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E.J. ASHWORTH

PARACONSISTENT LOGIC

A logic is paraconsistent if it does not validate the principle that from a pair of contradictory sentences, A and ~A, everything follows, as most orthodox logics do. If a theory has a paraconsistent underlying logic, it may be inconsistent without being trivial (that is, entailing everything). Sustained work in formal paraconsistent logics started in the early 1960s. A major motivating thought was that there are important naturally occurring inconsistent but non-trivial theories. Some logicians have gone further and claimed that some of these theories may be true. By the mid-1970s, details of the semantics and proof-theories of many paraconsistent logics were well understood. More recent research has focused on the applications of these logics and on their philosophical underpinnings and implications.

The idea that a contradiction implies everything (*ex contradictione quodlibet* – ECQ) has always been a contentious one in logic. Despite this, formal paraconsistent logics, which do not validate ECQ, are creatures of the twentieth century. The earliest ones were constructed in Russia by N.A. Vasil'ev *c.*1912 – an Aristotelian logic – and I.E. Orlov in 1929 – a relevance logic (see Anderson, Belnap and Dunn 1992). However, these had no impact at the time.

Work on formal paraconsistent logics did not begin in earnest until after the Second World War. Since then they have been proposed independently by many logicians, the earliest notable ones being S. Jaśkowski (in Poland) in 1948, F.G. Asenjo (Argentina) *c.*1954, N.C.A. da Costa (Brazil) *c.*1958 and T.J. Smiley (the UK) in 1959. Work on relevance logic by A.R. Anderson and N.D. Belnap (in the USA) also started in the late 1950s (see RELEVANCE LOGIC AND ENTAILMENT), and the specifically paraconsistent aspects of relevance logic were developed by R. Routley and others (in Australia) in the late 1960s and 1970s. (Note that a paraconsistent logic need not be relevant.) Since then, work on formal paraconsistent logic has continued apace in many places, but most notably in Brazil (under the leadership of da Costa) and Australia. (Some of the original work is rather difficult to come by or of a rather preliminary nature. The best access points in the literature are: Jaśkowski (1969); Asenjo (1966); da Costa (1974); Anderson and Belnap (1975); and Routley (1977).)

A major motivation behind the construction of formal paraconsistent logics has always been the idea that in many contexts we may have information that is inconsistent, but from which we want to draw conclusions in a controlled way. (The term 'paraconsistent' was coined by M. Quesada at the third Latin American Symposium on Mathematical Logic in 1976, to indicate just this.) Examples that are frequently appealed to are: evidence provided by different witnesses, constitutions and other legal documents, various scientific theories, numerous philosophical theories, and information in a computer database. In such contexts, even though the data are incorrect, if we are stuck with them (as we may well be), then the logic had better be paraconsistent. Moreover, inconsistent scientific theories, even if they are not correct, may still be useful, or good approximations to the truth.

Some paraconsistent logicians have claimed that inconsistent theories may actually be true (see, for example, Priest 1987). The view that some contradictions or contradictory theories are true is called 'dialeth(e)ism' – from 'dialetheia' (a term coined by Priest and Routley in 1982), which means a true statement of the form $A \& \sim A$. The most commonly cited examples of dialetheias are the paradoxes of self-reference, such as Russell's and the liar paradox (see PARADOXES OF SET AND PROPERTY; SEMANTIC PARADOXES AND THEORIES OF TRUTH). The failure to obtain consensus on any consistent account of the paradoxes gives this suggestion its appeal. Other suggested examples of dialetheias include: statements about objects on the borderline of some vague predicate; moral dilemmas; and dialectical contradictions in the tradition of Hegel and Marx.

Some approaches to paraconsistent logic, such as Smiley's, obtain a suitable inference relation by starting with the classical one and filtering out ECQ and other undesirables. Such logics typically give up the transitivity of entailment. A more common approach is to specify a notion of entailment semantically, defined in terms of truth-preservation over a class of interpretations. For this approach, it is necessary to have a mechanism whereby contradictory sentences may simultaneously hold in an interpretation. For this reason, one may think of paraconsistent logic as a kind of dual of intuitionist logic (see INTUITIONISM), the former violating the law of non-contradiction, the latter violating the law of the excluded middle. (Though it is quite possible to have a paraconsistent logic in which $\sim(A \& \sim A)$ is semantically valid.) Paraconsistent logics of this kind characteristically invalidate the Disjunctive Syllogism (DS): $A \vee B$ and $\sim A$ entail B. For both premises may be true in virtue of the properties of A, while B is not.

Various techniques have been proposed to achieve the required end. Jaśkowski's is to interpret 'true' as 'true in some possible world or other'. Da Costa's is to give up the truth-functionality of negation, so that if A is true, $\sim A$ may be either true or false. Routley's is to treat negation as an intensional operator, so that $\sim A$ is true at a world w if A is false at some associated world, w^*. Asenjo's suggestion (which can also be harnessed in the semantics of relevance logics) is to allow sentences to take a non-classical truth value, which may be thought of as *both true and false*, and which is a fixed point for negation.

By the mid-1970s the semantics and proof theories of many paraconsistent logics were well developed. More recently, much work has gone into their applications, both technical and philosophical. The technical applications all involve the investigation of inconsistent theories. Notable results in this area include a proof that naïve set theory with (an unrestricted comprehension axiom and) a suitable underlying paraconsistent logic is non-trivial (Brady 1989), and a proof (by R. Meyer) that there is a (consistent!) arithmetic that can prove its own non-triviality. The techniques involved in the latter involve the construction of models of full first-order arithmetic, many of which are finite (see Meyer and Mortensen 1984). It is also possible to construct non-trivial theories that contain both self-reference and epistemic operators, and that are semantically closed, in the sense of Tarski (see Priest 1991a).

The philosophical applications are more diffuse. Traditionally, consistency was thought to be the cornerstone of many important philosophical notions (for example, truth, rationality), but the viability of paraconsistent logic throws down a challenge to any such claim. In the light of this, a reappraisal both of such notions and of the significance of results that turn on consistency, for example, Gödel's incompleteness theorems, is called for and is in train.

Many would be prepared to concede the possibility of a limited use for paraconsistent logic, for example, as an inference engine for a computational database. The major criticism as far as this goes is that a paraconsistent logic is too weak to permit useful inference. In particular, a number of writers have argued that the DS is essential to most practical inference. In the author's opinion, this sort of objection carries little weight. For a start, the logical resources of the programming language PROLOG are validated in most paraconsistent logics when suitably interpreted. More generally, most paraconsistent logicians have been prepared to accept the usability of principles such as the DS in consistent ('normal'?) contexts. This idea has motivated the construction of formal non-monotonic logics in which the DS is a default inference (see Batens 1989; Priest 1991b).

Much of the criticism of paraconsistent logic has fallen on dialetheism. One of the chief general criticisms is that the paraconsistent semantics for negation fails to capture (true) negation, but this is difficult to make stick for the many-valued semantics. Some have argued that if contradictions are not to be rejected as logically unacceptable then logic is ruined as an instrument of rational criticism, but this objection trades on a confusion of what is possible according to formal logic and what is rationally possible. A third important general criticism is that if one is to accept A one must rationally reject $\sim A$. However, dialetheism aside, there are situations where we seem to have little rational option but to accept a contradiction (for example, the paradox of the preface). Most of the application-specific criticisms have tried to undercut a dialetheic solution to the self-referential paradoxes by arguing that paraconsistent solutions to these collapse into triviality – or, at least, fail to do so only by *ad hoc* manoeuvres of a kind that make consistent purported solutions so unsatisfactory. To date, such arguments have not hit the mark squarely. (On much of the above, see Smiley and Priest (1993).)

The possibility of violations of the law of non-contradiction was widely canvassed by pre-Aristotelian philosophers. Aristotle attacked this possibility in *Metaphysics* (1005b8–1009a5). His arguments are distinctly dubious, as was shown by Łukasiewicz (1971), but his authority has determined subsequent orthodoxy. Only a few (notably Hegel) have challenged it. Even fewer have produced a sustained defence of the law. The technical viability of paraconsistent logic and dialetheism therefore raises a challenge to contemporary philosophy of no little significance.

See also: LOGICAL AND MATHEMATICAL TERMS, GLOSSARY OF

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GRAHAM PRIEST

PARADIGMS *see* KUHN, THOMAS (§4)

see KUHN, THOMAS (§4)

PARADOXES, EPISTEMIC

The four primary epistemic paradoxes are the lottery, preface, knowability, and surprise examination paradoxes. The lottery paradox begins by imagining a fair lottery with a thousand tickets in it. Each ticket is so unlikely to win that we are justified in believing that it will lose. So we can infer that no ticket will win. Yet we know that some ticket will win. In the preface paradox, authors are justified in believing everything in their books. Some preface their book by claiming that, given human frailty, they are sure that errors remain. But then they justifiably believe both that everything in the book is true, and that something in it is false.

The knowability paradox results from accepting that some truths are not known, and that any truth is knowable. Since the first claim is a truth, it must be knowable. From these claims it follows that it is possible that there is some particular truth that is known to be true and known not to be true.

The final paradox concerns an announcement of a surprise test next week. A Friday test, since it can be predicted on Thursday evening, will not be a surprise yet, if the test cannot be on Friday, it cannot be on Thursday either. For if it has not been given by Wednesday night, and it cannot be a surprise on Friday, it will not be a surprise on Thursday. Similar reasoning rules out all other days of the week as well; hence, no surprise test can occur next week. On Wednesday, the teacher gives a test, and the students are taken completely by surprise.

1 Lottery and preface paradoxes

The lottery paradox – first developed in Kyburg (1961) – and the preface paradox – originally formulated in Makinson (1965) – have a similar structure, although some (for example, Pollock (1986)) hold that they have different solutions. Each hinges on a conflict between a rule of acceptance, a condition on the transfer of warrant, and an axiom about warrant.

Rule of acceptance: There is some threshold short of certainty where acceptance of a claim is warranted or justified.

Transfer condition: A set of warranted claims is closed under deduction. That is, a set of warranted claims includes all the deductive consequences of that set.

Warrant axiom: It is not possible to be warranted in believing p and, for the same time and the same individual, be warranted in believing not p.

One standard approach is to find some fault with the transfer condition. Denials of the transfer condition sometimes result from explicit consideration of these paradoxes, sometimes from more general considerations within the theory of knowledge. For example, Kyburg addresses the lottery paradox explicitly and holds that it relies on the faulty conjunction principle, the principle according to which a person is warranted in believing a conjunction $p\&q$ if they are warranted in believing p and warranted in believing q. In this way, outright contradictions are thought to be avoidable in the set of warranted beliefs for a person at a time, while still allowing that the set can be inconsistent, that is, be such as to deductively imply a contradiction. Other epistemologists (for example, Nozick (1981)) develop general theories of knowledge