Logic: One or Many?

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1 Introduction: Logical Monism and Logical Pluralism

Is there one logic or are there many? Traditionally, there was only one—the logic of Aristotle and the Stoics, as melded together in the middle ages—and the question never arose. This century, we have seen a plethora of logics: Frege/Russell (classical) logic, intuitionism, paraconsistent logic, quantum logic. Usually, advocates of these logics were still logical monists, in the sense that they took it that other logics were wrong. But the weight of the plethora has been too much for some people, who have decided that there is no one true logic: there are lots of things which may, with equal justification, be called logic. In logic, as in a multi-cultural society, pluralism must be endorsed. Is this right?[1]

The answer, as one might expect, is that it depends on what one means. There are certain senses in which the answer is clearly 'yes'. But in the case of central importance—I will explain what I take that to be in due course—I think that the answer is 'no'; and I will defend a monist position.

I think that much of the *initial* plausibility of pluralism in this central case arises from a confusion between the various senses of the question. The first topic on the agenda is, therefore, an appropriate disambiguation. One of the important notions that requires clarification is that of rivalry between logics. For pluralism arises in a serious form when there is such a rivalry, and we are called upon to adjudicate. Can we, in such circumstances, decide for more than one candidate? The notion of rivalry will therefore be one of our concerns.

A discussion of these issue in the first main part of the paper will clear the ground for considering serious objections to monism, which we will do in the

[1] An excellent preliminary discussion of the issue can be found in Haack (1978), ch.12. One should note, at the start, that pluralism is to be distinguished from *relativism*. Relativism equals pluralism plus the claim that each of the plurality is, in some relevant sense, equally good. Pluralists sometimes sail very close to the relativist wind though: 'There are different, equally good ways of ...[giving an account of validity]; they are different, equally good *logics'*. (Beall and Restall (1998), p.5; italics original.)

second. This will leave a few words to be said about inductive inference in the final brief section.

2 Some Basic Distinctions

2.1 Pure and Applied Logic

Let me, for a start, distinguish between pure and applied logic. I will explain what I mean by an analogy with geometry.[2] It is now an uncontentious fact that there are many pure geometries: Euclidean, Riemannian, spherical, etc. Each is a perfectly good mathematical structure, and can be formulated as an axiom system, with standard models, etc. There is no question of rivalry between geometries at this level. The question of rivalry occurs when one applies geometries for some purpose, say to provide an account of the physical geometry of the cosmos. Then the question of which geometry is right must be faced.

In the same way, there are many pure logics. I enumerated several of these above. Each is a well-defined mathematical structure with a proof-theory, model theory, etc. There is no question of rivalry between them at this level. This can occur only when one requires a logic for application to some end. Then the question of which logic is right arises.

If one is asking about pure logics, then, pluralism is uncontentiously correct. Plurality is an issue of substance only if one is asking about applied logics. In this case, there is the potential for rivalry, and whether one should be a monist or a pluralist about this rivalry is a question that must be faced.

2.2 Theoretical Pluralism

Let us turn, then, to applied logics. The first thing to note here is that pure logics can be applied for many purposes, such as simplifying electronic circuits, or analysing certain grammatical structures. And again, it is clear and uncontentious that different pure logics may be appropriate for each application. In the two examples given, for example, the appropriate logics are Boolean logic and the Lambek calculus.[3] We will have further examples, closer to home, later. Plurality is, then, an interesting issue only when we have one particular application in mind.

Fix, then, on some one application. A pure logic is applied by interpreting it in some way or other. It then becomes a *theory* of how the domain in which it is interpreted behaves; just as a pure geometry, when interpreted as a physical

[2]Which is appealed to by many parties in the debate. See, e.g., da Costa (1997), Beall and Restall (1998). The analogy is discussed in detail in Priest (199a).

[3]The Lambek calculus is a sub-structural logic with a clear application to grammatical categories.

geometry, is a theory about space. In such a situation there may well be disputes about which theory is correct. This has certainly happened in geometry. It also happens in logic, as we shall have several occasions to observe below.

Such disputes are to be resolved in logic, as in geometry and elsewhere, by the usual criteria of theoretical evaluation, such as adequacy to the data, and theoretical virtues, such as simplicity, unity, no *ad hoc*ness, etc. We do not need to go into the details here. The present relevance of all this is that we have the source of another kind of pluralism. Given a fixed application to some domain, there may be many different applied logics which constitute theories about the behaviour of that domain—and correspondingly, disputes about which theory is right. I will call this, for want of a better term, *theoretical pluralism*.

2.3 The Canonical Application of Logic

Let us now turn to the most important and traditional application of a pure logic, what might be called its *canonical application*: the application of a logic in the analysis of reasoning, which was also traditionally called 'logic', of course (just as 'geometry' was used ambiguously before Euclidean geometry and its canonical application were distinguished, to describe both). The central purpose of an analysis of reasoning is to determine what follows from what—what premises support what conclusions—and why. An argument where this is, in fact, the case may be called *valid*. It is traditional to distinguish between two notions of validity, deductive and inductive. So immediately it would appear that we have a pluralism here. As to whether this is indeed the case, I will return at the end of this essay. For most of this essay I will focus just on deductive validity.

Before we can discuss the issue of plurality for this application, we need to look more closely at how pure logics are applied in an analysis of reasoning.[4] In the first place, we reason in the vernacular. We establish by observation that a planet moves in a certain way, and wish to know if the description of its trajectory follows from our theory of motion; or we infer what would be the case if the Butler did it, to see whether these consequences do, in fact, obtain. Premises and conclusions are formulated in a natural language. By all means, this may be a natural language augmented with technical vocabulary, such as that of mathematics. The language employed, though, is still the language of use and communication.

Pure logics, at least as standardly conceived, do not concern the vernacular at all, but are couched in terms of formal languages. A definition of validity is provided for inferences expressed in one of these. Applying this to provide an account of validity for the vernacular, as in the application of any pure mathematical structure, requires an interpretation. And in the case of logic, the interpretation

[4]The topic is discussed at greater length in Priest (199a), section 10, and concerning negation, in particular, in Priest (1999), sections 2, 3.

is quite literally so: a translation procedure between the formal language in question and the vernacular. Such a procedure is rarely articulated at great length in logic texts. Rather, students are given hints about treating 'or' as $\vee$ (most of the time), etc., and then set loose on a bunch of exercises, through which they develop procedural knowledge. But given such a translation procedure, a pure logic provides an account of validity for vernacular inferences. A vernacular inference is valid iff its translation into the formal language is valid in the pure logic.

The result, moreover, often produces disagreement. According to some logics, 'there is an infinitude of primes' follows from 'it is not the case that there is not an infinitude of primes'; according to others, it does not. According to some logics, 'the sky is green' follows from 'the liar sentence is both true and false'; according to others, it does not. Such disagreements mean that we clearly have a case of theoretical pluralism. Is there a more serious kind of pluralism here? To address this question, let us turn to accounts of validity for formal languages.

2.4 Validity

There is a plurality of formal languages employed in logic. I am not talking here of the fact that different symbols are used in different languages. The fact that '$\neg$' is used for negation in one language, whilst '$\sim$' is used in another, is neither here nor there: as long as each plays the role of vernacular negation according to the appropriate translation manual, the two are, in effect, the same symbol. Rather, what is important is that one language may lack a symbol, in this sense, that another has. Thus, a modal language contains modal operators, which an extensional language lacks. Strictly speaking, then, the accounts of validity for two such languages must be different, and we have a pluralism. But this, again, is not a very interesting pluralism. For as far as the common set of symbols goes, the two accounts may be in perfect agreement; it is simply that one (or each) extends the other in some direction.

Given, then, a formal language, and a translation procedure between it and the vernacular, what should a formal theory of validity be like? I take the answer to be essentially as follows.[5] When we reason, we reason about various situations or state of affairs. These may be actual or hypothetical. We reason to establish what holds in these situations given what we know, or assume, about them. I will call this truth-preservation (forward),[6] though it is not actually truth that is in question unless the situation we are reasoning about is itself actual. The *point* of deduction, then, is to give us a set of canons that are guaranteed to preserve truth in this sense. A valid inference is therefore one such that in all the situations where the premises hold, the conclusion holds.

[5]The answer is discussed and defended in Priest (199b).

[6]Conversely, we may reason to establish what fails to hold in a situation, given what we know, or assume, about what fails to hold in it. This is untruth-preservation backwards.

This answer is, of course, nothing more than a gesture in the direction of an account of validity. Much remains to be done by way of spelling out what situations there are, and what it is to hold in them. And there are certainly different possibilities here: according to paraconsistent semantics, but not classical or intuitionist semantics, there are inconsistent situations. According to intuitionist semantics (and some paraconsistent semantics, but not classical semantics (and some other paraconsistent semantics), the definition of truth in a situation requires the truth conditions of negation to make reference to more than one possible world. There may therefore be different views on how to fill in the details required. But the familiar disputes about this, such as the intuitionist critique of classical logic, or the dialetheist critique of explosive logics, would seem to be cases of theoretical pluralism. Is there a more serious kind of pluralism here? So far, we have no reason to believe so.

3 Advocates of Pluralism

3.1 Domain Variation

So let us turn to some possible reasons. These have been advocated by a number of people, and we will look at six. The first is provided by da Costa (1997).[7] Da Costa's major argument for pluralism is to the effect that reasoning about different *kinds* of things may require different logics. The kinds in question are kinds like macro-objects, quantum objects, platonic objects, mental constructions, etc. What to say about this depends on how, exactly, it is envisaged that logic will vary.

One thing that da Costa envisages is that different kinds of objects have different properties. Thus, for example, he moots the possibility that quantum objects do not have the property of self-identity, which macro-objects have. Hence, micro-objects require a non-classical logic of identity, one where $\forall x\, x = x$ is absent.

In response to this, the obvious monist reply is that since validity is truth preservation in all situations, if there are situations in which objects may not be self-identical, then self-identity is not a logical law at all. This does not mean that when reasoning about, e.g., macro-objects one may not use the law, though. It is just a "contingent" property of certain domains, and may thus be invoked when reasoning about them. In a similar way, the intuitionist may invoke the law of excluded middle when reasoning about finite (or at least decidable) domains: this, plus intuitionist logic, gives classical logic. But the intuitionist has not changed logical allegiances. It is simply that classical validity can be recovered enthymematically, given the extra domain-specific premises.

[7] A review of this in English, which documents the claims attributed to da Costa in what follows, can be found in Priest (199c).

Da Costa's pluralism is more radical than I have so far indicated, though. He envisages not only that objects of different kinds may have different logical properties, but that different logical operators may also need to be used in reasoning about different kinds of objects. Thus, for example, classical negation is appropriate for dealing with platonic objects, and intuitionist negation is appropriate for dealing with mental constructions.

But, this cannot be right. Classical and intuitionist connectives have different meanings. (Indeed, some intuitionists, such as Dummett, even claim that classical connectives have no meaning at all). Thus, take negation as an example. Classical and intuitionist negations have different truth conditions.[8] But difference in truth conditions entails difference in meaning. Hence, the two connectives have different meanings.[9] Now, either vernacular negation is ambiguous or it is not. If it is not, then, since the different theories attribute it different meanings, they cannot both be right. We have a simple theoretical pluralism. The other possibility is that vernacular negation is ambiguous. Thus, it may be argued that vernacular negation sometimes means classical negation and sometimes means intuitionist negation. But if this were right, we would have two legitimate meanings of negation, and the correct way to treat this formally would be to have two corresponding negation signs in the formal language, the translation manual telling us how to disambiguate when translating into formalese. In exactly this way, it is often argued that the English conditional is ambiguous, between the subjunctive and indicative. This does not cause us to change logics, we simply have a formal language with two conditional symbols, say $\supset$ and $>$, and use both. This is pluralism in a sense, but the sense is just one of ambiguity.

In fact, I see no cogent reason to suppose that negation is ambiguous in this sense.[10] Indeed, negation cannot have different meanings when reasoning about different kinds of things. This is because we can reason about different kinds simultaneously. We can, for example, reason from the claim that it is not the case that a and b have some particular property in common, where a and b are of different kinds. Moreover, there are other reasons as to why vernacular negation cannot be ambiguous between intuitionist and classical negation. For if it were, as I have argued, we could have two formal negations. But it is well known that in the presence of classical negation, many other important intuitionist distinctions collapse. For example, the intuitionist conditional collapses into the classical conditional.

Of course, as I have already noted, the intuitionist logician can, in effect, use the full force of classical argument when reasoning about finite domains. But this is not because negation has changed its meaning. It is because they are entitled

[8]Intuitionist connectives are often given provability conditions, but since an intuitionist, in effect, identifies truth and provability, these are truth conditions.

[9]One might challenge this for reasons that we will come to in 3.5, and I will deal with there.

[10]There are some reasons as to why one might suppose that it is. I will deal with these in the next section.

to various extra premises, such as the law of excluded middle.

3.2 Context Dependence

The next notable advocate of pluralism that we look at is Batens (1985), (1990). According to him, different logics are required, not for different domains, but in different contexts, where a context is, in a nutshell, a problem-solving situation. Two of Batens' arguments for pluralism are particularly notable.

The first, given mainly in (1990), takes it for granted that a paraconsistent logic is correct in some contexts, but argues that it cannot be correct in others (e.g., a metatheoretic one), for which classical logic is required. Hence, pluralism. Now, just as an intuitionist may use what amounts to classical logic when reasoning about finite situations, so a paraconsistent logician may use what amounts to classical logic given appropriate information about the domain. For example, sufficient information is that for all α, $(\alpha \wedge \neg\alpha) \rightarrow \bot$, where $\rightarrow$ is an appropriate detachable conditional, and $\bot$ is a logical constant that entails everything. This extra information, together with the base paraconsistent logic, generates classical logic.[11]

But this reply might not satisfy Batens, for the following reason. As I have stressed, the meaning of a logical operator, such as negation, does not change simply in virtue of the extra information. But, Batens argues, a paraconsistent negation just cannot express certain things that need to be expressed in talking about such situations. In particular, $\neg\alpha$ cannot express the fact that α is *rejected*. What to say about this argument depends on how one interprets 'rejection', which may, in fact, mean several different things. I will not go into many details here, but the major sense of rejection that Batens has in mind is that to reject α is to commit oneself to something, which, in conjunction with α, produces triviality (p. 222). But a paraconsistent logician can do just that, by endorsing $(\alpha \wedge \neg\alpha) \rightarrow \bot$.[12]

The second of Batens' arguments (given mainly in (1985)) is that, notwithstanding the above, different contexts result in different meanings for logical operators, and hence deliver different logics (p. 338f.). The argument for meaning-variance is a familiar one, of a kind employed by Kuhn and Feyerabend in the philosophy of science. The first premise is that what is accepted may vary from context to context. The second is that what is accepted (partially) determines the meanings of the words involved. It follows from these that meanings change from context to context.

Now one might say a good deal about this argument, but its central failing is that a change of what is accepted does not (necessarily) result in a change of meaning. When a Christian loses their certitude, and comes to believe that God

[11] See Priest (1987), 8.5. There are less brute force ways of recovering classical logic in consistent situations. See Priest (1991).

[12] A fuller discussion of this, and of the other relevant senses of rejection, can be found in Priest (1999).

does not exist, the word 'God' has not changed its meaning. What they come to believe is the very opposite of what they believed before. Similarly, the classical logician believes that it is not the case that it is not the case that α entails α; the intuitionist believes otherwise. It does not follow that 'it is not the case that' means something different for the two of them. If it did, they would not be in disagreement, which they most certainly are. Intuitionist and classical negation do mean different things;[13] as I have argued, the point of disagreement between the two logicians is precisely which of those two negations it is that correctly characterises vernacular negation.

3.3 Logical Constants and Variables

The next three arguments we will look at are all to be found in Beall and Restall (1998), who give what is, perhaps, the most sustained defence of pluralism. B&R endorse an account of validity similar to that which I gave in 2.4;[14] but argue for pluralism, none the less. The three arguments are all to the effect that the details of the account may be filled in in different, equally legitimate, ways.

B&R's first argument proceeds as follows. There is a strong tradition in logic of treating certain words in premises and conclusions as parameters. This is done in the semantics of first order logic, e.g., where one treats predicates and constants as semantically variable. B&R observe that the account of validity can be parameterised in this way to give the usual model-theoretic (Tarskian) account of validity.[15] What are the consequences of this? A simple example illustrates. Consider the inference: a is red; hence a is coloured. This is valid on the unparameterised account. Any situation in which a is red, it is coloured (which is not to say that there might not be inconsistent situations, in which it is not coloured too). But the inference is not valid according to a parameterised definition of validity—at least if the parameters include predicates—for it is of the form: $Pa \vdash Qa$. What is at issue here is the question of what situations are to be admitted into our semantics. Are we to include, in our sweep, situations in which things can be in the extension of 'red' but not of 'coloured', etc., or are we not?

Now, it is true that the model-theoretic construction can be done in either way, parameterised or unparameterised, so giving different notions of validity. But as far as I can see, this is just another case of theoretical pluralism—or at least, B&R provide no reasons for thinking otherwise. They say that they do not think it '*fruitful* to debate which of ...[the two accounts] is *logic'* (p.9). I

[13]If they mean anything at all, which, as I have noted, may be disputed.

[14]Similar, but not identical. They think of situations as those of situation semantics. For me, a situation is (represented by) a structure of the kind normally thought of as an interpretation of the language.

[15]B&R appear to suggest that it is only the paramaterised account that can be applied to formal languages (p.7); this is certainly not true.

am really not sure what this means. It certainly cannot mean that there are no considerations that might persuade one to adjudicate for one theory over the other.[16] Many of the moves in this theoretical dispute are, in fact, well known.

The inference '*a* is red; hence *a* is coloured' is certainly *prima facie* valid. Hence, if the parametric account is to survive, this validity must be explained away. The standard move is to claim that the inference is, in fact, invalid, but that it *appears* to be valid because we confuse it with a valid enthymeme, with suppressed premise 'All red things are coloured', taken for granted. But this move is not very plausible when scrutinised. For if something about 'All red things are coloured' gave it the power to make an enthymeme, whose suppressed premise it was, appear valid, then the inference: 'Snow is white; hence snow is white and all red things are coloured' should equally appear valid—which it does not.

There are other reasons for being dissatisfied with the parametric account. For example, the parameterised definition of validity in effect allows the meanings of some words, but not others, to vary. But when we ask what follows from something's being red, we are not asking about what might be the case if 'red' meant something different. That's just changing the subject. It is essential, then, that meaning change is not allowed—even of so called logical variables.[17]

This is no place to pursue the ramifications of the dispute further. The main point for present purposes is just that parameterised validity is not real validity at all—just a false theory thereof.[18] This is not to say that the parameterised notion of validity is a useless notion. It is still true that every parametrically valid inference is valid. Hence, a conclusion that follows according to such a notion, still follows, and this may well allow us, in reasoning, to show what we need. But there is more to validity than formal validity.

3.4 Classes of Situation

B&R's next argument is to the effect that we can obtain equally legitimate notions of logical consequence by defining validity as truth preservation over different

[16]B&R agree: 'One might now wonder: Is there any basis upon which to choose between these two accounts? Is there any reason you might prefer one to the other? The answer is a resounding *yes*.' (B&R, p.8; italics original.)

[17]A third objection to the parametric account is that if it is to have any credibility, the distinction between logical constants and logical variables has to be drawable in some principled fashion, and I doubt that there is any such way. It is true that there is, in practice, a distinction of this kind handed down to us by tradition, which puts 'or' and 'not' on one side, and 'red' and 'coloured' on the other. But this, it seems to me, is simply an historical contingency.

[18]It is perhaps worth noting that in the standard semantics of first order logic, we assign predicates arbitrary extensions. Sometimes this is glossed informally as assigning each predicate a meaning. Sometimes, on the other hand, an interpretation is thought of as representing a "possible world", where the extension of each predicate, whose meaning is determined in other ways, is as assigned. These are not the same, as should now be clear. It is perhaps the confusion of these two things that lends the parameterised approach some credibility.

classes of situations. Thus, if we restrict ourselves to consistent and complete domains, classical logical consequence will result; if we take a broader class, it will not. If we include situations with empty domains we will have a free logic; otherwise not. We therefore have a plurality of validities: one for each appropriate class of situations—and all equally legitimate notions of validity.

The obvious reply to this argument is that it is only truth-preservation over *all* situations that is, strictly speaking, validity. One of the points about deductive logic is that it will work come what may: we do not have to worry about anything except the premises. As I have already observed, this is not to say that in practice one may not reason as if one were using a different, stronger, notion of validity, one appropriate to a more limited class of situations. But this is not because one has changed logical allegiances; it is because one is allowed to invoke contingent properties of the domain in question.

A possible reply here[19] is that the class of situations is so large and variegated that there are no inferences that are truth preserving over all of them, except, possibly, the trivial $\alpha \vdash \alpha$. Hence monism is vacuous. I have never seen any persuasive argument as to why one should suppose this to be the case, however. It may well be that the intersection of the valid principles in all *pure* logics is empty, but this is beside the point. We are not talking about pure logics here; we are talking about applied logics, and indeed, logics applied for one particular purpose.

Moreover, I think it just false that all principles of inference fail in some situation. For example, any situation in which a conjunction holds, the conjuncts hold, simply in virtue of the meaning of $\wedge$. Naturally, if one were allowed to vary the meanings of words as the class of situations is broadened, then this would not be the case. But when we ask about what is the case in situations where the sky is blue *and* the sun is shining, we are interested in just that: allowing 'and' to take on some other meaning is beside the point. It is certainly open to someone to claim that what I have just said gets the meaning of 'and' wrong, and that the standard truth conditions are not, in fact, delivered by its meaning. That is a substantial issue, and certainly gives rise to theoretical pluralism, but it is not germane here. As long as meanings *are* fixed, one can't vary them to dispose of valid inferences.

There is another possible move that might be made at this point. What, exactly, is the disagreement between the monist and the pluralist about here? The monist accepts that there is a core of universally correct inferences; but this may be augmented if we are reasoning about certain kinds of situations. The pluralist holds that different sorts of situations require different validities, but may accept—will accept, if the above argument is correct—that there is a core of inferences that are acceptable in all kinds of situations. What is the difference? None, it might be thought. The facts are agreed upon. What is at issue is only

[19] One that, in fact, B&R give in section 6.

how to describe them.[20]

But even this duck/rabbit pluralism would appear not to be correct. Suppose that one is a pluralist of the kind in question. We often reason about some situation of other; call it s; suppose that s is in different classes of situations, say, K_1 and K_2. Should one use the notion of validity appropriate for K_1 or for K_2? We cannot give the answer 'both' here. Take some inference that is valid in K_1 but not K_2, $\alpha \vdash \beta$, and suppose that we know (or assume) α; are we, or are we not entitled to accept β? Either we are or we are not.[21] A natural reply is that we should use the notion of validity appropriate to the smallest class of situations that s is in; in this case, presumably, $K_1 \cap K_2$. But if we should, indeed, apply the notion of validity appropriate to the smallest class that s is in, then we should apply the notion appropriate to $\{s\}$. Thus, the valid inferences are those that have a premise false in s, or whose conclusion is true in s. In other words, it is now pluralism that has become vacuous. Note how this is *not* a problem for the monist. For the monist, when reasoning about situation s we apply the uniquely correct logic. We are, of course, entitled to invoke any other information we have about s, enthymematically. But that is entirely as it should be.

3.5 Instrumentalism

B&R's final argument is that we may generate different logics by giving the truth conditions of the connectives in different ways. Thus, for example, we may give either intuitionist truth conditions or classical truth conditions. If we do the former, the result is a notion of validity that is constructive, that is, tighter than classical validity, but which it is perfectly legitimate to use for certain ends.

There are two aspects of this claim that need concern us. The first was covered essentially in 3.1. If we give different truth conditions for the connectives, we are giving the formal connectives different meanings. When we apply the logics to vernacular reasoning we are, therefore, giving different theories of the meanings of the vernacular connectives. We have a case of theoretical pluralism; and the theories cannot both be right—or if they are, we simply have a case of ambiguity, as we have already seen.

One might dispute this argument (as Restall did in correspondence). Classical and intuitionist connectives do not have different meanings. Truth conditions

[20] The argument is made in Priest (199a), sect. 15.

[21] Restall objected to this in correspondence, as follows. Entitlement is ambiguous, each sense corresponding to one of validity. So one can be entitled in one sense, but not another. Now, entitlement can certainly be ambiguous. For example, one can be entitled to something under one legal jurisdiction, say a Federal one, but not under another, say a State one. But the crucial question, then, is that of which jurisdiction is in force in the particular case at hand. Which one will prevail in court? Similarly with the case of logical entitlement. Which logical jurisdiction should inform my epistemic state? Am I to add β to it, or am I not? The matter has to be settled one way or the other.

are general and uniform. Apparently different truth conditions are just special cases. Thus, for example, the truth conditions for the connectives in a Kripke interpretation for intuitionist logic collapse into classical conditions at "classical" worlds, namely, those which access no worlds other than themselves. We do not, therefore, have connectives with different meanings. It is just that the intuitionist countenances more situations than their classical cousin.

If this move is right, then this third example of pluralism reduces to the previous one, which I have already dealt with. But I do not think it is right. For a dispute about meanings can be cashed out just as much in terms of what situations there are, as in terms of truth conditions. For example, to determine whether it is part of the meaning of 'to see' that the eyes be employed, we might consider whether there are any (maybe hypothetical) situations where someone could be described as seeing, even though they had no (functioning) eyes. One who holds that seeing involves having eyes will deny the possibility of such situations.

Exactly the same is true of the case at hand. For someone who takes classical truth conditions to govern an account of the connectives, say negation, for example, what it means to say that $\neg\alpha$ holds in a situation is just that α fails to hold there. It follows that there can be no situations where neither α nor $\neg\alpha$ holds. Suppose, for example,[22] that one thinks of a situation as a *part* of a possible world. Moreover, suppose that we have a situation of which JC is not a part. It follows that 'JC is reading' is not true of that situation. For classical negation, it will follow that 'JC is not reading' is true of that situation. It is not an option to say that neither of these two is true. The situation cannot be "incomplete". As B&R themselves put it (p.12), 'the classical account of negation fails *for situations*'.[23]

The second part of B&R's claim that we need to look at is the claim that, even if one is, say, a classical logician, one may legitimately accept the policy of restricting one's reasoning to the use of intuitionistically valid inferences, and so reason constructively, for certain ends.[24] Now this policy is certainly not intuitionism. For intuitionism justifies many mathematical principles that are classically inconsistent, and so intuitionist mathematics is not a proper part of classical mathematics. Neither is it a policy of reasoning only in accordance with principles that are constructive from a classical perspective. For this extends intuitionist logic. For example, it legitimates Markov's Principle in arithmetic: $(\forall x(A \vee \neg A) \wedge \neg\forall x \neg A) \rightarrow \exists x A$ (where the quantifiers are constructive). This is

[22]One of B&R's, p.12.

[23]The case is even clearer with respect to inconsistent situations (of a kind that are also invoked in situation semantics). If one subscribes to classical negation, *no* situation can be such that α *and* $\neg\alpha$ hold at it. This is a very part of what negation is taken to mean.

[24]B&R, p.17, italics original: 'The constructivism ... [of certain mathematicians] can best be described as mathematics *pursued in the context of intuitionist logic*'.

not intuitionistically valid.[25]

None the less, it is certainly the case that one could decide to operate with just intuitionist logic, or, more generally, with a constructive part of one's preferred logic (assuming that that logic is not itself constructive). One could do this simply as an exercise; but more importantly, there might be a perfectly sensible *point* to doing so. A point might be provided by the fact that conclusions obtained constructively contain more information, or more computational content, than conclusions proved non-constructively. Reasoning in this way may therefore be a useful *instrumental* technique. I noted that a pure logic may be applied for many purposes, and instrumental purposes are purposes. Hence this is simply a case of applying a logic for a different end, and we have already seen that different applications may require different logics. Note, though, that it does not follow from this that conclusions obtained using non-constructive principles of inference are themselves defective in any way. The things so proved are guaranteed, in fact, to hold in the situation about which we are reasoning, by the definition of validity.

Whilst we are on the instrumental use of logics, let me give one more example of this. Inference procedures are used in science. And sometimes in science, we have a theory that we know to be incorrect, but which we use, nonetheless, for instrumental purposes. If we are doing this, then we are clearly also at liberty to use whatever inferential processes we like if they satisfy those purposes.

An example of this is as follows. The Bohr theory of the atom was a very useful theory, which made strikingly accurate empirical predictions. Its inference procedures were highly non-standard, though, involving the chunking of principles, with limited flow of information between the chunks. It can be argued that this is best understood in terms of a non-adjunctive paraconsistent logic.[26] Now, no one ever thought that Bohr's theory was correct. It is commonly suggested that this is because it was inconsistent. I do not think that this is a good reason. What is a good reason is that if a truth-preserving inference mechanism had been used, the theory would have had all sorts of empirical consequences that are not observed.[27] The theory, then, cannot be true. Despite this, it was an important stepping stone *en route* to a more adequate theory. It was necessary to obtain insight into various principles and their interaction, so that more adequate principles could be formulated. The use of a logic that is not one of simple truth preservation was perfectly legitimate in this context.

A number of writers, working in the field of non-adjunctive paraconsistent logic in particular, have argued that it may be important to have an inference engine that preserves something other than (or as well as) truth, for certain

[25] Classically, if an arithmetic predicate is decidable, and we are assured that not everything fails to satisfy it, then we can test it for each number until we find one that satisfies it. This is a perfectly good classical construction, but not an intuitionist one. See Dummett (1977), p.22f.

[26] See Brown (1993).

[27] See Priest (199d).

purposes.[28] As I have already stressed, different applications may well require different logics. State your purpose and how it is to be achieved; then use a logic the application of which does this.

3.6 Underdetermination by the Data

Let us finish with one more argument for pluralism. In effect, an applied logic is, as I have said, a theory about how its domain of application behaves. Hence, a pure logic, when given the canonical application, provides a theory about the norms of correct (deductive) reasoning. It may be thought that there is insufficient information to determine which logic is the correct applied logic here; in particular, that our intuitions about the goodness and badness of particular inferences —which provide the data in this case—are insufficient to determine which theory is correct. Hence, there may be a plurality of equally correct answers.

Pluralism of this kind may be argued on very general grounds, familiar from the philosophy of science. The correct theory, it may be said, is always underdetermined by the data, which is, in any case, soft. In virtue of this, the argument sometimes continues, the correct theory is a purely conventional matter. And we can make whichever conventions are most convenient.

What is to be said about this? The first point is that indeterminism of the kind I have just described is, in general, quite over-rated. Adequacy to the data is just *one* of the criteria to be employed in assessing theories. Others, such as simplicity, unity, a low degree of *ad hoc*ness, etc., are familiar from the literature. Whilst it is clearly a possibility of some sort that different theories may yet tie—and not just *pro tem*, but in the long term—it is hard to come up with concrete examples of this in the history of intellectual inquiry. At any rate, I know of no argument to suppose that this is a real possibility in logic.

And even if it were, this is not an end of the matter. For this plurality is merely epistemic: we just cannot tell which of the theories is the correct one. Given competing theories, as long as one is a realist about their subject matter, at most one of them can be correct, even if we cannot tell which. There is therefore no alethic pluralism.

But should we be a realist about logic? The answer, I think, is yes. Validity is determined by the class of situations involved in truth-preservation, quite independently of our theory of validity. This answer has a certain ontological sting, however. For, as I observed, the situations about which we reason are not all actual: many are purely hypothetical. And one must be a realist about these too. There are numerous different sorts of realism that one might endorse here, many of which are familiar from debates about the nature of possible worlds. One may take hypothetical situations to be concrete non-actual situations; abstract objects, like sets of propositions or combinations of actual components; real but

[28] See, e.g., Brown (199a).

non-existent objects.[29] I will not address the question of which of these account is correct here.[30] Any of them will do, as long as they provide for an independent realm of situations; and hence a determinate answer to the question of which theory is correct (even if our theories do tie, epistemically).

4 Inductive Inference

We have now dealt with a number of important arguments for pluralism about deductive validity. Let me conclude the essay by returning to the issue of inductive validity, to see what it adds to the picture. The situation here is complicated by the fact that there is no theoretical agreement about the nature of inductive inference, even of the limited kind that exists for deductive inference. What I shall do, then, is say what I think is the correct account of inductive inference,[31] and take it from there.

The account is most simply explained with an example. Consider the inference: Bonzo is a dog; hence, Bonzo has four legs. The inference is not deductively valid: there are, after all, dogs who have lost legs in accidents. But it is inductively good; and this is so because normally dogs have four legs. In other words, in all normal situations where the premise is true, so is the conclusion. Normality comes by degrees, though, so it is more accurate to say that in all the *most* normal situations in which the premises are true, so is the conclusion.

The idea is handled formally by structuring situations with an ordering, taken to represent normality. A most normal situation where the premises are true is then a situation where the premises are true, such that there is no situation higher in the ordering where this is so; but we need not go into details of this for the following discussion. The important point to note at present is that when we reason inductively, what counts as normal is context-dependent. If we are reasoning about biological appendages, a dog with four legs is not normal; if we are reasoning about creatures' genotypes, the dog may be perfectly normal, provided that its lost leg is caused by a road accident, and not a genetic abnormality.

Now, how does this affect the issue of pluralism? First, the fact that we have two canons of inference, inductive and deductive, is not a case of pluralism. This is because, according to the above definition, all deductively valid inferences are inductively valid. (If truth is preserved in all interpretations, it is preserved in all most normal ones.) Hence, we do not have two canons of inference: we have one set of valid inferences, inside which there is a distinguished subset.

But the fact that normality is context-relative does give rise to a pluralism. What is inductively valid will depend on the normality ordering, which may vary

[29]Views of these kinds are held, concerning possible worlds, by Lewis, Stalnaker, Cresswell and Routley, respectively.

[30]My sympathies, in fact, lie with a Meinongian account.

[31]This is defended in Priest (199b).

from context to context. One might be tempted to reply here as I replied to the pluralist in 3.4. It is still the case that there is a core of inductive inferences that are valid in all contexts, namely those that hold whatever the normality ordering. Why not call those the valid inductive inferences, the others being reducible to those enthymematically? A main answer to this is that if one conceptualises the valid inductive inferences in this way, then there is no difference between inductive and deductive validity. The valid inductive inferences turn out to be indistinguishable from the valid deductive ones.[32] One can maintain a monism about inductive inference, but if one does, it is an entirely vacuous one.

5 Conclusion

We have now reviewed several reasons for logical pluralism, and several things that pluralism might mean. The issue of monism vs. pluralism has turned out to be a delicate one. As we have seen, there are many senses in which pluralism is uncontentiously correct. No one, for example, is going to argue about pluralism in pure logic, or theoretical pluralism in applied logic. The important, and hard, question is whether one should be a monist or a pluralist about the correct logic for the canonical application. As I have argued, monism is the correct answer.[33]

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[32]Just consider the degenerate ordering, where no situation compares with any other. The maximally normal models of a set of premises are then just its models.

[33]A version of this paper was given at the University of Tasmania. I would like to thank those present for a number of helpful points, but particulary, J.C.Beall, Jay Garfield and Winston Nesbitt. I'm also grateful to Greg Restall for his comments on earlier drafts of the paper. A version of the paper was also read at a meeting of the Society for Exact Philosophy held at the University of Lethbridge, May, 1999. I would like to thank those present for their helpful comments. In particular, the argument for pluralism of 3.6 was pressed on me by a number of people there, and especially Peter Woodruff.

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