Graham Priest

Minimally Inconsistent LP

Abstract. The paper explains how a paraconsistent logician can appropriate all classical reasoning. This is to take consistency as a default assumption, and hence to work within those models of the theory at hand which are minimally inconsistent. The paper spells out the formal application of this strategy to one paraconsistent logic, first-order LP. (See, Ch. 5 of: G. Priest, *In Contradiction*, Nijhoff, 1987.) The result is a strong non-monotonic paraconsistent logic agreeing with classical logic in consistent situations. It is shown that the logical closure of a theory under this logic is trivial only if its closure under LP is trivial.

1. Introduction: the Classical Recapture — Relevant Logic

Intuitionism it a revisionist philosophy. It sees a good part of the reasoning of classical mathematics, particularly that concerning infinite totalities, as quite fallacious. It has therefore wished to debunk it. The programme of paraconsistent logic has never been revisionist in the same sense. By and large, it has accepted that the reasoning of classical mathematics is correct. What it has wished to reject is the excrescence *ex contradictione quodibet*, which does not appear to be an integral part of classical reasoning, but merely leads to trouble when reasoning ventures into the transconsistent.

Since the early days of paraconsistent logic it has, however, been clear that the reject of *ex contradictione* is not possible without the rejection of other things which appear to be much more integral to classical reasoning. Crucially, the disjunctive syllogism is a casualty in most paraconsistent logics. The problem is therefore posed as to how to account for the apparently acceptable but invalid classical reasoning.

There are at least two strategies for trying to solve the problem.[1] The first is to note that the most crucial failures of the disjunctive syllogism appear to be those where the material conditional is attempting to play the role of a genuine conditional. One may therefore attempt to reconstruct the informal reasoning of classical mathematics (and similar areas) by producing a new account of the conditional to be grafted on to an underlying extensional paraconsistent logic (without ruining its paraconsistent properties)[2], and using this in the reconstruction.

[1] These are spelled out clearly in [4], Section IV.

[2] E. g., as in [5], Ch. 6, or [8], Ch. 3, 'Systems of Paraconsistent Logic'.

This is the route that standard relevant paraconsistent logic has taken. Several logicians (Brady, Meyer, Mortensen, Priest, Routley, *et al.*) have attempted to reconstruct various fragments of classical reasoning in this way. While the results are not definitive, they are not terribly encouraging. There appear to be classical arguments which defy reconstruction in this way. The most ambitious project in this direction was Meyer's attempt to reconstruct the reasoning of classical number theory in the relevant theory $R^{\#}$. This project has ended in failure.[3] And this is so where the underlying logic, R, is a very strong one, much stronger than is suitable for many paraconsistent purposes. Thus, though the aim of furnishing paraconsistent logic with a correct account of the conditional is a highly important — indeed, essential — enterprise, it would now appear that the aim of reconstructing sensible classical reasoning in this way is not likely to be realised.

2. The Classical Recapture —Limiting the Domain

The other way of attempting to recapture sensible classical reasoning stems from the observation that counter-examples to inferences such as the disjunctive syllogism occur only in the transconsistent. Hence provided we stay within the domain of the consistent, which classical reasoning of course does (by and large), classical logic is perfectly acceptable. (Similarly, for the intuitionism, classical logic is perfectly acceptable provided one stays within the domain of the decidable.)

Compared with the first strategy for appropriating classical reasoning, this strategy has little that can go wrong: nothing has to be reconstructed; the theory just legitimizes classical reasoning as it stands. The problems for this strategy are rather different. The first is to understand the exact import of the claim that 'provided we stay within the domain of the consistent, classical logic is perfectly acceptable'. This is not as easy as it appears, but I have discussed the matter at length elsewhere[4], so I will not take up the issue again. I merely report that an important upshot of that discussion is that we are justified in assuming consistency until and unless shown otherwise.

The second problem for this approach is to see whether it can be worked into an interesting formal theory of reasoning. It can; and that is the main topic of the paper.[5] The crucial insight here is due to Batens[6]; and is that given some information, from which we have to reason, we can cash out the

[3] See [3].

[4] [5], Ch. 8 and [7].

[5] A somewhat different way is given in [5], Ch. 8; but this now strikes me as contrived in comparison with the present approach.

[6] [1] and [2].

idea that the situation is no more inconsistent than we are forced to assume by restricting ourselves to those models of the information which are, in some sense, as consistent as possible, given the information; or as we will say, minimally inconsistent.

How, precisely, one is to understand minimal inconsistency, may depend on the underlying paraconsistent logic. For reasons I have explained elsewhere,[7] my preferred extensional paraconsistent logic is the system LP. Hence, I shall work with this (though clearly the techniques are more generally applicable). First, I will give a summary of LP semantics; then I will explain minimally inconsistent LP, LPm. I will then establish a number of its pleasing properties.

3. Semantics for LP

LP semantics are for a first-order language with connectives $\neg$ and $\wedge$, and quantifier $\forall$. $\exists$ and $\vee$ are defined in the usual way. There are constants and predicates, including the identity predicate. I will use lower case greeks from α on as schematic letters for formulas of the language, lower case romans from p on as schematic letters for atomic formulas, and upper case greeks for sets of formulas.

We will assume that there are no propositional parameters or function symbols in the language. This is largely a matter of simplicity. But also, in the present context, they are an irrelevancy. It is well known that they do not extend the expressive power of classical first-order logic. Hence, if the use of this logic can be justified, so is their use.

An interpretation, $\mathfrak{A}$, for the language is a pair $\langle D, I\rangle$, where D is the non-empty domain of quantification; I is a function which maps each individual constant, c, into D and each n-place predicate, P, into a pair $\langle I^+(P), I^-(P)\rangle$, where $I^+(P) \cup I^-(P) = D^n$. We also require that $I^+(=) = \{\langle x, x\rangle; x \in D\}$. (But note that since $I^+(=)$ and $I^-(=)$ need not be disjoint, things of the form $\langle x, x\rangle$ may be in $I^-(=)$ too.) The *language of* $\mathfrak{A}$ is the language augmented by a set of individual constants, one for each member of D. For simplicity we take the set to be D itself, and specify that for all $d \in D$ $I(d) = d$. (Thus, in the language of $\mathfrak{A}$, every member of the domain of $\mathfrak{A}$ has at least one name.)

Every formula in the language of $\mathfrak{A}$, α, is now assigned a truth value, $\nu(\alpha)$, in the set $\{\{1\}, \{0\}, \{1, 0\}\}$ by the following recursive clauses.

$$1 \in \nu(Pc_1 \ldots c_n) \Leftrightarrow \langle I(c_1) \ldots I(c_n)\rangle \in I^+(P)$$

[7] [4] and [5], Chs. 4, 5.

$$0 \in \nu(Pc_1 \dots c_n) \Leftrightarrow \langle (c_1) \dots I(c_n) \rangle \in I^-(P)$$

$$1 \in \nu(\neg\alpha) \Leftrightarrow 0 \in \nu(\alpha)$$
$$0 \in \nu(\neg\alpha) \Leftrightarrow 1 \in \nu(\alpha)$$

$$1 \in \nu(\alpha \wedge \beta) \Leftrightarrow 1 \in \nu(\alpha) \text{ and } 1 \in \nu(\beta)$$
$$0 \in \nu(\alpha \wedge \beta) \Leftrightarrow 0 \in \nu(\alpha) \text{ or } 0 \in \nu(\beta)$$

$$1 \in \nu(\forall x\alpha) \Leftrightarrow \text{for all } d \in D \;\; 1 \in \nu(\alpha(x/d))$$
$$0 \in \nu(\forall x\alpha) \Leftrightarrow \text{some } d \in D \;\; 0 \in \nu(\alpha(x/d))$$

where $\alpha(x/d)$ denotes α with all free occurrences of 'x' replaced 'd'.

Let $\mathfrak{A} = \langle D, I \rangle$ be an interpretation, then:

$\mathfrak{A}$ is a model for α ($\mathfrak{A} \vDash \alpha$) iff $1 \in \nu(\alpha)$

$\mathfrak{A}$ is a model for Σ ($\mathfrak{A} \vDash \Sigma$) iff for all $\beta \in \Sigma$ $\mathfrak{A} \vDash \beta$

α is an LP consequence of Σ ($\Sigma \vDash \alpha$) iff
every model of Σ is a model of α.

For propositional LP, an interpretation, ν, simply assigns each propositional parameter a truth value in $\{\{1\}, \{0\}.\{1, 0\}\}$. The rest is similar.

I will not pause long to discuss the logic LP, but I note that any standard classical interpretation is isomorphic to an LP interpretation in which all atomic formulas (and so all formulas) take the value $\{0\}$ or $\{1\}$. Consequently I will call such LP interpretations *classical interpretations*. It follows that every LP consequence of a set of formulas is also a classical consequence. What is not so obvious (but is true) is that the set of logical truths of LP is exactly the set of logical truths of first-order logic (with identity).[8]

One further property of LP will be useful in what follows. So I will state it now.

LEMMA. *Let* $\mathfrak{A}$ *be any interpretation. If for every atomic formula,* p, *in the language of* $\mathfrak{A}$ $\nu(\mathrm{p}) = \{1, 0\}$, *then for every formula,* α, $\nu(\alpha) = \{1, 0\}$.

PROOF. The proof is by a simple recursion over the structure of formulas. Details are omitted.

[8] Further details and proof can be found in [5], Ch. 5.

4. Semantics for LPm

Giving a precise definition of minimal inconsistency requires us to find some measure of degree of inconsistency, or, what comes to the same thing, a way of ordering interpretations with respect to their inconsistency. One might attempt this in a number of ways, but the one that appears to give the best results is as follows.[9] If α is a formula, let $\alpha!$ be $\alpha \wedge \neg\alpha$. Note that, given any interpretation, $1 \in \nu(\alpha!)$ iff $\nu(\alpha) = \{1,0\}$. If $\mathfrak{A} = \langle D, I\rangle$ is an interpretation, define the inconsistent part of $\mathfrak{A}$, $\mathfrak{A}!$, to be the set of atomic facts with value $\{1,0\}$ in $\mathfrak{A}$, i.e.:

$$\mathfrak{A}! = \{\text{p}\,;\ \text{For some } P, \text{ and } d_1 \ldots d_n \in D,\ \text{p} = Pd_1 \ldots d_n \text{ and } 1 \in \nu(\text{p}!)\}.$$

Note that this is a (in general, proper) subset of the set of true contradictory atomic formulas. (In the propositional case $\mathfrak{A}!$ is just the set of propositional parameters taking the value {1,0}.) $\mathfrak{A}!$ is an appropriate measure of the inconsistency of $\mathfrak{A}$. If $\mathfrak{A}$ is classical then $\mathfrak{A}!$ is clearly $\emptyset$, which is minimal. If every atomic formula (and so every formula) is inconsistent then $\mathfrak{A}!$ is maximal (relative to the domain).

We can now define a consistency ordering thus. Let $\mathfrak{A}_1 = \langle D_1, I_1\rangle$ and $\mathfrak{A}_2 = \langle D_2, I_2\rangle$:

$$\mathfrak{A}_1 < \mathfrak{A}_2 \text{ iff } \mathfrak{A}_1! \subset \mathfrak{A}_2! \text{ and } D_1 = D_2.$$

Here $\subset$ denotes strict inclusion. As may easily be checked, $<$ is a strict partial order. The reason for the second clause in the *definiens* may not be immediately apparent. (In the propositional case, the first clause on its own is quite sufficient.) I will return to this in a moment. $\mathfrak{A}_1 \leq \mathfrak{A}_2$ is defined in the obvious way as: $\mathfrak{A}_1 < \mathfrak{A}_2$ or $\mathfrak{A}_1 = \mathfrak{A}_2$.

We now define:

$\mathfrak{A}$ is a minimally inconsistent (mi) model of Σ ($\mathfrak{A} \models_m \Sigma$)
iff $\mathfrak{A} \models \Sigma$ and if $\mathfrak{A}' < \mathfrak{A}$ then $\mathfrak{A}' \not\models \Sigma$.

α is a mi consequence of Σ ($\Sigma \models_m \alpha$)
iff every mi model of Σ is a model of α.

Notice that if the second clause in the definition of $<$ were not present, minimising inconsistency would require making the domain as small as possible, which would give quite unintended results. For example, let $\Sigma = \{\forall x(Px!)\}$; if we omitted the second clause from the *definiens* then any mi

[9] For full discussion, see [6].

model, $\mathfrak{A}$, of Σ would have a signle-element domain (or we could make $\mathfrak{A}$ smaller by decreasing the size of the domain). Hence we would have, e.g., $\Sigma \models_m \forall x Qx \vee \forall x \neg Qx$, quite counter-intuitively.

5. Properties of LPm

The first thing to observe about LPm is that it is non-monotonic. For example, let Π be $\{p, \neg p \vee q\}$. Then $\Pi \models_m q$, since the mi models of Π are just the classical models. But $\Pi \cup \{p!\} \not\models_m q$, since there is a mi model of the premises where p has the value $\{1,0\}$ and q has the value $\{0\}$. (Note, however, that if r is a distinct atomic formula $\Pi \cup \{r!\} \models_m q$, since in mi models of the premises, only r takes the value $\{1,0\}$.) In effect LPm is a logic which implements the default assumption of consistency.

To state some more general properties of LPm, we need a little notation. Let Σ^{CL}, Σ^{LP} and Σ^m be the set of classical, LP and LPm consequences of Σ. Then:

FACT 1. $\Sigma^{LP} \subseteq \Sigma^m \subseteq \Sigma^{CL}$, *since every classical model of* Σ *is a mi model, and every mi model is an* LP *model.*

FACT 2. *In general, the inclusions in* Fact 1 *are proper, since* $\Pi \not\models q$, *but* $\Pi \models_m q$ (*where* Π *is as above*); *and* $\{p!\} \not\models_m q$, *but* q *is a classical consequence of* p!.

FACT 3. *If* Σ *is classically consistent* $\Sigma^m = \Sigma^{CL}$, *since if* Σ *is classically consistent its mi models just are its classical models.*

Hence LPm is a more generous notion of consequence then LP, which allows for classical inferences such as the disjunctive syllogism provided inconsistency does not "get in the way", and, in particular, is identical with classical logic in consistent situations. It thus gives a precise account of how it is that classical inferences are acceptable, paraconsisently, in consistent situations.

6. Reassurance: the Propositional Case

I have noted that LPm is a more generous inference engine than LP. The next question is 'how much more generous?' Can we, for example, prove more contradictions using LPm than using LP? The answer to this is 'In general, yes'. To see this, just note that $\{p!, q! \vee r\} \models_m (p \wedge r)!$. For in any mi model of the premises r must be true. Hence $p \wedge r$ must be true. But since p must be false $p \wedge r$ must also be false. However, $\{p!, q! \vee r\} \not\models (p \wedge r)!$, as a simple counter-model demonstrates.

This raises the possibility that Σ^m may collapse into triviality when Σ^{LP} does not. This would obviously be unfortunate since it would show that there are perfectly sensible (non-trivial) contexts where LPm could not be used. Its theoretical legitimacy would therefore have to be restricted, just as that of classical logic is. It would be very reassuring, therefore, if whenever Σ^{LP} is non-trivial so is Σ^m. Let us therefore call this property *Reassurance*.

Reassurance is not to be taken lightly. For example, suppose we augment LP with propositional quantifiers and a conditional connective, $\rightarrow$, satisfying (at least) *modus ponens*. Let $\Sigma = \{\exists p(p!)\} \cup \{p_i! \rightarrow p_{i+1}!;\ i$ a natural number$\}$ (where the natural numbers index the set of propositional variables). Σ^{LP} is non-trivial; for example, $\Sigma \not\models p_0$. But Σ has no mi model. If $\mathfrak{A}$ is a model of Σ then $\mathfrak{A}!$ must be non-empty. Let n be the least m such that p_m is inconsistent. Then $\mathfrak{A}'$, which is exactly the same except that p_{n+1} is the least m, is a less inconsistent model. Hence (vacuously) Σ^m is trivial.

It is, therefore, a welcome result that LP satisfies Reassurance (with one possible and not terribly important qualification). For propositional LP, Reassurance was proved where Σ is finite in Priest [1988]. The propositional case where Σ is infinite follows from the following lemma, whose proof is due to Fangzhen Lin (in conversation):

LIN'S LEMMA. *If ν is a model of Σ then there is a mi model of Σ, ν' such that $\nu' \leq \nu$.*

For suppose that Σ is non-trivial. Then there must be a ν and an α such that $\nu \models \Sigma$ and $\nu \not\models \alpha$. Hence there must be a propositional parameter, p, such that $\nu \not\models p!$, by the lemma of Section 3. By Lin's Lemma, there is a ν' such that ν' is a mi model of Σ and $\nu' \not\models p!$. Hence Σ^m is non-trivial.

The proof of Lin's Lemma goes as follows:

PROOF OF LIN'S LEMMA. In LP, every formula is logically equivalent to one in disjunctive normal form. Hence, without lose of generality we can assume that the members of Σ are of the form $\pm p_1 \vee \pm p_2 \vee \ldots \vee \pm p_n$, where $\pm$ is either $\neg$ or is nothing.

Consider the set $S = \{\nu'; \nu' \leq \nu \text{ and } \nu' \models \Sigma\}$. S is partially ordered by $<$. Let C be any chain in this ordering. We show that C is bounded below. If follows by Zorn's Lemma that C has a minimal element.

If C is finite, we are home; so suppose it is infinite. Let $C! = \bigcap\{\mu!; \mu \in C\}$. Define a subset, Σ', of Σ as follows:

$$\alpha \in \Sigma' \text{ iff } \alpha \in \Sigma, \alpha \text{ is } \pm \mathrm{p}_1 \vee \pm \mathrm{p}_2 \vee \ldots \vee \pm \mathrm{p}_n \text{ and for } 1 \leq i \leq n \ \mathrm{p}_i \notin C!$$

If $\alpha \in \Sigma'$ then for some ν in C $\nu \models \alpha$ and the value of each propositional parameter in ν is classical, since C is a chain. Hence α has a classical model. Similarly, if Σ'' is a *finite* subset of Σ', Σ'' has a classical model. By the classical Compactness Theorem Σ' has a classical model, μ.

Define an LP interpretation μ' as follows:

$$\mu'(\mathrm{p}) = \begin{cases} \{1,0\} & \text{if } \mathrm{p} \in C! \\ \mu(\mathrm{p}) & \text{otherwise.} \end{cases}$$

Clearly, $\mu' < \nu'$ for all $\nu' \in C$. It remains to show that $\mu' \models \Sigma$. If $\alpha \in \Sigma'$ then there is no propositional parameter in $C!$ which occurs in α. Hence $\mu' \models \alpha$ since $\mu \models \alpha$. If, on the other hand, $\alpha \in \Sigma - \Sigma'$ then there is a propositional parameter, p, in $C!$ which occurs in α. Hence $\mu' \models \alpha$.

7. Reassurance: the First Order Case

For first-order LP, the proof of Reassurance is slightly more complicated, and assumes that the number of predicates in the language is finite. (This is the qualification I alluded to above. The result in the completely general case is still open.) It depends on two more lemmas.

LEMMA 1. *Let $\mathfrak{A}$ a finite interpretation such that $\mathfrak{A} \models \Sigma$. Then there is a $\mathfrak{A}'$ such that $\mathfrak{A}' \leq \mathfrak{A}$ and $\mathfrak{A}' \models_m \Sigma$.*

PROOF. Since there are a finite number of predicates in the language and the domain of $\mathfrak{A}$ is finite $\mathfrak{A}!$ is finite. Thus, $\{\mathfrak{A}'; \mathfrak{A}' \leq \mathfrak{A} \text{ and } \mathfrak{A}' \models \Sigma\}$ is a finite set partially ordered by $<$. Hence there is a minimal member.

Note that this is the first-order analogue of Lin's Lemma, but restricted to the finite case. Its generalisation is still open (though I conjecture that it is true). If it could be proved, Reassurance in general would follows in the way that it does in the propositional case.

To state the next lemma we need a definition. Let $\mathfrak{A} = \langle D, I \rangle$ be an interpretation. Let $^\sim$ be an equivalence relation on D, and if $d \in D$, let $[d]$ be the equivalence class of d under $^\sim$. Let $\mathfrak{A}^\sim = \langle D^\sim, I^\sim \rangle$ be defined as follows. $D^\sim = \{[d]; d \in D\}$. For every constant, c, $I^\sim(c) = [I(c)]$, and if $a_1 \ldots a_n \in D^\sim$:

$$\langle a_1 \ldots a_n \rangle \in I^{\sim +}(P) \text{ iff } \exists x_1 \in a_1 \ldots x_n \in a_n, \langle x_1 \ldots x_n \rangle \in I^{+}(P)$$
$$\langle a_1 \ldots a_n \rangle \in I^{\sim -}(P) \text{ iff } \exists x_1 \in a_1 \ldots x_n \in a_n, \langle x_1 \ldots x_n \rangle \in I^{-}(P).$$

It is easy to check that $\langle I^{\sim +}(=), I^{\sim -}(=)\rangle$ satisfies the appropriate conditions. Hence the interpretation is well defined. In effect, the new interpretation identifies everything in an equivalence class, producing a composite individual with all the properties of the individuals of which it is composed. I note in passing that if there were function symbols in the language, there would be no natural way of defining their interpretation in $\mathfrak{A}^{\sim}$. For example, if f is a 1-place function symbol, one cannot define $I^{\sim}(f)[d]$ in the obvious way as $[I(f)(d)]$ since there is no guarantee that if $[d] = [e]$ then $[I(f)(d)] = [I(f)(e)]$. To that extent, the assumption that there are no function symbols in the language is necessary for the following proofs.

LEMMA 2 (the Collapsing Lemma). *For every formula α, in the language of $\mathfrak{A}$, $\nu^{\sim}(\alpha) \supseteq \nu(\alpha)$.*

PROOF. The proof is by recursion on the structure of α. I will give only the truth cases. The falsity cases are similar. For atomic sentences the argument is as follows:

$$\begin{aligned}
1 \in \nu(Pc_1 \ldots c_n) &\Rightarrow \langle I(c_1) \ldots I(c_n)\rangle \in I^{+}(P)\\
&\Rightarrow \exists x_1 \in [I(c_1)] \ldots x_n \in [I(c_n)] \langle x_1 \ldots x_n \rangle \in I^{+}(P)\\
&\Rightarrow \langle [I(c_1)] \ldots [I(c_n)]\rangle \in I^{\sim +}(P)\\
&\Rightarrow \langle I^{\sim}(c_1) \ldots I^{\sim}(c_n)\rangle \in I^{\sim +}(P)\\
&\Rightarrow 1 \in \nu^{\sim}(Pc_1 \ldots c_n).
\end{aligned}$$

The recursion case for $\wedge$ is as follows (that for $\neg$ is similar):

$$\begin{aligned}
1 \in \nu(\alpha \wedge \beta) &\Rightarrow 1 \in \nu(\alpha) \text{ and } 1 \in \nu(\beta)\\
&\Rightarrow 1 \in \nu^{\sim}(\alpha) \text{ and } 1 \in \nu^{\sim}(\beta)\\
&\Rightarrow 1 \in \nu^{\sim}(\alpha \wedge \beta).
\end{aligned}$$

The case for $\forall$ is as follows:

$$\begin{aligned}
1 \in \nu(\forall x \alpha) &\Rightarrow 1 \in \nu(\alpha(x/d)) \text{ for all } d \in D\\
&\Rightarrow 1 \in \nu^{\sim}(\alpha(x/d)) \text{ for all } [d] \in D^{\sim}\\
&\Rightarrow 1 \in \nu^{\sim}(\forall x \alpha).
\end{aligned}$$

We can now prove Reassurance.

PROOF. Suppose that Σ^{LP} is non-trivial. Then for some α $\Sigma \nvDash_{LP} \alpha$. Hence, there is a $\mathfrak{A} = \langle D, I\rangle$ such that $\mathfrak{A} \models \Sigma$ and $\mathfrak{A} \nvDash \alpha$. By the Lemma of Section 3, there is some predicate P and $d_1 \dots d_n \in D$ such that $\mathfrak{A} \nvDash Pd_1 \dots d_n!$. Define the equivalence relation $^\sim$, on D as follows:

$$x^\sim y \text{ iff } x = y = d_1 \text{ or } \dots \text{or } x = y = d_n \text{ or } x, y \notin \{d_1 \dots d_n\}.$$

In effect, $^\sim$ leaves $d_1 \dots d_n$ alone, but identifies all other members of D. Clearly $\mathfrak{A}^\sim$ is finite. Moreover, by the Collapsing Lemma $\mathfrak{A}^\sim \models \Sigma$. Further, it is obvious that $\mathfrak{A}^\sim \nvDash Pd_1 \dots d_n!$. By Lemma 1, there is a $\mathfrak{A}' \leq \mathfrak{A}^\sim$ such that $\mathfrak{A}' \models \Sigma$, and since $\mathfrak{A}' \leq \mathfrak{A}^\sim$, $\mathfrak{A}' \nvDash Pd_1 \dots d_n!$. This is not *quite* what we need to show, since this sentence belongs to the language of $\mathfrak{A}$, and not the original language. But it follows that $\mathfrak{A}' \nvDash \forall x_1 \dots \forall x_n(Px_1 \dots x_n!)$, a formula which is in the original language. Hence Σ^m is non-trivial.

8. Conclusion

The Collapsing Lemma, on which the above proof of Reassurance depends, is interesting in its own right. It may, initially, be rather surprising. After all, we can, given any model of a set of formulas, produce a model of any smaller cardinality simply by choosing an equivalence relation that identifies the appropriate number of objects. And this is true even though the set may contain formulas which appear to constrain cardinality, e.g., $\exists x \exists y\, x \neq y$. The reason why they do not do so in a paraconsistent context is, of course, that there is no guarantee that their negations are not also true.

The Reassurance Theorem provides the final piece of evidence that LPm provides a good theoretical account of how classical reasoning is possible in consistent domains, and, in a constrained way, in the transconsistent too. The next job is to look at the mi consequence of some interesting inconsistent theories, such as Naive Set Theory; but that is a whole new subject.

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DEPARTMENT OF PHILOSOPHY
UNIVERSITY OF QUEENSLAND
QUEENSLAND, 4072
AUSTRALIA

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