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P. H. Partridge (1967)

MOTION

The nature of motion and the philosophical problems surrounding it have been perennial issues in Western philosophy. Motion is a special case of change, and much discussion relevant to motion extends naturally to change in general (see Mortensen 2002).

Notable among the problems of motion are those provided by Zeno's paradoxes. Perhaps the hardest of these is the Arrow paradox. Consider an object in motion. At any instant of that motion, since it is an instant, the object makes no advance on its journey. But if it makes no advance in any instant of its journey, how can it make advance in all of them? The sum of a collection of nothings—even an infinite collection—is nothing. It would seem that it cannot move at all.

MOTION AND THE CALCULUS

Substantial progress concerning the topic of motion was made with the development of the calculus by Isaac Newton and Gottfried Wilhelm Leibniz in the seventeenth century. The velocity of an object at time t, $v(t)$ (with respect to a frame of reference), is given by the derivative of its spatial location, $x(t)$, with respect to time. That is, $v(t_0)$ is $dx(t)/\mathrm{d}t$, evaluated at t_0. An object is in motion at an instant if its velocity at that instant is nonzero; it is at rest if its velocity is zero.

The understanding of motion thus provided is, of course, parasitic on an understanding of the calculus itself and specifically on the notion of a derivative. In the eighteenth and early nineteenth centuries this depended on the notion of an infinitesimal; and infinitesimals behaved in a notoriously inconsistent fashion. Specifically, they were assumed to be nonzero (sometimes) and zero (sometimes).

HEGEL ON MOTION

Georg Wilhelm Friedrich Hegel, writing at the start of the nineteenth century, put the contradictory properties of the infinitesimal to the service of his dialectic. The continuous and the discrete are contradictory notions. There is, therefore, something that is their synthesis. This is a variable point: the infinitesimal. It has the property of being a point, so having zero extension, and being extended, so having nonzero extension.

This understanding allows him a particular view of the account of motion provided by the calculus. To be in motion at an instant is precisely to move an infinitesimal amount. Thus,

> [when a body is moving] there are three different places: the present place, the place about to be occupied and the place which has just been vacated; the vanishing of the dimension of time is paralyzed. But at the same time there is only *one* place, a universal of these places, which remains unchanged throughout all the changes [i.e., the variable point]; it is duration existing immediately in accordance with its notion, and as such it is motion. (Hegel 1970, p. 43)

That is, "Something moves not because at one moment of time it is here and at another there, but because at one and the same moment it is here and not here, because in this 'here' it at once is and is not" (Hegel 1969, p. 440). This provides Hegel with a simple solution to the Arrow paradox. The object advances on its journey because it does advance at each instant: It moves a tiny amount at each instant.

RUSSELL ON MOTION

Within fifty years Hegel's analysis of motion was rendered obsolete by new mathematical developments. Toward the end of the nineteenth century the notion of an infinitesimal disappeared from standard mathematics. This was because, through the work of Baron Augustin-Louis Cauchy, and particularly Karl Weierstrass, a different understanding of the derivative was developed. A derivative came to be understood simply as the limit of a certain ratio as some variable approaches a value. In particular, the velocity $v(t_0)$, that is, $dx(t)/dt$ as evaluated at t_0, came to be understood as the limit of $(x(t_0+\epsilon)-x(t_0))/\epsilon$ as ϵ approaches 0.

Therefore, the new interpretation of the calculus provided a different understanding of motion. This was spelled out by Bertrand Russell in *The Principles of Mathematics* as follows:

> [I]n consequence of the denial of the infinitesimal, and in consequence of the allied purely technical view of the derivative of a function, we must entirely reject the notion of a *state* of

> motion. Motion consists *merely* in the occupation of different places at different times.… There is no transition from place to place … no such thing as velocity except in the sense of a real number which is the limit of a certain set of quotients. (1938, p. 473)

The paradox of the Arrow can then be dismissed:

> In the case of motion, [Zeno's Arrow paradox] denies that there is such a thing as the *state* of motion. In the general case of a continuous variable, it may be taken as denying actual infinitesimals. For infinitesimals are an attempt to extend to the *values* of a variable the variability which belongs to it alone.… [The modern account of the variable has clarified this confusion, but] its absence in Zeno's day led him to suppose that continuous change was impossible without a state of change, which involves infinitesimals and the contradiction of a body's being where it is not. (Russell 1938, pp. 350–351)

PROBLEMS WITH THE ORTHODOX ACCOUNT

The view concerning motion expressed by Russell became the orthodox view of motion in the twentieth century. It is not without its problems, however. As Russell makes clear, according to this account there is no such thing as an intrinsic state of motion. That is, the instantaneous states of two objects, one in motion and one at rest at that instant, but at the same place, would be identical. Whether the object is in motion or at rest at that instant depends entirely on its states at neighboring instants. This is highly counterintuitive: Motion turns out to be a sequence (albeit a continuous one) of states that are indistinguishable from rest-states. There is no genuine flux. Motion occurs in much the same way as it appears to when successive stills in a cinema film are shown so fast that something seems to move. Indeed, one might call this the cinematic view of change. One way to bring home its oddity is as follows. Suppose that there is a particle that behaves as follows: At any time it exists simply at some place, but at any time it may disappear and reappear at some other place. Suppose that, by an accidental string of occurrences, the positions of the particle over a short period just happen to be a continuous function of time with a nonzero derivative. One would not, on this account, be inclined to say that the particle is in motion at each instant.

The cinematic account of change is not just counterintuitive. It has a number of other untoward consequences, as Russell himself notes (1938, p. 482). It is natural to take laws of nature to state causal relations between various quantities, such as velocity and its derivative, acceleration. Indeed, one normally takes it that the states of these quantities at a time are causal determinants of later states. If, in nature, there are no such things as these quantities, all this must be foregone—including the possibility of Laplacean determinism: the view that the intrinsic state of a system at any time determines its future states.

Further problems arise when one considers discontinuities of various kinds. Thus, suppose that an object is at rest before time t, and then starts to move with velocity *1*. That is, $x(t) = 0$ if $t<0$ and $x(t) = t$ if $t\geq0$. The object has no velocity at $t = 0$ (since $x(t)$ has no derivative there), and a fortiori no acceleration. Still, it would seem that it ought to, if the motion is the result of an impulse applied to the object at $t = 0$. Worse: suppose that the object moves instantaneously at $t = 0$ to some other position where it is at rest; so $x(t) = 0$ if $t<0$ and $x(t) = 1$ if $t\geq 0$. If $t \neq 0$, the velocity of the particle is 0; and if $t = 0$, the velocity is undefined. Hence, the particle has changed places at $t = 0$, yet it has never been in motion!

Finally, and Russell's protestations to the contrary notwithstanding, it would appear that he has not so much solved the Arrow paradox as ignored it. He accepts that no progress is made on the journey in an instant, but simply insists that, nonetheless, progress is made in the whole journey. This is not a solution, it is what must be explained.

TOOLEY'S ACCOUNT

These and other objections were leveled against the Russellean account by Graham Priest (1985, 1987) and Michael Tooley (1988), each of whom offers an account of motion according to which velocity (relative to a frame of reference) is an instantaneous property of an object.

According to Tooley velocity is a theoretical (i.e., unobservable) property of an object that is causally efficacious in determining its behavior. Specifically, it is a quantity, $v(t)$, satisfying the equations:

$$x(t_1) = x(t_0) + {}_0\int^1 v(t)dt$$

$$m(t_1).v(t_1) = m(t_0).v(t_0) + {}_0\int^1 F(t)dt$$

where $m(t)$ is the inertial mass of the object at t and $F(t)$ is the force acting on it at that time. These, note, are the two key laws in (relativistic) kinematics involving velocity. The first relates velocity to position; the second to the forces acting. The crucial point is that, on Tooley's view, these equations should be interpreted as stating relations between (instantaneous) physical quantities.

PRIEST'S ACCOUNT

Priest's account draws on Hegel. It does not resurrect Hegel's account of the categories; nor does it rehabilitate the notion of the infinitesimal. What it does do is take seriously the possibility that, at an instant, the position of a moving object may be spread out over a short (but non-infinitesimal) region. Because the object is in motion it may be impossible to localize it to any one position. This is called the spread hypothesis.

More specifically, let $x(t)$ be the locus of motion of an object, as it occurs in the laws of motion cited in the previous section. One can write r_t for the value of this function at t. For Russell, the state of the object at time t is characterized by the set of statements S_t = {'The object is at r_t'}∪ {'The object is not at r'; where $r \neq r_t$}. Given the spread hypothesis, one must suppose that there is an interval of times containing t, θ_t, such that the object is equally at $x(t')$ for all $t' \epsilon \theta_t$. The state of the object at t is therefore characterized by the set of all those statements in S_t' for $t' \epsilon \theta_t$. (What, exactly, θ_t is, is a matter to be determined by other consideration; possibly by nature itself. But it is not unnatural to suppose that the width of θ_t is proportional to $dx(t)/dt$ if this is defined.)

If $x(t')$ is constant for $t' \epsilon \theta_t$ (and, in particular, if θ_t contains just t), the state-description is identical to the Russellean state-description; in particular, it is consistent. But if $x(t')$ takes different values, r_1 and r_2, for $t' \epsilon \theta_t$, then it will be inconsistent: it will contain the statements that the object both is and is not at r_1 (and r_2).

To be in motion at an instant, then, according to this account, is to have an inconsistent state description at that instant. Objects in motion are at one place at one time, and another at another. But this is not sufficient. This would be equally true of an object at rest at each of these places. To be in motion at a time, an object must both be and not be at a place at that time.

THE ARROW AGAIN

If one is to have a theory according to which motion is an intrinsic property of an object, then the accounts of Tooley and Priest may not be the only ones; but they are the only two presently on offer. Therefore, it is natural to compare their relative merits.

One feature of Tooley's account, unlike Priest's, is that it is consistent. Priest's account (and Hegel's) presupposes that one can make sense of the possibility that the truth about a situation can be contradictory (dialetheism). It requires the use of a logic that is such that contradictions do not imply everything. One may take this to be a strong mark in Tooley's favor. Other objections against Priest can be found by consulting Tooley (1988). It appears that there are perfectly natural replies to these objections, but this is not the place to go into the matter.

On the other side, it is clear that Priest's account solves the Arrow paradox essentially as does Hegel's. The object, by occupying more than one point at an instant, does make progress during each instant, and so in the whole comprising them. Tooley's account would not appear to solve the paradox. It still leaves one with the fact that the object makes no progress during an instant of its journey. Russell, whether rightly or wrongly, took the problem to be solved by rejecting instantaneous states of motion. Even this step is not open to Tooley.

Doubtless, there is more to be said on these matters. Regardless, one thing is clear: Even after the development of the calculus, the theory of the limit, the understanding that it is possible to postulate unobservables in science, and even of paraconsistency, Zeno's paradox of the Arrow still haunts us.

See also Hegel, Georg Wilhelm Friedrich; Motion, A Historical Survey; Russell, Bertrand Arthur William; Zeno of Elea.

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Graham Priest (2005)

MOTION, A HISTORICAL SURVEY

"Motion," or "movement," in its modern meaning, is change—or more precisely, change of the relative positions of bodies. The concept of motion thus involves the ideas of space and time. Kinematics, in the nineteenth century usually called "kinetics" or "phoronomics," is the science that deals exclusively with the geometrical and chronometrical aspects of motion, in contrast to dynamics, which considers force and mass in relation to motion. In medieval terminology, following Aristotelian tradition, "motion" (*motus* or *kinesis*) had a much wider significance, denoting any continuous change in quality, quantity, or place.

EARLY CONCEPTS OF MOTION

Ever since the beginning of philosophical speculation and scientific analysis, the concept of motion has played a predominant role in Western thought. Anaximander of Miletus (sixth century BCE) saw in motion an eternal agent of the cosmos. For Heraclitus motion was a cosmological principle underlying all physical reality (*panta rhei*, "everything is in perpetual flow"). Yet in spite of their insistence on the universality of motion, neither Anaximander nor Heraclitus seems to have inquired into the nature of motion itself. The Eleatics were probably the first to do so, when they discovered the contradiction inherent in the idea of motion and consequently denied the reality of motion, relegating its appearance to the realm of illusions and deceptions. A body, they argued, can move neither where it is nor where it is not; hence, reality is motionless and unchanging. Zeno's famous antinomies (Aristotle, *Physics* 239), such as the "Arrow" and "Achilles," seem to have been aimed, at least in part, at a refutation of the possibility of motion. On the other hand, for the atomists, such as Democritus and Leucippus, motion was a fundamental property of the atoms. All changes in nature were reduced to the movements of atoms in the void, and with the eternity and uncreatedness of the atoms their motion was eternal and uncreated; this motion itself, in the atomists' view, was not further analyzable. It remained a primary concept until Epicurus searched for a causal explanation. This (according to Lucretius) he thought to have found in weight, the cause of the downward movements of atoms, and in their little "swerves," by which he explained the otherwise incomprehensible collisions and redistributions of atoms without which physical processes could not be accounted for.

ARISTOTLE. In Aristotle's natural philosophy the concept of motion played a decisive role, since for him nature was the principle of movement or change: "We must understand what motion is; for, if we do not know this, neither do we understand what nature is" (*Physics* 200b12), a statement recurrent in Peripatetic philosophy under the motto *Ignato motu, ignatur natura* ("To be ignorant of motion is to be ignorant of nature"). For Aristotle, in contrast to his predecessors, motion raised a profound problem—not merely from the logical point of view. Expressing the deeply rooted metaphysical conviction of Western thought that motion is neither logically nor ontologically self-sufficient but requires an explanation, Aristotle contended that motion is neither in the causal, or genetic, nor in the ontological sense a primary concept. Causally, every motion originates in another motion; only animate organisms possess an inherent power to move. Hence his famous dictum *Omne quod movetur ab aliquo movetur* ("All things that move are moved by something else"). To avoid infinite regression and to find a satisfactory explanation of the existence of motion, Aristotle reduced the ultimate origin of all movements to an eternal mover who is himself unmoved. (*Physics* 258b). Ontologically, Aristotle derived motion from the basic notions of his metaphysics of substance and form by defining it as "the progress of the realizing of a potentiality *qua* potentiality" (*Physics* 201a10). Motion as the actualization of that which exists in potentiality may produce a substantial form (*generatio*), may change qualities (*alteratio*) and quantities (*augmentatio* or *diminutio*), or, finally, may be a change of place (*motus localis*). Although Aristotle did not reduce qualitative differences to quantitative relations of size and position, as did the atomists, his physics is essentially a physics of qualities. He did regard local motion as of a more fundamental character than the other kinds of motion (*Physics* 208a31); it is "the primary and most general case of passage and prior to all other categories of change" (*Physics* 260b22). Yet in spite of this preferential status, local motion for Aristotle is only a necessary concomitant of change, not, as the mechanistic physicists of the post-Newtonian era maintained, the essential and exclusive constituent of change.