.[10]

To take just one very well estabished example: the Wason Card Test. There is a pack of cards. Each card has a letter on one side and a positive integer on the other. Four cards are laid out on the table so that a subject can see the following:

$$A \qquad K \qquad 4 \qquad 3$$

The subject is then given the following conditional concerning the displayed situation: If there is an A on one side of the card, there is an even number on the other. They are then asked which cards should be turned over (and only those) to check this hypothesis. The correct answer is: A and 3. But a majority of people (even those who have done a first course in logic!) tend to give one of the wrong answers: A, or A and 4.

Exactly what is going on here has occasioned an enormous literature, which we do not need to go into. The experiment, and ones like it, show that people can reason wrongly systematically. Of course, people are able to appreciate the error of their reasoning when it is pointed out to them. But how to draw a principled distiction between correcting a standard performance error, and revising an actual practice is not at all obvious.

Fortunately, we do not need to go into this here. I point these facts out only to bring home the point that *logica utens* is not a *descriptive* notion; it is a *normative* one. A *logica utens* is constituted by the norms of an inferential practice. Subjects in the Wason Card Test can see, when it is pointed out to them, that they have violated appropriate norms. How to understand the normativity involved here is a particularly hard question, which, fortunately, we also do not need to pursue. We have sufficient understanding to turn to the first of our three questions. Can a *logica utens* be revised?

3.2 Can It be Revised?

Clearly, different reasoning practices come with different sets of norms. Thus, the norms that govern reasoning in classical mathematics are different from those that govern reasoning in intuitionist mathematics. I was trained as a classical mathematician, and have no difficulty in reasoning in this way. But I have also studied intuitionist logic, and can reason (more falteringly) in

[10]See Wason and Johnston-Laird (1972).

this way too. Clearly, then, it is possible to move from one *logica utens* to another. I can reason like a classical mathematician on Mondays, Wednesdays, Fridays, and like a intuitionist on Tuesdays, Thursdays, and Saturdays. (And on Sundays flip a coin.) So practices can be changed.

At this point one might wonder about the nature of inference sketched in Wittgenstein's *Philosophical Investigations*. According to this, correct reasoning is simply how we feel compelled to go on after suitable training. If such is the case, then how can one change? The answer is that we must take the *suitable training* seriously. I can follow my training as a classical logician some days, and my training as an intuitionist on others—just as I can follow my training in cricket on some days, and my training in baseball on others.

3.3 Can it be Revised Rationally?

So *logica utens* can change. Can it be changed rationally? Unless one is a complete relativist about inferential practices, the answer must be yes: some practices are better than others. And to move from one that is less good to one that is more good for principled reasons is clearly rational.

Moreover, being a relativist about such practices is a hard pill to swallow. For we use reasoning to establish what is true, and what is not, about many things. A relativism about these practices therefore entails a relativism about truth. And such a relativism is problematic. To take an extreme example: suppose that reasoning in one way, we establish that the theory of evolution is correct, but that reasoning in another way, we establish that creationism is true and the theory of evolution false. Something, surely, must be wrong with one of these forms of reasoning.[11]

3.4 How is it Revised Rationally?

Assuming, then, that rational change is possible, how is this to be done? The answer to that is easy. We determine what the best theory of reasoning is (the best *docens*), and simply bring our practice (*utens*) into line with that. How else could one be rational about the matter?

[11] It is quite compatible with this point that *sometimes* truth may be internal to a practice—for example, within classical and intuitionist pure mathematics. See Priest (2012).

4 Logica Ens

4.1 Can it be Revised?

We now turn to what I think is the hardest of the three disambiguations: *logica ens*. These are the facts of what follows from what—or better, to avoid any problems with talk of facts: the truths of the form 'that so and so follows from that such and such'. Can these be revised? The matter is sensitive for a number of reasons.

As we have seen, our *logica docens*, with its canonical application, is a theory about what claims of this form *are* true. Now, if one changes one's theory of dynamics, the dymanics of the earth do not themselves change. Such realism about the physical world is simply common sense. But logic is not a natural science. It is a social science, and concerns human practices and cognition. When a theory changes in the social sciences, the object of the science may change as well. One has to look only at economics to see this. When free-market economics became dominant in the capitalist world in the 1980s, so did the way that the then deregulated economy functioned. So, in the social sciences one is not automatically entitled to the view that a change of theory does not entail a change of object.

But the object of a social scientific theory may not change when the theory does, for all that. (Many basic laws of psychology are, presumably, hard-wired in us by evolution.) Whether the truth of validity-claims can change will depend on what, exactly, consitutes validity. Let me illustrate. Suppose that one held a "divine command" theory of validity: something is valid just if God says so. Then God being constant and immutable, what is valid could not change. On the other hand, suppose that one were to subscribe to the "dentist endorsement" view of validity: what is valid is what 90% of dentists endorse. Clearly, that can change.

These theories are, of course, rather silly. But they make the point: the truth of validity-claims may or may not change, depending on what validity actually is. An adequate answer to our question would therefore require us to settle the issue of what validity is, that is, to determine the best theory of validity. That is far too big an issue to take on here.[12]

I shall restrict myself in what follows to some remarks concering the model-theoretic and proof-theoretic accounts of validity. According to the

[12] I have said what I think about the matter in Priest (2006), ch. 11.

first, an inference is valid iff every model of the premises is a model of the conclusion. But a model is a structured set, that is, an abstract object, the premises form a set, another abstract object, and the premises and conclusions themselves are normally taken to be sentence types, also abstract objects. According to the second, an inference is valid if there is a proof structure (sequence or tree), at every point of which there is a sentence related to the others in certain ways. But a proof structure is an abstract object, as, again, are the sentences.

In other words, validity, on these accounts, is a realtionships between abstract objects. As usual, we may take these all to be sets. If this is so, then, at least if one is a standard platonist about these things, the truth of claims about validity cannot change.[13] Claims about mathematical objects are not significantly tensed: if ever true true, always true.

4.2 Can Meanings Change?

That is not an end of the matter, though. The propositions about validity may not change their truth values. But we express these in language. It might be held that the words involved may change their meanings—and, moverover, do this in such a way that the truth values of the sentences involved may change. If this is the case, then the sentences expressing validity claims can change their truth values.

Can meanings change in such a way as to affect truth value? Of course they can. When Nietzsche wrote *The Gay Science*, it was a reference to the art of being a troubadour. Nowadays, one could hear it only as concerning a study of a certain sexual preference. In modern parlance, Nietzsche did not write a book about (the) gay science.

Now, could there be such change of meaning in the case we are concerned with? Arguably, yes. In both a proof-thoeretic and a model-theoretic account of validity, part of the machinery is taken as giving an account of meanings—notably, of the logical connectives (introduction or elimination rules, truth conditions). If we change our theory, then our understanding of these meanings will change. This does *not* mean that the meanings of the of the vernacular words corresponding to their formal counterparts changes. You can change your view about the meaning of a word, without the word

[13]Certain kinds of constructivists may, of course, hold that the truth about numbers and other mathematical entities may change—for example, as the result of our acquiring new proofs.

changing its meaning. However, if one revises one's theory, and then brings one's practice into line with it, in the way which we noted may happen, then the usage of the relevant words *is* liable to change. So, then, will their meanings—assuming that meaning supervenes on use (and some version of this view must surely be right). So the sentences used to express the validity claims, and maybe even which propositions the language is able to express, can change.[14]

It might be thought that this makes such a change a somewhat trivial matter. Suppose we have some logical constant, c, which has different truth or proof conditions according to two different theories. Can we not just use two words, c_1 and c_2, which correspond to these two different senses? Perhaps we can sometimes; but certainly not always: for meanings can interact. Let me illustrate. Suppose that our logic is intuitionist. Then "Peirce's law", $((A \to B) \to B) \to B$, is not logically valid. But suppose that we now decide to add a new negation sign to the language, which behaves as does classical negation. Then Peirce's law becomes provable. The extension is not conservative. Another case: given many relevant logics, the rules for classical negation can be added conservatively, as can the natural introduction and elimination rules for a truth predicate. But the addition of both (when appropriate self-reference is available) produces triviality. Meanings, then, are not always "separable".[15]

4.3 Can Meanings Change Rationally?

So meanings can change, and not necessarily in a straightforward way. Can this happen rationally, and if so, how? The answers to these questions are implicit in the preceding discussion. Suppose we change our *logica docens* to a rationally perferable one. Suppose that we then change our *logica utens* rationally to bring it in line with this. The the meanings of our logical constants, and so the language used to express the facts of validity, may also change. And the whole process is rational.

[14]A pertinent question at this point is whether the meaning of 'follows deductively from'—or however this is expressed—can itself change. Perhaps it can; and if it does, this adds a whole new dimension of complexity to our investigation. However, I see no evidence that the meaning of the phrase (as opposed to our *theories* of what follows from what) has changed over the course of Western philosophy. So I ignore this extra complexity here.

[15]On these matters, see Priest (2006), ch. 5.

5 Conclusion

Let me end by summarising the main conclusions we have reached, and making a final observation.

A *logica docens* may be revised rationally, and this happens by the standard mechanism of rational theory choice. A *logica utens* may be changed by bringing it into line with a *logica docens*; and if the *docens* is chosen rationally, so is the *utens*. The answer to the question of whether or not the *logica ens* may change depends on one's best answer to the question of what validity is. However, under the model- or proof-theoretic accounts of validity, the answer appears to be: no. This does not mean, however, that the sentences used to express these facts may not change. And a rational change of *logica utens* may occasion such a change.

Now the observation. The rational *logica utens* depends on the rational *logica docens*. The true *logica docens* depends on the facts of validity. And assuming a model- or proof-theoretic account of meaning, the language available to express these may depend on the *logica utens*. It is clear that we have a circle. If one were a foundationalist of some kind, one might see this circle as vicious: there is no privileged point where one can ground the entire enterprise, and from which one can build up everything else. However, I take it that all knowledge, about logic, as much as anything else, is situatated.[16] We are not, and could never be, *tabulae rasae*. We can start only from where we are. Rational revision of all kinds then has to proceed by an incremental and possibly (Hegel notwithstanding) never-ending process.

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[16] See Priest (200+).

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